# Two Topics in Chiral Perturbation Theory

Pyungwon Ko

(KAIST)

- Dipion invariant mass spectrum in $X(3872) \to J/\psi\pi\pi$

  [ KAIST-TH 2003/10 ]

- Chiral lagrangian for pentaquark baryons and its applications

  [Work with J. Lee (ANL), T. Lee (SNU) and J.H. Park (KAIST),

  hep-ph/0312147 and in progress ]

— Talk at YITP workshop on "Multiquark Hadrons" Feb. 17–19, 2004 —

## I. Dipion invariant mass spectrum in $X(3872) \to J/\psi\pi\pi$

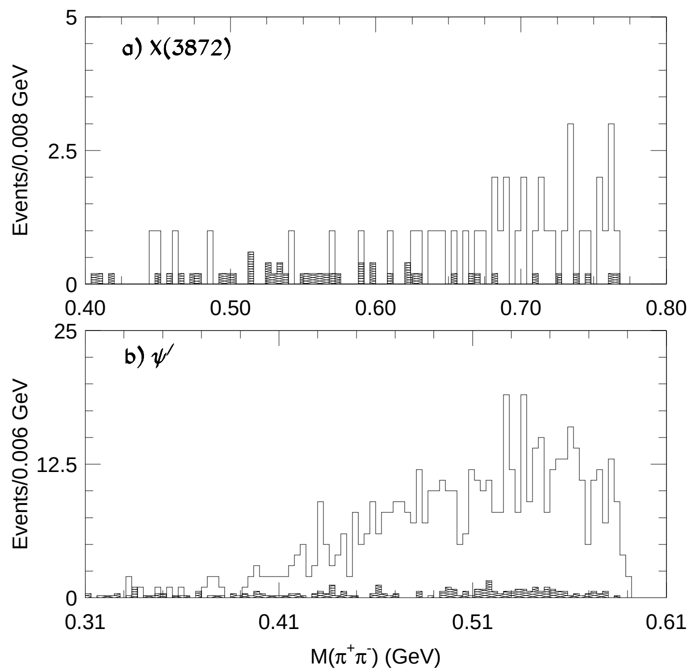
- $B$ meson system is a good place
  to study CP violation within the SM and its various extensions and also
  to look for new resonances
  - Resonance with quantum numbers such that they can not be reached from $n^3S_1$ states $(J/\psi, \psi^{'},\ldots)$ by cascade decays : $1^1P_1,\ldots$
    [Bodwin, Braaten, Lepage, Phys. Rev. D **46**, 3703 (1992) ]
  - Resonance above open charm threshold : $2^3P_J$, $^3D_J$, ....
    [ P. Ko, Phys. Rev. D **52** (1995) 3108 ;
    P. Ko, J. Lee and H. S. Song, Phys. Lett. B **395**, 107 (1997) ]
  - More extensive studies by Eichten, Lane and Quigg
    [ Phys. Rev. Lett. **89**, 162002 (2002) ]

- NRQCD $\to$ Reliable estimates of branching ratios for the inclusive $B$ decays into charmonia without infrared divergence problem
  [Bodwin, Braaten, Lepage, Phys. Rev. D **46**, 3703 (1992) ]

**Belle observed a new resonance $X(3872)$ in $B \to XK \to (J/\psi\pi\pi)K$**

[S.K. Choi et al. (2003), Belle Collaboration PRL ]

Talk by H. Yamamoto




Also confirmed by CDF later on

Many works already on the web !!

[ Pakvasa and Suzuki, Voloshin, N. A. Tornqvist, F. E. Close and P. R. Page,

C. Z. Yuan, X. H. Mo and P. Wang, C. Y. Wong, E. Braaten and M. Kusunoki ]

## Q: What is the nature of this new resonance ?

### Analysis by Pakvasa and Suzuki (hep-ph/0309294)

- $X(3872)$ has a very narrow width ($< 2.3$ MeV) :
  - If it is a charmonium, it should be either $1^3D_2(2^{--})$ or $2^1P_1(1^{+-})$.

    [ Notation : $n^{2S+1}L_J(J^{PC})$ ]
  - If $X(3872)$ is the $D\overline{D^*}$ molecular state, it should be either $J^{PC} = 1^{+-}$ with $I = 0$, or $J^{PC} = 1^{++}$ with $I = 1$.
- Dipion angular spectrum : useful to determine $J^{PC}$ quantum number

## My Point

- Dipion invariant mass spectrum provides independent information on the nature of $X(3872)$.
- Already eliminates a possibility that $X = {}^1P_1$ state.

## Hadronic Transition between Heavy Quarkonia

$$\psi' \to J/\psi\pi\pi, \quad J/\psi\eta, ....$$

- QCD multipole expansion [ Voloshin, Shifman, Kuang, Yan, Tuan, .... ]
- Chiral Perturbation Theory (ChPT) for Heavy Quarkonia

  [ See the review by Casalbuoni et al., Phys. Rept. **281**, 145 (1997) ]
- Equivalent except that the absolute normalization is unknown within ChPT
- Interested in spectra $\longrightarrow$ use ChPT

## Digression on ChPT

- QCD with 3 light quarks has spontaneously broken global chiral symmetry:

$$SU(3)_L \times SU(3)_R \to SU(3)_V$$

$q_L \to Lq_L, \quad q_R \to Rq_R$ ($L, R : SU(3)$ matrices)

- Pion field $U(x) \equiv exp(2i\pi(x)/f_\pi)$ transforms as

$$U(x) \to LU(x)R^\dagger$$

- Contruct Lagrangian which is inv. under chiral tr. and P and C.

$$
\begin{aligned}
P: \ \pi(t,\vec{x}) &\to \ -\pi(t,-\vec{x}) \\
U(t,\vec{x}) &\to \ U^{\dagger}(t,-\vec{x}) \\
C: \ \pi(t,\vec{x}) &\to \ \pi(t,\vec{x})^{T} \\
U(t,\vec{x}) &\to \ U(t,\vec{x})^{T}
\end{aligned}
$$

- The final charmonium is moving very slowly in the rest frame of the initial state $X(3872)$, we can use the heavy particle effective theory approach, by introducing a velocity dependent field $X_v(x) \equiv X e^{i m_X v \cdot x}$, and similarly for $J/\psi$ field $\psi_v(x)$.

- Then we can construct chiral invariant lagrangian using $X_v$, $\psi_v$, $\epsilon_{\mu\nu\alpha\beta}$ and the pion field $U(x)$. Transformation properties of $X_v$, $\psi_v$ and $v$ under parity and charge conjugation are given in Table 1.

- Transformation properties of $X_v$, $\psi_v$ and $v$ under $P$ and $C$

| Fields | $P$ | $C$ |
|---|---|---|
| $v^{\mu}$ | $v_{\mu} = (v^0, -\vec{v})$ | $v^{\mu} = (v^0, \vec{v})$ |
| $\psi_v^{\mu}$ | $\psi_{v\mu}$ | $-\psi_v^{\mu}$ |
| $X_v^{\mu\nu}(^3D_2)$ | $-X_{v\mu\nu}$ | $-X_v^{\mu\nu}$ |
| $X_v^{\mu}(^1P_1)$ | $-X_{v\mu}$ | $-X_v^{\mu}$ |
| $X_v^{\mu}(J^{PC} = 1^{++})$ | $-X_{v\mu}$ | $X_v^{\mu}$ |

## $1^3D_2(2^{--}) \to J/\psi\pi\pi$

- Relevant Chiral Lagrangian for this decay :

$$\mathcal{L} = g(^3D_2)\ \epsilon^{\mu\nu\alpha\rho}\ v_\mu \psi_{v,\nu} X_{v,\alpha\beta}\ \mathrm{Tr}\ \left[\partial_\rho U \partial^\beta U^\dagger\right] + h.c.$$

$*$ $g(^3D_2)$ : unknown coupling that should be determined by the data or could be calculated within QCD multipole expansion. [ See P. Moxhay, Phys. Rev. D **37**, 2557 (1988) for an explicit form of $g(^3D_2)$ ]

- Amplitude for this decay :

$$\mathcal{M} \sim \epsilon_{ijk}\ \epsilon_i^\psi \epsilon_{jl}^X\ \left(p_k p_l - q_k q_l\right),$$

$*$ $p \equiv p_1 - p_2$ and $q \equiv p_1 + p_2$

- Only one operator that contributes to the decay $1^3D_2(2^{--}) \to \pi\pi J/\psi$

$\to$ We can predict the $\pi\pi$ spectrum without any ambiguity

- The overall normalization is determined by the decay rate, but is irrelevant to our discussion for the $\pi\pi$ spectrum

- Calculate the $m_{\pi\pi}$ spectrum and compare with the data

## $X(1^{+-}, I=0) \to J/\psi\pi\pi$

- Relevant chiral lagrangian :

$$\mathcal{L} = g(^1P_1)\ \epsilon^{\mu\nu\alpha\beta}\ X_\mu \psi_\nu\ \mathrm{Tr}\ \partial_\alpha U \partial_\beta U^\dagger$$

- Amplitude for this case :

$$\mathcal{M} \sim \epsilon_{ijk} \epsilon_i^X \epsilon_j^\psi\ \left( E_1 p_{2,k} + E_2 p_{1,k} \right),$$

- If $X$ is a $D\overline{D^*}$ molecular state with $J^{PC} = 1^{+-}$, we cannot apply QCD multipole expansion. Still the chiral lagrangian approach will be applicable, and the above amplitude is still valid.

## $X(1^{++}, I=1) \to J/\psi\pi\pi$

- This case includes that the decaying state is $I=1$ $D\overline{D^*}$ molecular state.
- The final dipion is in $I=1$, dominated by $\rho$ meson
- $\rho^0 \to \pi^0\pi^0$ is forbidden by angular momentum conservation and Bose symmetry

  $\to \pi\pi$ in $X \to J/\psi\pi\pi$ should be charged pions in this case
- $m_{\pi\pi}$ spectrum : the Breit–Wigner profile of the $\rho$ resonance

## Comparison with ${}^3S_1 \to {}^3S_1^{'}\pi\pi$

[P. Ko et al. PRD48, 1205 (1993) ; PRD48, 1212 (1993) ; PRD50, 389 (1994)]

- Examples: $\psi^{'} \to J/\psi\pi\pi$ or $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$ , etc
- 4 Op.'s in chiral lagrangian approach, leading to

  $$\mathcal{M} \sim \epsilon_X \cdot \epsilon_\psi \left[ (p_1 + p_2)^2 + B E_1 E_2 + C m_\pi^2 \right] + D \left[ \epsilon_X \cdot p_1 \epsilon_\psi \cdot p_2 + \epsilon_X \cdot p_2 \epsilon_\psi \cdot p_1 \right]$$

  $\rightarrow$ For $D \neq 0$, spin flip possible between the initial and the final quarkonia

  (see ref.'s above for details)

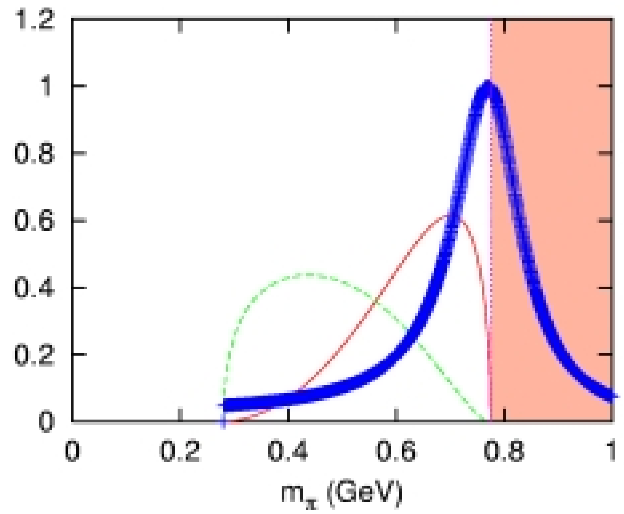
- QCD multipole expansion predicts $B = D = 0$ and $C \sim O(1)$ is determined from the $m_{\pi\pi}$ spectrum

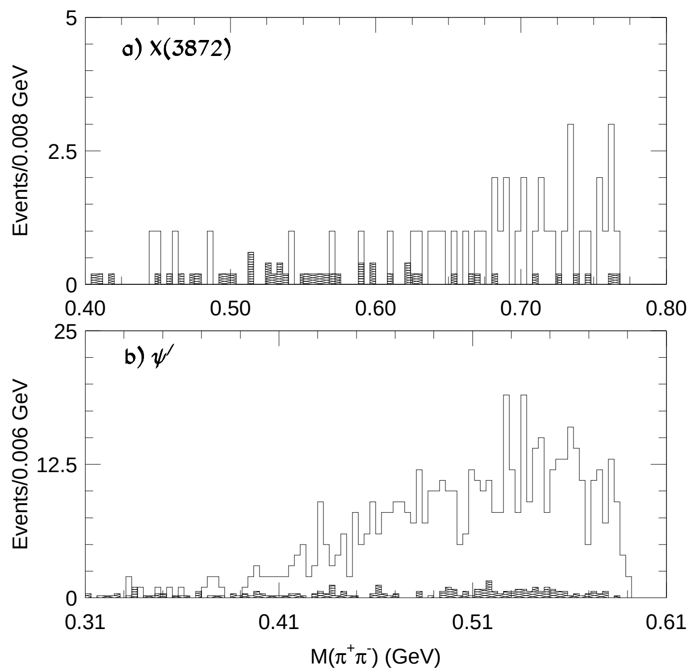
  $\rightarrow$ No spin flip between the initial and the final quarkonia

  cf. Could be relevant to the depolarization of high $p_T$ $\psi^{(')}$ and $\Upsilon(nS)$ produced by color octet mechanism at Tevatron

- No definite prediction for $m_{\pi\pi}$ spectrum possible in this case unlike the previous 3 cases !

## Back to our $X(3872) \to J/\psi\pi\pi$ !


1.2
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
$m_\pi$ (GeV)


a) X(3872)
Events/0.008 GeV
5
2.5
0
0.40
0.50
0.60
0.70
0.80
b) ψ′
Events/0.006 GeV
25
12.5
0
0.31
0.41
0.51
0.61
M(π⁺π⁻) (GeV)

## Conclusion

- Dipion invariant mass spectrum in $X(3872) \to J/\psi\pi\pi$:

  useful in determination of $J^{PC}$ quantum number of the newly observed

  resonance $X(3872)$

- Current preliminary data seems to already exclude the possibility $X = {}^1P_1$

- If $X = {}^3D_2(2^{--})$, then the $m_{\pi\pi}$ spectrum has a peak at high $m_{\pi\pi}$ region,

  which is consistent with the data.

- Interpretation in terms of $D\overline{D^*}$ molecular state predicts $m_{\pi\pi}$ spectrum to

  have a sharp peak near $m_{\pi\pi} \approx m_\rho$

- Useful to look for $X \to J/\psi\pi^0\pi^0$ and the angular correlations

  $\leftarrow$ forbidden if $\pi\pi$ come from $\rho^0$

- This will be easily distinguishable in the future when more data are

  accumulated

## II. Chiral Lagragian for pentaquark baryons and its applications [ Work with J. Lee, T. Lee and J.H. Park, hep-ph/0312147 ]

### Introduction

- QCD : underlying theory of strong interaction
  - Well tested up to $\sim 1\%$ level in high energy regime
  - Should explain the proerpties of observed hadrons (mesons and baryons)

    $\longrightarrow$ Difficult due to nonperturbative nature in low energy regime
- Observed hadrons: mesons $q\bar{q}$ and baryons $qqq$
- QCD predicts other exotics such as
  - Glueballs : $gg, ggg, \ldots$
  - Hybrid mesons : $q\bar{q}g, \ldots$
  - Multiquark hadrons : $qq\bar{q}\bar{q}, qqqq\bar{q}, qqqqqq, \ldots$

- Observations of a new baryonic state $\Theta^+(1540)$ with $S = +1$ and a very narrow width $< 5$ MeV
  - SPRING-8 in $\gamma n \to K^-(K^+ n)$ :

    $m = 1.54 \pm 0.01$ GeV, $\Gamma < 25$ MeV
  - DIANA (ITEP) in $K^+ Xe \to \Theta^+ Xe' \to (K^0 p) Xe'$ :

    $m = 1539 \pm 2$ MeV, $\Gamma < 9$ MeV
  - CLAS (JNL) in $\gamma d \to p K^-(K^+ n)$:

    $m = 1542 \pm 5$ MeV, $\Gamma < 21$ MeV
  - And more experiments confirming $\Theta^+$
- This $\Theta^+$ is likely to be a pentaquark state $(uudd\bar{s})$

  [ Diakonov et al. (1997) within chiral soliton model ]

$$\Theta^+ \to n(udd) K^+(u\bar{s})$$

- NA 49 reports a new resonance $\Xi(1860)$ observed via

  $\Xi^{--} \to \Xi^- \pi^-$ etc.
- Arguments based on quark models suggest that this state is a member of $\mathrm{SU}(3)$ antidecuplet with spin $J = \frac{1}{2}$ or $\frac{3}{2}$ with

  $P = -1$ (uncorrelated quark model) or $P = +1$ (diquark picture)

## Q: How to determine $J^P$ of $\Theta^+(1540)$ ?

- In this talk, we construct a chiral lagrangian for pentaquark baryons assuming they are $\mathrm{SU}(3)$ antidecuplet with $J = \frac{1}{2}$ and $P = +1$ or $-1$

  $*$ $J = \frac{3}{2}$ case can be discussed in a similar manner except that antidecuplets are described by Rarita–Schwinger fields

- Then we can study

  – Mass spectra of antidecuplets and their possible mixings with pentaquark octets

  – Decay rates of pentaquark antidecuplets

  – Cross sections for $K^+n \to \Theta^+ \to K^0p$, $K^+p \to \pi^+\Theta^+$, $\pi^-p \to K^-\Theta^+$, $K^+n \to \gamma\Theta^+$, and $\gamma n \to K^-\Theta^+$

    (NB: green color processes involve energetic $\pi/K$'s and chiral lagrangian approach becomes unreliable, but could be a guide for field theoretic models for hadrons which are used to consider dynamics of these particles)

- Discuss how to include light vector mesons in our framework, and how the low energy theorem is recovered in the soft pion limit

## Chiral lagrangian for a pentaquark baryon decuplet: w/o light vector mesons

- Relevant fields and their transformation Rules under chiral $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ :

  – Goldstone boson field by pion octet $\pi$ and $\Sigma \equiv \exp(2i\pi/f)$

$$\Sigma(x) \to L\Sigma(x)R^\dagger.$$

It is convenient to define another field $\xi(x)$ by $\Sigma(x) \equiv \xi^2(x)$, which transforms as

$$\xi(x) \to L\xi(x)U^\dagger(x) = U(x)\xi(x)R^\dagger.$$

The pion octet $\pi$ in $\xi(x) = exp(i\pi/f)$ is given by

$$\pi^i_j = \frac{1}{\sqrt{2}} \begin{pmatrix} \pi^0/\sqrt{2} + \eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2} + \eta/\sqrt{6} & K^0 \\ K^- & \overline{K}^0 & -2\eta/\sqrt{6} \end{pmatrix} \tag{1}$$

The $3 \times 3$ matrix field $U(x)$ depends on Goldstone fields $\pi(x)$ as well as the $\mathrm{SU}(3)$ transformation matrices $L$ and $R$. It is convenient to define two vector fields with following properties under chiral transformations:

$$\begin{aligned} V_\mu &= \tfrac{1}{2}\,(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger), \quad V_\mu \to UV_\mu U^\dagger + U\partial_\mu U^\dagger, \\ A_\mu &= \tfrac{i}{2}\,(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger), \quad A_\mu \to UA_\mu U^\dagger. \end{aligned} \tag{2}$$

Note that $V_\mu$ transforms like a gauge field

- Baryon octet $B$ including nucleons :

$$B^i{}_j \to U^i{}_a \, B^a{}_b \, U^{\dagger\, b}{}_j$$

$$B^i_j = \begin{pmatrix} \Sigma^0/\sqrt{2} + \Lambda/\sqrt{6} & \Sigma^+ & p \\ \Sigma^- & -\Sigma^0/\sqrt{2} + \Lambda/\sqrt{6} & n \\ \Xi^- & \Xi^0 & -2\Lambda/\sqrt{6} \end{pmatrix}$$

- Antidecuplet $\mathcal{P}_{abc} = \mathcal{P}_{(abc)}$ including $\Theta^+$ :

$$\mathcal{P}_{ijk} \to P_{abc} \, U^{\dagger\, a}{}_i \, U^{\dagger\, b}{}_j \, U^{\dagger\, c}{}_k,$$

$$\begin{aligned} \mathcal{P}_{333} &= \Theta^+ \\ \mathcal{P}_{133} = \frac{1}{\sqrt{3}}\, \tilde{N}^0, \qquad & \mathcal{P}_{233} = \frac{1}{\sqrt{3}}\, \tilde{N}^+ \\ \mathcal{P}_{113} = \frac{1}{\sqrt{3}}\, \tilde{\Sigma}^-, \quad \mathcal{P}_{123} &= \frac{1}{\sqrt{6}}\, \tilde{\Sigma}^0, \quad \mathcal{P}_{223} = \frac{1}{\sqrt{3}}\, \tilde{\Sigma}^+ \\ \mathcal{P}_{111} = \Xi^{--}_{3/2}, \quad \mathcal{P}_{112} = \frac{1}{\sqrt{3}}\, \Xi^-_{3/2}, \qquad & \mathcal{P}_{122} = \frac{1}{\sqrt{3}}\, \Xi^0_{3/2}, \quad \mathcal{P}_{222} = \Xi^+_{3/2} \end{aligned}$$

- Define a covariant derivative

$$\mathcal{D}_\mu B == \partial_\mu B + [V_\mu, B]$$

which transforms as

$$\mathcal{D}_\mu B \to U \mathcal{D}_\mu B U^\dagger$$

Similarly for pentaquark baryons $\mathcal{P}_{ijk}$

## Explicit chiral symmetry breaking

- Non-vanishing current-quark masses : Consider the quark-mass matrix

  $m = \mathrm{diag}\,(m_u, m_d, m_s)$ as a spurion with transformation property

  $m \to LmR^\dagger = RmL^\dagger$

  More convenient to use $\xi m \xi + \xi^\dagger m \xi^\dagger$, which transforms as an $\mathrm{SU}(3)$ octet

- Electromagnetic interactions :

  Electromagnetic interactions can be included by introducing photon field

  $\mathcal{A}_\mu$ and its field strength tensor $F_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$:

$$
\begin{aligned}
\partial_\mu \Sigma &\to \mathcal{D}_\mu \Sigma \equiv \partial_\mu \Sigma + ie\mathcal{A}_\mu [Q, \Sigma], \\
V_\mu &\to V_\mu + \tfrac{ie}{2} \mathcal{A}_\mu (\xi^\dagger Q \xi + \xi Q \xi^\dagger), \\
A_\mu &\to A_\mu - \tfrac{e}{2} \mathcal{A}_\mu (\xi^\dagger Q \xi - \xi Q \xi^\dagger),
\end{aligned}
$$

  where $Q \equiv \mathrm{diag}\ (2/3, -1/3, -1/3)$ is the electric-charge matrix for light

  quarks ($q = u, d, s$)

## Lowest order chiral lagrangian

- Symmetries: Poincare, Chiral, parity ($P$) and Charge Conjugation ($C$)

$$\mathcal{L} = \mathcal{L}_\Sigma + \mathcal{L}_B + \mathcal{L}_\mathcal{P},$$

where

$$\begin{aligned}
\mathcal{L}_\Sigma &= \frac{f_\pi^2}{4}\,\mathrm{Tr}\big[\mathcal{D}_\mu\Sigma^\dagger\mathcal{D}^\mu\Sigma - 2\mu m(\Sigma+\Sigma^\dagger)\big], \\
\mathcal{L}_B &= \mathrm{Tr}\,\overline{B}(i\not{\mathcal{D}} - m_B)B + D\,\mathrm{Tr}\,\overline{B}\gamma_5\{\not{A}, B\} \\
&\quad + F\,\mathrm{Tr}\,\overline{B}\gamma_5[\not{A}, B], \\
\mathcal{L}_\mathcal{P} &= \overline{\mathcal{P}}(i\not{\mathcal{D}} - m_\mathcal{P})\mathcal{P} + \mathcal{C}_{\mathcal{P}N}\big(\overline{\mathcal{P}}\Gamma_P\not{A}B + \overline{B}\Gamma_P\not{A}\mathcal{P}\big) \\
&\quad + \mathcal{H}_{\mathcal{P}N}\overline{\mathcal{P}}\gamma_5\not{A}\mathcal{P},
\end{aligned}$$

$*$ $P$ : Parity of $\Theta^+$

$*$ $\Gamma_+ = \gamma_5$, and $\Gamma_- = 1$

$*$ $m_\mathcal{P}$ is the average of the pentaquark decuplet mass.

- $\mathcal{H}_{\mathcal{P}N}$ term is a new ingredient in our chiral lagrangian

- Gell-Mann-Okubo formulae for pentaquark baryons from

$$\mathcal{L}_m = \alpha_m \overline{\mathcal{P}}(\xi m \xi + \xi^\dagger m \xi^\dagger)\mathcal{P}$$

Expanding this, we get $\Delta m_i \equiv m_i - m_{\mathcal{P}}$ within the antidecuplet:

$$\Delta m_\Theta = 2\alpha_m m_s, \tag{5a}$$

$$\Delta m_{\widetilde{N}} = \alpha_m \left(2\hat{m} + 4m_s\right)/3, \tag{5b}$$

$$\Delta m_{\widetilde{\Sigma}} = \alpha_m \left(4\hat{m} + 2m_s\right)/3, \tag{5c}$$

$$\Delta m_{\Xi_{3/2}} = 2\,\alpha_m \hat{m}, \tag{5d}$$

where $\hat{m} = m_u = m_d$ ignoring small isospin-breaking effects.

If the newly observed state at a mass $1862 \pm 2$ MeV is identified as $\Xi_{3/2}$, we find

$$m_{\widetilde{N}} = 1647\,\mathrm{MeV}, \quad m_{\widetilde{\Sigma}} = 1755\,\mathrm{MeV}. \tag{6}$$

- Mixing between pentaquark antidecuplet $\mathcal{P}_{abc}$ and pentaquark octet $\mathcal{O}^a{}_b$

[Jaffe and Wilczek, Diakonov, Oh et al.]

In chiral lagrangian approach, such a general mixing arises from

$$\beta_m \left[\overline{\mathcal{P}}(\xi m \xi + \xi^\dagger m \xi^\dagger)\mathcal{O} + \overline{\mathcal{O}}(\xi m \xi + \xi^\dagger m \xi^\dagger)\mathcal{P}\right]. \tag{7}$$

Expanding this leads to

$$\mathcal{L} = \mathcal{B}_m \left[\overline{p}\widetilde{N}^+ - \overline{n}\widetilde{N}^0 + \overline{\Sigma^0}\widetilde{\Sigma}^0 - \overline{\Sigma^-}\widetilde{\Sigma}^- + \overline{\Sigma^+}\widetilde{\Sigma}^+ + \mathrm{H.c.}\right], \tag{8}$$

where $\mathcal{B}_m = 2\beta_m(m_s - \hat{m})/\sqrt{3}$ and we borrowed baryon-octet notation for pentaquark octet states in Eq. (8).

$*$ Relative sign between $\overline{n}\widetilde{N}^0$ and $\overline{p}\widetilde{N}^+$ (and also $\overline{\Sigma^0}\widetilde{\Sigma}^0$) is negative, unlike the case in the paper by Diakonov. This is due to the $\overline{\mathbf{10}}$ nature of the $\mathcal{P}$.

- Ignore the mixing between pentaquark antidecuplet $\mathcal{P}_{abc}$ and the ordinary baryon octet $B$, since it is a mixing between $qqq$ and $qqqq\bar{q}$.

- The baryon decuplet can only couple to pentaquark octet $\mathcal{O}$, but not to pentaquark antidecuplet $\mathcal{P}$, since $\mathbf{10} \otimes \mathbf{8} \otimes \mathbf{10}$ does not contain $\mathrm{SU}(3)$ singlet.

$\rightarrow$ $N(1440)$ or $N(1710)$ cannot be pure pentaquark antidecuplets, because they have substantial branching ratios into $\Delta\pi$ final states. They could be mixed states of pentaquark octet and pentaquark antidecuplet, and their productions and decays will be more complicated than pure antidecuplet case. Since the current data on baryon sectors are not enough to study such mixings in details, we do not pursue the mixing further in the following.

- Numerical values for parameters above :

$*$ $m_B \approx 940\,$MeV is the nucleon mass,

$*$ $D \approx -0.81$ and $F \approx -0.47$ at tree level

$*$ Assume $\hat{m} = 0$ and $m_\eta^2 = (4/3)\ m_K^2$

- The coupling $\mathcal{C}_{\mathcal{P}N}$ is determined from the decay width of the $\Theta^+$

  which is dominated by $K^+n$ and $K^0p$ modes

$$\Gamma_\Theta = \frac{\mathcal{C}_{\mathcal{P}N}^2 |\mathbf{p}^*|}{8\pi f^2 m_\Theta^2}(m_\Theta \pm m_B)^2 \, [(m_\Theta \mp m_B)^2 - m_K^2],$$

  $*$ $\mathbf{p}^*$ is the kaon momentum in the $\Theta^+$ rest frame

  $*$ Two signs are for $P(\Theta^+) = \pm 1$

  $\rightarrow$ the $\mathcal{C}_{\mathcal{P}N}$ is determined as

$$\mathcal{C}_{\mathcal{P}N}^2(P = +, -) = (2.7, 0.90) \times \Gamma_\Theta/\mathrm{GeV}.$$

  Cahn and Trilling argues that $\Gamma_\Theta = (0.9 \pm 0.3)$MeV using the DIANA results: $\mathcal{C}_{\mathcal{P}N} = (0.05, 0.03)$

- Why is $\mathcal{C}_{\mathcal{P}N}$ so small ?

  A mystery !

- Decay rates for $\Xi^{--}_{3/2}$ and $\Xi^{0}_{3/2}$ relative to $\Gamma(\Theta^+)$:

| | $\frac{1}{2}^-$ | $\frac{1}{2}^+$ |
|---|---|---|
| $\Gamma(\Xi^{--}_{3/2} \to \Xi^- \pi^-)/\Gamma(\Theta^+)$ | 0.79 | 2.20 |
| $\Gamma(\Xi^{--}_{3/2} \to \Sigma^- K^-)/\Gamma(\Theta^+)$ | 0.87 | 1.19 |
| $\Gamma(\Xi^{0}_{3/2} \to \Xi^- \pi^+)/\Gamma(\Theta^+)$ | 0.26 | 0.73 |
| $\Gamma(\Xi^{0}_{3/2} \to \Sigma^+ K^-)/\Gamma(\Theta^+)$ | 0.30 | 0.43 |
| $\Gamma(\Xi^{0}_{3/2} \to \Xi^0 \pi^0)/\Gamma(\Theta^+)$ | 0.54 | 1.52 |
| $\Gamma(\Xi^{0}_{3/2} \to \Sigma^0 \overline{K^0})/\Gamma(\Theta^+)$ | 0.59 | 0.83 |
| $\Gamma(\Xi^{--}_{3/2} \to \Xi^- \pi^-)/\Gamma(\Xi^{--}_{3/2} \to \Sigma^- K^-)$ | 0.91 | 1.85 |
| $\Gamma(\Xi^{0}_{3/2} \to \Xi^- \pi^+)/\Gamma(\Xi^{0}_{3/2} \to \Sigma^+ K^-)$ | 0.87 | 1.72 |

Useful for determining $J^P$

(Similar results by Mehen and Schat in HBChPT)

- The coupling $\mathcal{H}_{\mathcal{P}N}$ is a new feature of our chiral largrangian, and is also unknown $\sim$ axial vector coupling $g_A$ of nucleons

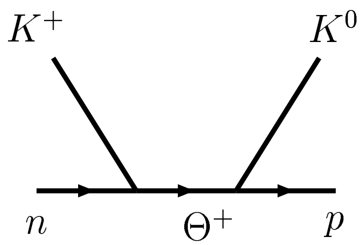
  $\mathcal{H}_{\mathcal{P}N}$ determines transition rates between pentaquark antidecuplets with pion or kaon emission. Unfortunately, such decays are all kinematically forbidden.

- However, we expect that $\mathcal{H}_{\mathcal{P}N} = O(1)$, without any suppression as in $\mathcal{C}_{\mathcal{P}N}$

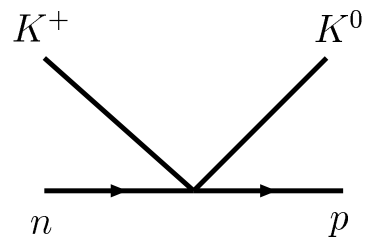
  With this remark in mind, we will assume $\mathcal{H}_{\mathcal{P}N}$ can vary between $-4$ and 4

## $K^+n \to \Theta^+ \to pK^0$

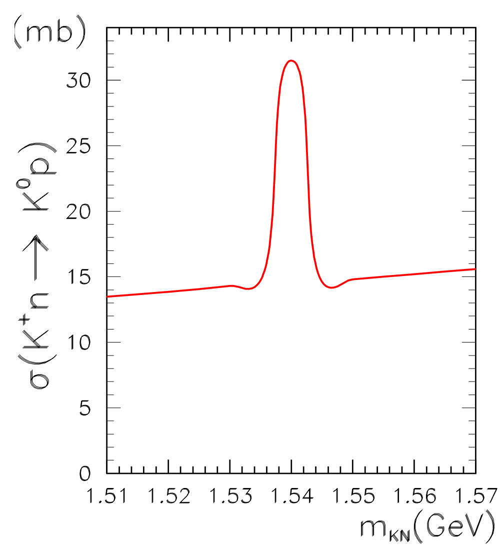
- DIANA observed $\Theta^+$ through this reaction in $Xe$
- Near threshold, $K$ momentum is small and our chiral lagrangian would give a good description for this process
- Contact terms present




(a)

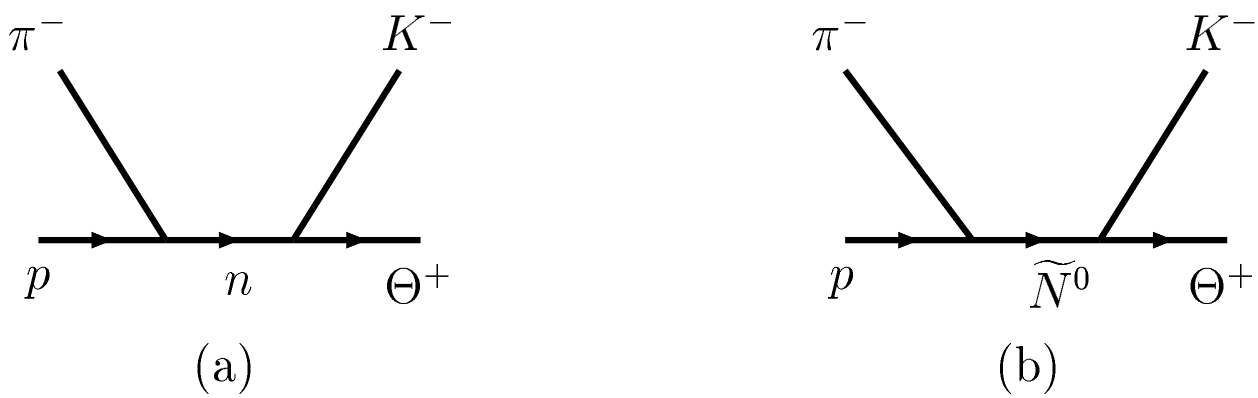



(d)



## $\pi^- p \to K^- \Theta^+$

- Feynman diagrams our chiral lagrangian for pentaquark baryons.



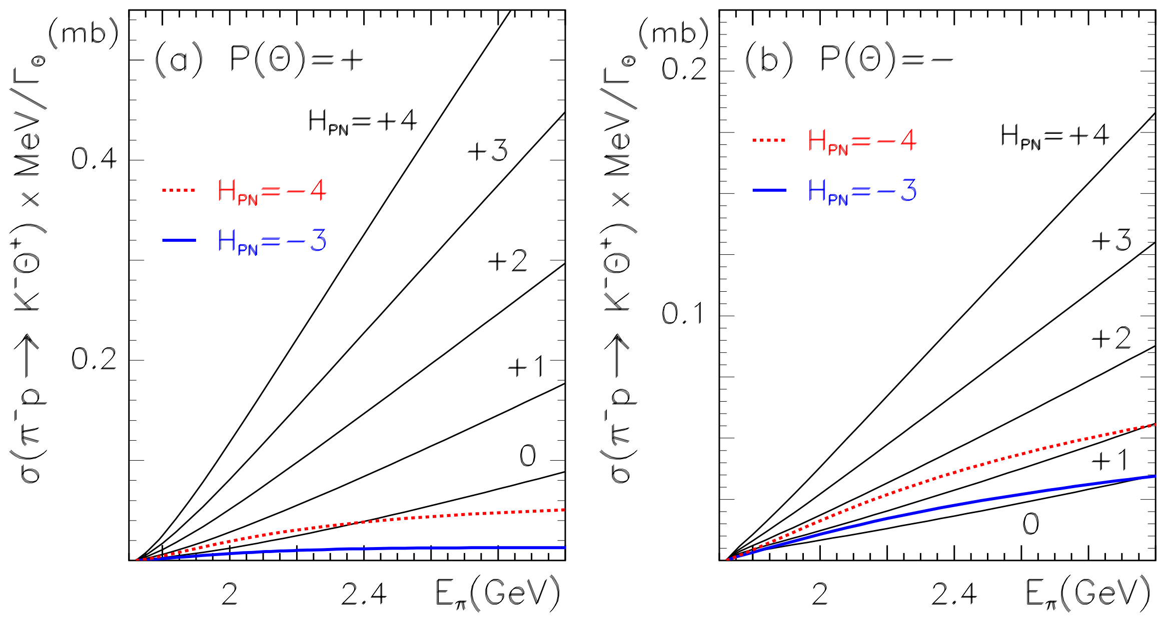

$*$ Only Fig. (a) was considered in the recent literature.

$*$ However, there is an $s-$channel $\widetilde{N}^0(1647)$ exchange diagram [ Fig. (b) ] in our chiral lagrangian from the $\mathcal{H}_{\mathcal{P}N}$ term, since $\Theta^+$ is not an $\mathrm{SU}(3)$ singlet, but belongs to the antidecuplet.

$*$ Therefore one has to keep both Figs. (a) and (b) in order to get an amplitude with correct $\mathrm{SU}(3)$ flavor symmetry

- The amplitude for $\pi^- p \to K^- \Theta^+$ :

$$\begin{aligned}\mathcal{M} &= \frac{\mathcal{C}_{\mathcal{P}N}}{2f^2}\bar{u}_{\Theta^+}\left(\Gamma_P \not{p}_{K^-}\frac{D+F}{\not{p}_{\pi^-}+\not{p}_p - m_B}\gamma_5\not{p}_{\pi^-}\right.\\ &\qquad \left. -\gamma_5\not{p}_{K^-}\frac{\mathcal{H}_{\mathcal{P}N}/3}{\not{p}_{\pi^-}+\not{p}_p - m_{\widetilde{N}}}\Gamma_P\not{p}_{\pi^-}\right)u_p. \qquad (9)\end{aligned}$$




- The sign of $\mathcal{H}_{\mathcal{P}N}$ is very important. If $\mathcal{H}_{\mathcal{P}N} > 0\,(< 0)$, two contributions will have constructive (destructive) interference. Thus our results differ from the previous results where only the $n$ contribution was included.

- The cross section is sensitive to $\mathcal{H}_{\mathcal{P}N}$, and may be useful to fix $\mathcal{H}_{\mathcal{P}N}$

- The even parity and the odd parity cases can be distinguished from the cross section for the parity-odd case is smaller than that for the parity-even case.

  NB: Since pion energy is so large, this result should be modified by form factors, etc.. $\rightarrow$ Model dependent

  Crossed channel $K^+p \rightarrow \pi^+\Theta^+$ shows similar behavior :

## Photo-production of $\Theta^+$

- Need to know the magnetic dipole interaction terms

- For the nucleon octet,

$$\mathcal{L} = -\frac{e}{4m_B}\mathrm{Tr}\Big[\overline{B}\sigma_{\mu\nu}F^{\mu\nu}\big(\kappa_D\{Q,B\} + \kappa_F[Q,B]\big)\Big].$$

The anomalous magnetic moments of nucleons are

$$\kappa_p = \kappa_F + \tfrac{1}{3}\kappa_D, \quad \kappa_n = -\tfrac{2}{3}\kappa_D,$$

at tree-level chiral lagrangian. Using $\kappa_p = 1.79$ and $\kappa_n = -1.91$, we get $\kappa_D = 2.87$ and $\kappa_F = 0.836$.

- For the pentaquark baryon $\mathcal{P}$, the relevant term is

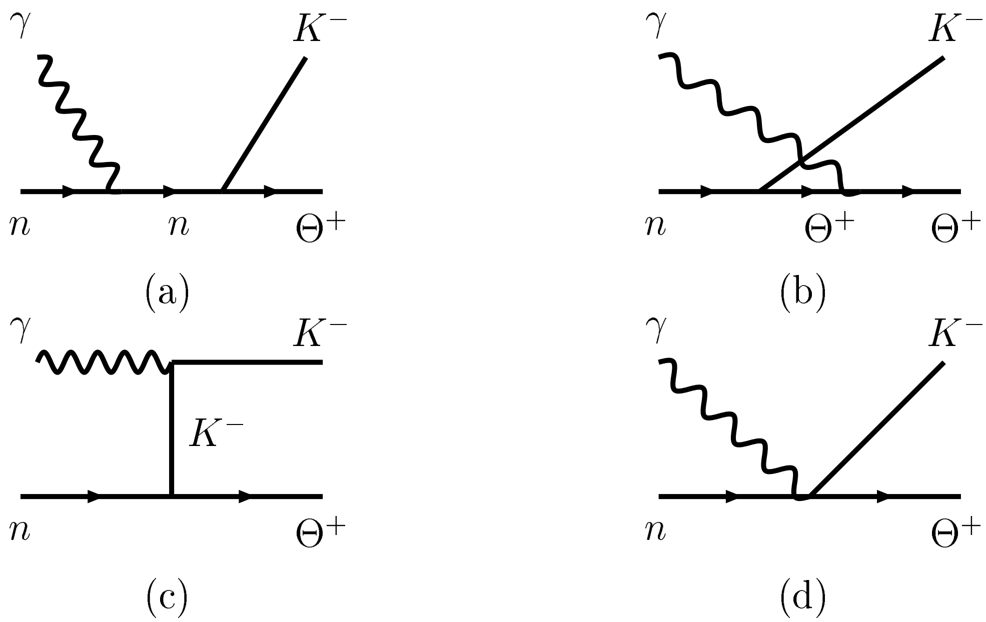
$$-\frac{e\kappa_{\mathcal{P}}}{4m_{\mathcal{P}}}\, q_i\overline{\mathcal{P}}_i\sigma_{\mu\nu}\; F^{\mu\nu}\; \mathcal{P}_i \to -\frac{e\kappa_\Theta}{4m_\Theta}\,\overline{\Theta^+}\sigma_{\mu\nu}F^{\mu\nu}\Theta^+.$$

We expect that $|\kappa_\Theta(\equiv\kappa_{\mathcal{P}})| \approx |\kappa_D| \approx |\kappa_F|$.

cf. a calculation in soliton picture predicts that $\kappa_\Theta \approx 0.3$ [ H. C. Kim (2003) ]. We vary $\kappa_\Theta$ between $-1$ and 1.

- We ignore transition magnetic moments between nucleon octet and pentaquark antidecuplet, since this transition involves $qqq$ and $qqqq\bar{q}$.

- Feynman diagrams for $\gamma n \to K^- \Theta^+$ :

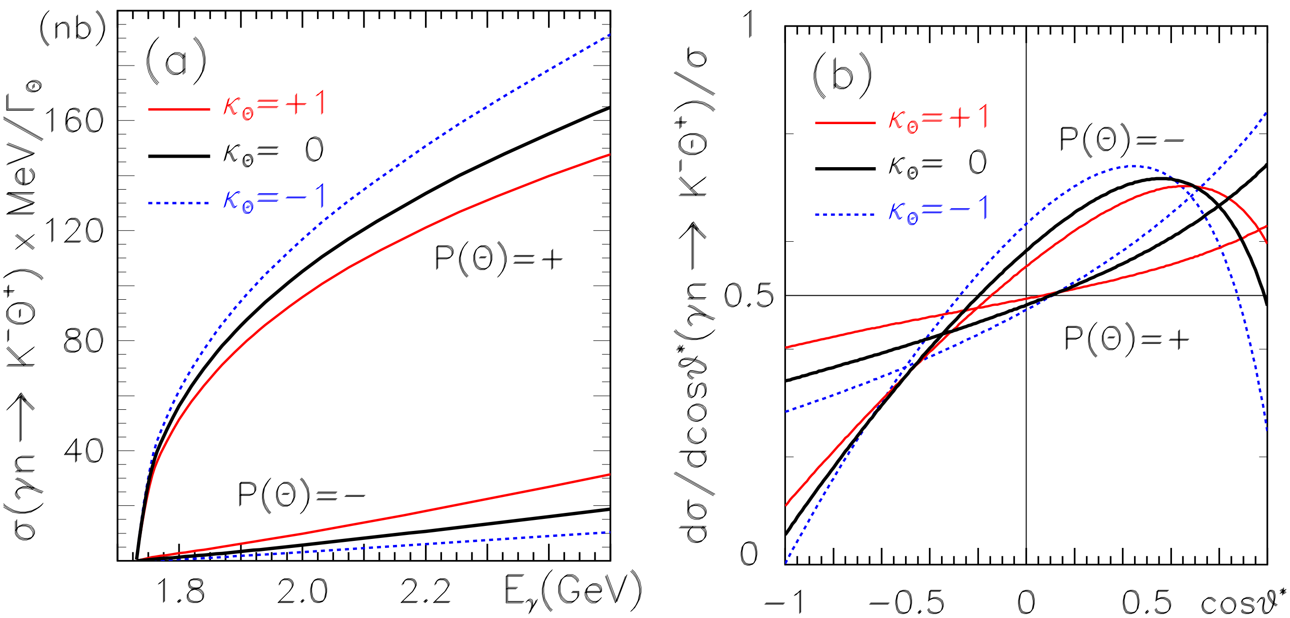



One salient feature of our approach based on chiral perturbation theory is the existence of a contact term for $\gamma K^- n \Theta^+$ vertex (d), which is necessary to recover $\mathrm{U}(1)_{\mathrm{em}}$ gauge invariance within spontaneously broken global chiral symmetries.

(See also Hosaka et al..

Ours corresponds to the PV model in their paper)

- Cross sections and angular distributions :




$*$ The parity-even case has larger cross section, and has a sharp rise near the threshold

$*$ The angular distribution shows that the forward/backward scattering is suppressed in the negative parity case, whereas the forward peak is present in the positive parity case.

$\to$ The angular distribution could be another useful tool to determine the parity of $\Theta^+$.

- Once $\mathcal{C}^2_{\mathcal{P}N}$ is determined from $\Gamma_\Theta$, one could determine the parity of $\Theta^+$

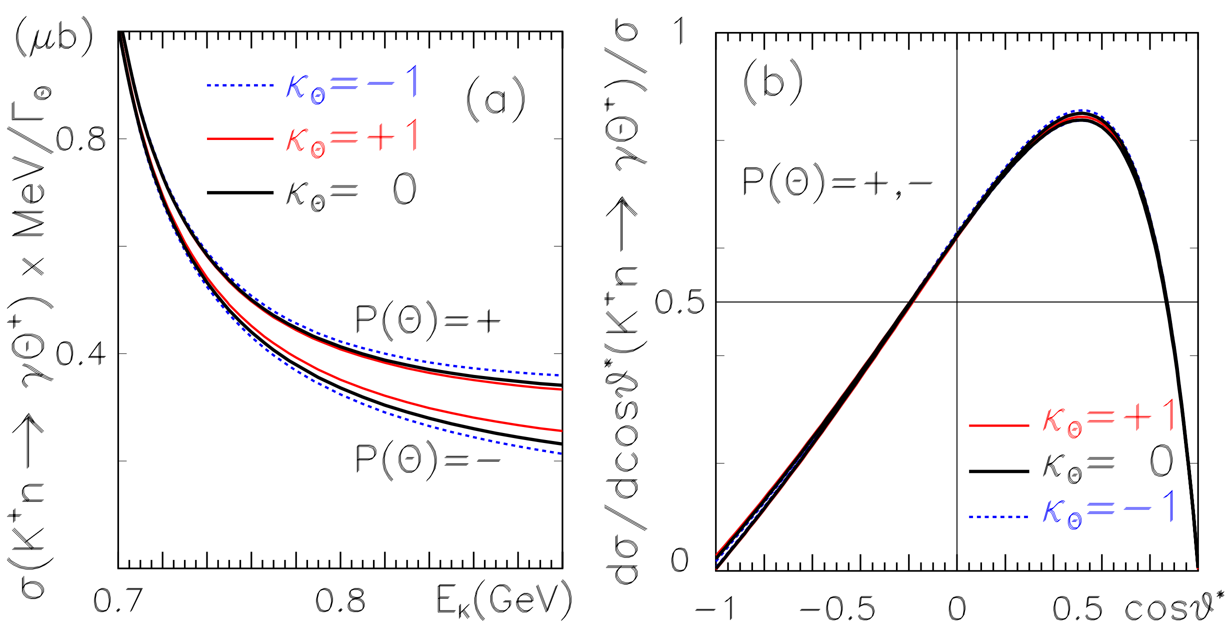
## $nK^+ \to \Theta^+\gamma$ (Inverse process of photoproduction)

- Near threshold, the s-channel $\Theta^+$ contribution is dominant

  $\to$ $\kappa_\theta$ is most important

  $\to$ No distinct difference between positive and negative parity

- At higher kaon energy, the angular distributions differ for positive/negative parity, but our lagrangian becomes less reliable



## Including light vector mesons

- Introduce light vector mesons $\rho_\mu$, which transforms as

$$\rho_\mu(x) \to U(x)\rho_\mu(x)U^\dagger(x) + U(x)\partial_\mu U^\dagger(x),$$

under global chiral transformations [ Bando et al., Hidden local symmetry ]

- $\rho_\mu(x)$ transforms as a gauge field under local $\mathrm{SU}(3)$'s, as $V_\mu(x)$ does

$\to$ The covariant derivative $\mathcal{D}_\mu$ can be defined using $\rho_\mu$ instead of $V_\mu$

- Note: $(\rho_\mu - V_\mu) \to U(x)(\rho_\mu - V_\mu)U^\dagger(x)$

- Straightforward to construct chiral invariant lagrangian using this new field

$$\begin{aligned}\mathcal{L}_\rho &= -\frac{1}{2}\,\mathrm{Tr}(\rho_{\mu\nu}\rho^{\mu\nu}) + \frac{1}{2}\,m_\rho^2\,\mathrm{Tr}(\rho_\mu - V_\mu)^2 \\ &+ \ \alpha\left[\overline{\mathcal{P}}(\not{\rho} - \not{V})B + \overline{B}(\not{\rho} - \not{V})\mathcal{P}\right] + \ldots\end{aligned}$$

- Important to notice that $N\Theta^+K^*$ coupling should be highly suppressed, since it can appear only in combination of $(\rho_\mu - V_\mu)$, which vanishes in the low-energy limit.

$*$ The low-energy theorem is violated if one includes only $n\Theta^+K^*$ diagram, without including the $n\Theta^+K\pi$ contact term arising from the $\not{V}$ term.

$\to$ Should be cautious about claiming that the $K^*$ exchange is important in $\pi^-p \to K^-\Theta^+$

## Conclusion

- We constructed a chiral lagrangian involving pentaquark baryon antidecuplet and octet, the ordinary nucleon octet and Goldstone bosons
- Derived Gell-Mann-Okubo formula, the decay rates for $\Xi$ and $\Theta$, and the cross sections for $K^+ n \to \Theta^+ \to K^0 p$, and
  $\pi^- p \to K^- \Theta^+$ and $\gamma n \to K^- \Theta^+$, etc. for $J^P = \frac{1}{2}^{\pm}$
- Important to respect chiral symmetry properly in order to get correct amplitudes for these processes
- Photo-production data : could be useful in identifying the parity of $\Theta^+$, because the threshold behavior of the cross section and its angular distributions strongly depend on the parity
- Once the coupling $\mathcal{C}_{\mathcal{P}N}$ is determined from the decay width of $\Theta^+$, then the parity and other couplings $\mathcal{H}_{\mathcal{P}N}$ and $\kappa_\Theta$ could be determined from the hadro/photo-production cross sections for $\Theta^+$
- We have outlined how to incorporate the vector meson degrees of freedom in our scheme, and how the low ernergy theorem is recovered.
- Parity of pentaquark will tell us which picture is correct: uncorrelated quark model $(P = -1)$ or diquark picture $(P = +1)$.