YITP workshop on
``Multi-quark Hadrons; Four, Five and More?''

Pentaquark baryons from lattice QCD

Shoichi Sasaki
Univ. of Tokyo

S. Sasaki, hep-lat/0310014

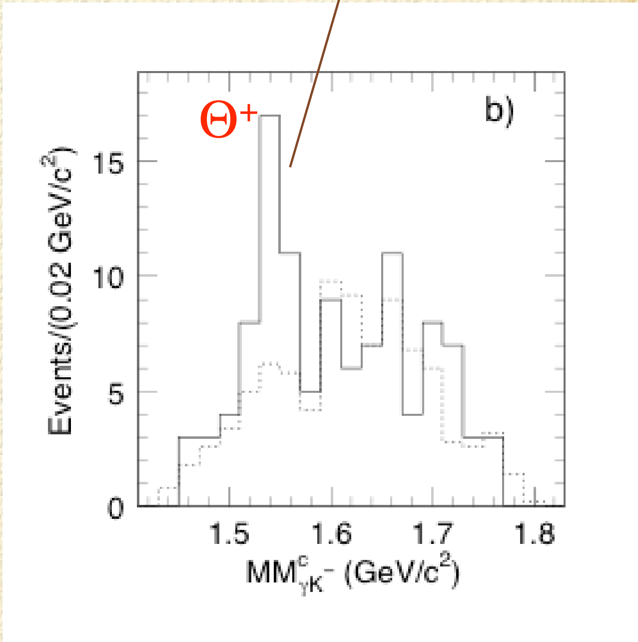
Discovery of Exotic S=+1 Baryon

- T. Nakano et al.
  Phys.Rev.Lett.91 (2003) 012002
  Laser-Electron Photon facility (LEPS)@Spring-8

$$\gamma n \to \Theta^+ K^- \to n K^+ K^-$$

- ☑ Positive Strangeness $(uudd\bar{s})$
- ☑ Very narrow width
- ☑ Spin and Parity are undetermined.

$Mass = 1540 \pm 10$ MeV
$Width \le 25$ MeV



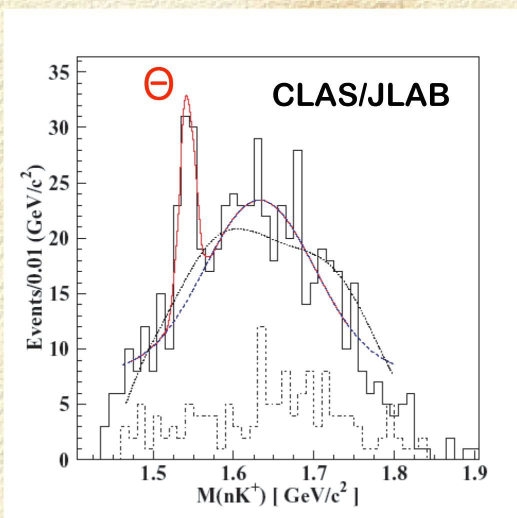
Confirmation from other experiments

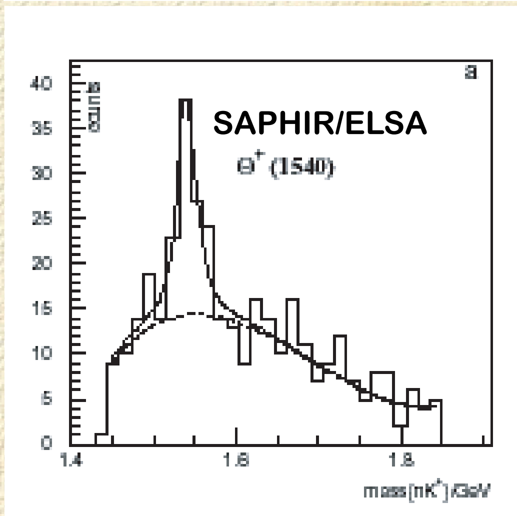
- **DIANA/ITEP (hep-ex/0304040)**
  Mass = 1539 ± 2 MeV,
  Width < 9 MeV

- **CLAS/JLAB (hep-ex/0307018)**
  Mass = 1542 ± 5 MeV,
  Width < 21 MeV

- **SAPHIR/ELSA (hep-ex/0307083)**
  Mass = 1540 ± 4 MeV,
  Width < 25 MeV

- **HERMES/DESY (hep-ex/0312044)**
  Mass = 1528 ± 2.6 MeV,
  Width < 19 ± 5 MeV





But, spin and parity are still undetermined.

The existence of the Θ has been established.

Exotic anti-decuplet baryons

- A narrow exotic S=+1 baryon Θ^+(Z^+) predicted by the chiral quark-soliton model

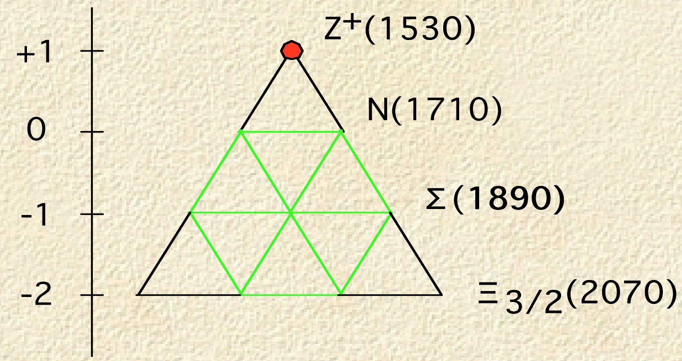
Diakonov et al. Z. Phys. A359 (97) 305

"Bound state" of octet baryons with octet mesons

$$8_f \times 8_f = 1_f + 8_f + 8_f + 10_f + 10_f^* + 27_f$$

Exotic S=+1 state in the 10*(I=0) and the 27(I=1)

- ☑ Exotic: S=+1 in the10*(I=0)
- ☑ Low mass:1530 MeV
- ☑ Narrow width: <15 MeV
- ☑ $J^P=1/2^+$



What can lattice QCD say?

The discovery of the $\Theta^+(1540)$ triggered many model predictions.

- What is spin, parity and isospin of the $\Theta^+(1540)$?
- Existence of the charm (bottom) pentaquark state

$$(uudd\bar{s}) \rightarrow (uudd\bar{c}) \text{ or } (uudd\bar{b})$$

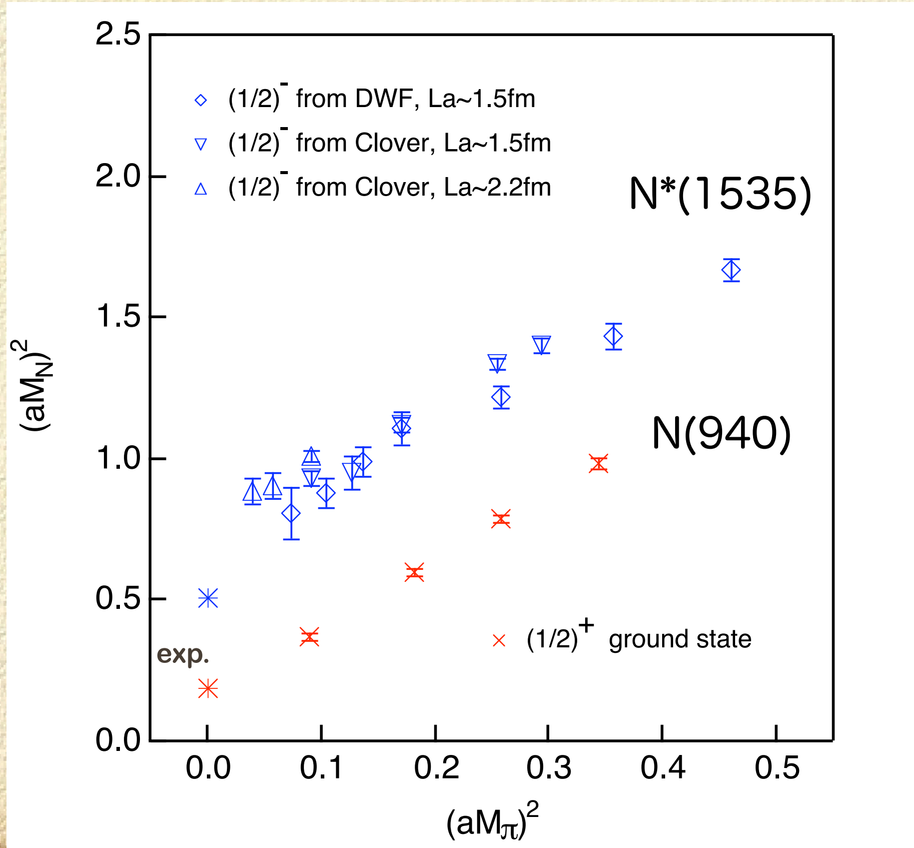
Maximal knowledge about those matters is essential to understanding the structure of the pentaquark state.

Lattice QCD can answer both of them before experimental efforts

Lattice studies of N* spectrum (1)

- **Lee-Leinweber**
  D χ 34 action hep-lat/9809095, D234 action, hep-lat/0011060, 0110164.
- **Sasaki-Blum-Ohta (RIKEN-BNL)**
  Domain wall fermion, hep-lat/9909093, Phys. Rev. D65 (2002) 074503.
- **Richards et al (UKQCD-QCDSF-LHPC)**
  Clover fermion, hep-lat/0011025, Phys. Lett. B532 (2002) 63.
- **Melnitchouk et al (Adelaide)**
  Fat-link clover fermion, hep-lat/0202022, Phys. Rev. D67 (2003) 114506.
- **Nemoto-Nakajima-Matsufuru-Suganuma**
  Clover fermion & anisotropic action, hep-lat/0204014, Phys.Rev.D68 (2003) 094505.
- **Bern-Graz-Regensburg Collaboration**
  Chirally improved fermion, hep-ph/0307073

Lattice studies of N* spectrum (2)



Large mass splitting between N and N* is well reproduced.

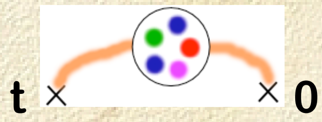
Some difficulty of lattice study ?

- A simple minded study of pentaquark state with
- $$\Theta^{+} \sim \boxed{\varepsilon_{abc} d_a d_b u_c} \times \boxed{\bar{s}_e u_e}$$

N K

How can we distinguish between

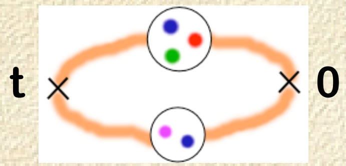
the mass of the pentaquark state



and

the total energy of the interacting KN two-body system

The 2-pt function $\langle \Theta(t)\overline{\Theta}(0) \rangle$ should be dominated by the latter if $M_{\Theta} > M_N + M_K$

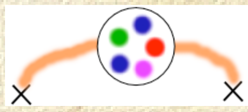


Some difficulty of lattice study ?

- A simple minded study of pentaquark state with
- $$\Theta^+ \sim \underbrace{\varepsilon_{abc} d_a d_b u_c}_{N} \times \underbrace{\bar{s}_e u_e}_{K}$$

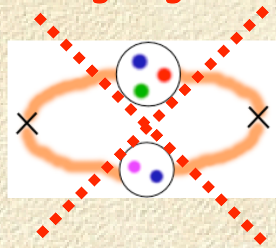
How can we distinguish between

the mass of the pentaquark state

t  0

and

the total energy of the interacting KN two-body system

t  0

Choose a specific operator with as little overlap with the KN scattering state as possible

$$|\langle \Theta^+ | \mathcal{O} | 0 \rangle| \gg |\langle K + N | \mathcal{O} | 0 \rangle|$$

Exotic pentaquark operator (1)

An exotic description of S=+1 state ($uudd\bar{s}$) can be described by

$$\Theta^+ \sim (\bar{s})_{qq}(\bar{s})_{qq}\bar{s}$$

using the flavor antitriplet diquark $(\bar{q}_i)_{qq} = \varepsilon_{ijk} q_j q_k$

flavor: $3_f^* \times 3_f^* \times 3_f^* = 1_f + 8_f + 8_f + 10_f^*$

For the color singlet state, above diquark should be in the color antitriplet as well

color: $3_c^* \times 3_c^* \times 3_c^* = 1_c + 8_c + 8_c + 10_c^*$

Recently, many authors remarked importance of exotic descriptions as diquark-diquark-antiquark

Karliner-Lipkin(hep-ph/0307019), Jaffe-Wilczek(hep-ph/0307341), Carlson et al.(hep-ph/0307396), Glozman(hep-ph/0308232)

Exotic pentaquark operator (2)

- The isospin zero and color 3* diquark field can be defined by

$$\Phi_{\Gamma}^{a}(x) = \varepsilon_{ij}\varepsilon_{abc} q_{i,b}^{T}(x) C\Gamma q_{j,c}(x)$$

where Γ is any of the 16 possible Dirac γ-matrices.

- Accounting for both color and flavor antisymmetries,

Γs are restricted within 1, γ_5 and $\gamma_5 \gamma_\mu$

which satisfy the relation $(C\Gamma)^T = -C\Gamma$

Three types of diquark: 0^+ (γ_5), 0^- (1), 1^- ($\gamma_5 \gamma_\mu$) can be allowed.

Exotic pentaquark operator (3)

- The color singlet state can be constructed by the color antisymmetric part of di-diquark with a strange anti-quark as

$$\varepsilon_{abc}\Phi^a_\Gamma(x)\Phi^b_{\Gamma'}(x)C\bar{s}^T_c(x) \quad \text{for} \quad \Gamma \neq \Gamma'$$

- Three types of exotic pentaquark operators are yielded

$$\Theta_+(x) = \varepsilon_{abc}\Phi^a_1(x)\Phi^b_{\gamma_5}(x)C\bar{s}^T_c(x) \qquad J = \frac{1}{2}$$

$$\left.\begin{aligned} \Theta_1{}^\mu(x) &= \varepsilon_{abc}\Phi^a_1(x)\Phi^b_{\gamma_5\gamma_\mu}(x)C\bar{s}^T_c(x) \\ \Theta_2{}^\mu(x) &= \varepsilon_{abc}\Phi^a_{\gamma_5}(x)\Phi^b_{\gamma_5\gamma_\mu}(x)C\bar{s}^T_c(x) \end{aligned}\right\} \quad J = \frac{1}{2} \text{ and } \frac{3}{2}$$

Exotic pentaquark operator (4)

- The parity of the spin-1/2, isosinglet Θ operator is positive

$$\Theta_+ = \varepsilon_{abc}\varepsilon_{aef}\varepsilon_{bgh}(u_e^T C d_f)(u_g^T C\gamma_5 d_h)C\bar{s}_c^T$$

0^- x 0^+ x $1/2^-$ = $1/2^+$

Multiplying the left hand side of Θ_+ by γ_5

$$\Theta_- = \gamma_5\Theta_+$$
$$= \varepsilon_{abc}\varepsilon_{aef}\varepsilon_{bgh}(u_e^T C d_f)(u_g^T C\gamma_5 d_h)\gamma_5 C\bar{s}_c^T$$

It turns out that $\langle\Theta_-(t)\overline{\Theta}_-(0)\rangle = -\gamma_5\langle\Theta_+(t)\overline{\Theta}_+(0)\rangle\gamma_5$

For details of the parity projection, see Sasaki-Blum-Ohta PRD65 (2002) 074503.

Details of the simulation

Gauge: Standard plaquette action

β =6.2, $a^{-1} \approx$ 3 GeV

lattice sizes 32^3 x 48, V≈(2.2 fm)^3,

statistics 135 configs

Fermion: Wilson fermions

5 quark masses (M_π > 600MeV) with charm mass

K=0.1520, 0.1515, 0.1506, 0.1489, 0.1480, 0.1360

Point source - Point sink (t_{src}= 6)

P.B.C. + A.P.B.C. for the temporal direction

Basic results

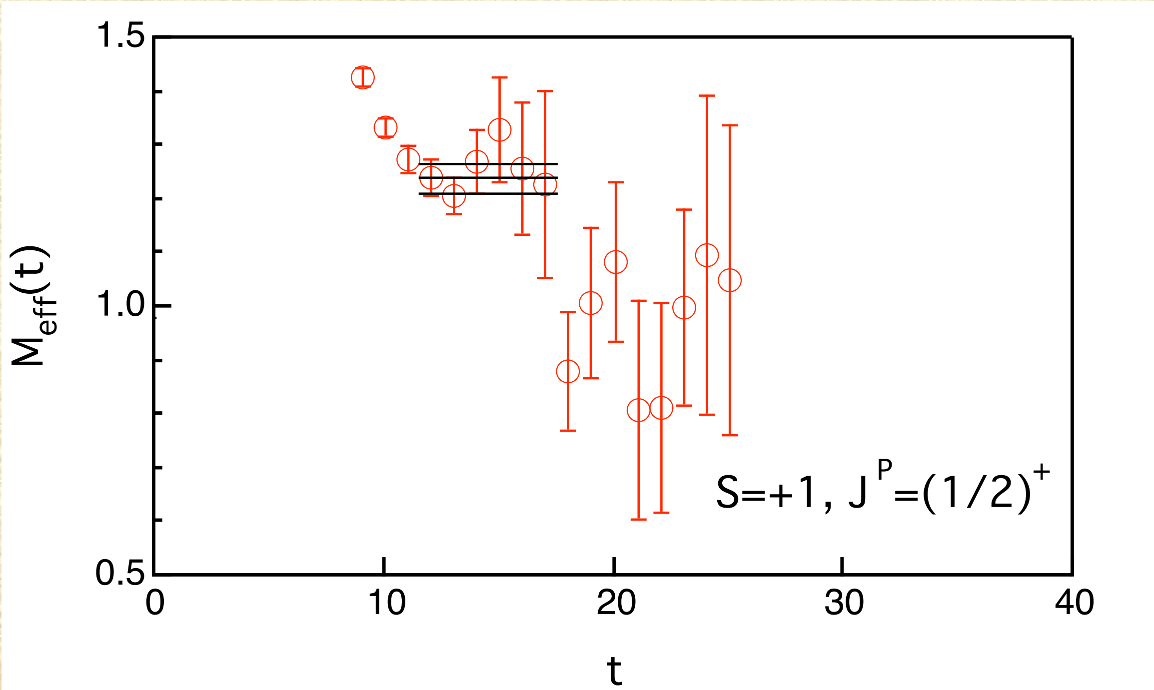
A lattice scale is set by the gluonic scale: $a = 0.0677$ fm, (a^{-1}=2.94 GeV)

- ✓ "strange": at K = 0.1515 aM_{Vector} = 0.335 (4) ~ 0.98 GeV ~ ϕ(1020)
- ✓ "charm": at K = 0.1360 aM_{Vector} = 1.031 (2) ~ 3.04 GeV ~ J/ψ(3097)
- ✓ chiral extrapolated values:

aM_ρ = 0.235 (6)	~ 0.69 GeV	11%↘	(0.77 GeV)
aM_N = 0.361 (10)	~ 1.06 GeV	12%↗	(0.94 GeV)
aM_K = 0.179 (2)	~ 0.53 GeV	8%↗	(0.49 GeV)
aM_Σ = 0.440 (8)	~ 1.30 GeV	8%↗	(1.20 GeV)
aM_Ξ = 0.486 (7)	~ 1.43 GeV	8%↗	(1.32 GeV)
aM_D = 0.641 (2)	~ 1.88 GeV	<1%	(1.89 GeV)
aM_{Σ_c} = 0.842 (13)	~ 2.48 GeV	<1%	(2.46 GeV)

(uudd ^{bar}s) state with positive parity

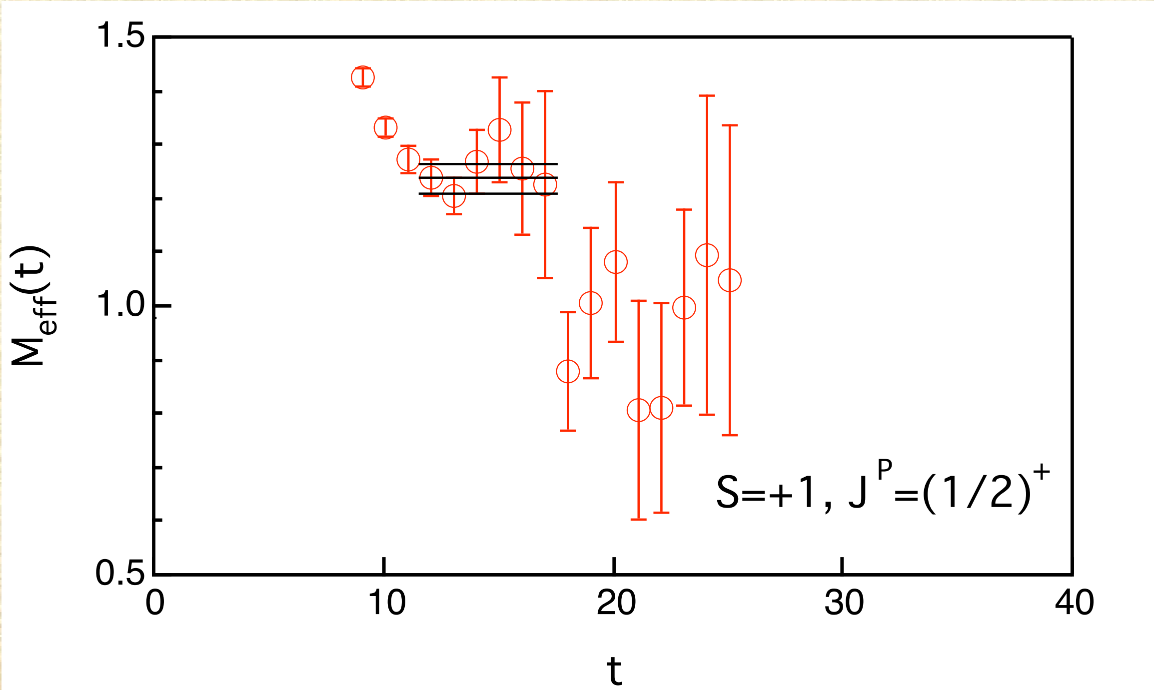
$\Theta(1/2^{+}) \rightarrow (KN)_{\text{P-wave}}$ $\sqrt{M_N^2 + p_{\min}^2} + \sqrt{M_K^2 + p_{\min}^2} \quad (|\vec{p}_{\min}| = 2\pi/L)$



$$M_{\text{eff}}(t) = \ln\{G(t)/G(t+1)\} \propto M \qquad (G(t) \propto e^{-Mt})$$

(uudd ^{bar}s) state with positive parity

$\Theta(1/2^+) \rightarrow (KN)_{\text{P-wave}}$ $\sqrt{M_N^2 + p_{\min}^2} + \sqrt{M_K^2 + p_{\min}^2} \quad (|\vec{p}_{\min}| = 2\pi/L)$

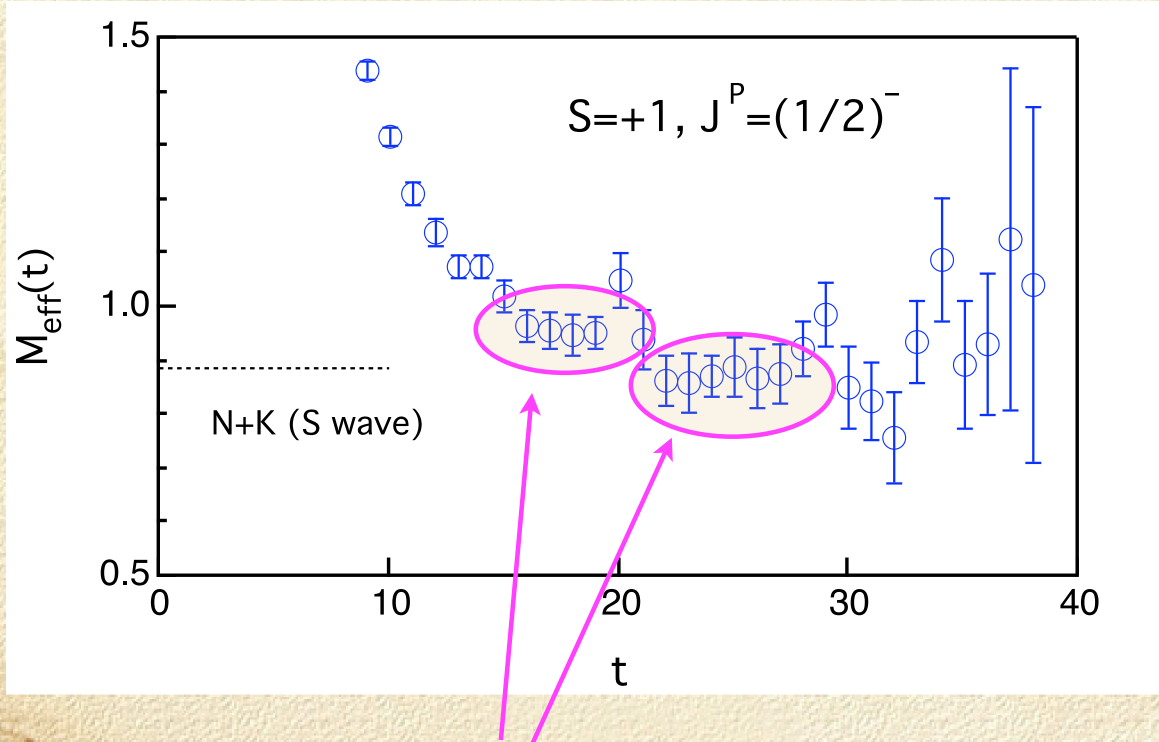


No clear signal for the KN scattering state

An expected feature: $|\langle \Theta^+ | \mathcal{O} | 0 \rangle| \gg |\langle K + N | \mathcal{O} | 0 \rangle|$

(uudd ^{bar}s) state with negative parity

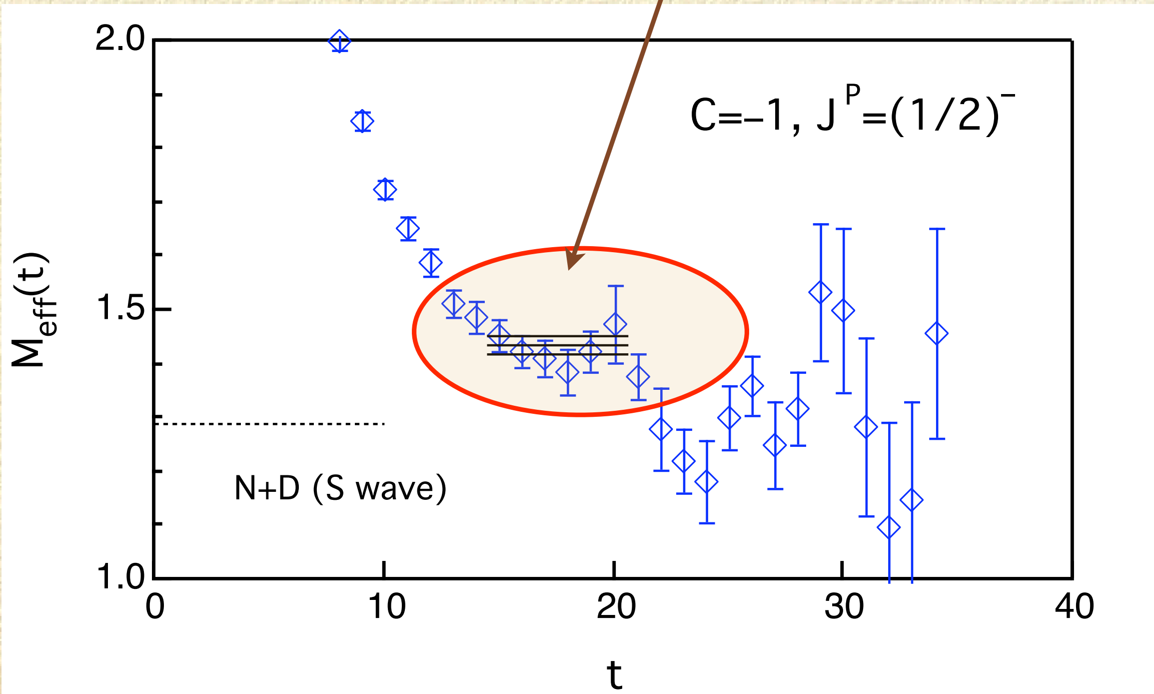
$\Theta(1/2^-) \rightarrow (KN)_{S\text{-wave}}$



Two distinct plateaus ?

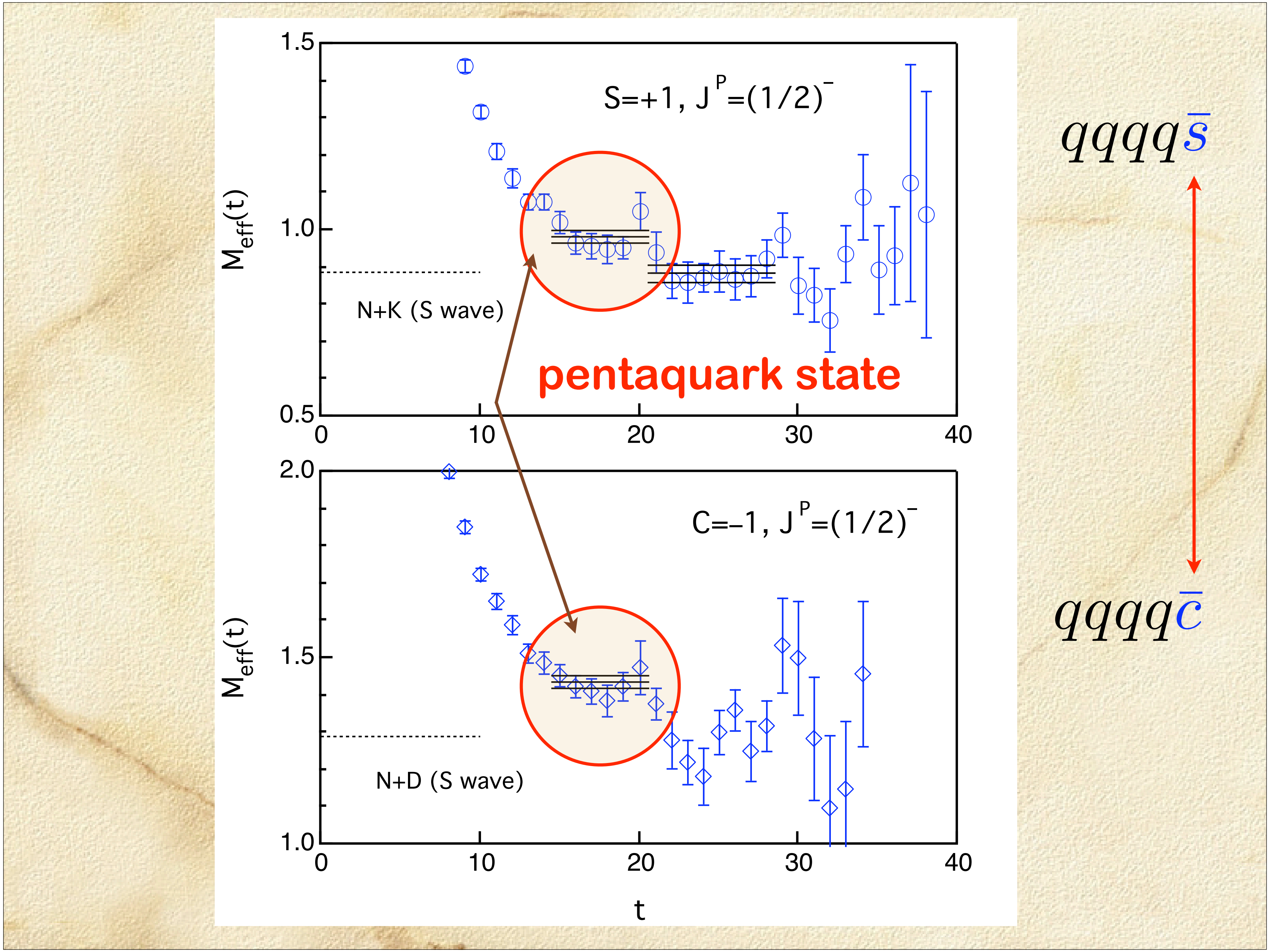
(uudd $^{\text{bar}}$c) state with negative parity

$(uudds) \to (uuddc)$ pentaquark state !!

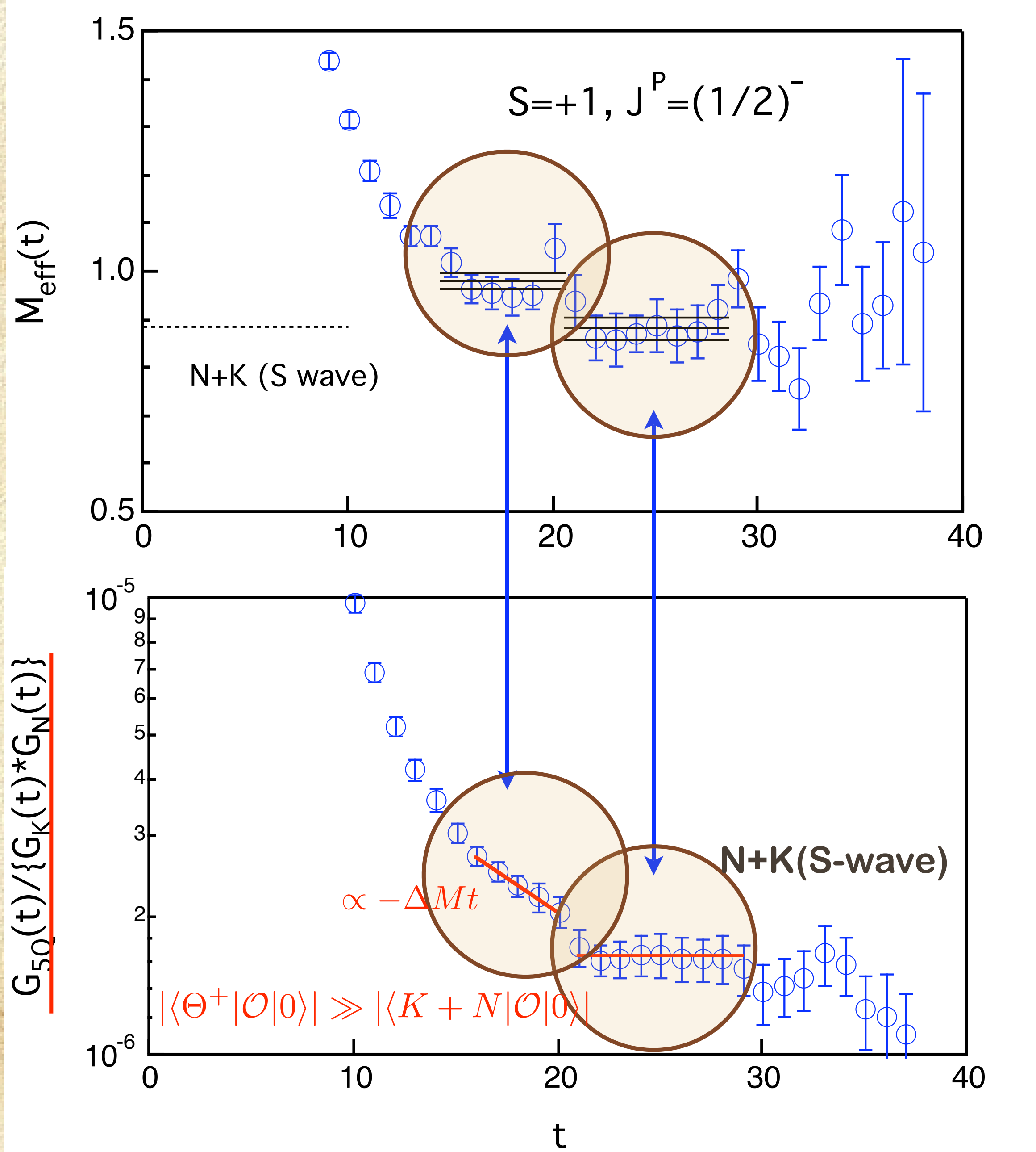


No clear signal for the DN scattering state

An expected feature: $|\langle \Theta_c^0 | \mathcal{O} | 0 \rangle| \gg |\langle D + N | \mathcal{O} | 0 \rangle|$

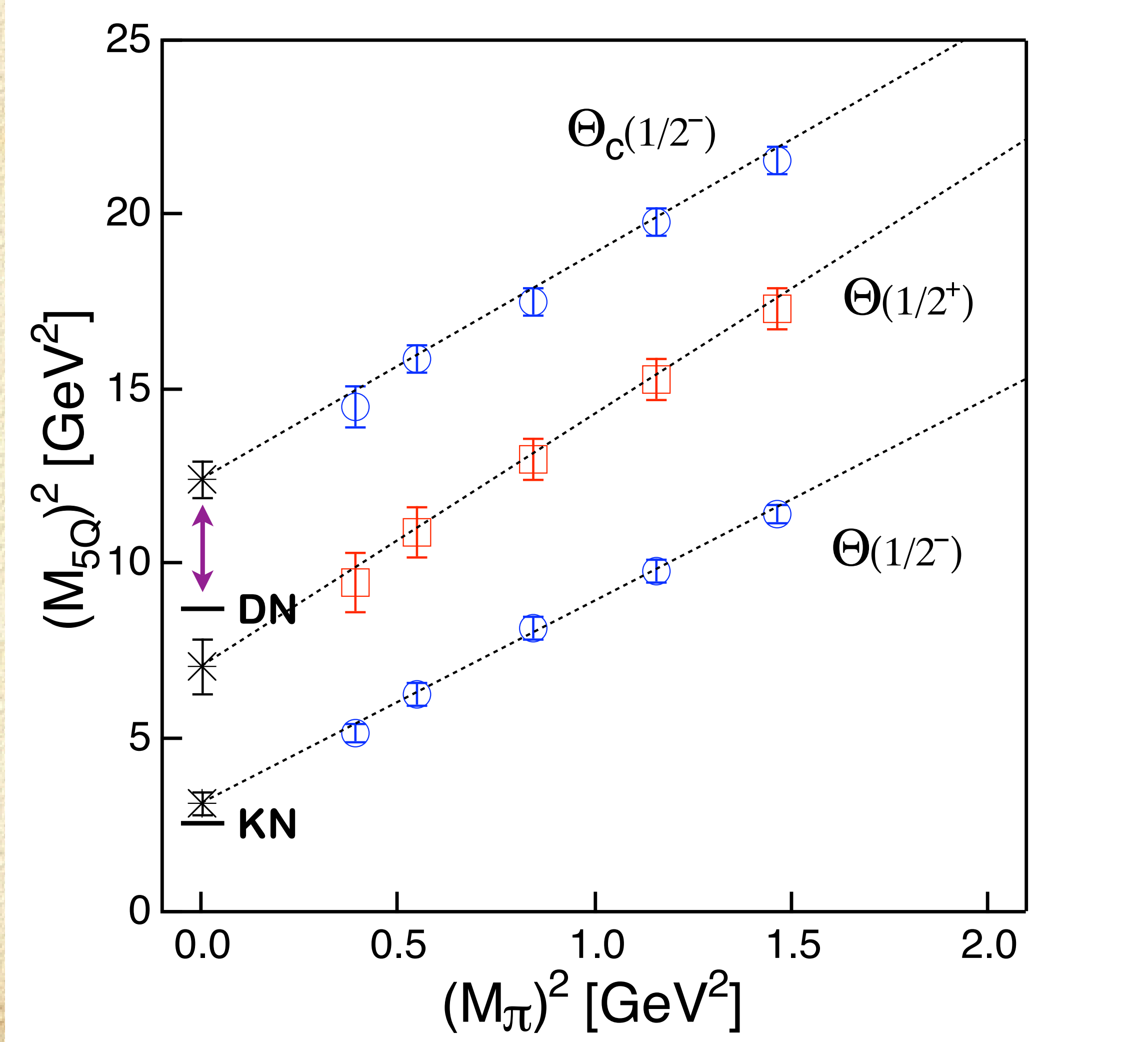


S=+1, J^P=(1/2)^-
N+K (S wave)
pentaquark state
C=-1, J^P=(1/2)^-
N+D (S wave)
M_eff(t)
t
qqqqs̄
qqqqc̄

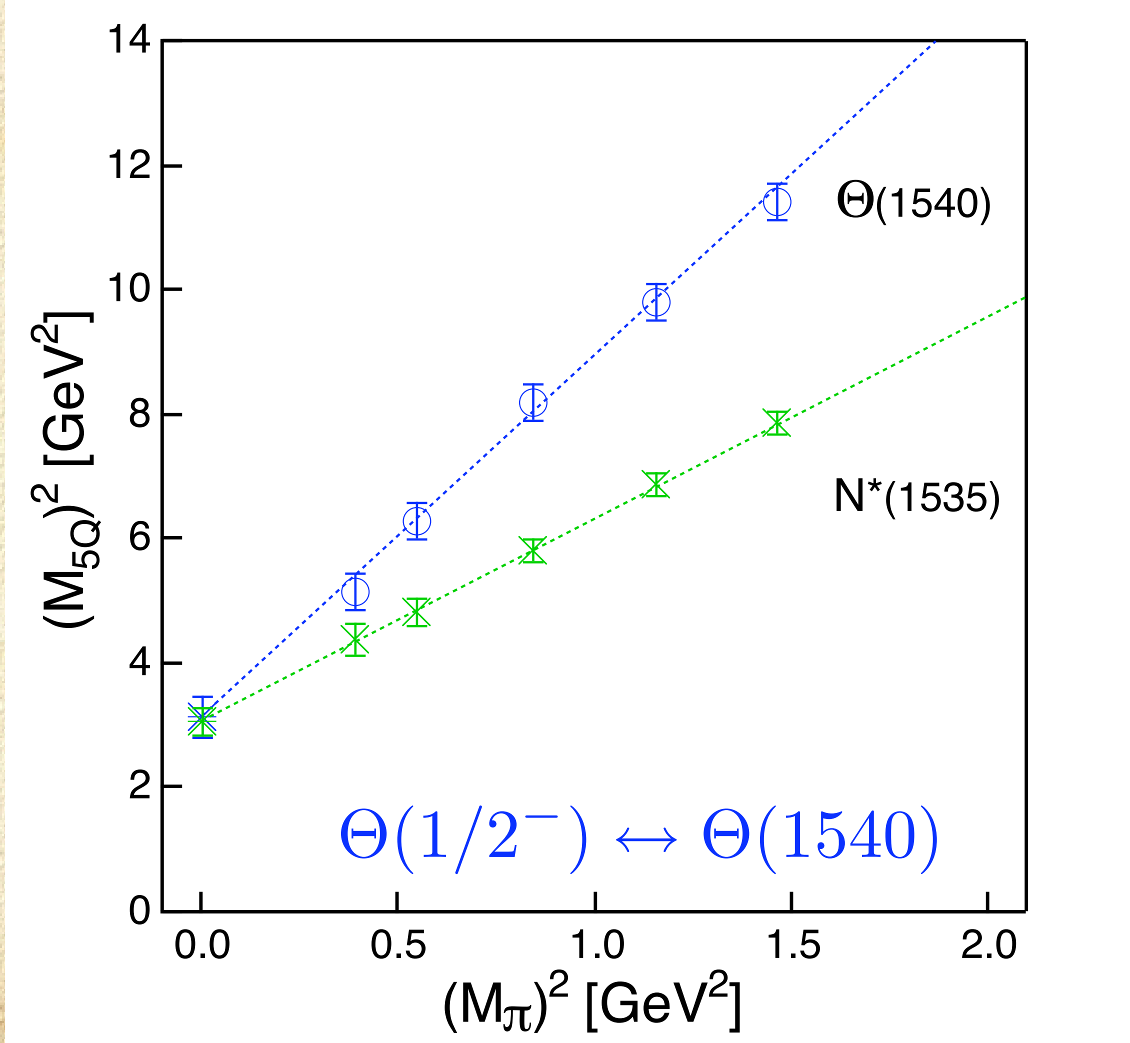


S=+1, J^P=(1/2)^-
M_eff(t)
N+K (S wave)
1.5
1.0
0.5
0
10
20
30
40
G_5Q(t)/{G_K(t)*G_N(t)}
N+K(S-wave)
∝ −ΔMt
|⟨Θ+|O|0⟩| ≫ |⟨K + N|O|0⟩|
10^-5
10^-6
t

the charm-pentaquark lies much higher than the DN threshold



the lowest pentaquark state has negative parity



$(M_{5Q})^2$ [GeV^2]
$(M_\pi)^2$ [GeV^2]
Θ(1540)
N*(1535)
$\Theta(1/2^-) \leftrightarrow \Theta(1540)$

Summary

We study the mass spectrum of pentaquark states in quenched lattice QCD with the newly proposed interpolating operator.

- Formulate and classify the exotic pentaquark interpolating operators.
  - ✓ $3_c^* \times 3_c^*$ diquark cluster with anti-quark ⤔ three types
  - ✓ Can study spin-3/2 states of the pentaquark as well as spin-1/2 states.
  - ✓ Couple weakly to the KN two-body system.
- Several important observations to understand the structure of $\Theta^+(1540)$
  - ✓ The J^P assignment of the lowest isosinglet Θ state is most likely $1/2^-$
  - ✓ The uudd ^{bar}c pentaquark with $J^P=1/2^-$ lies much higher than the DN threshold. (~ 3.5 GeV).
    - Exclude the possibility of the charm analog Θ state like a very narrow resonance or a bound state.

Other related studies

Other lattice study

- Csikor, Fodar, Katz, Kovacs, hep-lat/0309090.v2
- Other operator:
- $$\Theta \sim \varepsilon_{abc}(u_a^T C\gamma_5 d_b)\{u_e \bar{s}_e \gamma_5 d_c - (u \leftrightarrow d)\}$$

QCD sum rules

- Sugiyama, Doi, Oka, hep-ph/0309271
- Same exotic diquark-diquark-antiquark operator
- $m_0^2 = \frac{\langle \bar{s} g_s \sigma \cdot G s\rangle}{\langle \bar{s} s\rangle} > 0.4\ \mathrm{GeV}^2 \quad (m_0^2 = 0.8 \pm 0.2\ \mathrm{GeV}^2)$

the parity of the Θ^+ is most likely negative

If the Θ^+ really exists,

its parity is most likely negative.

But, this conclusion contradicts

the Skyrme model and the Jaffe-Wilczek model

The parity question should be interesting to settle experimentally.

Outlook

- ☑ The possible spin-orbit partner of the Θ state (J=3/2)
- ☑ Cross correlation between Θ and KN
- ☑ Identify the levels of the KN scattering state precisely
- ☑ Other types of diquark-diquark-antiquark
  - Jaffe-Wilczek type: S-wave diquark + P-wave diquark

    Phys. Rev. Lett. 91 (2003) 232003, hep-ph/0401034 .
  - Glozman type: $3_c^* \times 6_c$ diquark-diquark cluster

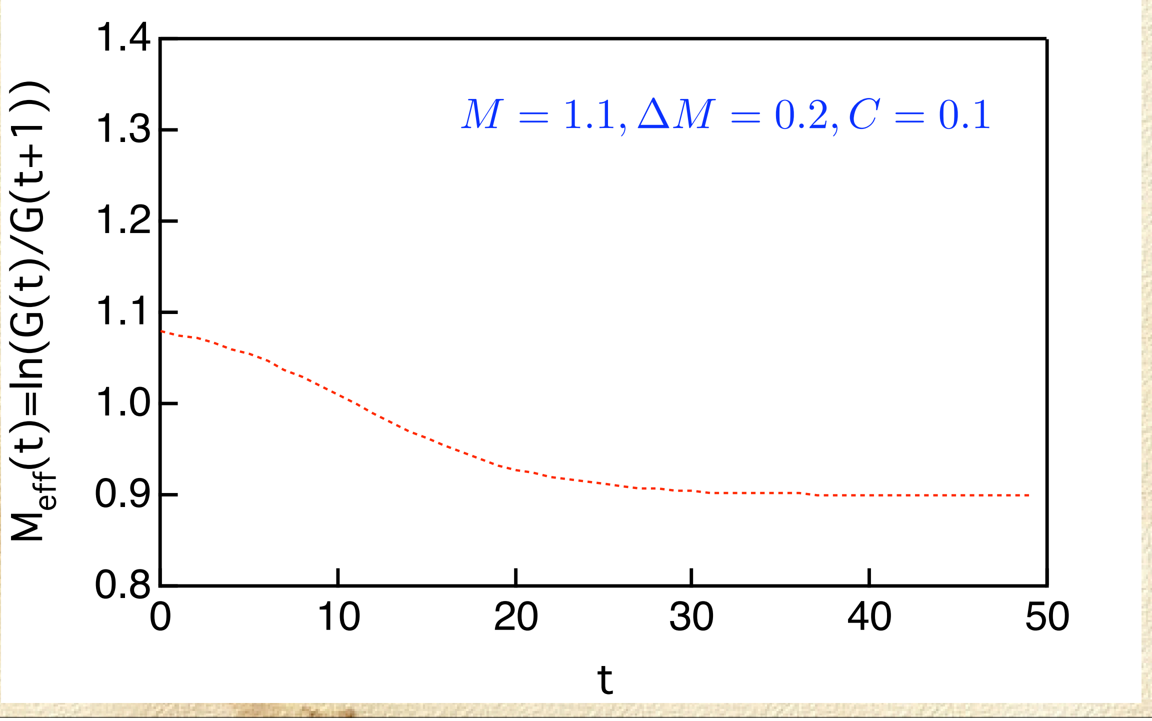
    Phys. Lett. B575 (2003) 18.

Reply to a criticism on two plateaus

Criticism:

Double exponentials can not reproduce two plateaus in effective mass plot.

$$G(t) = e^{-Mt}\left(1 + C \cdot e^{+\Delta M t}\right)$$



Reply to a criticism on two plateaus

Unstable particle in euclidean time $(\Delta M \gg \gamma, \Delta M t \gg 1)$

$$G(t) = e^{-Mt}\left(\cos(\gamma t) + \frac{\gamma}{\pi \Delta M^2 t}\cdot e^{+\Delta M t}\right)$$

$M = 1.1, \Delta M = 0.2, \gamma = 0.1$

C. Michael NPB327 (89) 515