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Magnetic Moments of the exotic pentaquark Baryons in the chiral quark-soliton model

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I. Introduction

Let me skip the current state of the art. We have listened to it enough in the present workshop. I do not want to bother you by repeating it.

Motivation

In order to describe photo-processes such as $\gamma n \to K^- \Theta^+$, it is essential to know information about the magnetic moment of Θ^+.

Aim

To investigate μ_{Θ^+}, based on the chiral quark-soliton model with model parameters fixed by experiments (octet baryon magnetic moments).

II. Formalism

Matrix elements for the magnetic moments of the baryon antidecuplet

$$\langle B_{\overline{10}} | \bar{\psi}(z) \gamma_\mu \hat{Q} \psi(z) | B_{\overline{10}} \rangle ,$$

where

$$\hat{Q} = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix} = \frac{1}{2}\left(\lambda^3 + \frac{1}{\sqrt{3}}\lambda^8\right) .$$

The Sachs form factors

$$\langle B_{\overline{10}}(p') | \bar{\psi}(z) \gamma_0 \hat{Q} \psi(z) | B_{\overline{10}}(p) \rangle = G_E^{B_{\overline{10}}}(Q^2)$$

$$\langle B_{\overline{10}}(p') | \bar{\psi}(z) \gamma_i \hat{Q} \psi(z) | B_{\overline{10}}(p) \rangle = \frac{1}{2M_N} G_M^{B_{\overline{10}}}(Q^2) i\epsilon_{ijk} q^j \langle s' | \sigma_k | s \rangle .$$

$$\mu_{B_{\overline{10}}} = G_M^{B_{\overline{10}}}(0)$$

The collective operator of the magnetic moment

$$\hat{\mu} = v_1 D^{(8)}_{Q3} \; + \; v_2 d_{pq3} D^{(8)}_{Qp} \cdot \hat{J}_q \; + \; \frac{v_3}{\sqrt{3}} D^{(8)}_{Q8} \hat{J}_3.$$

v_i in the χQSM – the moments of inertia expressed generically by

$$\sum_{m,n} \langle n|\Gamma_1|m\rangle \langle m|\Gamma_2|n\rangle \mathcal{R}(E_n, E_m, \Lambda)$$

with the quark states and their eigenenergies

$$H|n\rangle = E_n|n\rangle$$

and the cut-off parameter Λ.

The magnetic moments of the baryon antidecuplet

$$\mu_{B_{\overline{10}}} = \int dR \psi^*_{B_{\overline{10}}}(R) \hat{\mu}(R) \psi_{B_{\overline{10}}}(R)$$

with the collective wave function:

$$\psi_{B_\nu}(R) = \sqrt{\dim(\nu)} (-1)^{J_3 - 1/2} D^{(\nu)*}_{Y,T,T_3;Y',J,-J_3}(R).$$

- $\nu \rightarrow$ The allowed irreducible representations of the SU(3) flavor group: $\nu = 8, 10, \overline{10}, \cdots$
- $Y, T, T_3 \rightarrow$ the corresponding hypercharge, isospin, and its third component, respectively.
- $Y', J, J_3 \rightarrow$ constrained to be unity for the physical spin states for which J and J_3 are spin and its third component.

Remark on the collective wave functions

The action of left (flavor) generators $\hat{T}_\alpha = -D^{(8)}_{\alpha\beta}\hat{J}_\beta \; \psi_{B_\mathcal{R}}$ transforms like a tensor in representation $\mathcal{R}$, while under the right generators $\hat{J}_\alpha$ like a tensor in $\mathcal{R}^*$ rather than $\mathcal{R}$.

$$\Downarrow$$

This is the reason why operators like the one multiplied by v_2 have different matrix elements for the decuplet (which is spin $3/2$) and antidecuplet (which is spin $1/2$).

D-function algebra

First term:

$$\left\langle \overline{10}, B, \frac{1}{2}, J_3 \right| D^{(8)}_{Q3}(R) \left| \overline{10}, B, \frac{1}{2}, J_3 \right\rangle = -\frac{1}{12} Q_B J_3,$$

Second term:

$$\left\langle \overline{10}, B, \frac{1}{2}, J_3 \right| d_{3bc} D^{(8)}_{Qb} J_c \left| \overline{10}, B, \frac{1}{2}, S_3 \right\rangle = -\frac{5}{24} Q_{B_{\overline{10}}} J_3$$

Remark: For the decuplet, we would have

$$\left\langle 10, B, \frac{3}{2}, J_3 \right| d_{3bc} D^{(8)}_{Qb} J_c \left| 10, B, \frac{3}{2}, J_3 \right\rangle = \frac{1}{24} Q_{B_{10}} J_3.$$

Third term:

$$\frac{1}{\sqrt{3}} \left\langle 10, B, \frac{3}{2}, J_3 \right| D^{(8)}_{Q8} \hat{J}_3 \left| \overline{10}, B, \frac{1}{2}, S_3 \right\rangle = \frac{1}{24} Q_{B_{\overline{10}}} J_3$$

The difference between the magnetic moments of the baryon antidecuplet and those of the decuplet arise from this second term!

III. Expression of the antidecuplet magnetic moments

$$\left\langle \overline{10}, B \left| \hat{\mu} \right| \overline{10}, B \right\rangle \Big|_{m_s=0} = -\frac{1}{12}\left(v_1 + \frac{5}{2}v_2 - \frac{1}{2}v_3\right) Q_B J_3.$$

For comparison, the magnetic moments of the decuplet are:

$$\langle 10, B \left| \hat{\mu} \right| 10, B \rangle|_{m_s=0} = -\frac{1}{12}\left(v_1 - \frac{1}{2}v_2 - \frac{1}{2}v_3\right) Q_B J_3$$

IV. Results

Sum rules (Similar to the Colman-Glashow sum rules)

$$\begin{aligned}
\mu_{\Sigma^0_{\overline{10}}} &= \frac{1}{2}\left(\mu_{\Sigma^+_{\overline{10}}} + \mu_{\Sigma^-_{\overline{10}}}\right), \\
\mu_{\Xi^+_{3/2}} + \mu_{\Xi^{--}_{3/2}} &= \mu_{\Xi^0_{3/2}} + \mu_{\Xi^-_{3/2}}, \\
\sum \mu_{B_{\overline{10}}} &= 0, \\
\mu_{\Theta^+} - \mu_{\Xi^{--}_{3/2}} &= \frac{1}{4}(2\mu_{\rm p} + \mu_{\rm n} + \mu_{\Sigma^+} - \mu_{\Sigma^-} - \mu_{\Xi^0} - 2\mu_{\Xi^-}).
\end{aligned}$$

Numerical Results

New set of variables:

$$v = \frac{1}{60}\left(v_1 - \frac{1}{2}v_2\right), \quad w = \frac{1}{120}\,v_3$$

$$\begin{aligned}
\mu_p &= \mu_{\Sigma^+} = -8v + 4w, \\
\mu_n &= \mu_{\Xi^0} = 6v + 2w, \\
\mu_\Lambda &= -\mu_{\Sigma^0} = 3v + w, \\
\mu_{\Sigma^-} &= \mu_{\Xi^-} = 2v - 6w, \\
\mu_{B_{10}} &= \frac{15}{2}\left(-v + w\right)\, Q_{B_{10}}
\end{aligned}$$

Antidecuplet magnetic moments:

$$\mu_{B_{\overline{10}}} = \left[\frac{5}{2}\left(-v + w\right) - \frac{1}{8}v_2\right] Q_{B_{\overline{10}}}.$$

Decuplet magnetic moments:

$$\mu_{B_{\overline{10}}} = -\frac{3}{2}(v + w)Q_{B_{10}}$$

The input: The experimental data of the baryon octet magnetic moments.

Fit I: The proton and neutron magnetic moments as input $\longrightarrow$

$$\begin{aligned} v &= (2\mu_{\rm n} - \mu_{\rm p})/20 &&= -0.331, \\ w &= (4\mu_{\rm n} + 3\mu_{\rm p})/20 &&= 0.037, \\ v_2 &\sim 0.5 \quad \text{from the } \chi\text{QSM}. \end{aligned}$$

Fit II: Average values of the octet magnetic moments $\longrightarrow$

$$\begin{aligned} v &= \left(2\mu_{\rm n} - \mu_{\rm p} + 3\mu_{\Xi^0} + \mu_{\Xi^-} - 2\mu_{\Sigma^-} - 3\mu_{\Sigma^+}\right)/60 &&= -0.268, \\ w &= \left(3\mu_{\rm p} + 4\mu_{\rm n} + \mu_{\Xi^0} - 3\mu_{\Xi^-} - 4\mu_{\Sigma^-} - \mu_{\Sigma^+}\right)/60 &&= 0.060, \\ v_2 &\sim 0.5 \quad \text{from the } \chi\text{QSM}. \end{aligned}$$

Remark: For the baryon antidecuplet we cannot predict the magnetic moments unambigously because of the new parameter v_2.

	exp.	fit I	fit II	χQSM
p	2.79	input	2.39	2.27
n	-1.91	input	-1.49	-1.55
Λ	-0.61	-0.96	-0.74	-0.78
Σ^+	2.46	2.79	2.38	2.27
Σ^0	(0.65)	0.96	0.74	0.78
Σ^-	-1.16	-0.89	-0.90	-0.71
Ξ^0	-1.25	-1.91	-1.49	-1.55
Ξ^-	-0.65	-0.89	-0.90	-0.71
Δ^{++}	4.52	5.52	4.92	4.47
Ω^-	-2.02	-2.76	-2.46	-2.23
Θ^+	?	0.30	0.20	0.12

V. Nonlreativistic limit

The χQSM reduces to the free valence quarks which, however, "remember" the soliton structure, as the soliton size $r_0 \to 0$.

In the NRQM limit:

$$v = -\frac{7}{90}K, \quad w = \frac{1}{90}K, \quad v_3 = \frac{4}{3}K,$$

with

$$K \sim \int_0^D drr^2 j_1(kr) r j_0(kr).$$

In this limit, the proton and magnetic moments are:

$$\mu_p = \frac{2}{3}K, \quad \mu_n = -\frac{4}{9}K.$$

$$\frac{\mu_p}{\mu_n} = -\frac{2}{3}.$$

For antidecuplet magnetic moments, we get

$$\mu_{B_{\overline{10}}} = -\frac{1}{3} K Q_{B_{\overline{10}}}$$

It becomes negative!

This is caused by the large value of v_3 in the quark-model limit.

VI. Conclusion and Outlook

- The magnetic moments of the exotic pentaquark baryons were calculated within the framework of the chiral quark-soliton model in the chiral limit.

- The results show that the magnetic moments of the antidecuplet are rather small, compared to typical values of the magnetic moments of the charged baryon octet and decuplet.

- In the limit of the nonrelativistic quark model, *i.e.* when the soliton size goes to zero, the magnetic moments of the baryon antidecuplet in the chiral limit become negative.

- The effect of SU(3) symmetry breaking will be expected to be around 25% from our experience and it will be soon reported.