Theory Overview of the Pentaquark Baryons

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YITP workshop on Multi-quark Hadrons; Four, Five and More?

Contents

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2. Theories of Θ^+
3. Quark Models
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Discovery of Θ^+ at SPring-8

T. Nakano et. al. (LEPS collaboration), Phys. Rev. Lett. 91, 012002 (2003)

$\gamma\, n \rightarrow K^- \Theta^+$

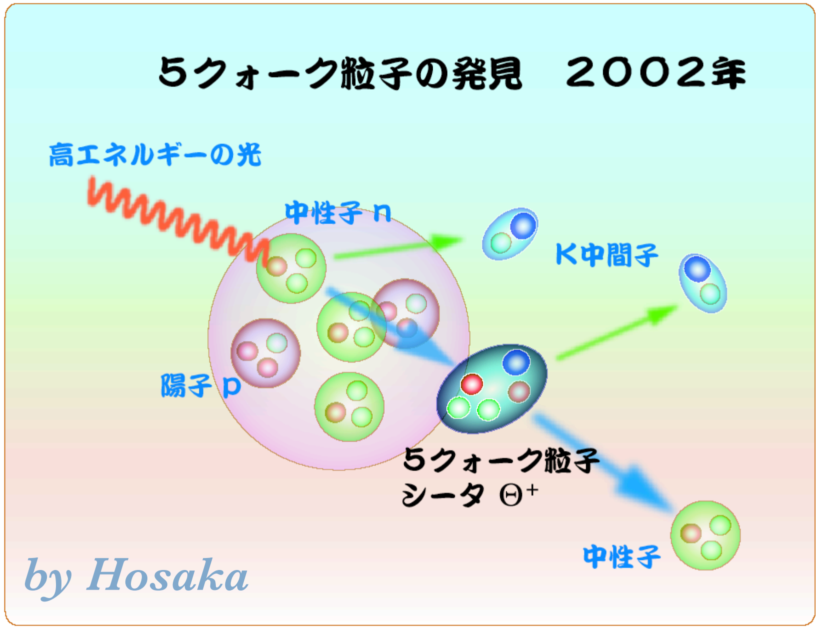
$\rightarrow K^- K^+ n$

SPring-8 tagged γ ($E_\gamma < 2.4$ GeV)

Penta-quark $u^2 d^2 \bar{s}$

$M = 1540 \pm 10$ MeV

$\Gamma < 25$ MeV (4.6σ)



Confirmed by ITEP (DIANA), JLAB (CLAS), ELSA (SAPHIR), ...

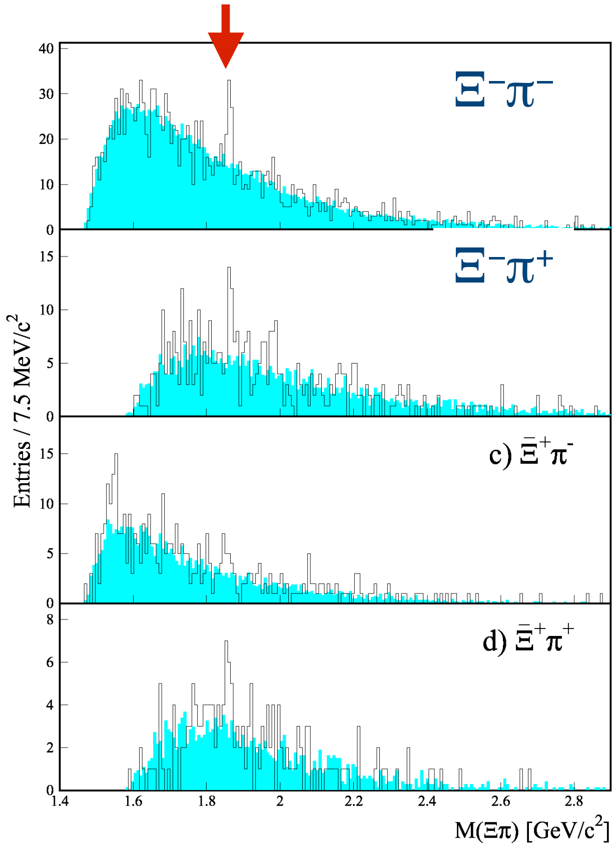
Ξ (I=3/2)

Ξ^{--} found by NA49 (CERN)

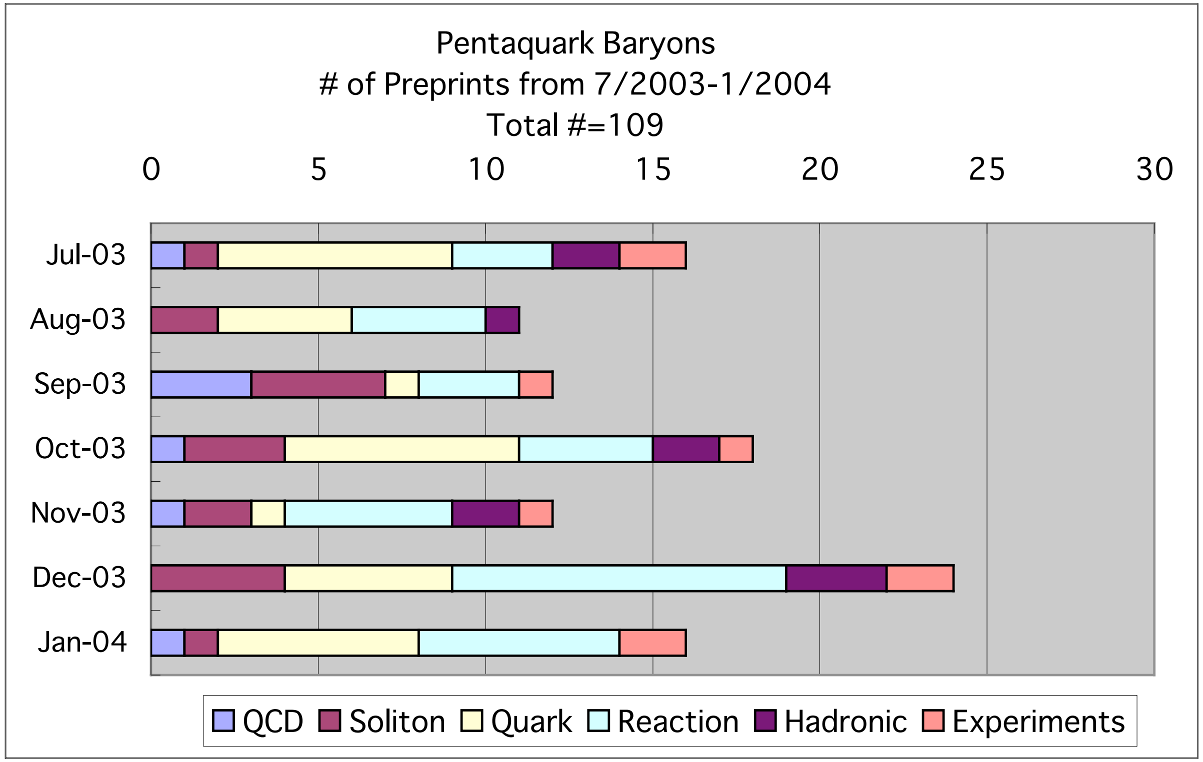
hep-ex/0310014

Ξ^{--} ($ssdd\bar{u}$) decays into $\Xi^{-}\pi^{-}$

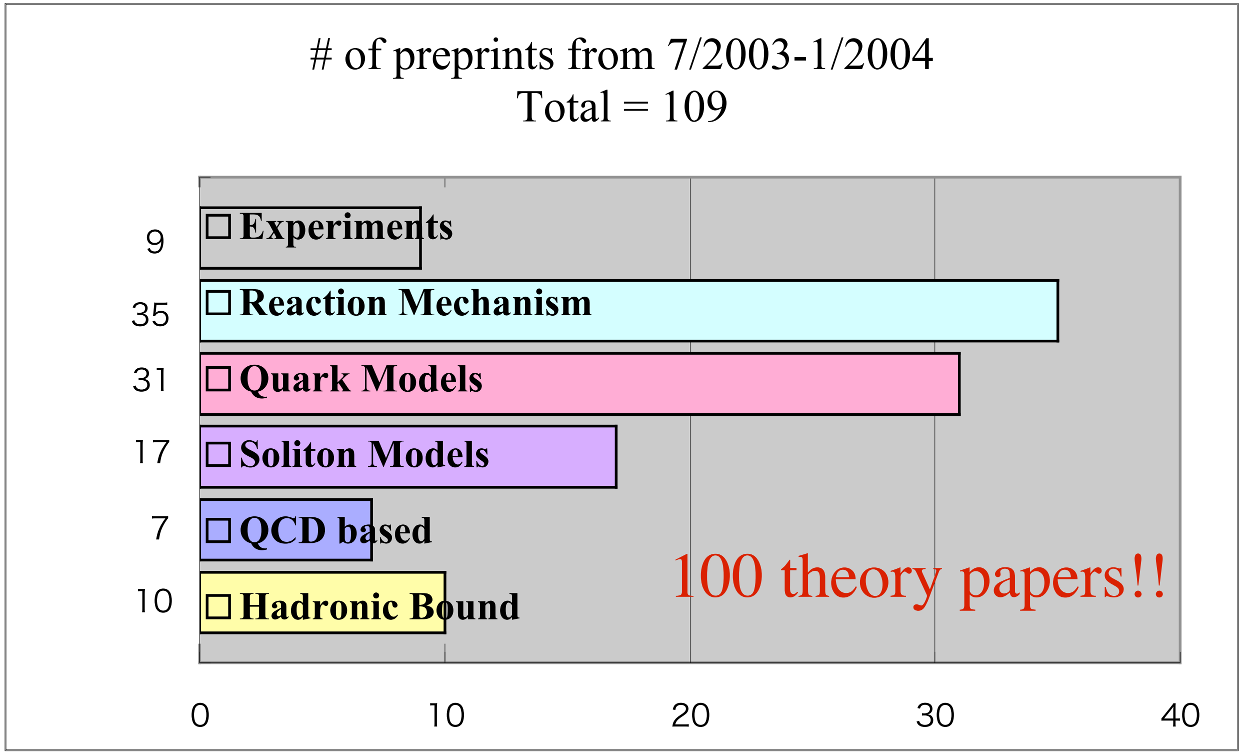
$M = 1.862$ GeV, $\Gamma < 18$ MeV



Papers


Pentaquark Baryons
# of Preprints from 7/2003-1/2004
Total #=109
0
5
10
15
20
25
30
Jul-03
Aug-03
Sep-03
Oct-03
Nov-03
Dec-03
Jan-04
QCD
Soliton
Quark
Reaction
Hadronic
Experiments

Papers



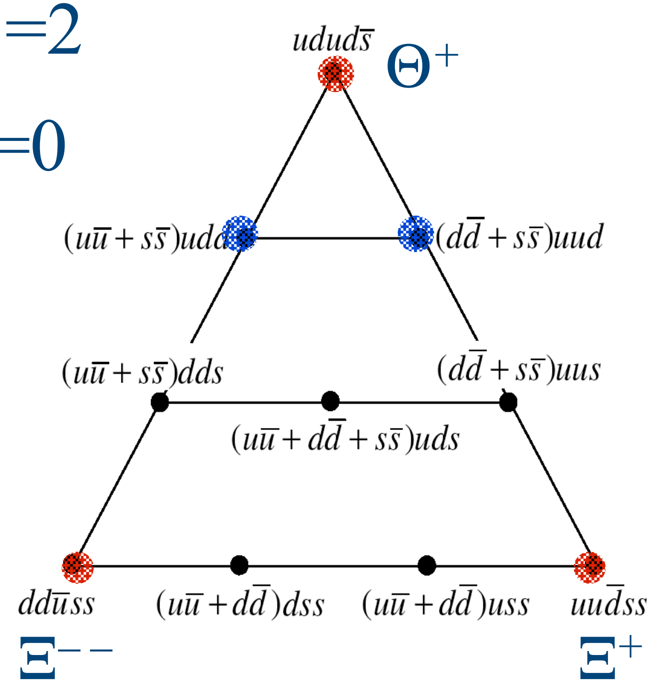
# of preprints from 7/2003-1/2004
Total = 109
9
35
31
17
7
10
Experiments
Reaction Mechanism
Quark Models
Soliton Models
QCD based
Hadronic Bound
100 theory papers!!
0
10
20
30
40

Facts of Θ^+

- $M \approx 1540$MeV, $\Gamma < 9$ MeV
- Strangeness= +1, B=1 ➡ Y=2
- No pK^+ (I_3= +1) state ➡ I=0
- No lower Str = +1 states

  This is the ground state!
- Simplest SU(3) irrep. ➡ 10*

  unique for ≤ 5 quarks



Problems

- Mass too low

  Most *conventional* calculations of $u^2d^2\bar{s}$ give higher masses

- Width (to KN) too narrow

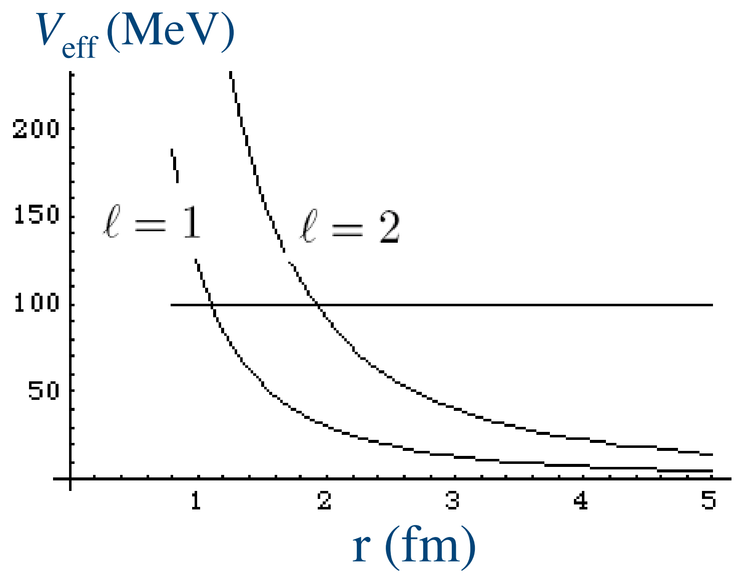
  100MeV above the threshold

  centrifugal barrier

$$V_{\rm eff}^{KN} \sim \frac{\ell(\ell+1)}{2\mu r^2} \simeq \frac{k^2}{2\mu}$$

$\mu(KN) \sim 324\text{MeV} \qquad k \sim 1.3\text{fm}^{-1}$

kr =1 at *r* ~ 0.8 fm



Theories for Θ^+

- Soliton models (*aka* Skyrmion) *17 papers*
  - Excited states in SU(3) Skyrmion: 10* irrep.
    Diakonov, Petrov, Polyakov, Zeit. Phys. A359 (1997) 305
    positive parity: $1/2^+$
    Θ^+ =1530 MeV, Γ < ~~15 MeV~~ → 30 MeV (Jaffe)
  - K^+ Skyrmion BS approach : compared to rigid rotor approach
    Itzhaki, Klebanov, Ouyang, Rastelli, hep-ph/0309305
    WZW term is repulsive (attractive) for K^+ (K^-) Callan, Klebanov (1985)
    $1/2^+$ most likely, but not enough to make a bound state
  - In general, many (sharp) resonances predicted
  - Significant $1/N_c$ corrections expected
    esp. for excited states as they are subleading in $1/N_c$

Theories for Θ^+

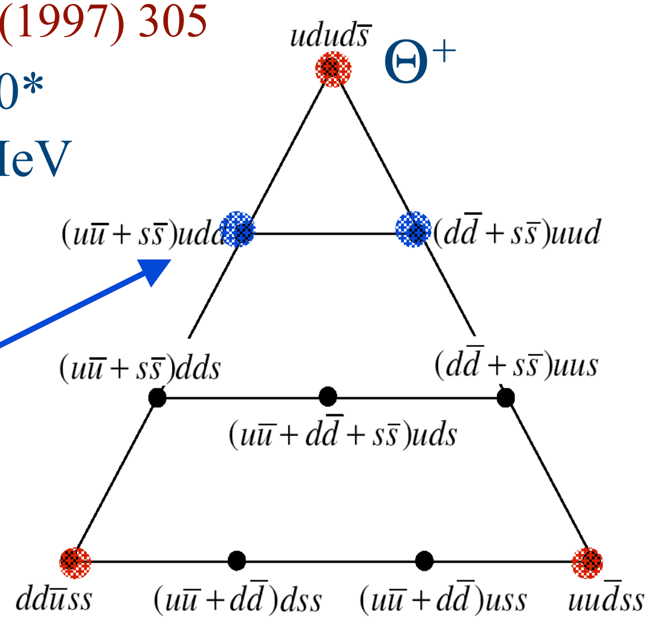
Chiral soliton model

Diakonov, Petrov, Polyakov, Zeit. Phys. A359 (1997) 305

N*(1710) is *assumed* to be in the $J^{\pi}=1/2^{+}$: 10*

$M = (1890 - 180\ Y)$ MeV → Θ^+ =1530 MeV

$\Gamma <$ ~~15 MeV~~ → 30 MeV (Jaffe)



SU(3) symmetry requires

a penta-quark N* (I =1/2, Y =1, 10*)

(2/3) $s\bar{s}$ + (1/3) ($u\bar{u}+d\bar{d}$)

Other candidates

$(1/2)^-$: 1535/ 1650 $(1/2)^+$: 1440/ 1710

ideal mixing with an octet (Jaffe, Wilczek, PRL 91,55)

Theories for Θ^+

- Quark Models *31 papers*
  - pentaquark spectroscopy
  - conventional or exotic (ex. diquarks)
- QCD *7 papers*
  - Lattice QCD
  - QCD sum rule
  - Others (ex. χPT, $1/N_c$ expansion)
- Production Mechanisms *35 papers*
  - determining the quantum numbers, ex spin, parity
  - cross sections, polarization
  - new reactions to produce pentaquarks

Theories for Θ⁺

- Hadronic bound/resonances *10 papers*

KN (K^+n, $K^0 p$) $I = 0$, $L = 0$ interaction is weak and repulsive

K^+n weak repulsive $$a_0^{K^+n} = -\frac{M_N m_K}{8\pi f_\pi^2 (M_N + m_K)} + O(m_K^2)$$

K^-p strong attractive $$a_0^{K^-p} = \frac{M_N m_K}{4\pi f_\pi^2 (M_N + m_K)} + O(m_K^2)$$

Other possibilities

NKπ bound state $E_{th} = 1.57$ GeV

Width must be strongly suppressed for L=0 bound states.

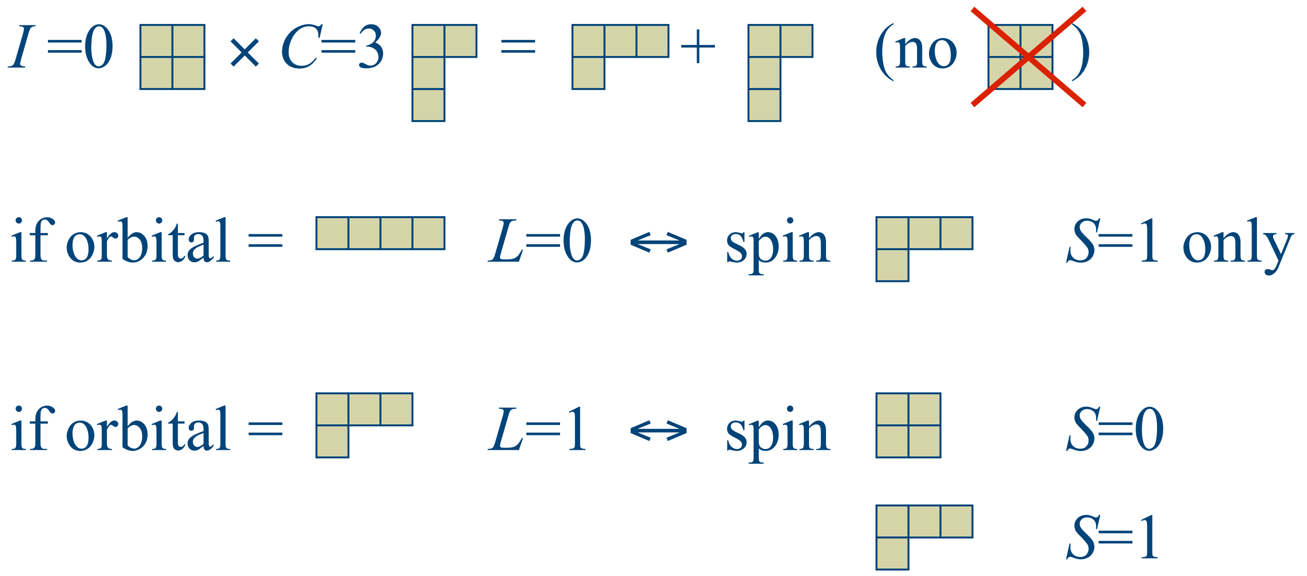
Bicudo, Marques, hep-ph/0308073 did not find a bound state.

Quark Models

- Symmetry
- Dynamics
  - Conventional (*traditional, uncorrelated*)
  - Exotic ideas (*correlated, chiral*)

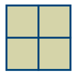
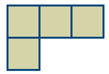
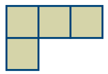
Θ^+ Symmetry

- quatre-quark $(u^2d^2)_{I=0,\ C=3}$



Θ^+ Symmetry

- possible low energy $(u^2d^2)_{I=0,\ C=3}$ states

		spin-color	*spin-flavor*
$L=0, S=1, J=1$	1^+	 +. . .	 +. . .
$L=1, S=0, J=1$	1^-	 +. . .	 +. . .

- possible quantum numbers for Θ^+

$u^2d^2(1^+) + \bar{s}\,(1/2^-) \;=\; 1/2^-, 3/2^- \leftrightarrow$ KN *S*-wave

$u^2d^2(1^-) + \bar{s}\,(1/2^-) \;=\; 1/2^+, 3/2^+ \leftrightarrow$ KN *P*-wave

Quark Models

- Orbital excitation energy

  How large is $\Delta E(\Delta L=1) = E(L=1) - E(L=0)$?

- Spin-dependent interaction(s)

  hyperfine (HF) interaction

  How large is $\Delta E(\Delta S=1) = E(S=1) - E(S=0)$?

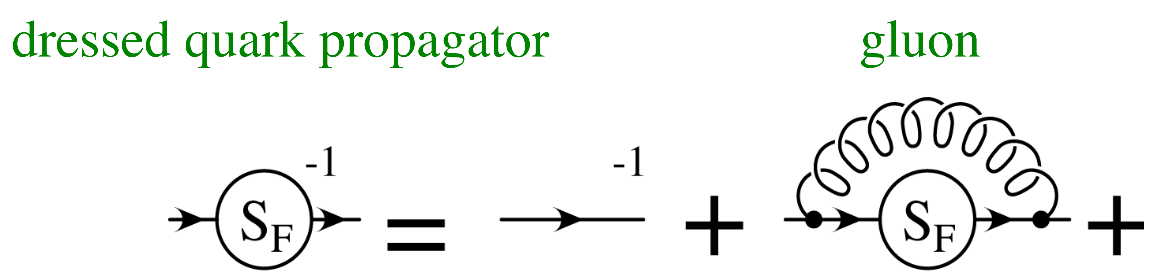
Constituent Quark

- Dyson-Schwinger equation

$$S_F^{-1}(p) = S_{F0}^{-1}(p) - \Sigma(p)$$
$$S_{F0}^{-1} = \not{p} - m$$
$$S_F^{-1} = A(p)\not{p} - \boxed{B(p)}$$

→ effective mass generated

dynamical chiral symmetry breaking



Constituent Quark

- Conserved currents are not renormalized.

  I , Y , C charges do not change.

- Constituent quark mass (~ single particle energy in the bag)

$$m_{\mathrm{u,\,d}} \approx 360\ \mathrm{MeV}$$

$$m_{\mathrm{s}} \approx 540\ \mathrm{MeV}$$

$$\left.\begin{aligned}\Sigma\, m_{\mathrm{q}} &= 1080\ \mathrm{MeV\ for\ N}/\Delta \\ &= 1260\ \mathrm{MeV\ for\ }\Lambda/\Sigma/\Sigma^{*}\end{aligned}\right\} + \text{Hyperfine interaction (spin dependent)}$$

$$= 1980\ \mathrm{MeV\ for\ u^2d^2s}$$

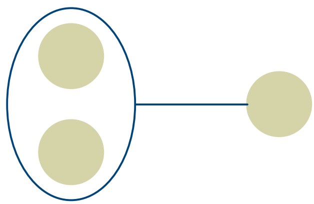
- Assume that the residual interactions are weak.

Quark Models

- Estimate of orbital excitation energy

  N*(1535) - N(940) : $\Delta E \sim 600$ MeV (incl. difference in HF int.)

  Kinetic energy contribution for ΔL=1

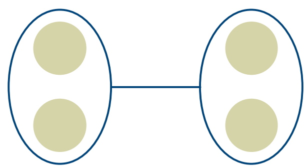


720MeV 360MeV

μ = 240 MeV

$$\left\langle \frac{\ell(\ell+1)}{2\mu r^2} \right\rangle \simeq 450\text{MeV} \quad \text{for } \langle r^2 \rangle^{1/2} \sim 0.6 \text{ fm}$$

ex. di-quark models



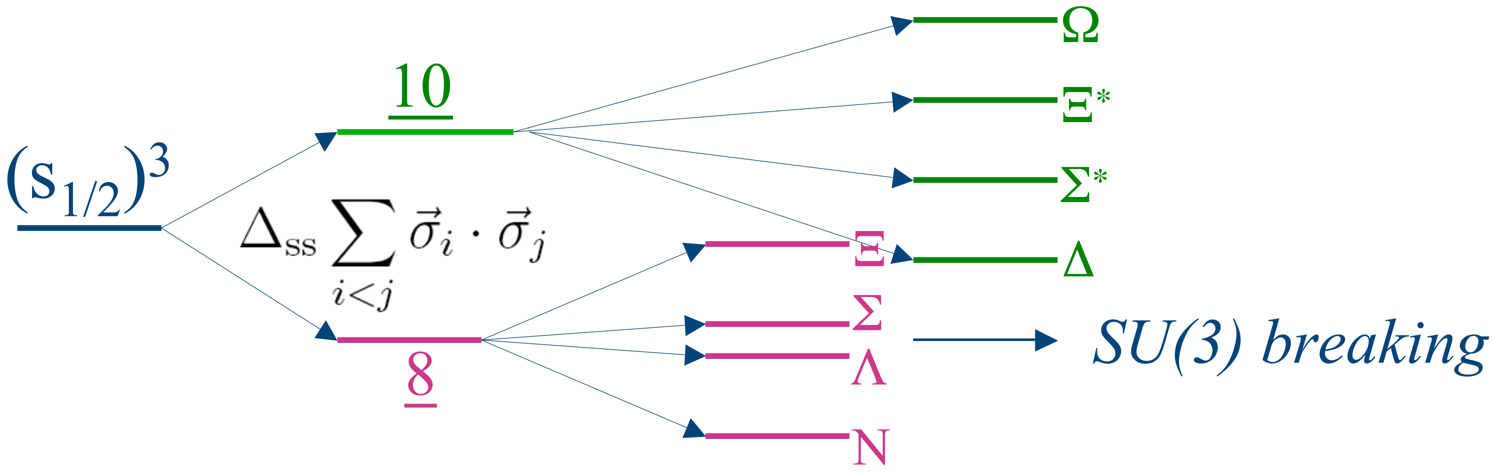
420MeV 420MeV

μ = 210 MeV

HF interaction in Baryon

Single particle motion

$(s_{1/2})^3$	$J = 1/2$	$\underline{8}$
	$J = 3/2$	$\underline{10}$



HF interaction in Baryon

N-Δ mass splitting (300 MeV) ↔ Δ_{ss} ~ 50 MeV

Λ–Σ mass splitting (~77 MeV) from SU(3) breaking

$$\Sigma_{\rm HF} = \boxed{\Delta_{ss}}\{\vec{\sigma}_u \cdot \vec{\sigma}_d + \boxed{\xi} \times \vec{\sigma}_s \cdot (\vec{\sigma}_d + \vec{\sigma}_u)\}$$

Δ_{ss} → 50 MeV

Λ $(ud)_{I=0,S=0}$ s 50MeV × [(–3) + 0 * ξ]

Σ $(ud)_{I=1,S=1}$ s 50MeV × [1 + (–4) * ξ]

ξ - factor: s-u, s-d HF interaction is weaker than u-d.

for ξ = 3/5 → Σ – Λ = (8/15) ×150 MeV = 80 MeV

Origin of HF Interaction

(1) One gluon exchange

or color-magnetic (CM) interaction

Breit-Fermi, DeRujula-Georgi-Glashow (1975)

$$\alpha_s \lambda_i \frac{\vec{\sigma}_i \cdot \vec{q}}{m_i} \frac{1}{q^2} \lambda_j \frac{\vec{\sigma}_j \cdot \vec{q}}{m_j} \simeq \frac{\alpha_s}{m_i m_j} (\lambda_i \cdot \lambda_j)(\vec{\sigma}_i \cdot \vec{\sigma}_j) \delta(\vec{r}_{ij})$$

SU(3) breaking $m_u/m_s \sim 3/5$

$$\Sigma_{\rm CM} = -\Delta_{\rm CM} \Sigma_{i<j} \xi_{ij} (\lambda_i^c \cdot \lambda_j^c)(\vec{\sigma}_i \cdot \vec{\sigma}_j)$$

N-Δ mass splitting (300 MeV) ↔ $\Delta_{\rm CM} \sim 18.75$ MeV

for Θ^+ **Carlson et al., hep-ph/0307396**

Origin of HF Interaction

(2) Pseudoscalar meson exchange
or flavor dependent (FD) interaction

Glozman, Riska, Phys. Rep. 208 (1996)

$$\Sigma_{\rm FD} = -\Delta_{\rm FD}\Sigma_{i<j}\xi_{ij}(\lambda_i^f \cdot \lambda_j^f)(\vec{\sigma}_i \cdot \vec{\sigma}_j)$$

N-Δ mass splitting (300 MeV) ↔ $\Delta_{\rm FD} \sim 30$ MeV

for Θ^+ **Stancu, Riska, hep-ph/0307010, 0402044**

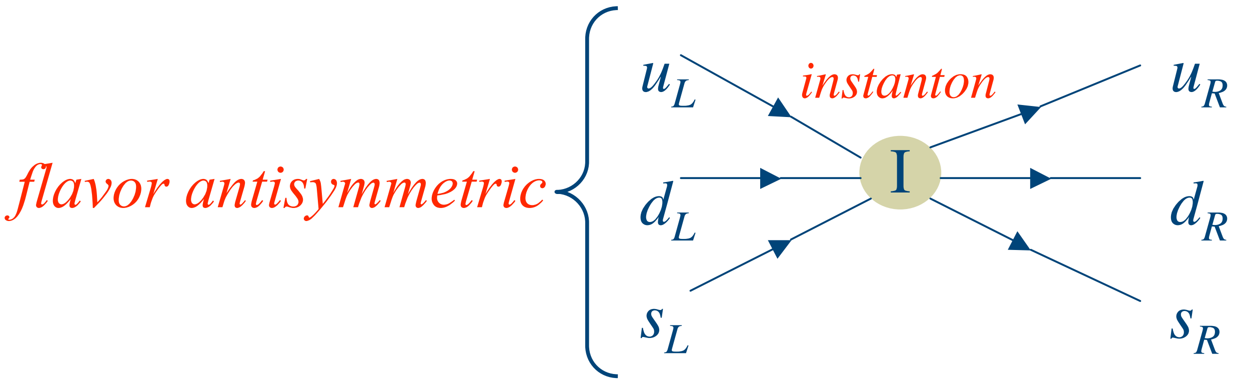
CM and FD **Jennings, Maltman, hep-ph/0308286**
Bijker et al., hep-ph/0310281

Origin of HF Interaction

(3) Instanton-induced-interaction (I I I)

aka Kobayashi-Maskawa-'t Hooft (KMT)

instanton-light-quark couplings



Origin of HF Interaction

(3) Instanton-induced-interaction (I I I)

flavor antisymmetric u-d-s 3-body repulsion

flavor antisymmetric 2-body attraction

$$V^{(3)}_{\mathrm{III}} = V^{(3)} \sum_{(ijk)} \mathcal{A}^f \left[1 - \frac{1}{7} \left(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + \boldsymbol{\sigma}_j \cdot \boldsymbol{\sigma}_k + \boldsymbol{\sigma}_k \cdot \boldsymbol{\sigma}_i \right) \right] \delta(\boldsymbol{r}_{ij}) \delta(\boldsymbol{r}_{jk})$$

$$V^{(2)}_{\mathrm{III}} = V^{(2)} \sum_{i<j} \mathcal{A}^f \left[1 - \frac{1}{5} (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \right] \delta(\boldsymbol{r}_{ij})$$

$$= V_{ij}^{(2)} \, (2/5) \, (1 - \boxed{\sigma_i \cdot \sigma_j}) \, \delta(r_{ij})$$ in the baryon

proportional to $1/m_i m_j$

Shuryak-Rosner (1989)
Takeuchi-Oka (1989)

Origin of HF Interaction

(3) Instanton-induced-interaction (I I I) (2-body part)

$$\Sigma_{\rm III} = \Delta_{\rm III}\Sigma_{i<j}\mathcal{A}^f_{ij}\xi_{ij}[1 - \frac{1}{5}(\vec{\sigma}_i \cdot \vec{\sigma}_j)]$$

N-Δ mass splitting (300 MeV) ↔ $\Delta_{\rm III} \sim 125$ MeV

It is quite likely that the realistic HF interaction is a <u>combination</u> of $\Sigma_{\rm CM}$, $\Sigma_{\rm FD}$, and $\Sigma_{\rm III}$.

Quark Models for Θ^+

- Crude estimates of the baryon masses
  - CM interactions

$$\Sigma_{CM} = -\Delta_{CM}\,\Sigma_{i<j}(\lambda_i \cdot \lambda_j)\,(\sigma_i \cdot \sigma_j) \qquad \Delta_{CM} = 150/8 \text{ MeV}$$

$$M_N = 3\,m_q + \langle\Sigma_{CM}\rangle_N = 360\times 3 - 150 \approx 930 \text{ MeV}$$

$$M_\Delta = 3\,m_q + \langle\Sigma_{CM}\rangle_\Delta = 360\times 3 + 150 \approx 1230 \text{ MeV}$$

$$M_\Lambda = 3\,m_q + \Delta m + \langle\Sigma_{CM}\rangle_{\Lambda\Sigma} = 360\times 3 + 180 - 150 \approx 1110 \text{ MeV}$$

$$\Delta m = m_s - m_q \sim 540 - 360 = 180 \text{ MeV}$$

Quark Models for Θ^+

- H dibaryon **Jaffe** (1977)

  Strangeness = –2, B =2 ΛΛ threshold = 2231 MeV

  $M_H = 6\ m_q + 2\Delta m + \langle\Sigma_{CM}\rangle_H = 360\times6 + 2\times180 - 450 \approx 2070$ MeV

  The 20-year searches for the bound state were not successful.

  The 3-body (u-d-s) III gives ~160 MeV repulsion.

  Takeuchi, Oka (1991)

Quark Models for Θ^+

- Θ^+ in Σ_{CM} $(1/2^-: q^4\ (L=0, S=1) + \bar{s} : J=1/2)$

  $M_\Theta = 5\ m_q + \Delta m + \langle\Sigma_{CM}\rangle_{5q}$ $(1/2^-: q^4\ L=0, S=1, J=1/2)$

  $= 360\times5 + 180\ \underline{-\ 250} \approx 1730$ MeV exp. 1540 MeV

- ~200 MeV extra attraction is necessary for $\Theta^+(1/2^-)$ in Σ_{CM}
- Other interactions?

  FD interaction slightly less repulsive.

  III (2-body) is strongly attractive

  $M_\Theta = 5\ m_q + \Delta m + \langle\Sigma_{III}\rangle_{5q} = 360\times5 + 180\ \underline{-660} \approx 1320$ MeV

  III (3-body) ~ +50 MeV repulsive

Shinozaki, Takeuchi, Oka *(preliminary)*

Quark Models for Θ^+

Evaluation of HF interactions for $\Theta^+(1/2^-)$

SU(3) limit	$\langle\Sigma_{CM}\rangle$ $\Delta_{CM} \sim 18.7\text{MeV}$	$\langle\Sigma_{FD}\rangle$ $\Delta_{FD} \sim 30\text{MeV}$	$\langle\Sigma_{III}\rangle$ (2-body) $\Delta_{III} \sim 125\text{MeV}$
N	$-8\Delta_{CM}$	$-14\Delta_{FD}$	$-\frac{12}{5}\Delta_{III}$
Δ	$+8\Delta_{CM}$	$-4\Delta_{FD}$	$0\Delta_{III}$
K	$-16\Delta_{CM}$	$2\Delta_{FD}$	$-\frac{8}{5}\Delta_{III}$
$\Theta^+(1/2^-)$	$-\frac{56}{3}\Delta_{CM}$	$-\frac{28}{3}\Delta_{FD}$	$-\frac{36}{5}\Delta_{III}$
$\Theta^+ - (N+K)$	$+100$ MeV	$+80$ MeV	-400 MeV

Quark Models for Θ^+

- Θ^+ (1/2$^+$) (1/2$^+$: q^4 (L=1, S=0) + $\bar{s}$: J=1/2)

  $M_\Theta = 5\, m_q + \Delta m + \langle\Sigma_{CM}\rangle_{5q} + \Delta E(\Delta L=1)$

  $= 360\times5 + 180 \underline{- 620 + 450} \approx 1810$ MeV

  $M_\Theta = 5\, m_q + \Delta m + \langle\Sigma_{FD}\rangle_{5q} + \Delta E(\Delta L=1)$

  $= 360\times5 + 180 \underline{- 660 + 450} \approx 1770$ MeV

- It is *unlikely* to reverse the $1/2^- \leftrightarrow 1/2^+$ ordering in conventional constituent quark model.

Quark Models for Θ^+

Evaluation of HF interactions for $\Theta^+(1/2^+)$

	$\langle \Sigma_{\rm CM} \rangle$	$\langle \Sigma_{\rm FD} \rangle$
SU(3) limit	$\Delta_{\rm CM} \sim 18.7\text{MeV}$	$\Delta_{\rm FD} \sim 30\text{MeV}$
$\Theta^+(1/2^+)$	$(-28 \sim -33)\Delta_{\rm CM}$	$-22\Delta_{\rm FD}$
$\Theta^+ - (N+K)$	$(-75 \sim -170)$ MeV	-300 MeV
$\Theta^+(1/2^-) - \Theta^+(1/2^+)$	$175 \sim 270$ MeV	380 MeV

depends on wave function/ range of the interactions

Jennings, Maltman, hep-ph/0308286

Quark Models for Θ^+

- Those who predict $1/2^+$ in conventional quark model

Stancu hep-ph/0402044

FD with subtract 510 MeV to fit $\Theta^+(1540)$

Jennings, Maltman hep-ph/0308286

CM and FD compared

assume ΔE (ΔL=1) = 210 - 250 MeV

Missing attraction

Exotic possibilities

- Diquarks

Jaffe, Wilczek, PRL 91 (2003) 232003

Karliner, Lipkin, hep-ph/0307232

	ud diquarks	color	spin	
$I = 1$	uu, {ud}, dd	3*	1^+	
		6	0^+	
$I = 0$	$[ud]_0$	3*	0^+	strongly attractive ~ 420 MeV
		6	1^+	
$I = 0$	$[ud]_T$ tensor diquark	3*	1^-	attractive in the instanton vacuum ~ 570 MeV

Shuryak, Zahed hep-ph/0310270

Missing attraction

Jaffe, Wilczek   Phys. Rev. Lett. 91 (2003) 232003

2 diquarks + $\overline{s}$ model

$[ud]_0 \times [ud]_0$: color antisymmetric

$L = 1 \Rightarrow 1/2^+, 3/2^+$ $M = 840+560+\Delta E(\Delta L) \sim 1400+\Delta E(\Delta L)$

However, note $\Delta E(\Delta L=1) \sim 400$–$450$ MeV

Karliner, Lipkin   hep-ph/0307232

diquark + triquark $L = 1$

$\Delta E(\Delta L=1) \sim 209$ MeV ?

from $\Delta\,[D_s(0^+, 2317) - D_s(0^-, 1969)]$ – HF

Shuryak, Zahed   hep-ph/0310270

$[ud]_0\ (0^+, 420\ \text{MeV}) + [ud]_T\ (1^-, 570\text{MeV}) + \overline{s}\ (560\ \text{MeV})$

$L = 0 \Rightarrow 1/2^+, 3/2^+$

Diquark spectrum

- Diquark spectrum in lattice QCD (Landau gauge)

  Wetzorke, Karsch, hep-lat/0008008

(FSC) [GeV]

Diquark state	$(\bar{3}0\bar{3})$	$(61\bar{3})$	$(\bar{3}16)$	(606)
$ma_{(FSC)}$: MEM result	0.60(2)	0.70(3)	0.74(9)	—
$ma_{(FSC)}$: 2-exp. fit	0.62(2)	0.73(4)	0.77(17)	0.50(15)

maxial entropy method

- Bethe-Salpeter calculation in the rainbow-ladder QCD

  Maris, nucl-th/0204020

  diquark mass $m(0^+) \sim 800$ MeV

Diquark spectrum

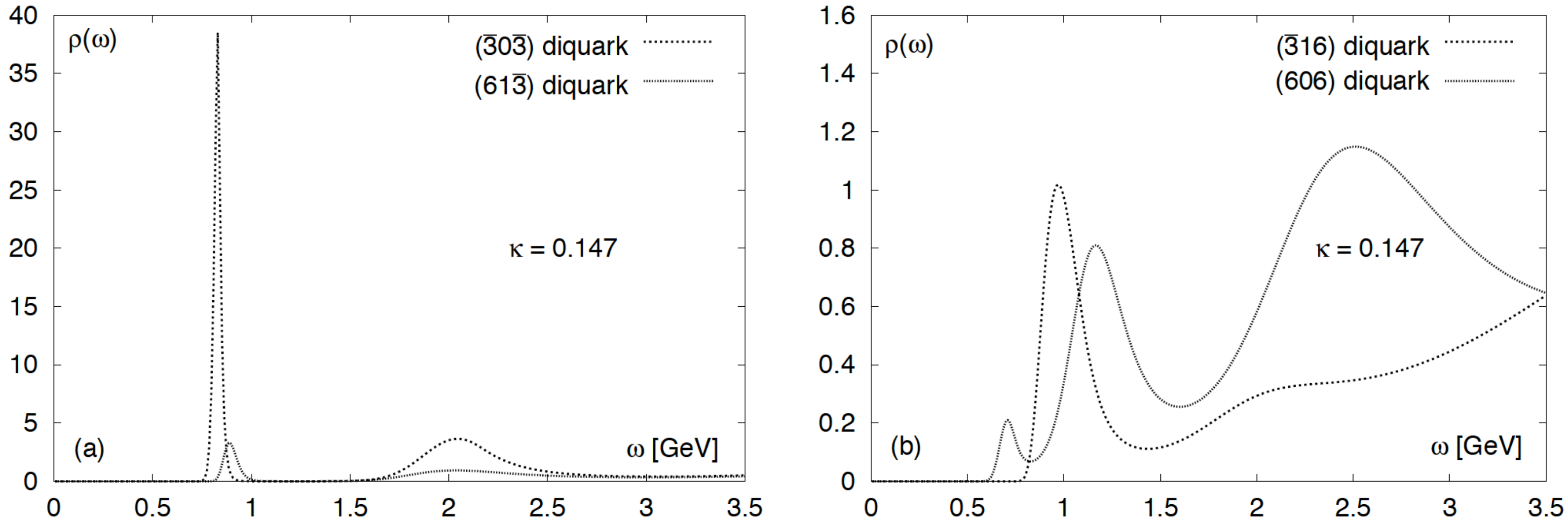


Figure 2: Spectral functions for color anti-triplet (a) and sextet diquark states (b).

Missing attraction

- Diquark in the nucleon (and the other gs baryons)

  Diquark enhancement breaks SU(6) symmetry.

  ex.

  $p = [ud]_0\, u \qquad n = [ud]_0\, d$

  $\mu_p / \mu_n = e_p / e_n = -2$

  if the quarks do not have anomalous moments.

Missing attraction

- Chiral bag model

  Hosaka, Phys. Lett. B571 (2003) 55

  uudd (I =0, J^π =1$^-$) + $\bar{s}$ (J^π =1/2$^-$)

  => 1/2$^+$, 3/2$^+$

- Bag excitation energy

  $E\,(p_{3/2}) - E\,(s_{1/2}) = 1.16/R \sim 230$ MeV

  seems too small?

  Can the bag model describe excited states?

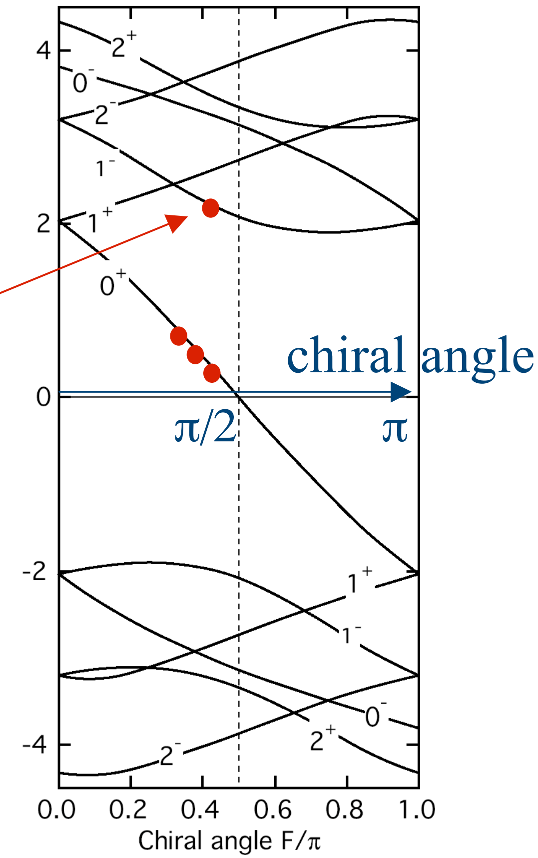


Figure 2: Eigenenergies of the hedgehog quark in the chiral bag model as functions of the chiral angle F.

QCD prediction

- QCD sum rule

  Zhu, Phys. Rev. Lett. 91 (2003) 232002

  Matheus et al., Phys. Lett. B578 (2004) 323

  No parity projection was made.
  Only the chiral even terms are considered.

  Sugiyama, Doi, Oka, Phys. Lett. B581 (2004) 167

  determine mass and parity

- Lattice QCD

  Csikor, Fodor, Katz, Kovacs, JHEP 0311 (2003) 070

  Sasaki, hep-lat/0310014 *Sasaki's talk*

  Kentucky group, Lee et al., (Lattice03, HYP2003) *Mathur's talk*

QCD Sum Rule for Θ^+

J. Sugiyama, T. Doi, M. O. PLB581 (2004) 167

- Assume J =1/2, I =0
- 5-quark local operator for J = 1/2 baryon

$$J_{\Theta^+}(x) \; = \; \epsilon^{abc}\epsilon^{def}\epsilon^{cfg}\{u_a^T(x)Cd_b(x)\}\{u_d^T(x)C\gamma_5 d_e(x)\}C\bar{s}_g^T(x)$$

	0^- diquark		0^+ diquark		$\bar{s}$
color	3_c^*	$\times$	3_c^*	$\times$	3_c^*
flavor	3_f^*	$\times$	3_f^*	$\times$	3_f^*

QCD Sum Rule for Θ⁺

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- Parity projection
- Retarded Green's function at rest, ***q*** = 0.

$$\Pi(q^0) = \int d^4q\, e^{iq\cdot x} i\langle 0|\theta(x^0)\, J(x)\bar{J}(0)|0\rangle|_{\vec{q}=0}$$

$$-\frac{1}{\pi}\mathrm{Im}\Pi(q^0) = \boxed{A(q^0)}\gamma^0 + \boxed{B(q^0)}$$

$$A(q^0) = \frac{1}{2}\left(\rho^+(q^0) + \rho^-(q^0)\right)$$

$$B(q^0) = \frac{1}{2}\left(\rho^+(q^0) - \rho^-(q^0)\right)$$

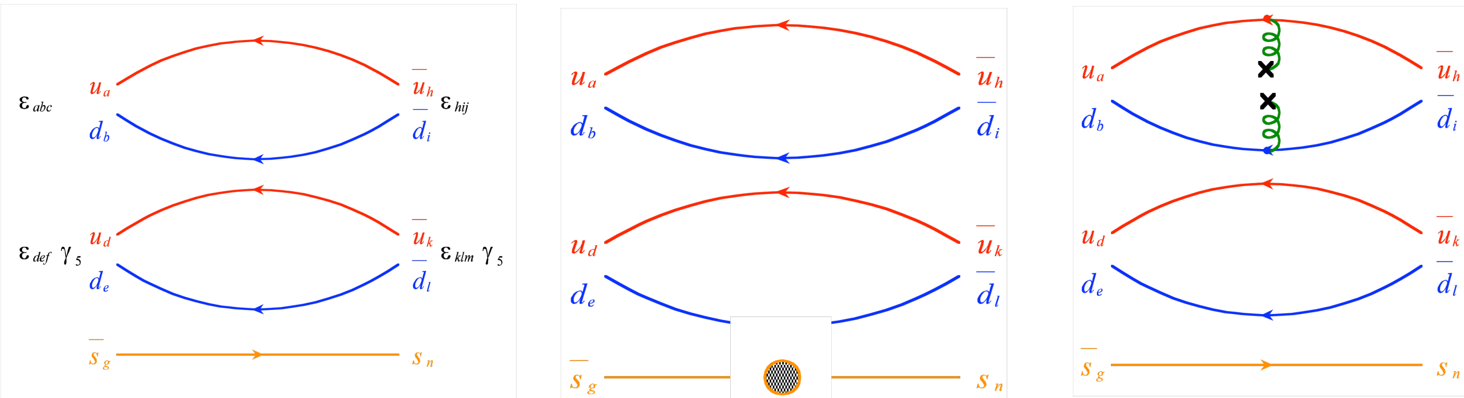
QCD Sum Rule for Θ^+

$$-\frac{1}{\pi}\mathrm{Im}\Pi(q^0) = A(q^0)\gamma^0 + B(q^0)$$

$$A_{\rm OPE}(q_0) = \frac{q_0^{11}}{5!\,5!\,2^{10}7\pi^8} + \frac{q_0^7}{3!\,5!\,2^8\pi^6}m_s\langle\bar{s}s\rangle + \frac{q_0^7}{5!\,3!\,2^{10}\pi^6}\langle\frac{\alpha_s}{\pi}G^2\rangle - \frac{q_0^5}{4!\,3!\,2^9\pi^6}m_s\langle\bar{s}g_s\sigma\cdot Gs\rangle$$

chiral odd

$$B_{\rm OPE}(q_0) = \frac{q_0^{10}m_s}{5!\,5!\,2^{10}\pi^8} - \frac{q_0^8}{4!\,5!\,2^7\pi^6}\langle\bar{s}s\rangle + \frac{q_0^6}{3!\,4!\,2^9\pi^6}\langle\bar{s}g_s\sigma\cdot Gs\rangle$$



QCD Sum Rule for Θ+

- Spectral function
- assume the continuum part is equal to the OPE

$$\rho^{\pm}_{\rm ph}(q_0) \quad = \quad |\lambda^2_{\pm}|\delta(q_0 - m_{\pm}) + \theta(q_0 - \sqrt{s_{\rm th}})\rho^{\pm}_{\rm OPE}(q_0)$$

- suppress $q_0 > M$ contribution by a weight function

$$W(q_0) = \exp\left(-\frac{q_0^2}{M^2}\right)$$

$$\int \rho^{\pm}_{\rm ph}(q_0)W(q_0)dq_0 \quad = \quad \int \rho^{\pm}_{\rm OPE}(q_0)W(q_0)dq_0$$

QCD Sum Rule for Θ+

- Sum rule *The chiral odd terms distinguish parity.*

$$|\lambda_\pm|^2 e^{-\frac{{m_\pm}^2}{M^2}} = \frac{1}{3!4!2^7\pi^6}\Big[\frac{1}{5600\pi^2}I_{11}(M,s_{\rm th}) \pm \frac{1}{800\pi^2}I_{10}(M,s_{\rm th})m_s$$
$$\pm\frac{1}{20}I_8(M,s_{\rm th})\langle\bar{s}s\rangle + \frac{1}{10}I_7(M,s_{\rm th})m_s\langle\bar{s}s\rangle$$
$$+\frac{1}{40}I_7(M,s_{\rm th})\langle\frac{\alpha_s}{\pi}G^2\rangle \pm \frac{1}{4}I_6(M,s_{\rm th})\langle\bar{s}g_s\sigma\cdot Gs\rangle$$
$$-\frac{1}{4}I_5(M,s_{\rm th})\,m_s\langle\bar{s}g_s\sigma\cdot Gs\rangle\Big],$$

$$I_n(M,s_{\rm th}) \equiv \int_0^{\sqrt{s_{\rm th}}} dq_0\, q_0^n e^{-\frac{q_0^2}{M^2}}$$

- Differentiate wrt $1/M^2$

QCD Sum Rule for Θ⁺

- QCD parameters: condensates, mass, …
  determined so as to reproduce the masses of the octet baryons

$$
\begin{aligned}
\langle \bar{q}q \rangle &= (-0.23[\mathrm{GeV}])^3 \\
\langle \bar{s}s \rangle &= 0.8 \times \langle \bar{q}q \rangle \\
\langle \frac{\alpha_s}{\pi} G^2 \rangle &= (0.33[\mathrm{GeV}])^4 \\
\langle \bar{s} g \sigma G s \rangle &= {m_0}^2 \langle \bar{s}s \rangle \\
{m_0}^2 &= 0.8[\mathrm{GeV}^2] \\
m_s &= 0.12[\mathrm{GeV}]
\end{aligned}
$$

QCD Sum Rule for Θ⁺

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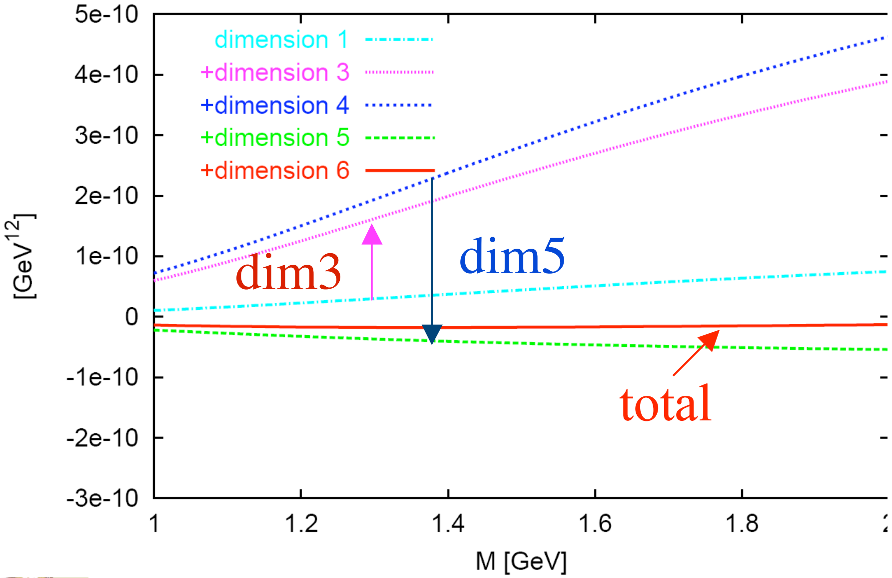
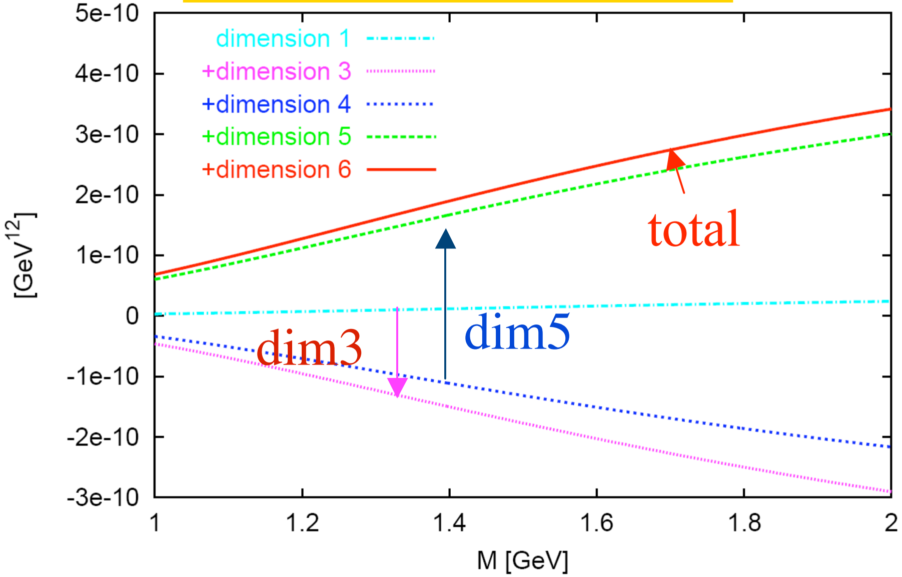
- positivity of $|\lambda_\pm|^2 e^{-\frac{m_\pm^2}{M^2}}$

positive parity



negative parity



QCD Sum Rule for Θ+

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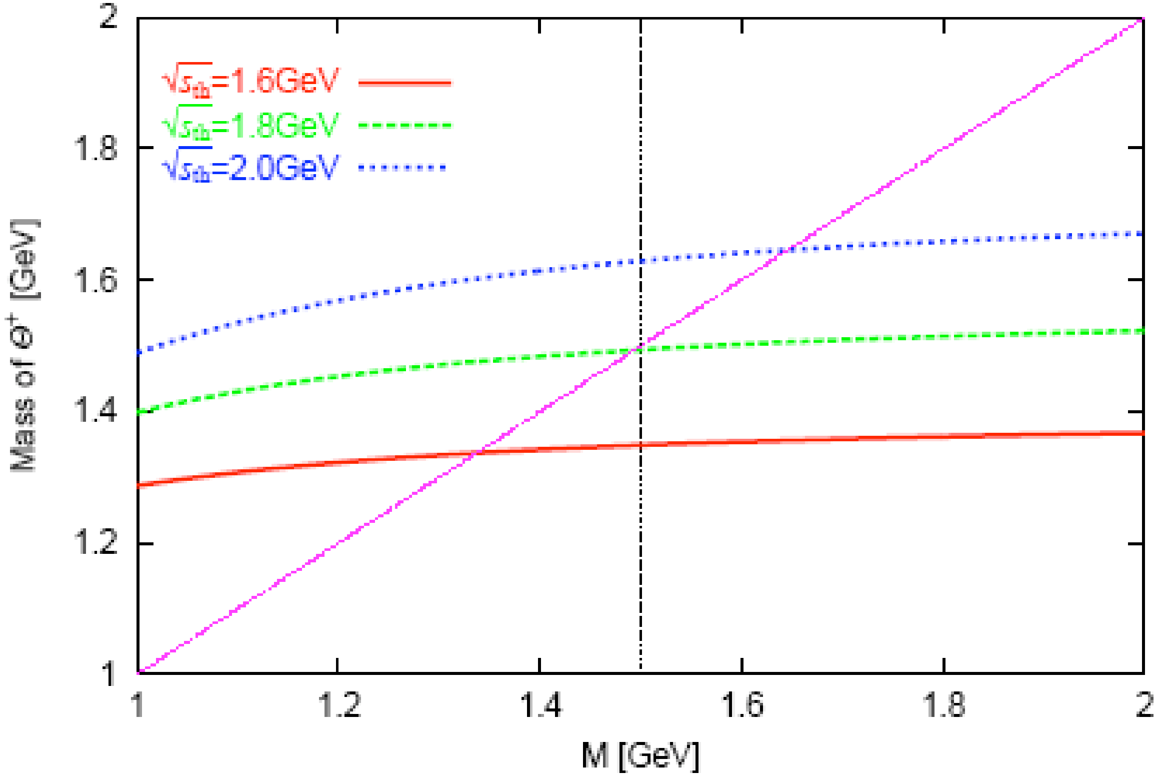

QCD Sum Rule for Θ^+

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- Conclusion
  - Parity splitting caused by chiral symmetry breaking by m_s and $<\bar{q}q>$ and $<\bar{q}\sigma Gq>$ condensates
  - Parity determined by the positivity of the correlation function => $1/2^-$
  - Mass $1/2^-$ Θ^+ baryon ≈ 1.3 – 1.7 GeV

Lattice QCD for Θ^+

- Lattice QCD calculations
  - F. Csikor, et al., JHEP 0311 (2003) 070
  - S. Sasaki, hep-lat/0310014
  - Kentucky group

  Each uses a different operator for Θ^+.

  All agree that the ground state has

  J =1/2, I =0, negative parity

  and the positive parity state is far high above.

Conclusion

- QCD predicts a (maybe) $1/2^-$, $I = 0$, Θ^+.

  It may not be easy to distinguish Θ^+ from the KN threshold.
- Many models suggest positive parity baryons.

  It seems unlikely to have $1/2^-$ higher than $1/2^+$ in the quark model.
- Mass prediction

  The masses are overestimated by ≥ 100 MeV- 200 MeV.

  The discrepancy is larger for $1/2^+$.

  Ideas of diquark correlations need careful confirmation.
- Width

  Arndt et al. nucl-th/0308012

  $\Gamma < 1$ MeV in order to be consistent with KN PSA.

Width

- How natural is the narrow width?

$$\mathcal{L}_{\text{int}}^{(-)} = g^- \bar{\psi}_N \psi_\Theta \phi_K \qquad \Gamma(1/2^-) = \frac{(g^-)^2}{4\pi} \frac{M_N + E_N}{M_\Theta} p$$

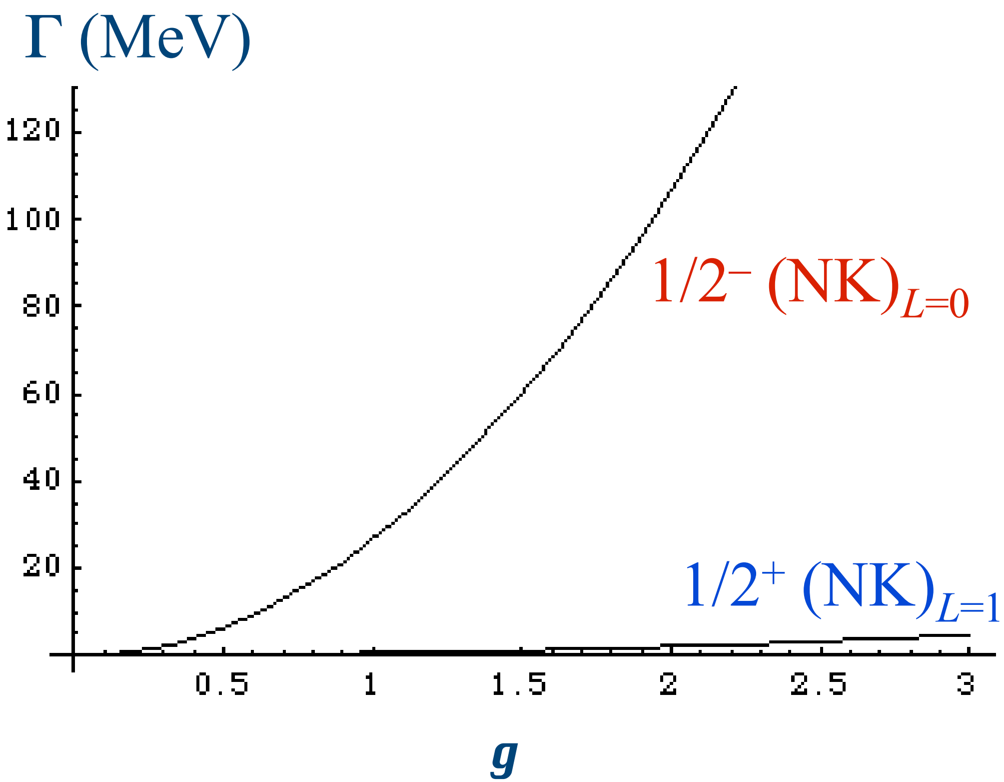
$$\mathcal{L}_{\text{int}}^{(+)} = g^+ \bar{\psi}_N \gamma^5 \psi_\Theta \phi_K \qquad \Gamma(1/2^+) = \frac{(g^+)^2}{4\pi} \frac{M_N - E_N}{M_\Theta} p$$

$$p = 269 MeV/c$$

$$\frac{\Gamma(1/2^-)}{\Gamma(1/2^+)} = 51 \left(\frac{g^-}{g^+}\right)^2$$

phase space difference for *L*=0/1

Width



KN phase shifts

K^+-nucleon scattering and exotic S=+1 Baryons

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PRC68(2003)04220

$\Gamma \leq 1$ MeV => $g^+ = 1.38$ for $1/2^+$
$g^- = 0.19$ for $1/2^-$

A mechanism of strong suppression of the decay is necessary.
S wave resonance might be very exotic.

Conclusion

- We need more experimental data
  - spin and parity
  - width
  - 10* members in order to confirm "pentaquark" structure.
  - excited states ex. J =3/2 states, I =1 states?
- and more theoretical ideas
  - How to suppress the width?
  - Is this related to other narrow hadron states? ex. D_s^*, X
    Does the heavy quark play a key role, or a spectator?
  - more exotic states? ex. molecular states