Multi-Quark Hadrons; Four, Five and More...

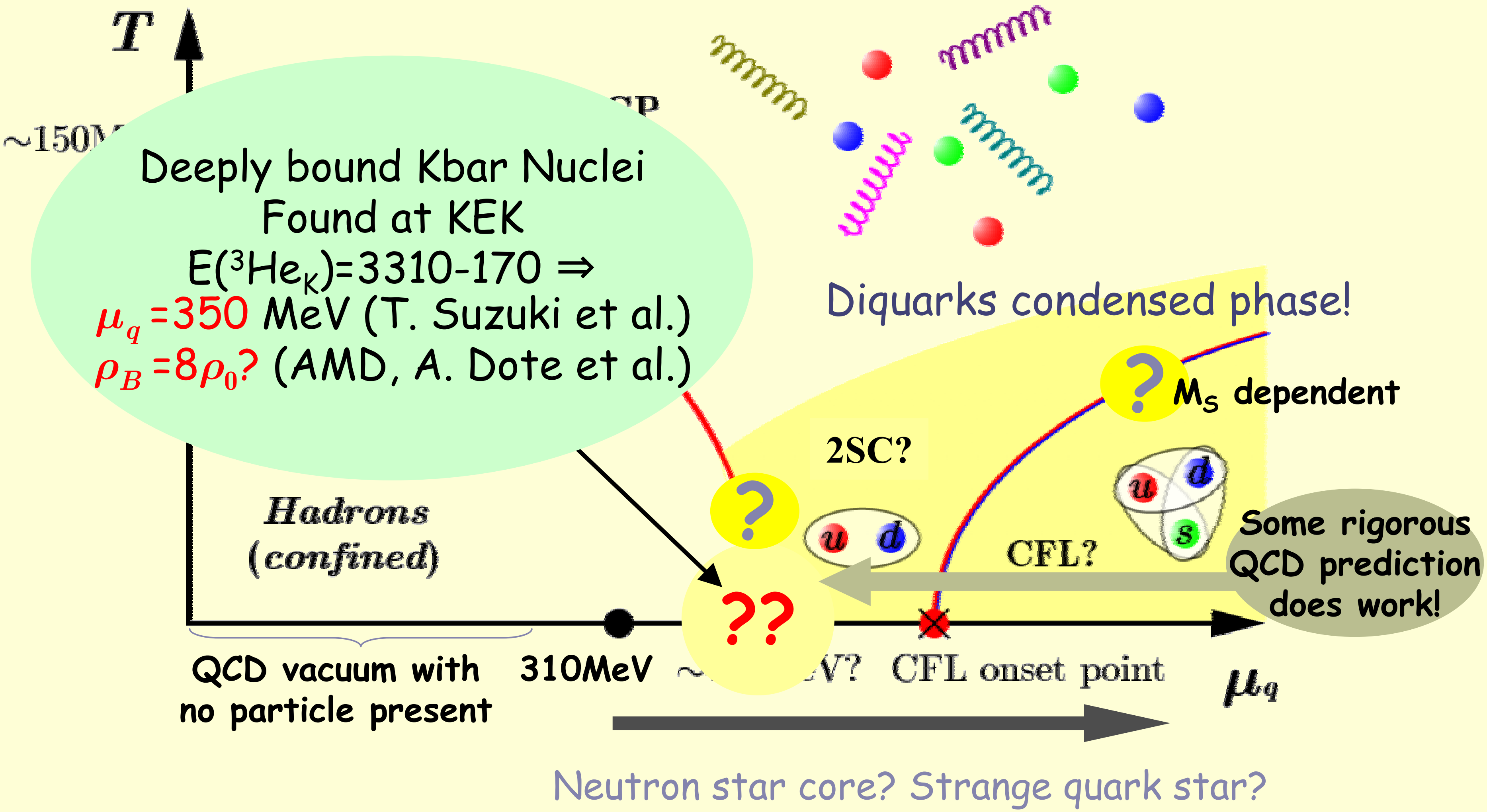
...Extremely Multi-Quark System:

Color SuperConductivity

Hiroaki Abuki (YITP, Kyoto Univ., Japan)

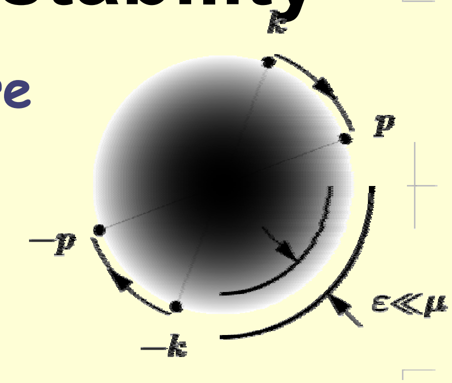
Contents:

Schematic QCD Phase Diagram



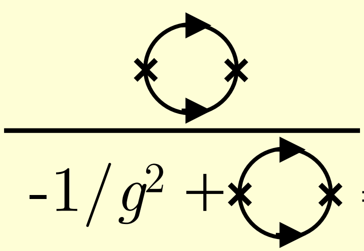
Fermi surface and Cooper instability

Our starting point : there is Large Fermi sphere



- Pauli-Blocking
- Low Energy Modes with Large Density of State

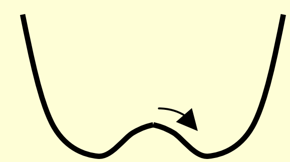
Results in non-perturbative IR dynamics



= 1PI vertex

$$\Gamma^{(2)}(p) = -\Delta_F^{-1}(p)$$

negative curvature of effective potential



low p RPA mode is *tachyonic* at T=0 !

Cooper instability !

leads to reorganization of Fermi ball
into BCS state with non-vanishing qq condensate

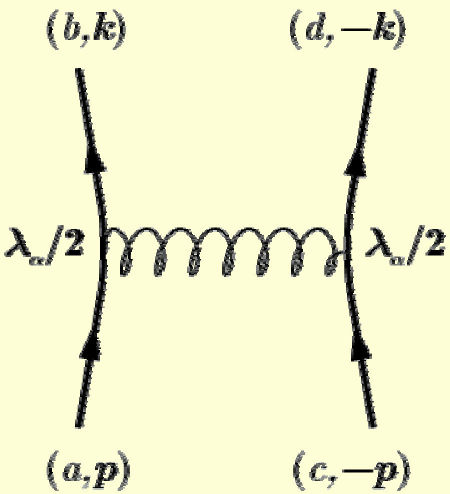
BCS mechanism $\Delta \sim \omega_D e^{-1/Ng^2}$

Attraction in color-antitriplet channel

$$\sum_\alpha (\lambda_\alpha/2)_{ab}(\lambda_\alpha/2)_{cd} = -\frac{N+1}{2N}(\delta_{ab}\delta_{cd} - \delta_{ad}\delta_{bc}) + \frac{N-1}{2N}(\delta_{ab}\delta_{cd} + \delta_{ad}\delta_{bc})$$

$\bar{3}_c$(anti-triplet) — Attraction!

6_c(sextet) — Repulsive



Attraction in anti-triplet channels

Dynamical Formation of $\langle q_a q_b \rangle \approx \varepsilon_{abc}\Delta^c$?

Bailin and Love, Phys. Rept. 107, 325 (1984)

- Ginzburg-Landau Approach
- Phenomenological IR cut off
- Small gap (a few hundred keV)

(i) 2 flavor pairing (2SC)

Bailin-Love, '80
Iwasaki-Iwado, '95, etc…

Order : $\left\langle q_i^a q_j^b \right\rangle_{2SC} = \Delta \varepsilon^{ab,g} \varepsilon_{s,ij}$

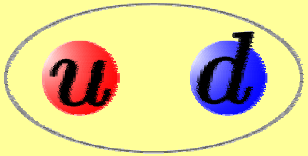
Rank of matrix: **4**

Gapless Quarks : **5** (sr, sb, sg, ug, dg)

Symmetry : $SU(2)_c \times SU(2)_f$

Eigenvalue (degeneracy) :

$$\Delta\ ,\ SU(2)_f \times SU(2)_c \ \text{doublet}^2 \ (4)$$
$$= \left[2_{(r,b)}, 2_{(u,d)} \right]$$



Correlation works between 4 quarks out of 9

(ii) Color-Flavor Locking (CFL)

M. Alford, K. Rajagopal, F. Wilczek, Nucl.Phys.B537, '99

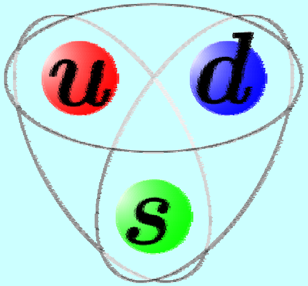
$$\left\langle q_i^a q_j^b \right\rangle_{CFL} = \sum_L \Delta_A \varepsilon^{abL} \varepsilon_{Lij}$$

9 (full rank)

0

$SU(3)_{C+V}$: δ_i^a **remains invariant**

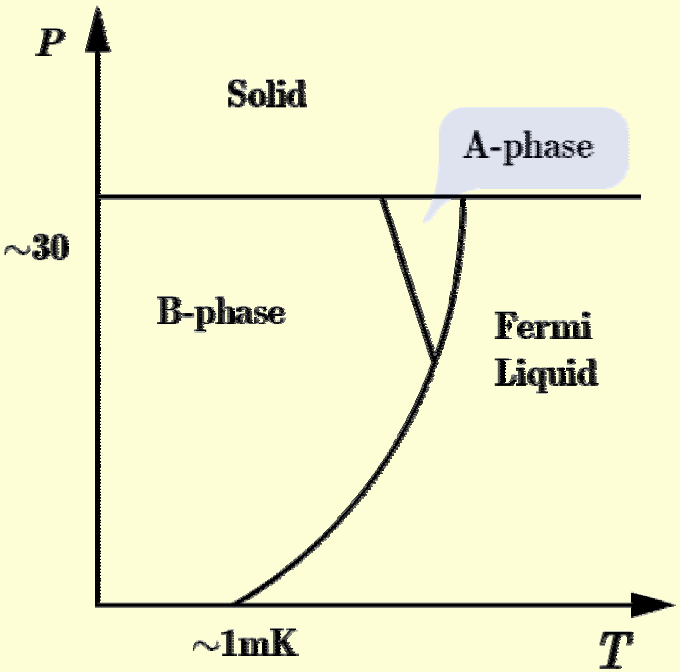
$$\begin{cases} \Delta_1 = 2\Delta_A\ ,\ SU(3)_{C+V} \ \text{singlet (1)} \\ \Delta_8 = -\Delta_A\ ,\ SU(3)_{C+V} \ \text{octet (8)} \end{cases}$$



Correlation works among all 9 quarks

Analogy with He³ system

$$\left\langle \psi_\alpha(\vec{q})\psi_\beta(-\vec{q})\right\rangle = \sum_{a,i} \hat{q}_i \left[\sigma^a \sigma_2\right]_{\alpha\beta} \boldsymbol{d}_a^i$$



: General L=1,S=1 state
with 3*3 matrix $\boldsymbol{d}_a^i$ (a:spin,i:spatial)

B-phase (BW state;Balian-Werthaner)

$\boldsymbol{d}_a^i = \Delta_{\mathrm{BW}} \delta_a^i$ (S and L are locked!!)

- Gap is equal on the entire Fermi surface

A-phase(ABM state;Anderson-Morel)

$\boldsymbol{d}_a^i = \Delta_{\mathrm{ABM}} \delta_a^3 \left(\boldsymbol{e}_1 + i\boldsymbol{e}_2\right)_i$

- Gap is zero at South and North poles

Low energy modes and Quark-Hadron continuity

(1) Gauge bosons

Flavor space

$$Q = \frac{1}{2}\lambda_3^f + \frac{1}{2\sqrt{3}}\lambda_8^f = \begin{pmatrix} 2/3 & & \\ & -1/3 & \\ & & -1/3 \end{pmatrix},$$

Color space

$$T \equiv -T_3^c - \frac{1}{\sqrt{3}}T_8^c = \begin{pmatrix} -2/3 & & \\ & 1/3 & \\ & & 1/3 \end{pmatrix}$$

$$\tilde{Q} = Q \times 1_c + 1_f \times T$$ This generator leaves the CFL ground state unaffected : $\tilde{Q}\left[\delta_i^a\right] = 0$

$$\tilde{\gamma} = \frac{g\gamma + eG}{\sqrt{e^2 + g^2}}$$ Propagating in CFL : Zero magnetic screening mass

$$\tilde{G} = \frac{gG - e\gamma}{\sqrt{e^2 + g^2}}$$ New gluon in the CFL : Meissner magnetic mass O(gμ)

Rischke, PRD62, '00

Partial Meissner effect : $\gamma = \frac{g}{\sqrt{e^2 + g^2}}\tilde{\gamma} - \frac{e}{\sqrt{e^2 + g^2}}\tilde{G}$

(2) Quasi-quark spectrum and its EM property

All 9-quark quarks having color charge acquire finite gaps (Δ_1, Δ_8)

$$\tilde{\gamma} = \frac{g\gamma + eG}{\sqrt{e^2 + g^2}}$$

Field couples electron with strength $-\tilde{e} = \dfrac{-eg}{\sqrt{e^2+g^2}}$

This set Charge unit in CFL medium

$$\tilde{\gamma} = \frac{g\gamma + eG}{\sqrt{e^2 + g^2}}$$

Field couples quasi-quark with f-c index (i, a)

$$\gamma \underset{\text{couple}}{\rightarrow} eQ = \begin{pmatrix} 2e/3 & & \\ & -e/3 & \\ & & -e/3 \end{pmatrix}, \quad G \underset{\text{couple}}{\rightarrow} gT = \begin{pmatrix} -2g/3 & & \\ & g/3 & \\ & & g/3 \end{pmatrix}$$

$$(1,1) = (2,2) = (3,3) = (2,3) = (3,2) : 0, \ (1,2) = (1,3) : +\tilde{e}, \ (2,1) = (3,1) : -\tilde{e}$$

$$\left[(u,r), \ (d,b), \ (s,g), \ (d,g), \ (s,b)\ \right] \qquad \left[(u,b), \ (u,s)\right] \qquad \left[(d,g), \ (s,r)\right]$$

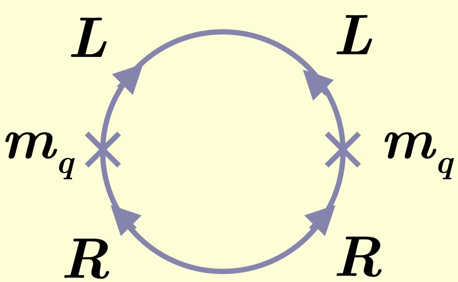
$\tilde{Q}\left[q_i^a\right] = n_i^a \tilde{e}\left[q_i^a\right]$ with integer $n_i^a = (-1, 0, +1)$

(3) NG bosons in the CFL phase

Chiral field : $\Sigma_{ij} = \phi_L^i \phi_R^{\dagger j} \sim \left| \langle \phi \rangle_{\text{CFL}} \right|^2 \exp\left[i\lambda_a \pi^a \right]$, with $a = 0 \sim 8$

with $\left(\phi_L\right)_{ai} = \left[q_L q_L \right]_{ai}^{\bar{3}_c \times \bar{3}_f, 0^+}$, $\left(\phi_R\right)_{ai} = \left[q_R q_R \right]_{ai}^{\bar{3}_c \times \bar{3}_f, 0^+}$

$a = 0 \sim 8$ **Chiral Symmetry breaking :** 0^- **octet and** η $O(m_q^2)$



$$\left.\begin{aligned} m_\pi^2 &= C(m_u + m_d) m_s + .. \\ m_{K^\pm}^2 &= C(m_d + m_s) m_d + .. \\ m_{K^0}^2 &= C(m_u + m_s) m_d + .. \end{aligned}\right\}$$

$$C = \frac{1}{3} \frac{51 + 32 \ln 2}{21 - 8 \ln 2} \sim 1.58$$

inverse meson mass ordering!

Son-Stephanov, PRD62, '00
Casalbuoni-Gatto, PLB464, '99

matching with

$L_{eff}^{M_q} \simeq \left(\det M_q\right) Tr\left[M_q^{-1} \Sigma \right]$: by Symmetry

Δ dependence is studied in
D-K. Hong et al, PLB477 '99

$$C \propto \left(\Delta^2 / \mu^2\right)\left(\ln \mu^2 / \Delta^2\right) \underset{\mu \to \infty}{\to} 0$$

These light bosons with no color index dominate thermodynamics of CFL phase

1. Colored excitation (q, g) have gap larger than Δ_{CFL}
2. All Elementary excitations have integer $\tilde{Q}$ (q, g, NG bosons)
3. Pseudo-Scaler Octet mesons in the low mass spectra

All Similar to physics in Hadron Phase

Do we really have phase transition
from Hadronic phase to CFL phase
with increasing Baryon-density??

"Quark-Hadron Continuity" conjectured by

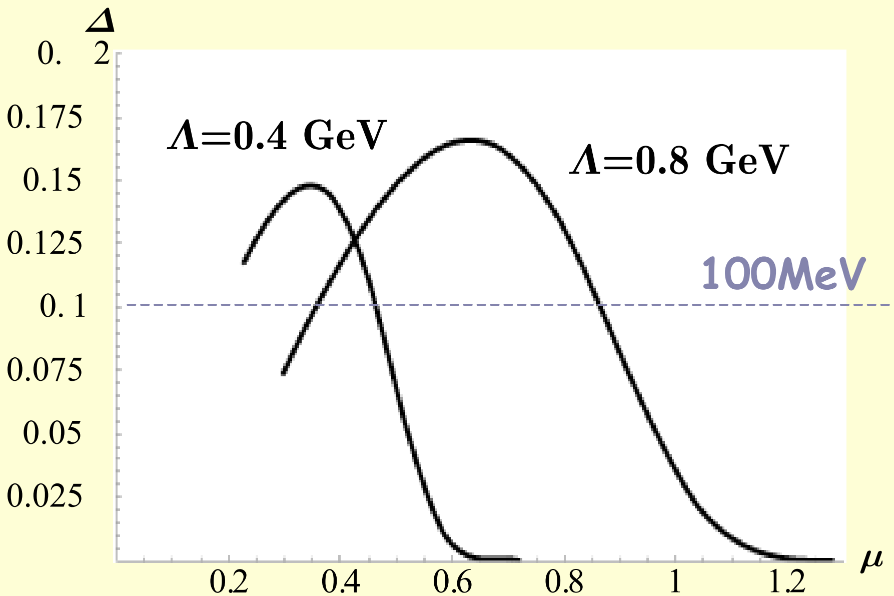
Schafer-Wilczek, PRL82, '99 , Schafer-Wilczek, PRD60, '99

Pairing at high density

Keywords:

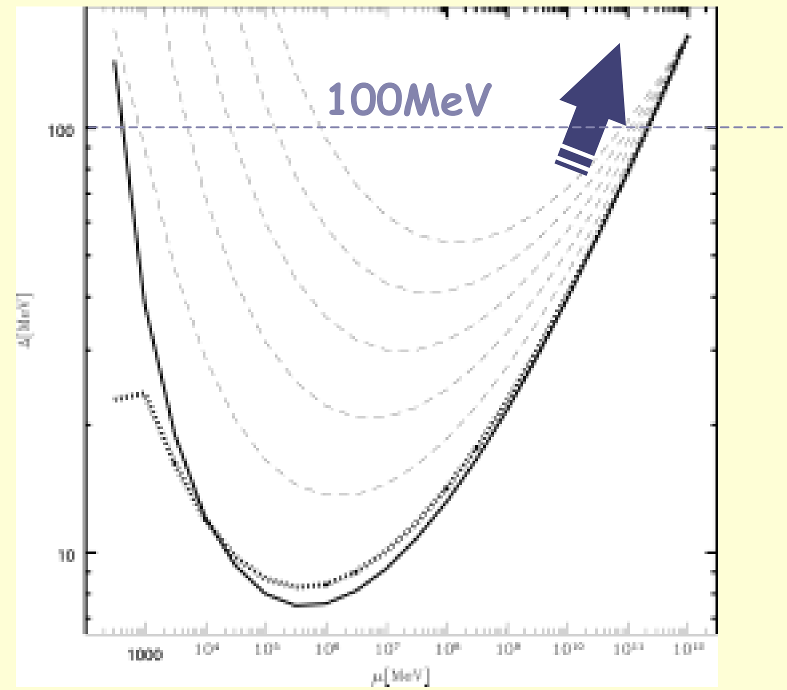
- Role of Magnetic gluons
- weak coupling expansion

Low density regime



From : Alford-Rajagopal-Wilczek, '98

High Density regime



From : Schaefer-Wilczek, '98

Effective 4-fermi model

Alford-Rajagopal-Wilczek, '98
Rapp, Schaefer-Shuryak, '98, etc…

Instanton Liquid, Random matrix

Carter-Diakonov, '00
Vanderhevden-Jackson, '99

Schwinger-Dyson approach

Iwasaki-Iwado, '95, Schaefer-Wilczek, '98
Hong-Miransiky-Shovkovy-Wijewardhana, '00
Pisarski-Rischke, '00, etc…

BS equation and Thouless criterion

Brown-Liu-Ren, '99

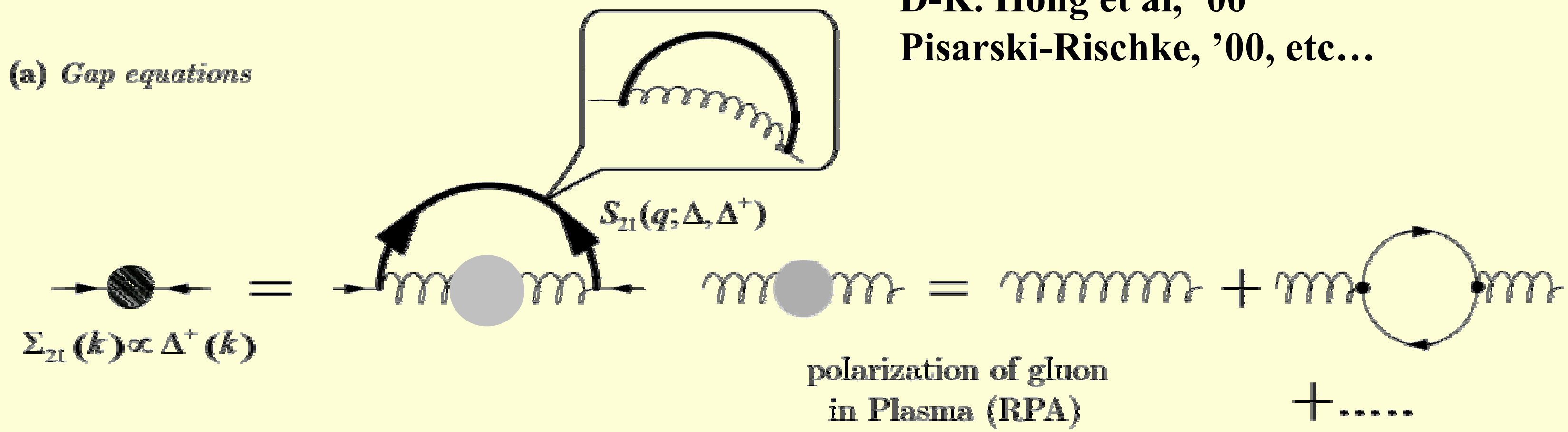
Asymptotic enhancement of gap and long-range nature of magnetic interaction

Indicated first by
RG analysis for 2-body T-matrix

D.T. Son, Phys.Rev.D59, 094019 (1999)

Confirmed by
Schwinger-Dyson approach

Schaefer-Wilczek, '98
D-K. Hong et al, '00
Pisarski-Rischke, '00, etc…



Hard dense loop propagator

$$:\ -iD_{\mu\nu}(p) = \frac{P^L_{\mu\nu}}{\vec{p}^2 + m_D^2} + \frac{P^T_{\mu\nu}}{\vec{p}^2 + M^2\,|p_0/p|} - \xi\frac{p_\mu p_\nu}{\vec{p}^4},\ \text{with } M^2 = \frac{\pi m_D^2}{4}$$

Debye screening

Dynamically screened
by Landau damping

Asymptotic enhancement of gap and long-range nature of magnetic interaction

$$\Delta(p_0) = \frac{g^2}{18\pi^2}\int dq_0 \frac{\Delta(q_0)}{\sqrt{q_0^2+\Delta(q_0)^2}}\left\{\log\left(1+\frac{64\pi^2}{N_f g^2\,|p_0-q_0|}\right)+\frac{3}{2}\log\left(1+\frac{8\pi^2}{N_f g^2}\right)\right\}$$

Magnetic electric

Note: Gauge parameter independent at high density !

leads to $\frac{12\pi^2}{g^2} \propto \log\left[\frac{2\Lambda}{\Delta}\right]\log\left[\frac{2\Lambda}{\Delta}\right]$: Double log structure

$$\Delta \propto \exp\left(-\frac{c\Lambda^2}{N_F g^2}\right) \mapsto \Delta \propto \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right)$$

BCS gap Gap in CSC

Long range nature of magnetic force leads to asymptotic enhancement of gap !

Weak Coupling Expansion of the gap in quasi-quark dispersion

$$\log\left(\frac{\Delta}{\mu}\right) = -\frac{3\pi^2}{\sqrt{2}g}$$: long ranged magnetic gluon

D.T. Son, Phys.Rev.D59, 094019 (1999)

$-5\log g$: electric gluon and static sector of magnetic gluon

Schaefer-Wilczek, '98, D-K. Hong et al, '00
Pisarski-Rischke, '00, Brown-Liu-Ren, '99, etc…

$+a+gb(g)+\ldots$: still has not yet been fixed

Wavefunction Renormalization
Brown-Liu-Ren, '99 (reduces magnitude about 1/4)

Local gauge (respect WT-identity)
D-K, Hong et al, Phys. Lett. B565, 153 (2003)
(increases magnitude about factor 1.6)
and else…? (Meissner effect,…), Still controversial…

Strange Quark Mass and CFL/2SC transition

Keywords:

- Unlocking transition
- BCS/BEC crossover?

Strange quark mass Ms and CFL/2SC transition

CFL → **2SC**

$$\left\langle q_i^a q_j^b \right\rangle_{\text{CFL}} \simeq \Delta_A \varepsilon^{abL} \varepsilon_{Lij}$$

$$\left\langle q_i^a q_j^b \right\rangle_{\text{2SC}} = \Delta \varepsilon^{ab,g} \varepsilon_{s,ij}$$

$M_s = 0$ (Chiral limit) M_s^c $M_s = \infty$

Unlocking transition

$$\left\langle us - su \right\rangle_{\text{CFL}} = \left\langle ds - sd \right\rangle_{\text{CFL}} \to 0$$

$p_F^{u,d}$

p_F^{s}

$$p_F^{u,d} - p_F^{s} \cong \frac{M_s^2}{4\mu} \begin{cases} \leq \Delta_{CFL}(M_s = 0) & : \text{CFL} \\ \geq \Delta_{CFL}(M_s = 0) & : \text{Unlocked into 2SC} \end{cases}$$

Simple **kinematical** Criterion for transition

Alford-Berges-Rajagopal,'99; Schaefer-Wilczek,'99

What about dynamical effect of Ms on quark pairing?

$$\left\langle q_i^a q_j^b \right\rangle_{CFL} = \Delta_A \varepsilon^{abL} \varepsilon_{Lij} = \frac{1}{2} \sum_{\alpha,\beta=1}^{9} \Delta_{\alpha\beta} \left(\lambda_\alpha\right)_{ai} \left(\lambda_\beta\right)_{bj}$$

$$\Delta_{\alpha\beta} = \begin{pmatrix} \Delta_8 \mathbf{1}_8 & \\ & \Delta_1 \end{pmatrix} \quad : \quad \textit{Pure}\ \text{CFL (Chiral limit)}$$

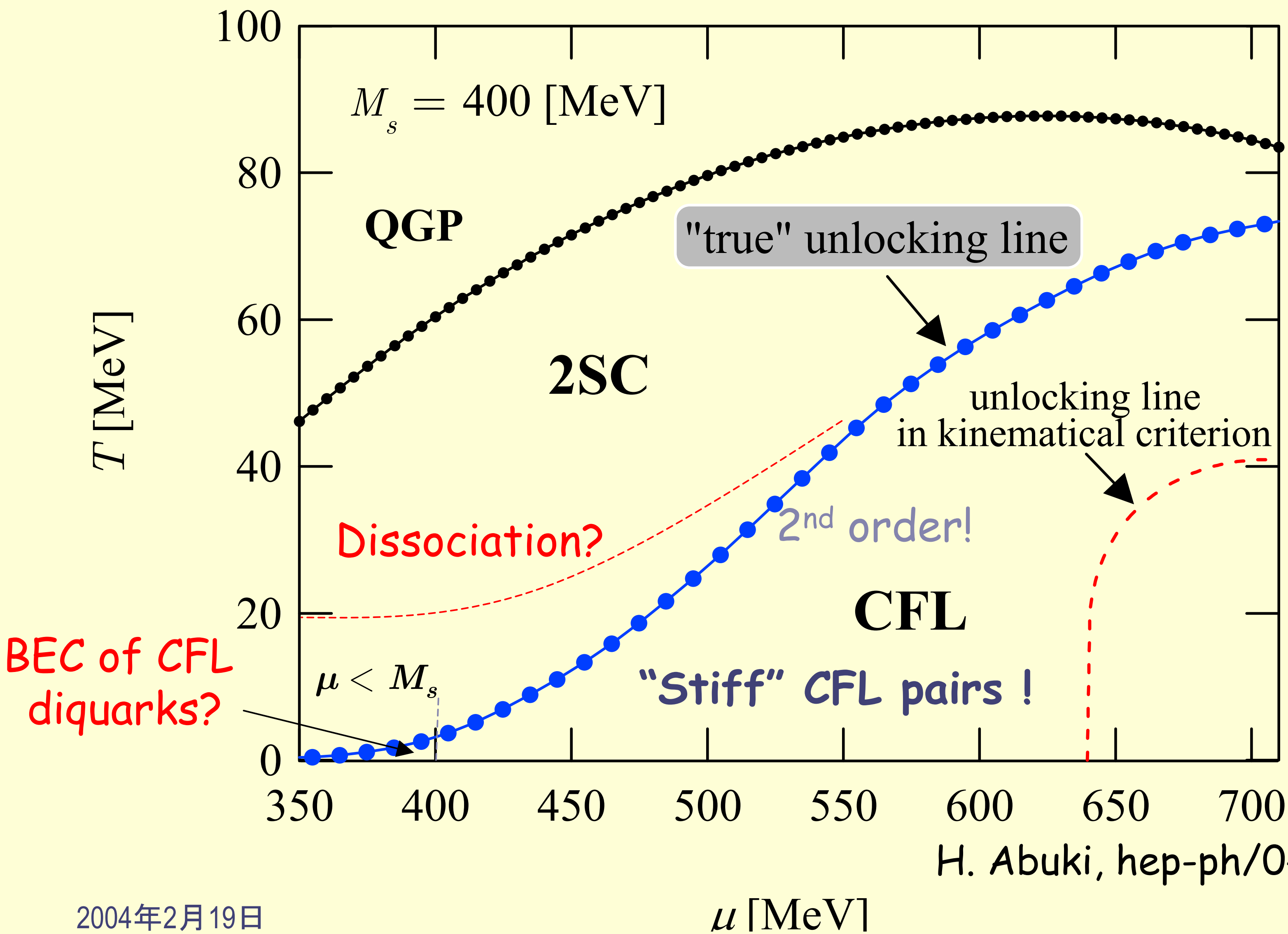
$$\Rightarrow \Delta_{\alpha\beta} = \begin{pmatrix} \Delta_8^3 \mathbf{1}_3 & & & \\ & \Delta_8^2 \mathbf{1}_4 & & \\ & & \Delta_8^1 & \\ & & & \Delta_1^1 \end{pmatrix} \quad : \quad \textit{Distorted}\ \text{CFL}\ (m_s \le m_s^c)$$

$$\Rightarrow \Delta_{\alpha\beta} = \begin{pmatrix} \Delta \mathbf{1}_3 & & & \\ & \mathbf{0}_4 & & \\ & & -\Delta/3 & -\sqrt{2}\Delta/3 \\ & & -\sqrt{2}\Delta/3 & -2\Delta/3 \end{pmatrix} \quad : \quad 2SC\ (m_s \ge m_s^c)$$

Pure CFL and Distorted CFL: $9 = 3 + 2^2 + 1 + 1$

2SC: $4 = 3 + 1 = 2 + 2$

Phase diagram for Ms=400MeV



H. Abuki, hep-ph/0401245

What is BCS/BEC crossover?

P. Nozieres and Schmitt-Rink, J. Low Temp. Phys. 59. 195, '85,
A.J. Leggett. in *Modern Trends in the Theory of Condensed Matter*, p. 13, '80,
Lambardo-Nozieres-Schuck-Schulze-Sedrakian,PRC64, '01.

In non-relativistic case : BEC (strong, dilute limit) ⇔ BCS,(weak, dense)

Weak coupling gap equation :

$$\Delta(p) = \sum_k V(p,k)\left[1 - 2n_F(\varepsilon_k)\right]\frac{\Delta(k)}{2\varepsilon_k[\Delta]}, \text{ with } \begin{cases} \varepsilon_k[\Delta] \equiv \sqrt{E_k^2 + \Delta(k)^2} \\ E_k = \left(\hbar^2 k^2 / 2m - \mu\right) \\ f_F(\varepsilon) = 1/(1 + e^{\varepsilon/T}) \end{cases}$$

is equivalent to the following wave equation :

$$\frac{p^2}{m}\varphi(p) - (1 - n_p)\sum_k V(p,k)\varphi(p) = 2\mu\varphi(p), \text{ with } \begin{cases} \varphi(p) = \langle a_p a_{-p} \rangle_{\text{BCS}} = (1 - 2f_F(\varepsilon_k))\dfrac{\Delta(k)}{2\varepsilon_k[\Delta(k)]} \\ n(p) = \langle a_p^+ a_p \rangle_{\text{BCS}} = \dfrac{1}{2} - (1 - 2f_F(\varepsilon_k))\dfrac{E_p}{2\varepsilon_p[\Delta]} \end{cases}$$

(Pauli-blocked) 2-body Schodinger (BS) equation
If in Dilute limit $\rho \sim 0$, $2\mu < 0$ signals appearance of Bound state! (BEC)

Color superconductivity in Finite size systems?

Keywords:

- **Strangelets,**
- **Bound Nuclear Kbar state**

Color Superconductivity in Finite Systems

Where CSC can be realized?

In Laboratories?

Deeply Bound Nuclear Kbar state at KEK?
$^4He(K^-,n)$ (Akaishi-Yamazaki matter)

$^4He(K^-,n)^3He_K$

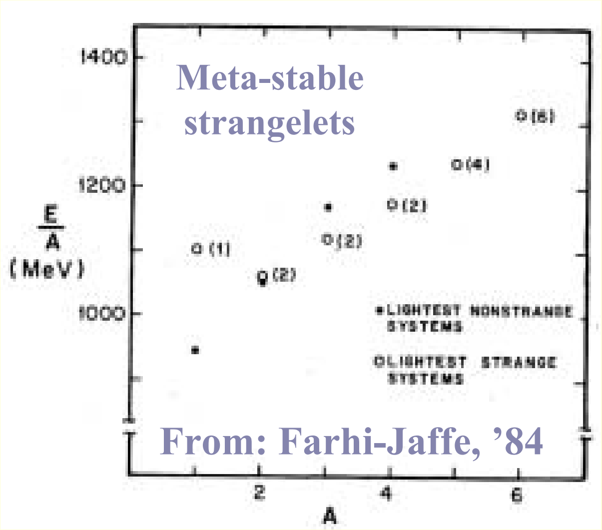
T. Suzuki et al., nucl-ex/0310018

- E_B/A is as large as 60MeV
- ρ can be possibly $8\rho_0$? (Dote et al)

Possible formation of Strangelets?

Witten, '84, Farhi-Jaffe, '84
Barger-Jaffe, '87, Gilson-Jaffe, '71,
R. Tamagaki, '91, Madsen, '93

^3He_K as $(u^4d^4s)_{A=3,Z=1}$ state?
S=1, E/A=1050MeV
Possible realization of CSC?



CSC in Finite Systems

Color-Flavor Locked strangelets : **O. Kiriyama, hep-ph/0401075**

$$L_{\text{int}} = G_1 \sum_{a=0}^{8} \left[\left(\bar{q} \lambda_a q \right)^2 + \left(\bar{q} i \gamma_5 \lambda_a q \right)^2 \right] + G_2 \,(\text{Pairing interaction})$$

And making use of MRE (multiple reflection expansion)
smeared density of state, **Balian-Bloch, Ann. Phys. 60, '70.**

$$6\int \frac{d^3k}{(2\pi)^3} \rightarrow 6\int \frac{d^3k}{(2\pi)^3} \left[1 + f_S\left(m_f / k\right) \frac{1}{kR} + f_C\left(m_f / k\right) \frac{1}{k^2 R^2} + \ldots \right]$$

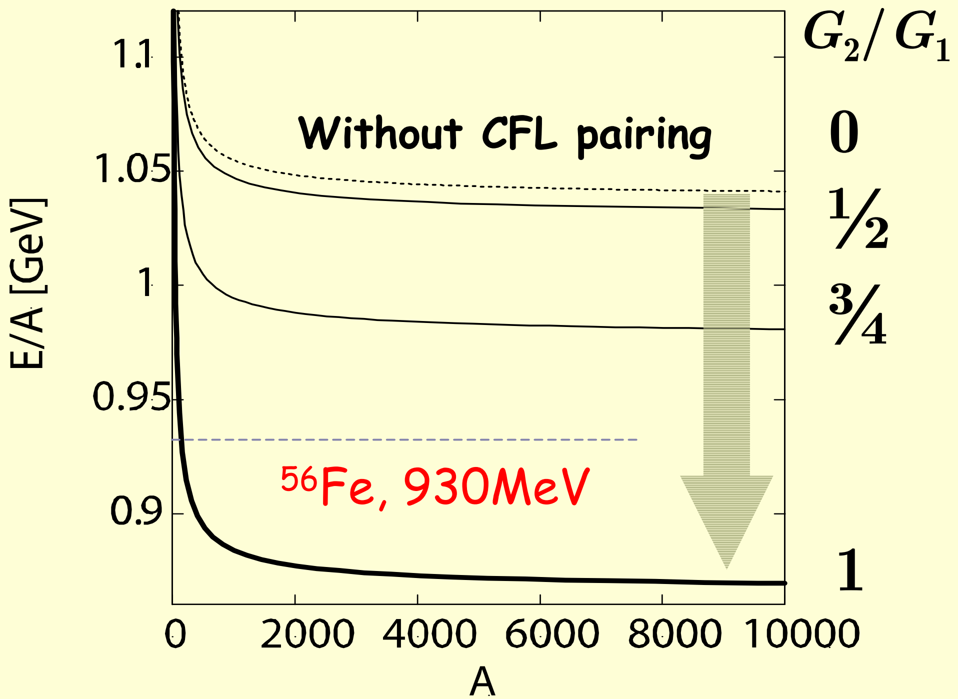
surface term curvature term

Berger-Jaffe, PRC35,'87
Madsen, PRD50,'94

Finite size effect is incorporated by reduction of DOS

CSC in Finite Systems

Color-Flavor Locked strangelets : **O. Kiriyama, hep-ph/0401075**



1. CFL strangelets are much more stable than normal strangelets
2. Strangelets with A>160 are absolutely stable ! (for G2=G1)
3. Density becomes larger than 4 times ρ_0 for A<160 (for G2=G1)

Color superconductivity in Finite Systems

What's a difference between CFL strangelets and normal one?

1. Energy-Mass relation: Strongly favors CFL-pairing in strangelets

   J. Madsen, PRL 87, 172003 ('01)., O. Kiriyama, hep-ph/0401075.

2. Charge-Mass relation (Z-A ratio)

   CFL strangelet : $Z \approx 0.3 A^{2/3}$: Small Z/A Ratio !

   J. Madsen, PRL 87, 172003 ('01)

   Normal one : $Z \approx 0.1 A$: for small strangelets A<170

   Farhi-Jaffe, RPD 30, '84, Berger-Jaffe, PRC 35, '87

Why? In the CFL phase in bulk, charge neutrality is automatically guaranteed for small m_s or μ_e: **Rajagopal-Wilczek, PRL 86, '01**

$n^u = n^d = n^s$ as long as $\left| \frac{m_s^2}{4\mu} - \delta\mu \right| < \Delta_{CFL}$ ⇒ Net charge is distributed only near surface! $Z \sim R^2$

Summary and possible future directions

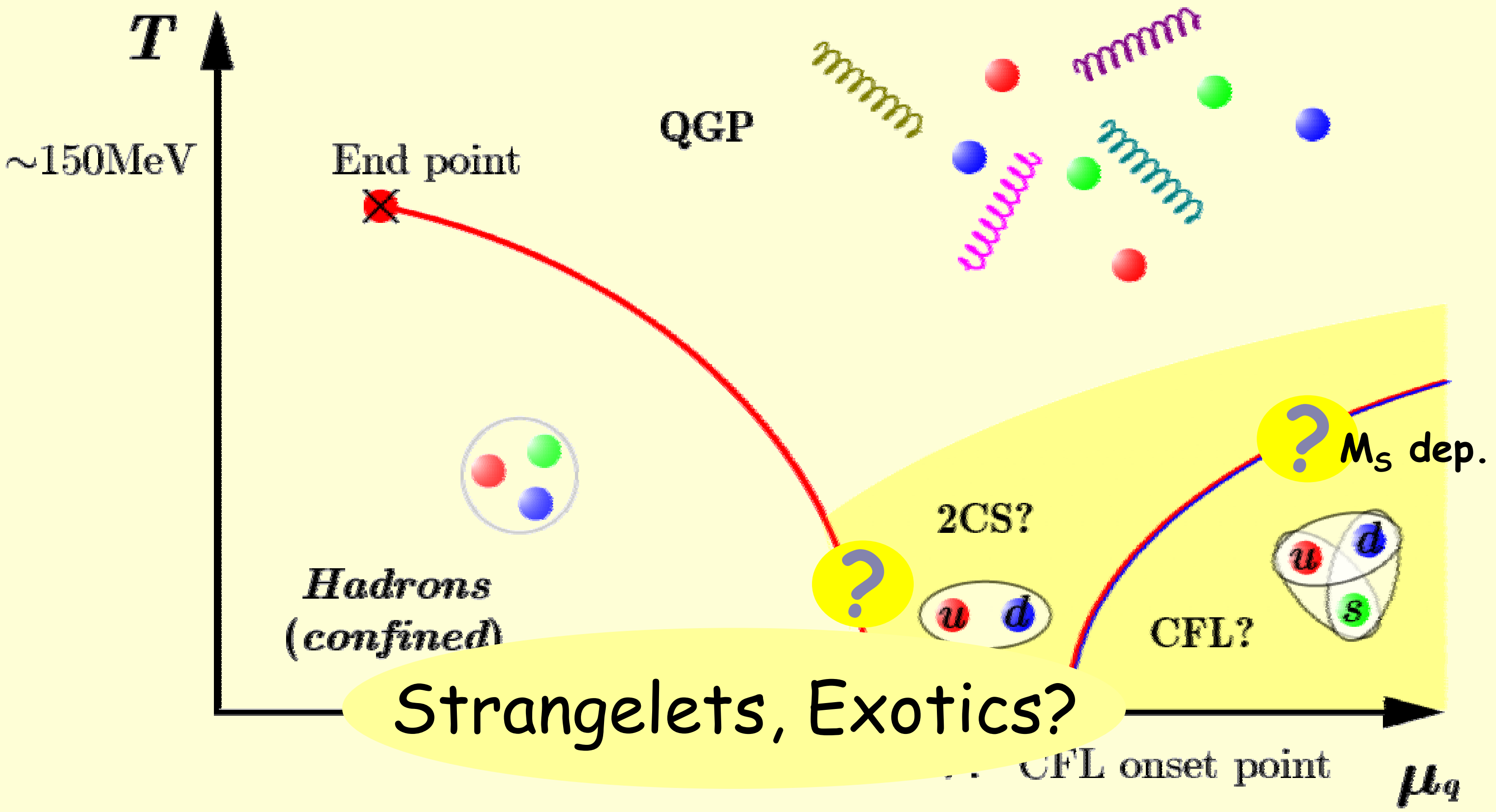
1. **Color Superconductivity: CFL, 2SC, …Q-H continuity**
2. **Pairing from high density to low density**
   - Role of magnetic gluon, weak coupling expansion
3. **Strange quark mass and 2SC/CFL unlocking**
   - Continuous unlocking, Kinematical picture?
   - CFL-diquark BEC at low strange density?
4. **CFL strangelets**
   - CFL pairing broadens model parameter space for realization of low mass strangelets (A>160, 3-4ρ_0)
   - CFL pairing reduces electric charge in strangelets

Summary and possible future directions

1. **Towards low density and finite system in more detail**
   - Diquarks in Exotic Hadrons: Diquarks in Pentaquarks?
   - Diquarks in Exotic Nuclei: Possible strangelets?
2. **Beyond mean field approximation (include fluctuation)**
   - Pseudogap phase ? **Kitazawa-Koide-Kunihiro-Nemoto, hep-ph/0309026, PRD65, '02**
   - Fluctuation induced 1st order transition to QGP phase ?

     Matsuura-Iida-Hatsuda, hep-ph/0312042

Schematic QCD Phase Diagram



Backup slides

Proposed pairing patterns in QCD (1)

2SC (2 flavor ansatz)

1. **Color** Attractive anti-symmetric channel

   ε_{abc} : Color anti-triplet

2. **Dirac** attractive in total $J=0$ (8 Dirac operator)
   and aligned chirality $(m/\mu \ll 0)$ (4 Dirac operator)

   $\left\{C\gamma_5,\ C\gamma_5\gamma_0\vec{\gamma}\cdot\hat{q}\right\}$: Parity $(+)$ ← **Favored by instanton**

   $\left\{C,\ C\gamma_0\vec{\gamma}\cdot\hat{q}\right\}$: Parity $(-)$??????

3. **Flavor** Anti-symmetric (purely from Pauli principle)

   ε_{ij} : $SU(2)_f$ flavor singlet

Proposed pairing patterns in QCD (2)

CFL (Color-Flavor Locking)

M. Alford, K. Rajagopal, F. Wilczek,
Nucl.Phys.B537:443-458,1999

1. Dirac structure is the same as in 2SC

$$\left\{C\gamma_5,\ C\gamma_5\gamma_0\vec{\gamma}\cdot\hat{q}\right\}:\ J=0^+ \text{ and aligned chirality}$$

anti-symmetric with respect to spinor indices

2. CFL ansatz for Flavor and Color structure

$$\left\langle q_i^a q_j^b \right\rangle_{\mathrm{CFL}} = \Delta_A \varepsilon^{abI}\varepsilon_{Iij} + \Delta_S\left(\delta_i^a\delta_j^b + \delta_j^a\delta_i^b\right)$$

$$\left(\bar{3}_c \times \bar{3}_f\right) \qquad \left(6_c \times 6_f\right)$$

(Attractive) (Repulsive) in naïve OGE level

symmetric under $(a,i) \Leftrightarrow (b,j)$ Pauli principle is OK.

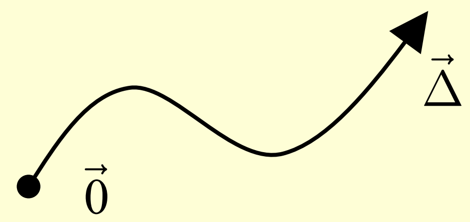
c.f. *B*-phase in He^3

Effective potential for multi-gap parameters

Pauli trick has to be carefully done in the case that many couplings enters in gap parameter space

$$g^2 \frac{\partial \Omega_{eff}}{\partial \Delta_i} \neq \left[\Delta_i - g^2 K_i[\vec{\Delta}] \right]$$

M. Alford et al., Nucl. Phys. B558,'99;



$$g^2 \frac{\partial \Omega_{eff}}{\partial \Delta_i} = \sum C^{ij} \left[\Delta_j - g^2 K_j[\vec{\Delta}] \right] : linear\ combination$$

C^{ij} *relate condensates* $\phi \sim \langle qq \rangle$ *and gap parameters* Δ *as*

$$symbolically\ g^2 \phi^i = \sum C^{ij} \Delta_j, \quad (g^2 tr(S(\Sigma)\Sigma) \sim C^{ij} \Delta_i \Delta_j)$$

$$\Omega_{eff}(\vec{\Delta}) = \frac{1}{2g^2} C^{ij} \Delta_i \Delta_j - \int_0^1 dt\ C^{ij} \Delta_i K_j[t\vec{\Delta}]$$

Derivation of effective potential

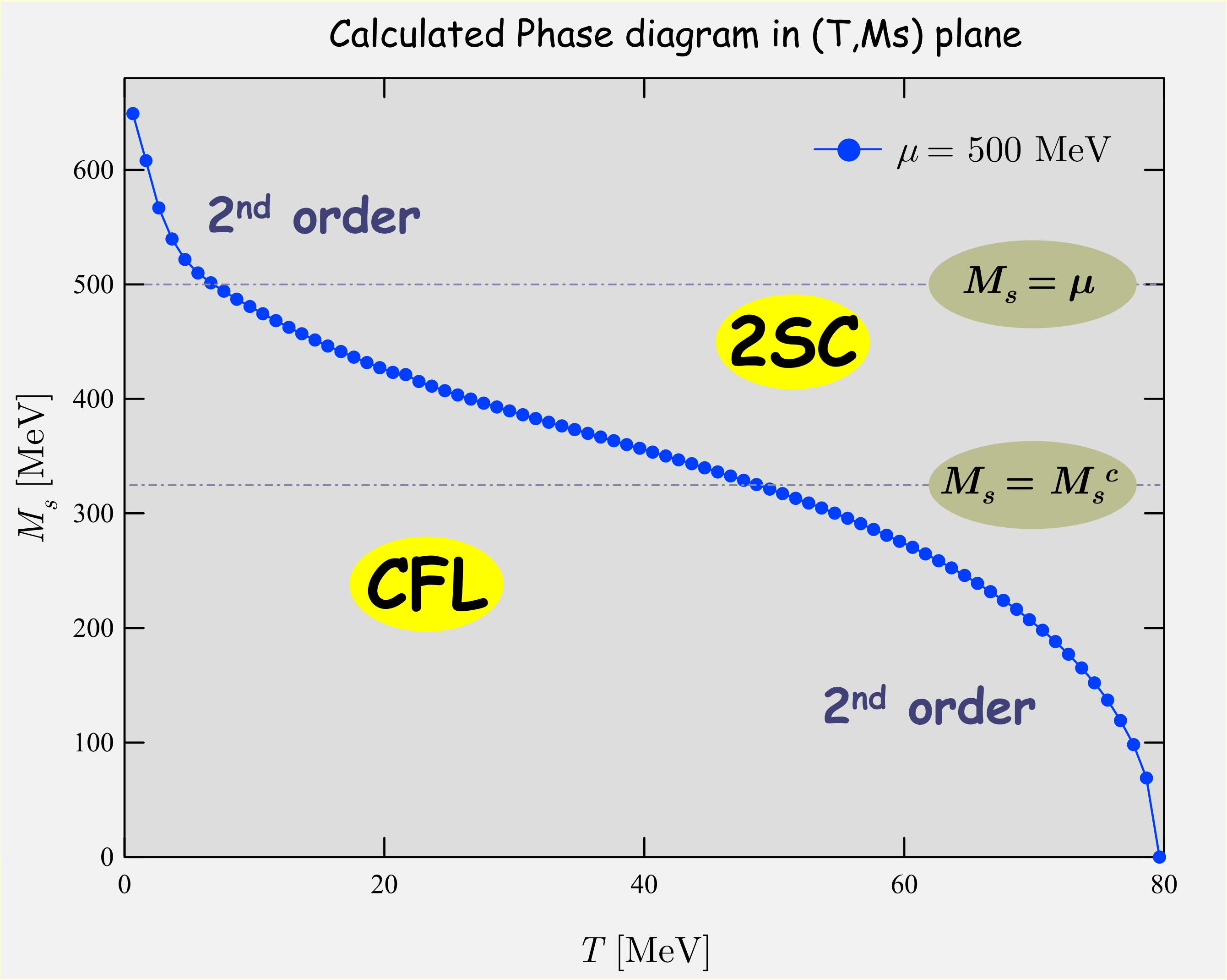
$$\frac{\partial\Omega_{eff}}{\partial g^2} = \frac{1}{g^2}\langle H_{\text{int.}}\rangle = \frac{1}{2g^2} tr\left[S(\Sigma)\Sigma\right] \quad \textit{with definition}$$

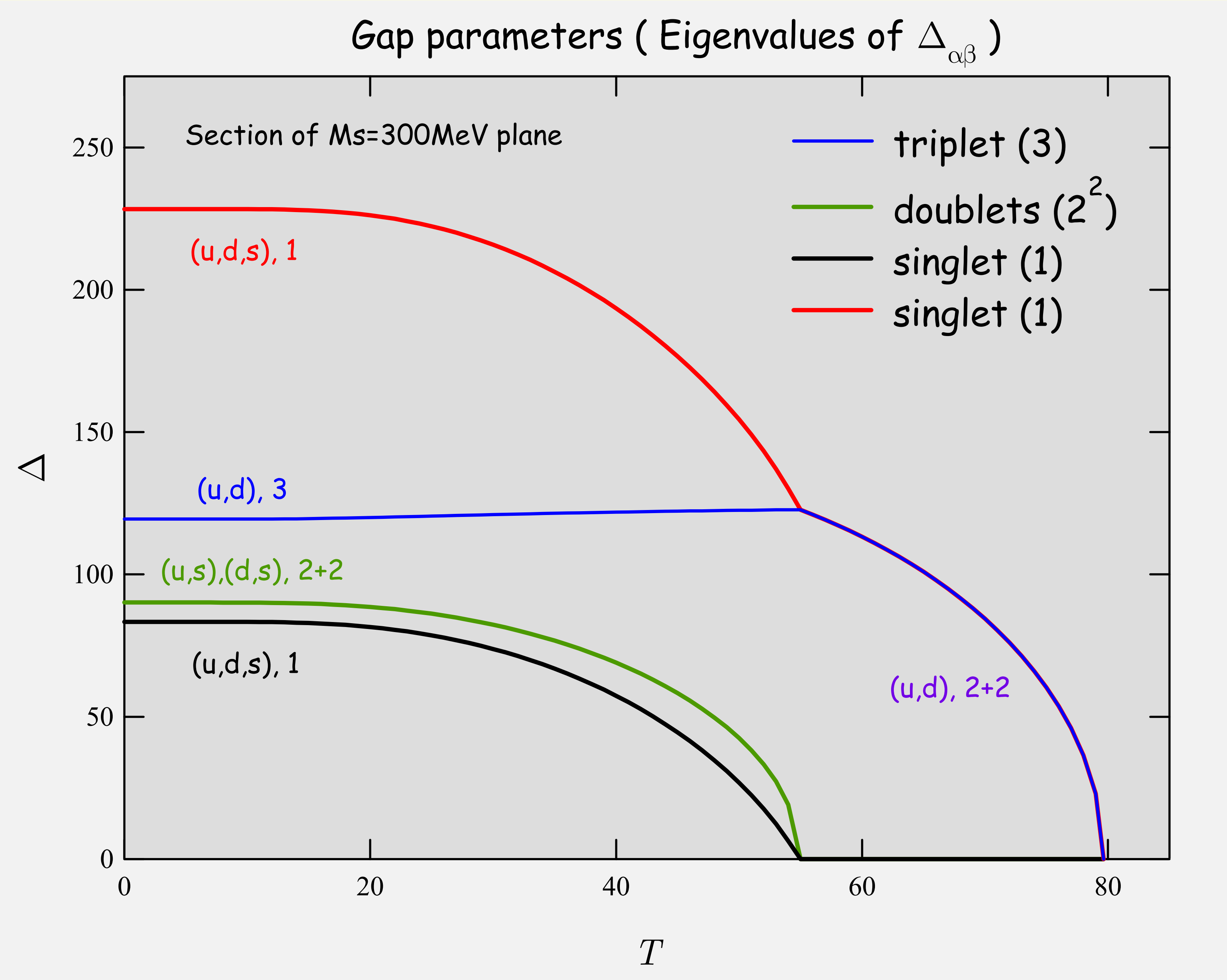
$$\Sigma_{\alpha\beta} = g^2 T_{\gamma\alpha} S^{\gamma\delta} T_{\delta\beta}, \quad S = \begin{pmatrix} \phi^1 E_3 & & & \\ & \phi^2 E_4 & & \\ & & \phi^3 & \phi^4 \\ & & \phi^4 & \phi^5 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \Delta_1 E_3 & & & \\ & \Delta_2 E_4 & & \\ & & \Delta_3 & \Delta_4 \\ & & \Delta_4 & \Delta_5 \end{pmatrix}.$$

$T_{\gamma\alpha}$ *is quark-gluon vertex in color-flavor mixed base. We have*

$$\frac{\partial\Omega_{eff}}{\partial g^2} = \frac{1}{2g^4} C^{ij}\Delta_i\Delta_j, \quad \textit{with} \quad C = \begin{pmatrix} 3 & 0 & -3 & -6\sqrt{2} & -6 \\ 0 & -8 & 8 & 4\sqrt{2} & -8 \\ -3 & 8 & -5 & 2\sqrt{2} & -2 \\ -6\sqrt{2} & 4\sqrt{2} & 2\sqrt{2} & -12 & 0 \\ -6 & -8 & -2 & 0 & -4 \end{pmatrix}$$

Phase Diagram in (T, Ms) plane





Gap parameters (Eigenvalues of $\Delta_{\alpha\beta}$)
Section of Ms=300MeV plane
triplet (3)
doublets (2^2)
singlet (1)
singlet (1)
(u,d,s), 1
(u,d), 3
(u,s),(d,s), 2+2
(u,d,s), 1
(u,d), 2+2
250
200
150
100
50
0
Δ
0
20
40
60
80
T

Detail of kernel

$$K_{83}[\Delta]=\int\frac{d^3q}{(2\pi)^3}\frac{1}{4}\left(\tanh\left(\frac{E_{0+}[\Delta]}{2T}\right)\frac{\Delta}{E_{0+}[\Delta]}+\tanh\left(\frac{E_{0-}[\Delta]}{2T}\right)\frac{\Delta}{E_{0-}[\Delta]}\right)\bar{g}^2(q,k)\left[g_{\mu\nu}D_{\mu\nu}(q-k)\right]$$

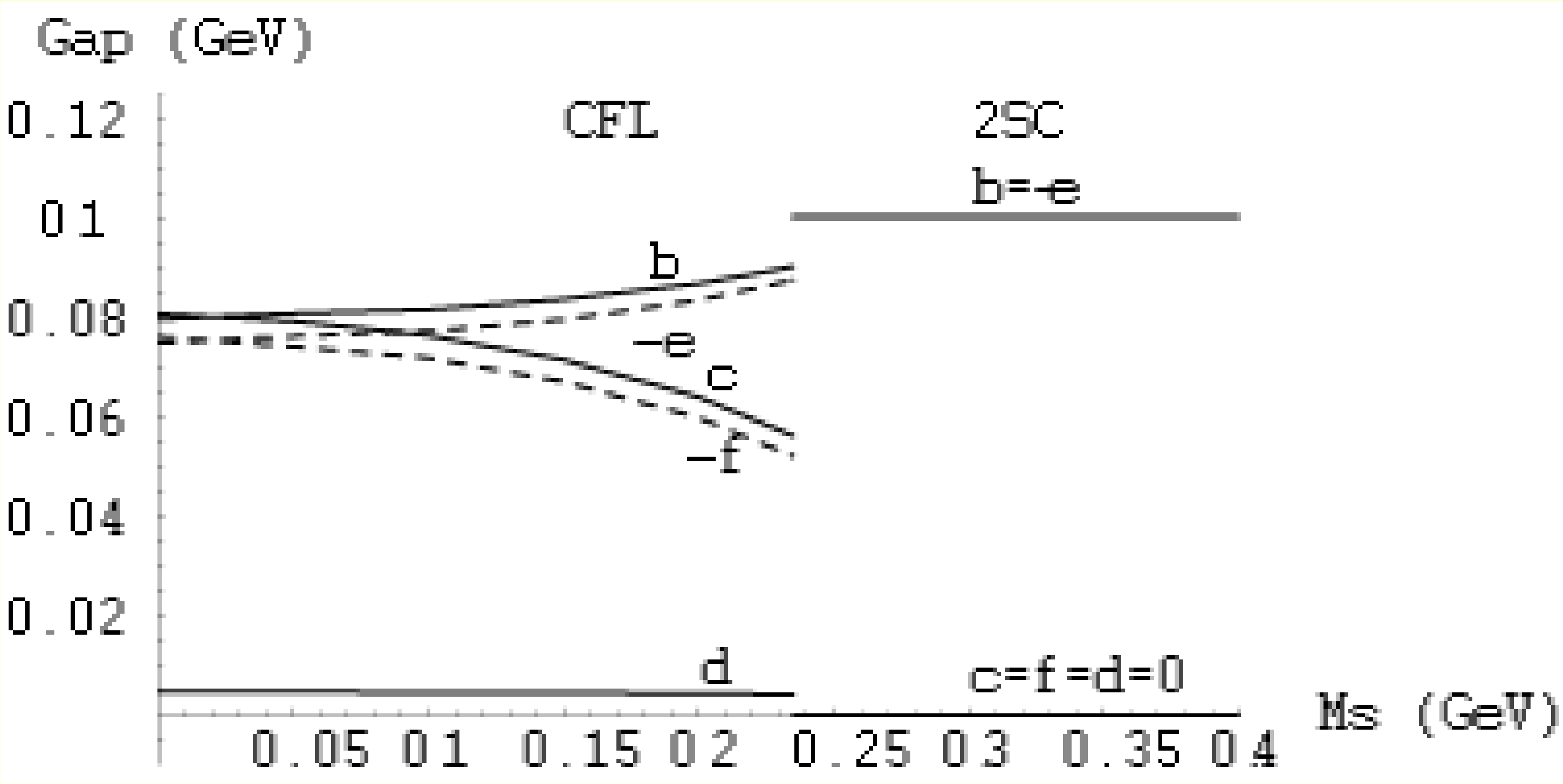
$$K_{82}[\Delta]=\int\frac{d^3q}{(2\pi)^3}\frac{1}{8}\left(\begin{array}{l}+\tanh\left(\frac{E_{0+}[\Delta]}{2T}\right)\frac{\Delta}{E_{0+}[\Delta]}+\tanh\left(\frac{E_{0-}[\Delta]}{2T}\right)\frac{\Delta}{E_{0-}[\Delta]}\\+\tanh\left(\frac{E_{M+}[\Delta]}{2T}\right)\frac{\Delta}{E_{M+}[\Delta]}+\tanh\left(\frac{E_{M-}[\Delta]}{2T}\right)\frac{\Delta}{E_{M-}[\Delta]}\end{array}\right)\bar{g}^2(q,k)\left[g_{\mu\nu}D_{\mu\nu}(q-k)\right]$$

$$K_{81}[\Delta_1,\Delta_2,\Delta_3]=\{\text{ultra super complicated form...}\}$$

$$K_{11}[\Delta_1,\Delta_2,\Delta_3]=\{\text{ultra super complicated form...}\}$$

$$K[\Delta_1,\Delta_2,\Delta_3]=\{\text{ultra super complicated form...}\}$$

$$E_{M\pm}[\Delta]=\sqrt{\left(\sqrt{q^2+M^2}\mp\mu\right)^2+\Delta^2}$$: **Quasi-Quark and Quasi-AntiQuark energy**



From : M. Alford et al., Nucl. Phys. B558,'99;

CSC in Finite Systems

Trials in Past

Amore-Birse-McGovern-Walet, PRD 65, 074005, ('02).

2SC in finite system is considered
Baryon # & Color singlet projection, Energy shell structure
pressur balance is not considered

Madsen, PRL 87, 172003 ('01).

CFL strangelet,
MRE (multiple reflection expansion) smeared density of State,

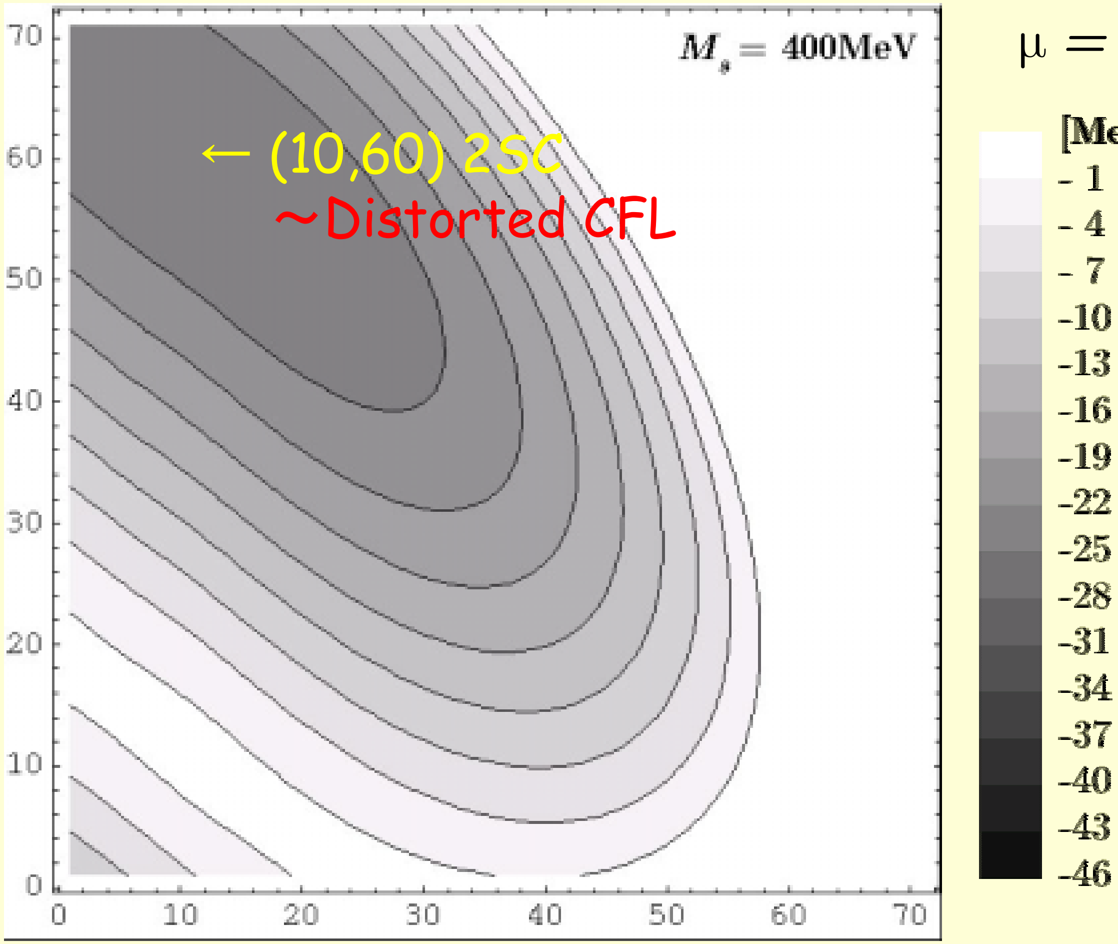
$\Omega_{CFL} = -3\Delta^2\mu^2/\pi^2$ with $\Delta = 100 MeV$: fixed

and pressure balance (Bag, CFL pressure), Ignore Coulomb

O. Kiriyama, hep-ph/0401075 ('04).

NJL model, MRE smeared density of State, Ms=0 fixed
$\Delta(A)$ and E/A are solved under pressure balance condition

Effective potential in five gap parameter space



$\mu = 400 \text{ MeV}, T = 0 \text{ MeV}$

2SC is saddle point
Always a solution of gap equation

Distorted CFL state moves towards the 2SC state and Condensation energy gets reduced

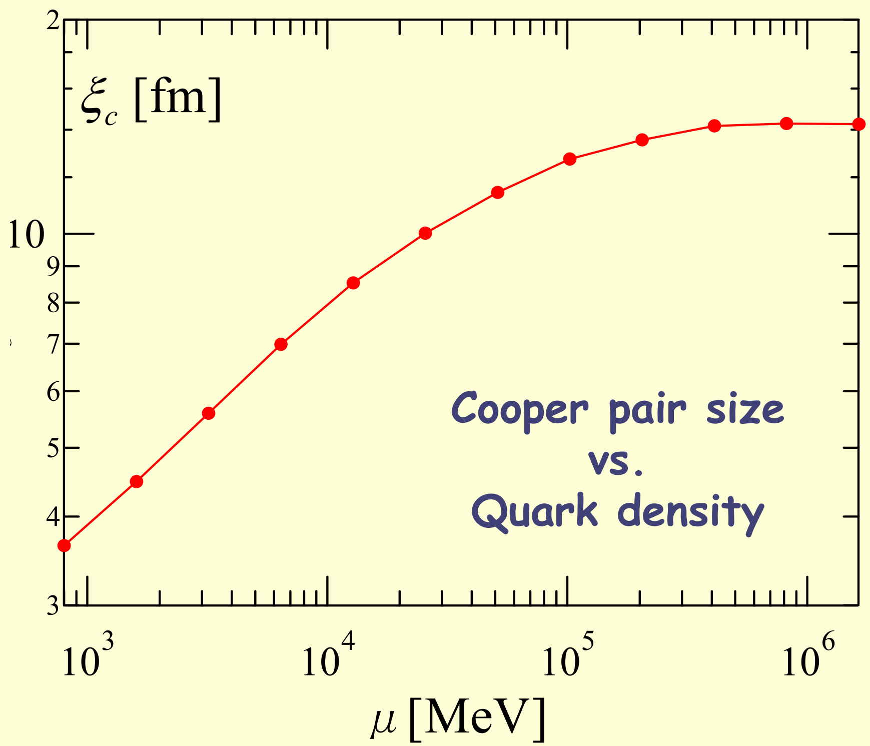
CFL state disappears into 2SC state as $M_s \to \mu$

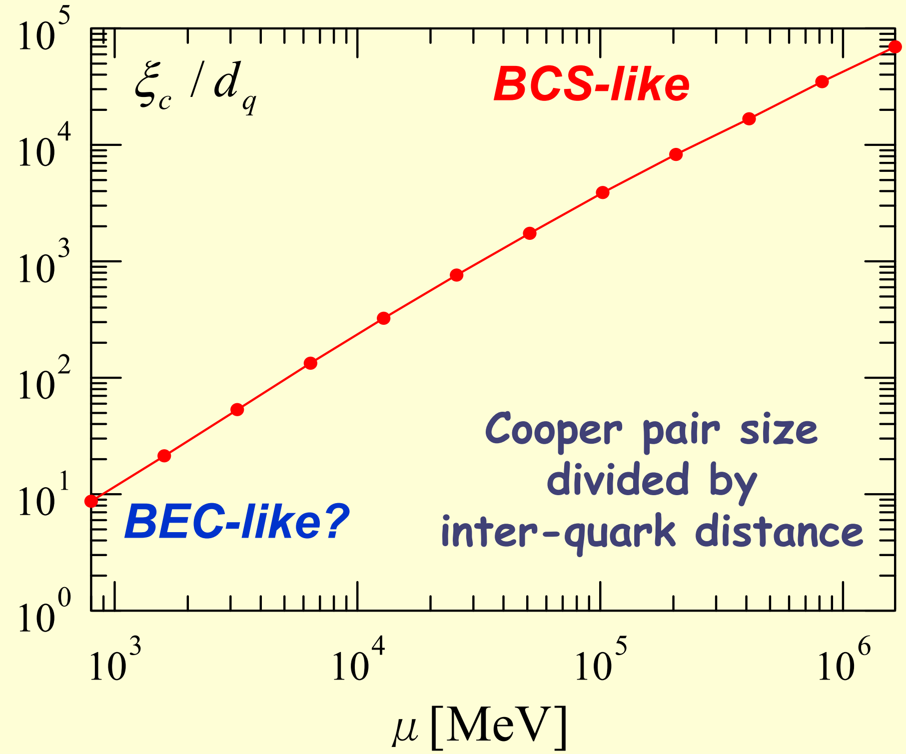
2nd order transition?

What happens when we goes towards low density?

Pair "wavefunction" : $\varphi(\vec{r}) = \langle q(t,\vec{r})\, q(t,0) \rangle \underset{\mu r \gg 1}{\rightarrow} N \frac{\sin(\mu r)}{(\mu r)^{3/2}} e^{-\Delta r}$

What about internal structure of Cooper pairs?





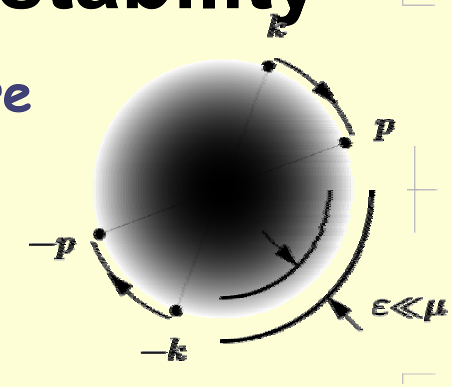
Improved ladder Schwinger-Dyson approach
Abuki-Itakura-Hatsuda, Phys. Rev. D65, (2002)

Tightly bound diquarks at low density?

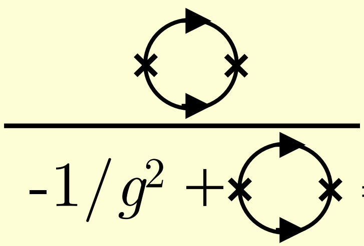
Fermi surface and Cooper instability

Our starting point : there is Large Fermi sphere

- Pauli-Blocking
- Low Energy Modes with Large Density of State



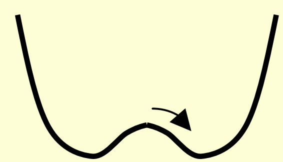
Results in non-perturbative IR dynamics



$\Gamma^{(2)}(p) = -\Delta_F^{-1}(p)$

for **small** p : $-\Delta_F(p) = \dfrac{Z}{p_0^2 - \alpha p^2 + M^2} \underset{p\to 0}{>} 0$:

negative curvature of effective potential



$$\frac{\partial^2 \Omega_{eff}}{\partial \Delta^2} = -\Gamma^{(2)}(0) = -\frac{M^2}{Z} < 0$$

low p RPA mode is *tachyonic* at T=0 !

Cooper instability !

leads to reorganization of Fermi ball
into BCS state with non-vanishing qq condensate

BCS mechanism $\Delta \sim \omega_D e^{-1/Ng^2}$

Symmetry of CFL

$$\left\langle q_{Li}^{a} q_{Lj}^{b} \right\rangle_{\mathrm{CFL}} = \sum_{I} \Delta_{A} \varepsilon^{abI} \varepsilon_{Iij} = -\left\langle q_{Ri}^{a} q_{Rj}^{b} \right\rangle_{\mathrm{CFL}}$$

Invariant under $SU(3)_{\mathrm{C+L}}$ $SU(3)_{\mathrm{C+R}}$

$$\underbrace{SU(3)_c \times SU(3)_L \times SU(3)_R}_{\supset U(1)_{\mathrm{EM}}} \times U(1)_B \rightarrow \underbrace{SU(3)_{C+V}}_{\supset \tilde{U}(1)_{\mathrm{EM}}}$$

- color symmetry is broken : 8 massive gluons
- chiral symmetry is broken : octet NG bosons
- baryon number is broken : superfluid mode (H)