

# Charmed scalar mesons and related

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- Motivation to four-quark mesons

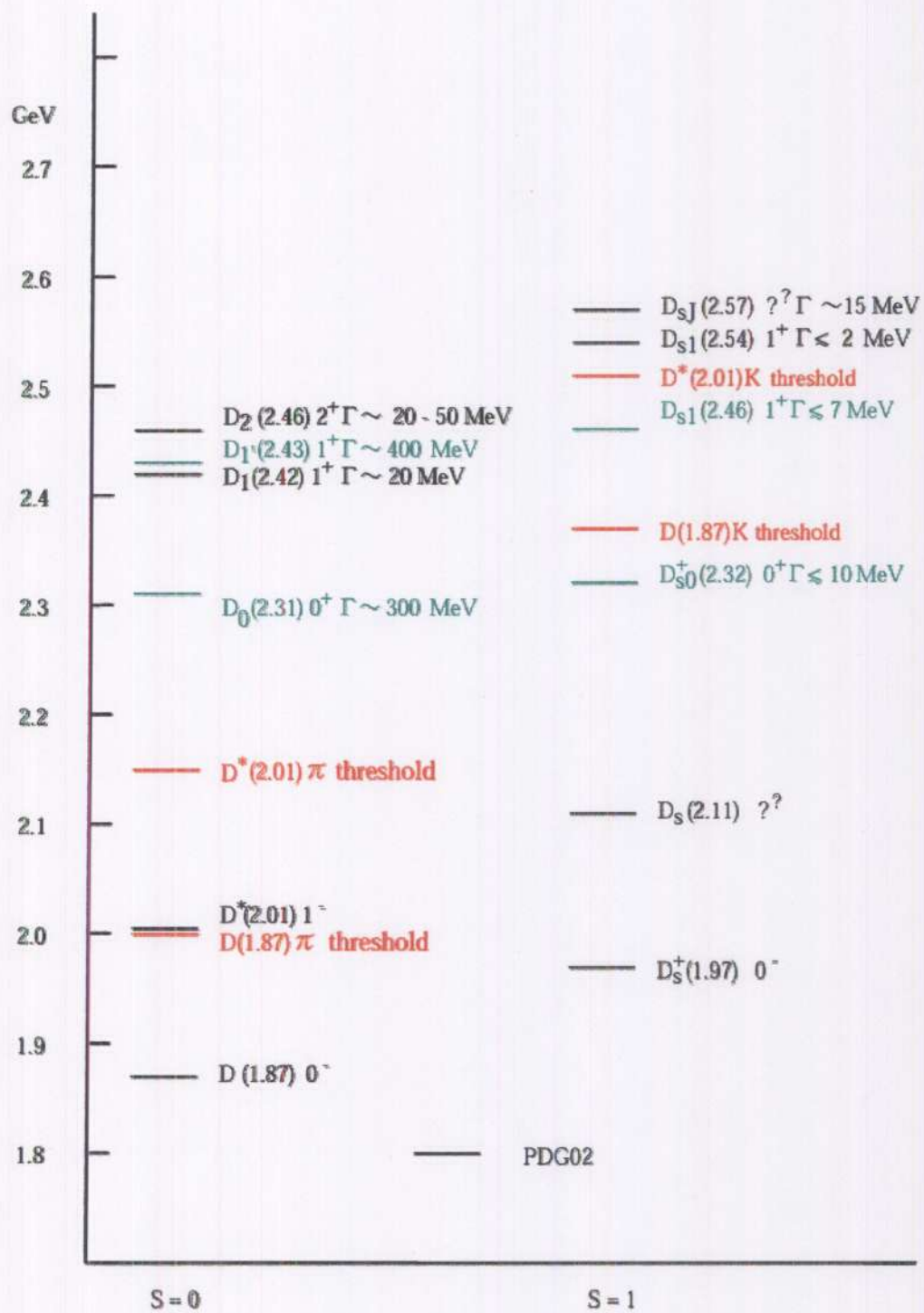
- An important role of four-quark mesons in hadronic weak decays of charm mesons, in particular, in the long standing puzzle,

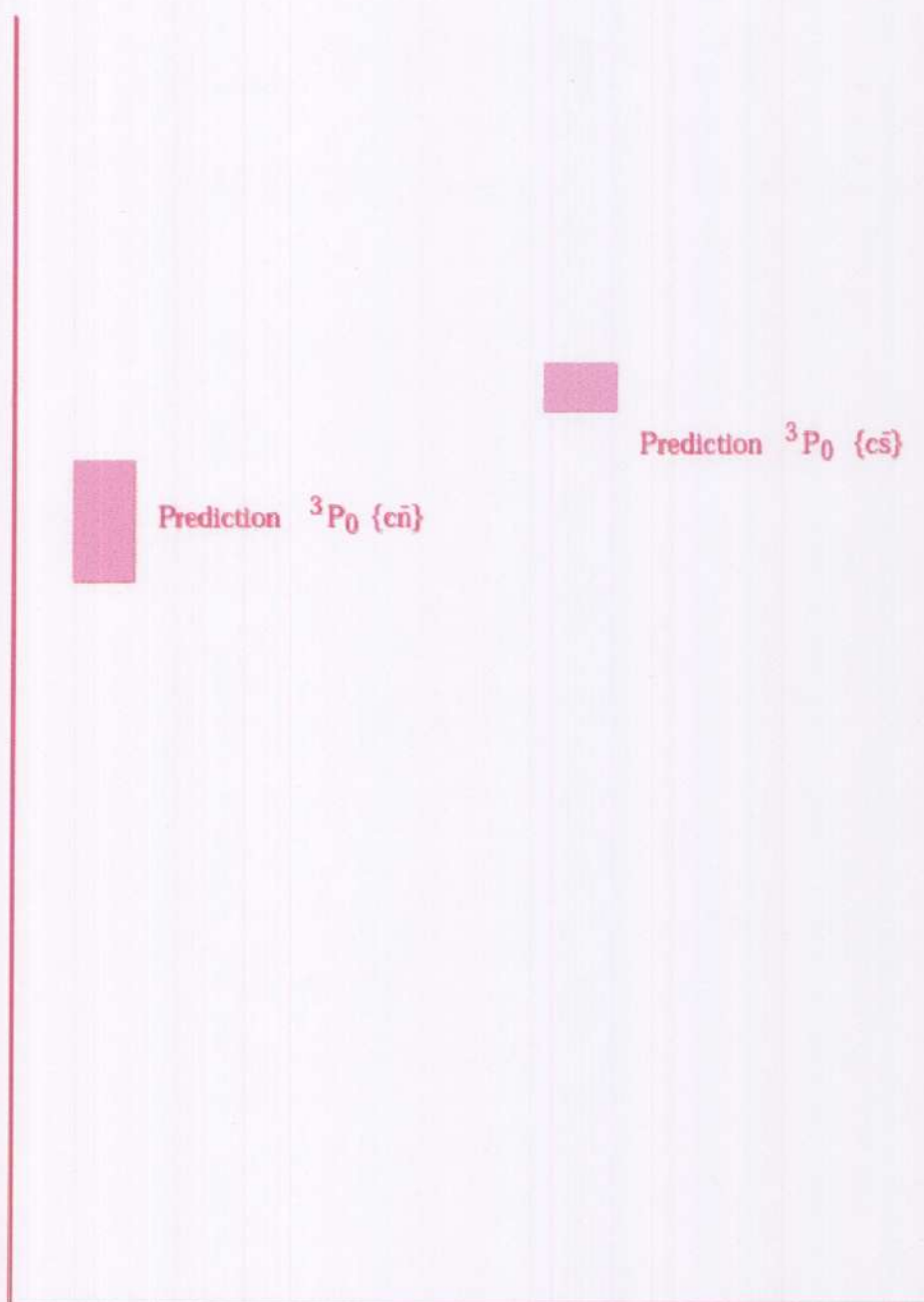
$$\frac{\Gamma(D^0 \rightarrow K^+K^-)}{\Gamma(D^0 \rightarrow \pi^+\pi^-)} = 2.88 \pm 0.15.$$

( PDG02, K. Hagiwara et al., )  
( P.R.D 66, 010001 (2002). )

- The new resonance  $D_{s0}^+(2.32)$  as a component of iso-triplet four-quark mesons
  - Extra narrow scalar states

## §1. Spectrum of charmed mesons







## §2. Existing models of $D_{s0}^+$ (2.32) [tentative]

Assignments	Comments
<ul style="list-style-type: none"> <li>• Scalar <math>\{c\bar{s}\}</math> (a)</li> <li>• <math>\chi</math>-partner of <math>D_s^+</math> (b)</li> </ul>	<ul style="list-style-type: none"> <li>• <math>m_{\text{pot, quench}} &gt; 2.4 \text{ GeV}</math></li> <li>• Only one scalar</li> <li>• Production rates ?</li> </ul>
<ul style="list-style-type: none"> <li>• Iso-singlet <math>DK</math> molecule (c)</li> <li>• Mixed state of <math>\{c\bar{s}\}</math> and <math>\{cq\bar{q}s\}</math> (d)</li> </ul>	<ul style="list-style-type: none"> <li>• <math>\Gamma_{D_{s0}} \ll 10 \text{ MeV}</math> (iso-spin violating)</li> <li>• Coupling with the <math>\{c\bar{s}\}</math></li> <li>• Extra states ?</li> <li>• Color degree of freedom ?</li> </ul>
<ul style="list-style-type: none"> <li>• Iso-singlet <math>\{cn\bar{n}s\}</math> (e)</li> </ul>	<ul style="list-style-type: none"> <li>• Narrow <math>\tilde{D}_{0s}^+</math> peak on a broad bump of <math>\tilde{D}_{1s}^+</math></li> <li>• Exotic : <math>\tilde{D}_{0\bar{s}}^0</math></li> </ul>
<ul style="list-style-type: none"> <li>• A component of iso-triplet <math>[cn][\bar{s}\bar{n}]</math> (f)</li> </ul>	$\Gamma_{\hat{F}_I^+} \simeq \Gamma(\hat{F}_I^+ \rightarrow D_s^+ \pi^0) \sim 10 \text{ MeV (input)}$ <ul style="list-style-type: none"> <li>• Narrow <math>\hat{F}_I, \hat{D}, \hat{D}^s</math></li> <li>• <math>\Gamma_{\hat{F}_0^+} \ll 10 \text{ MeV}</math> (iso-spin viol.)</li> <li>• Exotic : <math>\hat{E}^0</math></li> </ul>
<ul style="list-style-type: none"> <li>• Pole of <math>DK (D\pi)</math> amplitude (g)</li> </ul>	<ul style="list-style-type: none"> <li>• Unitarized amplitude,</li> <li>• Chiral Lagrangian</li> </ul>

## References (tentative):

- Assignments:
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  - (b) M. A. Nowak, M. Rho and J. Zahed, P.R.D **48**, 4370 (1993); W. A. Bardeen and C. T. Hill, P.R.D **49**, 409 (1994).
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  - (d) T. Browder, S. Pakvasa and A. A. Petrov, hep-ph/0307054.
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  - Potential model;  
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    - \* Quench  
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    - \*  $N_f = 2$   
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- MIT bag model;  
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  - Quark-meson model;  
A. Deandrea, G. Nardulli and A. D. Polosa,  
P.R.D **68**, 097501 (2003).
  - QCD sum rule;  
Y.-B. Dai, C.-S. Huang, C. Liu and S.-L. Zhu,  
P.R.D **68**, 114011 (2003).
  - Lightcone oscillator model;  
S. -G. Zhou and H.- C. Pauli, hep-ph/0310330.
  - HQET sum rule;  
Y.-B. Dai, C.-S. Huang, C. Liu S.-L. Zhu,  
hep-ph/0401142.
- Production rates:
    - C.-H. Chen and H. Li, hep-ph/0307075;
    - A. Datta and P. J. O'Dnnell, P.L. **B572**, 164  
(2003); hep-ph/0312160;
    - M. Suzuki, hep-ph/0307118.
  - Exotic  $\hat{E}^0$  (or  $\tilde{D}_{0\bar{s}}^0$ ):
    - H. J. Lipkin, P.L. **70B**, 113 (1977);
    - M. Suzuki and S. F. Tuan, P.L. **133B**,  
125 (1983).

### §3. Four-quark mesons

- Potentials mediated by vector mesons with  $SU(3)$  "color": S. Hori, P.T.P. **36**, 131 (1966)

$$V_{qq}(\mathbf{r}) = \sum \Lambda_i \Lambda_i v(\mathbf{r}), \quad V_{q\bar{q}}(\mathbf{r}) = - \sum \Lambda_i \Lambda_i v(\mathbf{r}).$$

Their expectation values:

$qq$	$\bar{3}$	$6$		
$q\bar{q}$			$8$	$1$
$\langle V_{qq} \rangle$	$-\frac{8}{3}\langle v \rangle$	$\frac{4}{3}\langle v \rangle$		
$\langle V_{q\bar{q}} \rangle$			$\frac{2}{3}\langle v \rangle$	$-\frac{16}{3}\langle v \rangle$

- Four-quark meson :

R. L. Jaffe, P.R.D **15**, 267 and 281 (1977)

$$\{qq\bar{q}\bar{q}\} = \underbrace{[qq][\bar{q}\bar{q}] \oplus (qq)(\bar{q}\bar{q})}_{J^P=0^+, 1^+, 2^+} \oplus \underbrace{\{[qq](\bar{q}\bar{q}) \pm (qq)[\bar{q}\bar{q}]\}}_{J^P=1^+}$$



Two ways to obtain color-singlet states

(dominated by)  $\bar{3} \otimes 3$        $6 \otimes \bar{6}$  [of  $SU_c(3)$ ]



{lower}  $\oplus$  {higher}

(without \*)      (with \*)

Mass difference  $> 500$  MeV ?

Bag potential  $\oplus$  spin-spin force:

$$H_g \sim -\frac{\alpha_c}{R} \sum_{a=1}^8 \sum_{i>j} \vec{\sigma}_i \cdot \vec{\sigma}_j \lambda_i^a \lambda_j^a M\left(\frac{n_s m_s}{N}, \frac{n_s m_s}{N}\right)$$

○ Ideally mixed  $9$  and  $9^*$  of scalar  $[qq][\bar{q}\bar{q}]$

$S$	$I = 1$	$I = \frac{1}{2}$	$I = 0$	Mass (GeV)	
				$9$	$9^*$
1		$\hat{\kappa}, \hat{\kappa}^*$		0.90	1.60
0			$\hat{\sigma}, \hat{\sigma}^*$	0.65	1.45
0	$\hat{\delta}^s, \hat{\delta}^{s*}$		$\hat{\sigma}^s, \hat{\sigma}^{s*}$	1.10	1.80

○ Ideally mixed 36 [and 36\*] of scalar  $(qq)(\bar{q}\bar{q})$

$S$	$I$					Mass (GeV) 36 [36*]
	2	$\frac{3}{2}$	1	$\frac{1}{2}$	0	
2			$E_{KK}$ [ $E_{KK}^*$ ]			1.55 [2.10]
1		$E_{\pi K}$ [ $E_{\pi K}^*$ ]		$C_K$ [ $C_K^*$ ] $C_K^s$ [ $C_K^{s*}$ ]		1.35 [1.95] 1.75 [2.20]
0	$E_{\pi\pi}$ [ $E_{\pi\pi}^*$ ]		$C_\pi$ [ $C_\pi^*$ ] $C_\pi^s$ [ $C_\pi^{s*}$ ]		$C$ [ $C^*$ ] $C^s$ [ $C^{s*}$ ] $C^{ss}$ [ $C^{ss*}$ ]	1.15 [1.80] 1.55 [2.10] 1.95 [2.35]



- Ideally mixed  $\bar{3} \oplus 6$  [and  $\bar{3}^* \oplus 6^*$ ]  
of charmed scalar  $[cq][\bar{q}\bar{q}]$  mesons

$S$	$I = 1$	$I = \frac{1}{2}$	$I = 0$	Mass(‡) (GeV)
1	$\hat{F}_I$ [ $\hat{F}_I^*$ ]		$\hat{F}_0^+$ [ $\hat{F}_0^{*+}$ ]	2.32(†) [3.1]
0		$\hat{D}$ [ $\hat{D}^*$ ] $\hat{D}^s$ [ $\hat{D}^{s*}$ ]		2.22 [3.0] 2.42 [3.2]
-1			$\hat{E}^0$ [ $\hat{E}^{*0}$ ]	2.32 [3.1]

(†) Input data:  $D_{s0}^+(2.32) \Rightarrow \hat{F}_I^+(I_z = 0)$

(‡) Quark counting with

$$\Delta m_s = m_s - m_n \simeq 0.1 \text{ GeV}$$

$$\left[ \begin{array}{l} m_{\hat{K}^*} = 1.6 \text{ GeV as the input data,} \\ \Delta m_c = m_c - m_n \simeq 1.5 \text{ GeV} \end{array} \right]$$

#### §4. Isospin conserving decays

- Decay rate for  $A(\mathbf{p}) \rightarrow B(\mathbf{p}')\pi(\mathbf{q})$  :

$$\begin{aligned} \Gamma(A \rightarrow B\pi) &= \left( \frac{1}{2J_A + 1} \right) \frac{q_c}{8\pi m_A^2} \sum_{\text{spins}} |M(A \rightarrow B\pi)|^2. \end{aligned}$$

- Hard pion technique (with PCAC) in the IMF:

$$\begin{aligned} M(A \rightarrow B\pi) &\simeq \lim_{\substack{\mathbf{p} \rightarrow \infty \\ \mathbf{q} \rightarrow 0}} M(A \rightarrow B\pi) \\ &\simeq \left( \frac{m_A^2 - m_B^2}{f_\pi} \right) \langle B | A_{\bar{\pi}} | A \rangle. \end{aligned}$$

where  $A_\pi$  is the axial counterpart of isospin  $V_\pi$ .

$\langle B' | A_\pi | B \rangle$  is given by

$$\begin{aligned} &\langle B'(\mathbf{p}') | A_\pi | B(\mathbf{p}) \rangle \\ &= (2\pi)^3 \delta^{(3)}(\mathbf{p} - \mathbf{p}') \langle B' | A_\pi | B \rangle \sqrt{N_{B'} N_B} \end{aligned}$$

in the IMF.

- Asymptotic flavor symmetry

Flavor symmetry of *asymptotic matrix elements*  
(matrix elements taken between single hadron states with infinite momentum)

- Its breaking:

Deviations of values of  $f_+(0)$  from unity

–  $SU_f(3)$ :

$$f_+^{(\pi^- K^0)}(0) = 0.961 \pm 0.008,$$

H. Leutwyler and M. Roos,  
Z.Phys.C **25**, 91 (1984)

–  $SU_f(4)$  :

$$f_+^{(\bar{K}D)}(0) = 0.74 \pm 0.03,$$

[PDG96, PRD54, 1(1996)],

$$\left| \frac{f_+^{(\pi D)}(0)}{f_+^{(\bar{K}D)}(0)} \right| = 1.00 \pm 0.13,$$

[E687, PLB382, 312(1996)],

$$= 0.99 \pm 0.08,$$

[CLEO, PLB405, 373(1997)]



– Example:

Input data;

$$* \Gamma(\rho \rightarrow \pi\pi) \simeq 149 \text{ MeV}, \quad (\text{PDG03})$$

$$* \Gamma_{D^{*+}} = (96 \pm 4 \pm 22) \text{ keV}, \quad (\text{CLEO})$$

$$* \langle \pi^- | A_{\pi^-} | \rho^0 \rangle = 2\sqrt{2} \langle D^+ | A_{\pi^0} | D^{*+} \rangle \\ = -\sqrt{2} \langle D^0 | A_{\pi^-} | D^{*+} \rangle$$

$$\Rightarrow \begin{cases} \Gamma(D^{*+} \rightarrow D^0 \pi^+)_{as} \simeq 96 \text{ keV} \\ \Gamma(D^{*+} \rightarrow D^+ \pi^0)_{as} \simeq 42 \text{ keV} \end{cases}$$

$$c.f. \begin{cases} \Gamma(D^{*+} \rightarrow D^0 \pi^+)_{exp} = 65 \pm 18 \text{ keV} \\ \Gamma(D^{*+} \rightarrow D^+ \pi^0)_{exp} = 30 \pm 8 \text{ keV} \end{cases}$$

from (PDG03)

$$Br(D^{*+} \rightarrow D^0 \pi^+)_{exp} = 67.7 \pm 0.5 \%$$

$$Br(D^{*+} \rightarrow D^+ \pi^0)_{exp} = 30.7 \pm 0.5 \%$$

and  $\Gamma_{D^{*+}}$  by CLEO

$$\Rightarrow \sqrt{\frac{\Gamma(D^{*+} \rightarrow D^0 \pi^+)_{as}}{\Gamma(D^{*+} \rightarrow D^0 \pi^+)_{exp}}} \simeq 1.2$$

↓

$$\left| \frac{\langle D^0 | A_{\pi^-} | D^{*+} \rangle_{as}}{\langle D^0 | A_{\pi^-} | D^{*+} \rangle_{ph}} \right| \simeq 1.2$$



- Parametrization of the asymptotic matrix elements of  $A_\pi$  :

$$\begin{aligned}
 \langle D_s^+ | A_{\pi^-} | \hat{F}_I^{++} \rangle &= \sqrt{2} \langle D_s^+ | A_{\pi^0} | \hat{F}_I^+ \rangle \\
 &= \langle D_s^+ | A_{\pi^+} | \hat{F}_I^0 \rangle = -\langle D^0 | A_{\pi^-} | \hat{D}^+ \rangle \\
 &= 2 \langle D^+ | A_{\pi^0} | \hat{D}^+ \rangle = -2 \langle D^0 | A_{\pi^0} | \hat{D}^0 \rangle \\
 &= -\langle D^+ | A_{\pi^+} | \hat{D}^0 \rangle.
 \end{aligned}$$

- Input data :

$$\Gamma(\hat{F}_I^+) \simeq \Gamma(\hat{F}_I^+ \rightarrow D_s^+ \pi^0) \sim 10 \text{ MeV}$$

(as an example)

– Why so narrow ?

- \* Small probability to find colorless " $D_s^+$ " and " $\pi^0$ " in the  $\hat{F}_I^+$

↑

( Crossing matrices for color and spin )  
 R. L. Jaffe, P.R.D **15**, 281 (1977).

TABLE IV. Crossing matrix for color.

	$ (Q\bar{Q})^1(Q\bar{Q})^1\rangle^1$	$ (Q\bar{Q})^8(Q\bar{Q})^8\rangle^1$
$ (Q^2)^6(\bar{Q}^2)^6\rangle^1$	$\begin{bmatrix} (\frac{2}{3})^{1/2} \\ (\frac{1}{3})^{1/2} \end{bmatrix}$	$\begin{bmatrix} -(\frac{1}{3})^{1/2} \\ +(\frac{2}{3})^{1/2} \end{bmatrix}$
$ (Q^2)^3(\bar{Q}^2)^3\rangle^1$		

TABLE V. Crossing matrices for spin.

	$ (Q\bar{Q})^3(Q\bar{Q})^3\rangle^1$	$ (Q\bar{Q})^1(Q\bar{Q})^1\rangle^1$	
$ (Q^2)^3(\bar{Q}^2)^3\rangle^1$	$\begin{bmatrix} (\frac{1}{4})^{1/2} \\ (\frac{3}{4})^{1/2} \end{bmatrix}$	$\begin{bmatrix} (\frac{3}{4})^{1/2} \\ -(\frac{1}{4})^{1/2} \end{bmatrix}$	
$ (Q^2)^1(\bar{Q}^2)^1\rangle^1$			
	$ (Q\bar{Q})^3(Q\bar{Q})^3\rangle^3$	$ (Q\bar{Q})^3(Q\bar{Q})^1\rangle^3$	$ (Q\bar{Q})^1(Q\bar{Q})^3\rangle^3$
$ (Q^2)^3(\bar{Q}^2)^3\rangle^3$	$\begin{bmatrix} 0 \\ (\frac{1}{2})^{1/2} \\ (\frac{1}{2})^{1/2} \end{bmatrix}$	$\begin{bmatrix} -(\frac{1}{2})^{1/2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$	$\begin{bmatrix} -(\frac{1}{2})^{1/2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$
$ (Q^2)^3(\bar{Q}^2)^1\rangle^3$			
$ (Q^2)^1(\bar{Q}^2)^3\rangle^3$			
		$ (Q\bar{Q})^3(Q\bar{Q})^3\rangle^5$	
$ (Q^2)^3(\bar{Q}^2)^3\rangle^5$		(1)	

R. L. Jaffe, P.R.D 15, 281 (1977).

Dominant decays of scalar  $[cq][\bar{q}\bar{q}]$  mesons and their estimated widths.

$\Gamma(\hat{F}_I^+) \simeq \Gamma(\hat{F}_I^+ \rightarrow D_s^+ \pi^0) \sim 10 \text{ MeV}$  is used as the input data.

Parent (Mass in GeV)	Final State	Width (MeV)
$\hat{F}_I^{++}(2.32)$	$D_s^+ \pi^+$	
$\hat{F}_I^+(2.32)$	$D_s^+ \pi^0$	$\sim 10$
$\hat{F}_I^0(2.32)$	$D_s^+ \pi^-$	
$\hat{D}^+(2.22)$	$D^0 \pi^+$	$\sim 10$
	$D^+ \pi^0$	$\sim 5$
$\hat{D}^0(2.22)$	$D^+ \pi^-$	$\sim 10$
	$D^0 \pi^0$	$\sim 5$
$\hat{D}^s(2.42)$	$D\eta$	$(\ll 10)$
$\hat{F}_0^+(2.32)$	$D_s^+ \pi^0$	(isospin viol.)
$\hat{E}^0(2.32)$	$\langle D_s \bar{K} \rangle$	(weak int.)



## §5. Ordinary charmed scalar mesons

Ordinary charmed scalar mesons:

$$D_0^* \sim \{c\bar{n}\}, \quad D_{s0}^* \sim \{c\bar{s}\}$$

Compare with the  $K_0^*(1.43) \sim {}^3P_0 \{n\bar{s}\}$  :

$$m_{K_0^*} = 1412 \pm 6 \text{ MeV},$$

$$\Gamma_{K_0^*} = 294 \pm 23 \text{ MeV},$$

$$\text{Br}(K_0^* \rightarrow K\pi) = 93 \pm 10 \%$$

$$\Rightarrow |\langle K^+ | A_{\pi^+} | K_0^{*0} \rangle| \simeq 0.29.$$

Asymptotic  $SU_f(4)$  symmetry breaking:

Input data;

- $\Gamma(\rho \rightarrow \pi\pi) \simeq 149 \text{ MeV}$ , (PDG03)
- $\Gamma_{D^{*+}} = (96 \pm 4 \pm 22) \text{ keV}$ , (CLEO)

↓ Asymptotic  $SU_f(4)$  symmetry

$$\begin{cases} \Gamma(D^{*+} \rightarrow D^0\pi^+)_{as} \simeq 96 \text{ keV} \\ \Gamma(D^{*+} \rightarrow D^+\pi^0)_{as} \simeq 42 \text{ keV} \end{cases}$$

$$c.f. \begin{cases} \Gamma(D^{*+} \rightarrow D^0\pi^+)_{exp} = 65 \pm 18 \text{ keV} \\ \Gamma(D^{*+} \rightarrow D^+\pi^0)_{exp} = 30 \pm 8 \text{ keV} \end{cases}$$

$$\Rightarrow \sqrt{\frac{\Gamma(D^{*+} \rightarrow D^0\pi^+)_{as}}{\Gamma(D^{*+} \rightarrow D^+\pi^0)_{exp}}} \simeq 1.2$$



- $D_0^* \sim \{c\bar{n}\}$

- Mass:  $m_{D_0^*} \simeq 2.35 \text{ GeV}$   
 ( tentative but in the region  
 of the predicted values )

- Width:

Asymptotic  $SU_f(4)$  symmetry:

$$|\langle D^+ | A_{\pi^+} | D_0^{*0} \rangle| = |\langle K^+ | A_{\pi^+} | K_0^{*0} \rangle| (\times 0.8)$$

$$\simeq 0.29 (\times 0.8)$$

$$\Rightarrow \Gamma_{D_0^*} \sim 90 (\times 0.8^2) \text{ MeV}$$

- $D_{s0}^* \sim \{c\bar{s}\}$

- Mass:  $m_{D_{s0}^*} \simeq 2.45 \text{ GeV}$   
 ( Quark counting with  $\Delta_s \simeq 0.1 \text{ GeV}$   
 and  $m_{D_0^*} \simeq 2.35 \text{ MeV}$  )

- Width ( $D_{s0}^{*+} \rightarrow DK$ ):

Asymptotic  $SU_f(4)$  symmetry:

$$|\langle D^0 | A_{K^-} | D_{s0}^{*0} \rangle| = |\langle D^+ | A_{\bar{K}^0} | D_{s0}^{*+} \rangle|$$

$$= |\langle K^+ | A_{\pi^+} | K_0^{*0} \rangle| (\times 0.8) \simeq 0.29 (\times 0.8)$$

$$\Rightarrow \Gamma_{D_{s0}^*} \simeq 70 (\times 0.8^2) \text{ MeV}$$

Mini-summary on the scalar mesons:

- Ordinary  $\{c\bar{q}\}$

$$K_0^*: m_{K_0^*} \simeq 1.43 \text{ GeV}, \Gamma_{K_0^*} \simeq 290 \text{ MeV (input)}$$

$$D_0^*: m_{D_0^*} \simeq 2.35 \text{ GeV}, \Gamma_{D_0^*} \simeq 90(\times 0.8^2) \text{ MeV}$$

$$D_{s0}^*: m_{D_{s0}^*} \simeq 2.45 \text{ GeV}, \Gamma_{D_{s0}^*} \simeq 70(\times 0.8^2) \text{ MeV}$$

- Salar  $[cq][\bar{q}\bar{q}]$ :

$$\hat{F}_I^+: m_{\hat{F}_I} \simeq 2.32 \text{ GeV}, \Gamma_{\hat{F}_I} \simeq 10 \text{ MeV (input)}$$

$$\hat{D}: m_{\hat{D}} \simeq 2.22 \text{ GeV}, \Gamma_{\hat{D}} \simeq 15 \text{ MeV}$$

$$\hat{D}^s: m_{\hat{D}^s} \simeq 2.42 \text{ GeV}, \Gamma_{\hat{D}^s} \ll 15 \text{ MeV}$$

$$\hat{F}_I: m_{\hat{F}_I} \simeq 2.32 \text{ GeV}, \Gamma_{\hat{F}_I} \simeq 10 \text{ MeV}$$

$$\hat{F}_0: m_{\hat{F}_0} \simeq 2.32 \text{ GeV}, \text{ (iso-spin viol.)}$$

$$\hat{E}^0: m_{\hat{E}^0} \simeq 2.32 \text{ GeV}, \text{ (weak decay)}$$

- Results by the BELLE Collaboration

BELLE Collaboration, hep-ex/0307021

– From  $D\pi$  mass distribution (Fig. 3),

$$\begin{aligned} m_{D_0^0} &= (2308 \pm 17 \pm 15 \pm 28) \text{ MeV}/c^2, \\ \Gamma_{D_0^0} &= (276 \pm 21 \pm 18 \pm 60) \text{ MeV}, \end{aligned}$$

$$\begin{aligned} m_{D_2^0} &= (2461.6 \pm 2.1 \pm 0.5 \pm 3.3) \text{ MeV}/c^2, \\ \Gamma_{D_2^0} &= (45.6 \pm 4.4 \pm 6.5 \pm 1.6) \text{ MeV}, \end{aligned}$$

(See also Table I.)

– From  $D^*\pi$  mass distribution (Fig. 8),

$$\begin{aligned} m_{D_1^0} &= (2421.4 \pm 1.5 \pm 0.4 \pm 0.8) \text{ MeV}/c^2, \\ \Gamma_{D_1^0} &= (23.7 \pm 2.7 \pm 0.2 \pm 4.0) \text{ MeV}, \\ m_{D_1^{\prime 0}} &= (2427 \pm 26 \pm 20 \pm 15) \text{ MeV}/c^2, \\ \Gamma_{D_1^{\prime 0}} &= (384_{-75}^{+107} \pm 24 \pm 70) \text{ MeV}, \end{aligned}$$



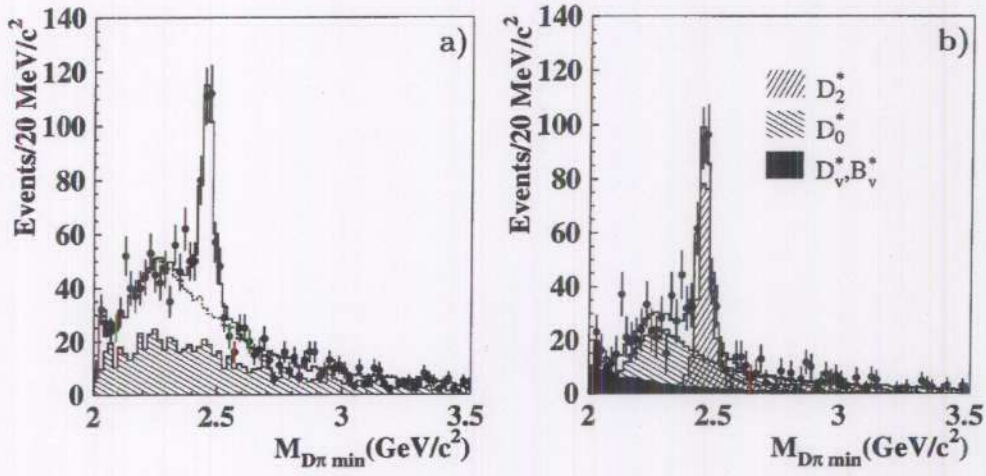


FIG. 3: a) The minimal  $D\pi$  mass distribution of  $B^- \rightarrow D^+\pi^-\pi^-$  candidates. The points with error bars correspond to the signal box events, while the hatched histogram shows the background obtained from the sidebands. The open histogram is the result of a fit while the dashed one shows the fit function in the case when the narrow resonance amplitude is set to zero. b) The background-subtracted  $D\pi$  mass distribution. The points with error bars correspond to the signal box events, hatched histograms show different contributions, the open histogram shows the coherent sum of all contributions.

Parameters	I $D_2^*, D_0^*$	II $D_2^*, D_0^*, D_v^*$	III $D_2^*, D_0^*, D_v^*, B_v^*$	IV $D_2^*, D_0^*, D_v^*, B_v^*,$ ph.sp( $a_3$ )
$Br_{D_2^*}(10^{-4})$	$3.21 \pm 0.24$	$3.26 \pm 0.26$	$3.38 \pm 0.31$	$3.47 \pm 0.37$
$Br_{D_0^*}(10^{-4})$	$6.09 \pm 0.42$	$4.96 \pm 0.47$	$6.12 \pm 0.57$	$8.35 \pm 0.94$
$\phi_{D_0^*}$	$-2.01 \pm 0.10$	$-2.35 \pm 0.11$	$-2.37 \pm 0.11$	$-2.31 \pm 0.14$
$Br_{D_v^*}(10^{-4})$	-	$1.46 \pm 0.23$	$2.21 \pm 0.27$	$2.23 \pm 0.32$
$\phi_{D_v^*}$	-	$0.03 \pm 0.15$	$-0.25 \pm 0.15$	$-0.33 \pm 0.19$
$Br_{B_v^*}(10^{-4})$	-	-	$0.67 \pm 0.04$	$0.72 \pm 0.04$
$\phi_{B_v^*}$	-	-	$-0.27 \pm 0.28$	$-0.39 \pm 0.24$
$M_{D_2^{*0}}(MeV/c^2)$	$2454.6 \pm 2.1$	$2458.9 \pm 2.1$	$2461.6 \pm 2.1$	$2462.7 \pm 2.2$
$\Gamma_{D_2^{*0}}(MeV)$	$43.8 \pm 4.0$	$44.2 \pm 4.1$	$45.6 \pm 4.4$	$46.1 \pm 4.5$
$M_{D_0^{*0}}(MeV/c^2)$	$2268 \pm 18$	$2280 \pm 19$	$2308 \pm 17$	$2326 \pm 19$
$\Gamma_{D_0^{*0}}(MeV)$	$324 \pm 26$	$281 \pm 23$	$276 \pm 21$	$333 \pm 37$
$a_3 \times 10^5$	-	-	-	$0.38 \pm 0.65$
$\phi_3$	-	-	-	$-0.10 \pm 0.93$
$N_{sig}$	$1058 \pm 47$	$1007 \pm 44$	$1056 \pm 46$	$1068 \pm 47$
$-2 \ln \mathcal{L}/\mathcal{L}$	115	26	0	-7
$\chi^2/N$	253.9/129	185.2/127	166.5/125	158.5/123

TABLE I: Fit results for different models. The model used to obtain the results includes amplitudes for  $D_2^*$ ,  $D_0^*$ ,  $D_v^*$ ,  $B_v^*$  intermediate resonances. Adding the constant term (ph.sp( $a_3$ )) does not significantly improve the likelihood.



- $DK$  mass distribution :

CLEO Collaboration, P.R.L. 72, 1974 (1994).

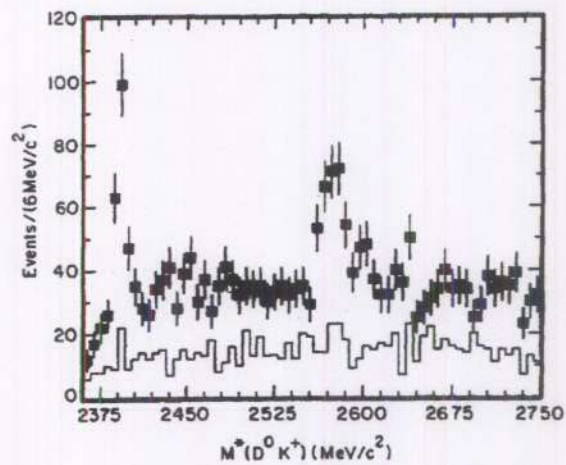
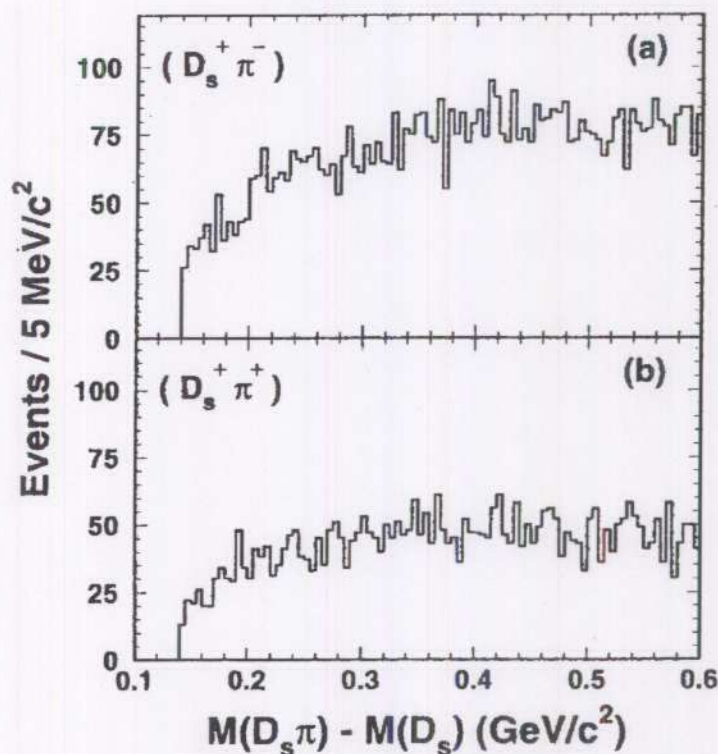


FIG. 1.  $M^*$ , "corrected" invariant mass, of  $(K^- \pi^+ [\pi^0]) K^+$  combinations. Data points are for  $K^- \pi^+ [\pi^0]$  combinations in the  $D^0$  signal region; the histogram shows  $M^*$  for  $(K^- \pi^+ [\pi^0]) K^+$  combinations where the  $K^- \pi^+ [\pi^0]$  combinations were chosen in  $D^0$  sidebands.

- Clear peak at  $\sim 2.57$  GeV  $(\Rightarrow D_{s2}^+)$
- A false peak at  $\sim 2.4$  GeV from  $D_{11}^*(2.54) \rightarrow D^*[\pi^0]K$
- A peak around  $\sim 2.45$  GeV from  $D_{s0}^* \rightarrow DK ?$

- Production rates of  $D_{s0}$  and  $D_{s1}$  :  
(Review by J. Wang, hep-ex/0312039)
  - Factorization **disfavors** the  $\{c\bar{q}\}$  but four-quark or molecule models are **consistent with experiments**.
  - No evidence for a peak in the  $D_s^+ \pi^\mp$   
(CLEO, hep-ph/0308166)

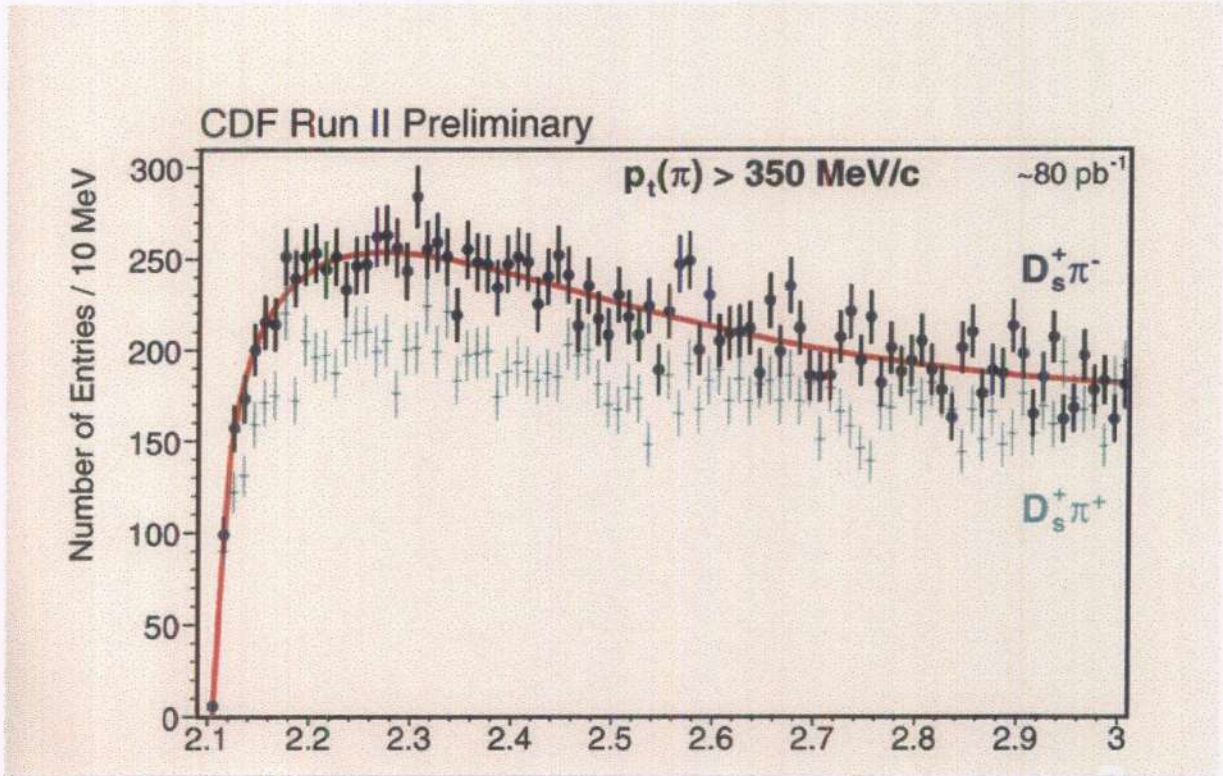
$\hat{F}_I^+$



- Spectra of the  $D_s\pi$  :

(Review by F. C. Porter, hep-ex/0312019)

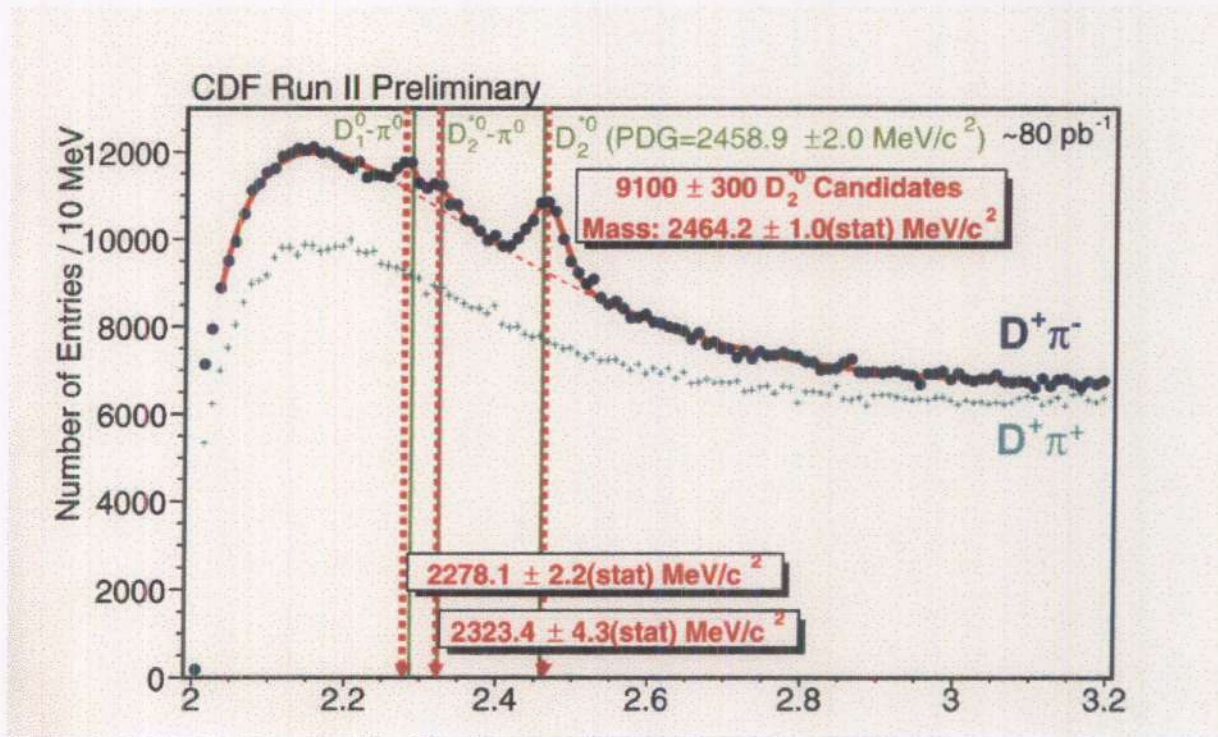
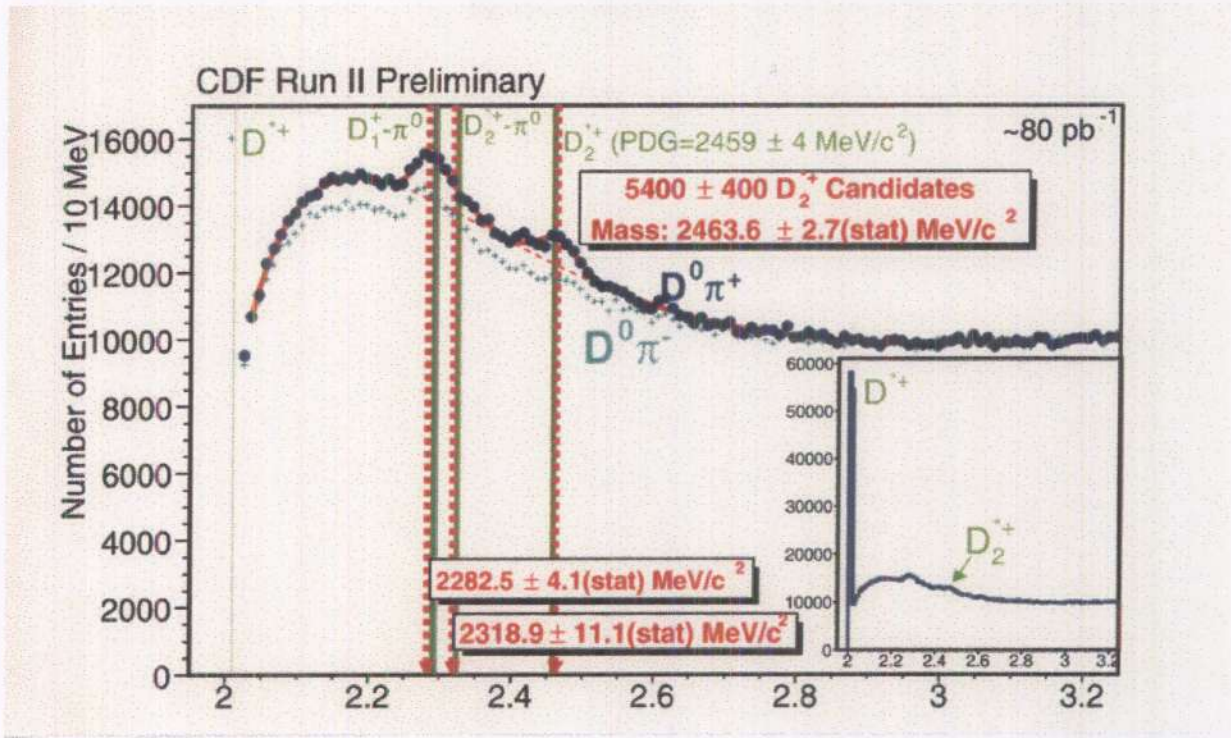
- No evidence for a peak in the  $D_s^\pm\pi^\pm$   
(CDF, M. Shapiro)



$\uparrow$   
 $\hat{F}_I$



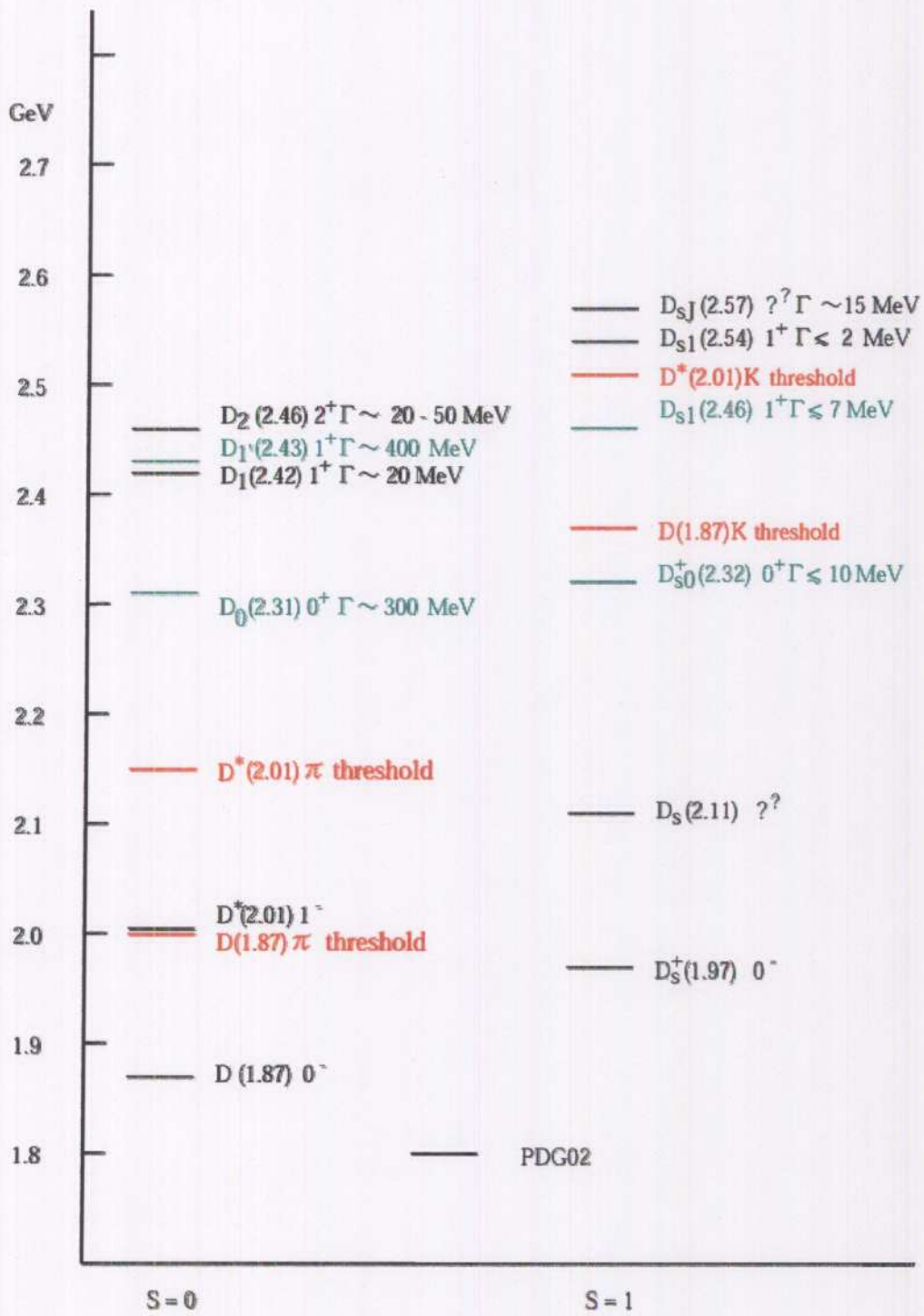
- Spectra of the  $D\pi$   
(CDF, M. Shapiro)

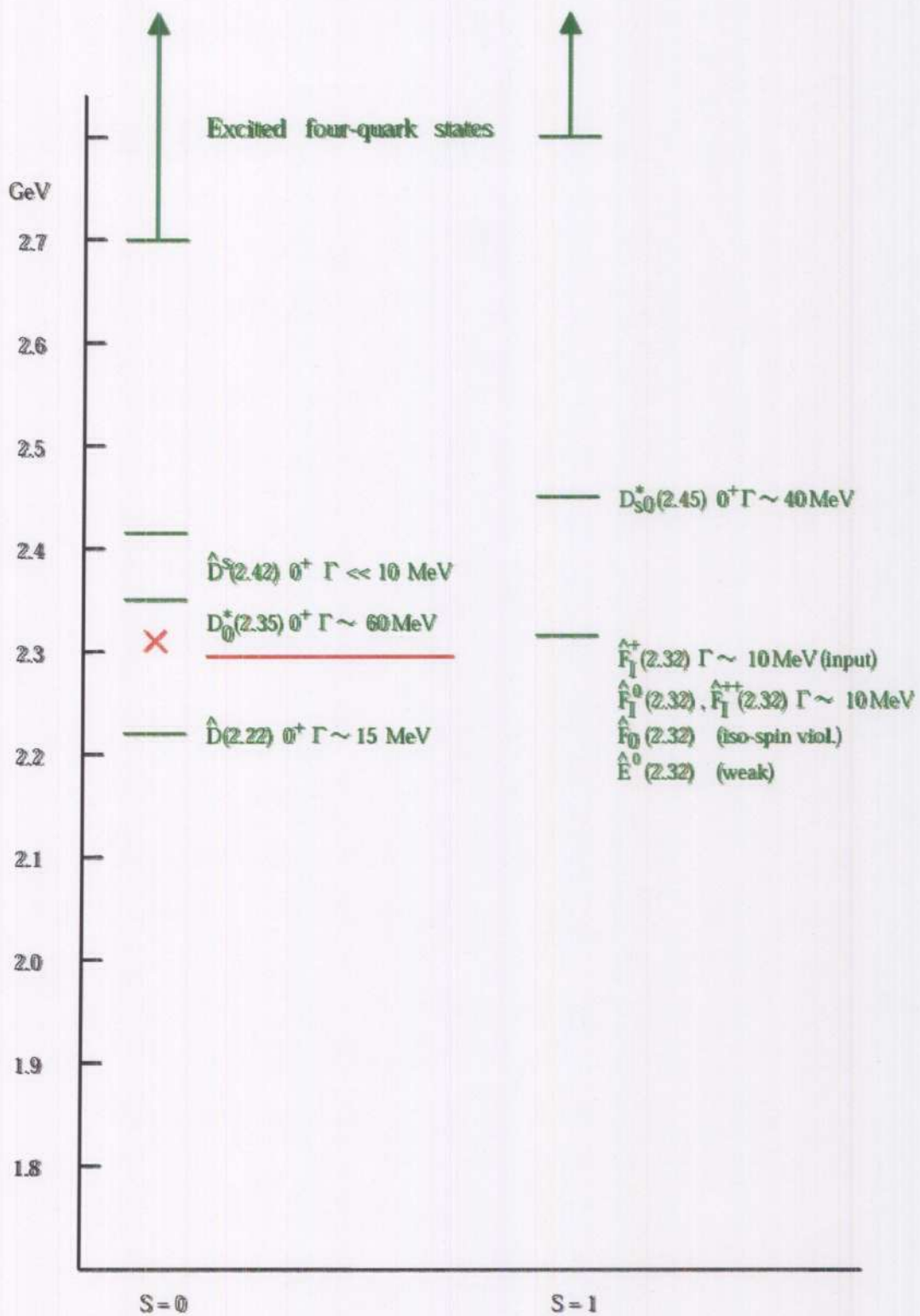


↑ ↑  
 $\hat{D}$   $D_0^*$



§6. Summary on **PART I**





- Charmed meson spectroscopy

$J^P$	$0^-$	$1^-$	$0^+$	$1^+$	$1'^+$	$2^+$
$\{c\bar{n}\}$	$D$	$D^*$	$D_0^*$	$D_1^*$	$D_1'^*$	$D_2^*$
$\{c\bar{s}\}$	$D_s$	$D_s^*$	$D_{s0}^*$	$D_{s1}^*$	$D_{s1}'^*$	$D_{s2}^*$
$[cn][\bar{u}\bar{d}]$			$\hat{D}$			
$[cn][\bar{s}\bar{n}]$			$\hat{F}_I, \hat{F}_0$			
$[cs][\bar{u}\bar{d}]$			$\hat{E}^0$			
$[cn][\bar{u}\bar{d}]$			$\hat{D}^s$			

- Unexpectedly small mass of  $D_{s0}^+(2.32)$ :  
 $\{m_{D_{s0}}\}_{\text{obs}} < \{m_{D_{s0}^*}\}_{\text{pot}}, \{m_{D_{s0}^*}\}_{\text{quench}}$
- Very small mass difference:  
 $\Delta m_s \gg \{m_{D_{s0}(2.32)}\}_{\text{obs}} - \{m_{D_0(2.31)}\}_{\text{obs}}$   
 $\sim 10 \text{ MeV},$   
 $\Downarrow$
- Scalar mesons of different structure ?
  - \*  $D_0^*(2.35) \oplus \hat{D}(2.22)$   
 in the broad bump around  $\sim 2.31 \text{ GeV}$ :
  - \*  $D_{s0}^*(2.45)$   
 in the  $DK$  invariant mass distribution  
 as the strange counterpart of  $D_0^*(2.35)$



- No evidence for a peak has been observed

in the  $\left\{ \begin{array}{l} D_s^+ \pi^\pm \text{ (CLEO)} \\ \text{and} \\ D_s^\pm \pi^\pm \text{ (CDF)} \end{array} \right\}$  mass distributions



$\{c\bar{s}\}$  or four-quark (or molecule) ?

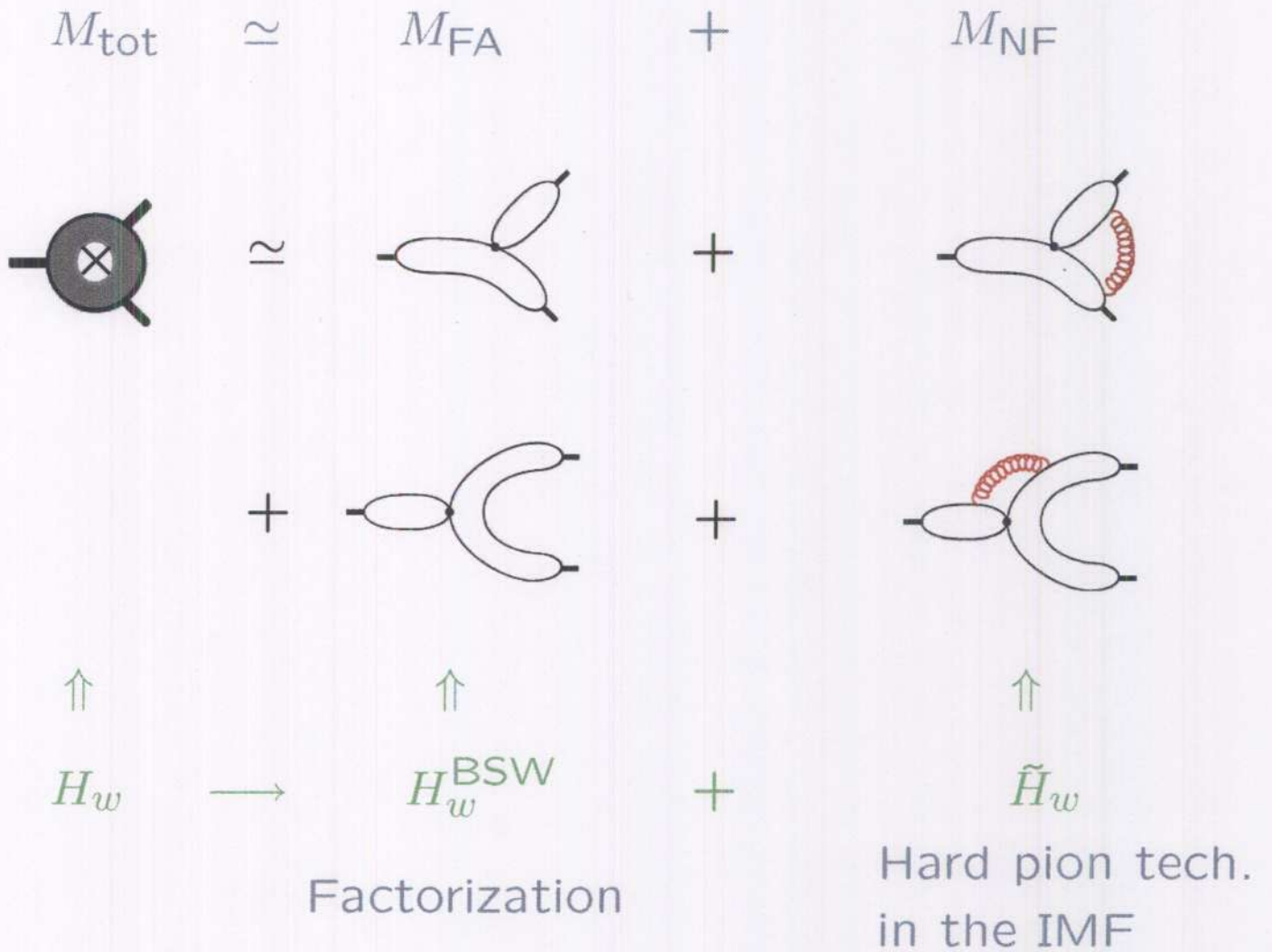
- Production mechanism  
Factorization disfavors the  $\{c\bar{s}\}$ .
- Broad  $\hat{F}_I$  ?  
(Why broad ? — crossing matrices)
- Experiments with higher statistics and resolution

More theoretical and experimental studies will be needed.

**PART II: Role of four-quark mesons  
in hadronic weak decays**

# §7. Introduction to Part II

— A hybrid perspective



- : weak vertex with hard gluon corrections
- : soft gluon(s)
- : weak interaction

Effective Hamiltonian (*Charm* decays):

$$H_w \simeq \frac{G_F}{\sqrt{2}} \{c_1 Q_1^{(s'c)} + c_2 Q_2^{(s'c)} + \dots\} + h.c.$$

$$Q_1^{(s'c)} =: (\bar{u}d')_L (\bar{s}'c)_L :$$

$$Q_2^{(s'c)} =: (\bar{s}'d')_L (\bar{u}c)_L :$$

$$(\bar{q}q)_L = \bar{q}\gamma_\mu(1 - \gamma_5)q$$

↓ Fierz identity (Bauer, Stech & Wirbel)

$$H_w^{\text{BSW}} \simeq \frac{G_F}{\sqrt{2}} \{a_1 Q_1^{(s'c)} + a_2 Q_2^{(s'c)} + \dots\} + h.c.$$

$$a_1 = c_1 + \frac{c_2}{N_c} \gg a_2 = c_2 + \frac{c_1}{N_c}$$

+

$$\tilde{H}_w \simeq \frac{G_F}{\sqrt{2}} \{c_2 \tilde{Q}_1^{(s'c)} + c_1 \tilde{Q}_2^{(s'c)} + \dots\} + h.c.$$

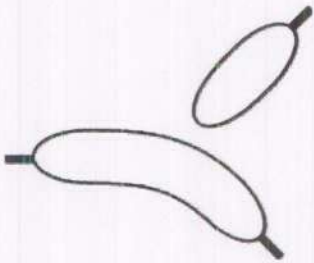
$$\tilde{Q}_1^{(s'c)} = 2 \sum_a : (\bar{u}t^a d')_L (\bar{s}'t^a c)_L :$$

$$\tilde{Q}_2^{(s'c)} = 2 \sum_a : (\bar{s}'t^a d')_L (\bar{u}t^a c)_L :$$

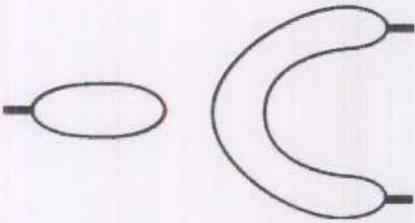


§8. Factorization

$$M_{FA} \sim \text{weak factor with hard corrections} \times \{$$



+



helicity suppression }

Factorizable amplitudes (charm decays) :

- Factorization: (Bauer, Stech & Wirbel)

– Two-body decays ( $D \rightarrow PP'$ ):

$$\begin{aligned} M_{\text{FA}}(D^+ \rightarrow \bar{K}^0 \pi^+) &= \frac{G_F}{\sqrt{2}} V_{cs} V_{ud} \left\{ a_1 \langle \pi^+ | (\bar{u}d)_L | 0 \rangle \langle \bar{K}^0 | (\bar{s}c)_L | D^+ \rangle \right. \\ &\quad \left. + a_2 \langle \bar{K}^0 | (\bar{s}d)_L | 0 \rangle \langle \pi^+ | (\bar{u}c)_L | D^+ \rangle \right\} \end{aligned}$$

Matrix elements of currents:

$$\langle \pi(q) | A_\mu | 0 \rangle = -i f_\pi q_\mu$$

$$\langle P'(p') | V_\mu | P(p) \rangle$$

$$= (p + p')_\mu f_+^{(P'P)}(q^2) + q_\mu f_-^{(P'P)}(q^2)$$

where  $q = p - p'$ .

Factorized amplitudes for two-body decays of charm mesons.

Decay	$A_{FA}$
$D^+ \rightarrow \bar{K}^0 \pi^+$	$i \frac{G_F}{\sqrt{2}} V_{cs} V_{ud} a_1 f_\pi (m_D^2 - m_K^2) f_+^{(\bar{K}D)}(m_\pi^2) \left[ 1 + \left( \frac{a_2}{a_1} \right) \left( \frac{f_K}{f_\pi} \right) \left( \frac{m_D^2 - m_\pi^2}{m_D^2 - m_K^2} \right) \frac{f_+^{(\pi D)}(m_K^2)}{f_+^{(\bar{K}D)}(m_\pi^2)} \right]$
$D^0 \rightarrow \bar{K}^0 \pi^0$	$-i \frac{G_F}{2} V_{cs} V_{ud} a_2 f_K (m_D^2 - m_\pi^2) f_+^{(\pi D)}(m_K^2)$
$D^0 \rightarrow K^- \pi^+$	$i \frac{G_F}{\sqrt{2}} V_{cs} V_{ud} a_1 f_\pi (m_B^2 - m_D^2) f_+^{(\bar{K}D)}(m_\pi^2)$
$D_s^+ \rightarrow K^+ \bar{K}^0$	$-i \frac{G_F}{\sqrt{2}} V_{cs} V_{ud} a_2 f_K (m_{D_s}^2 - m_K^2) f_+^{(KD_s)}(m_K^2)$
$D^0 \rightarrow \pi^+ \pi^-$	$-i \frac{G_F}{\sqrt{2}} V_{cs} V_{us} a_1 f_\pi (m_D^2 - m_\pi^2) f_+^{(\pi D)}(m_\pi^2)$
$D^0 \rightarrow \pi^0 \pi^0$	0
$D^+ \rightarrow \pi^+ \pi^0$	$-i \frac{G_F}{2} V_{cs} V_{us} a_1 f_\pi (m_D^2 - m_\pi^2) f_+^{(\pi D)}(m_\pi^2)$
$D^0 \rightarrow K^0 \bar{K}^0$	0
$D^0 \rightarrow K^+ K^-$	$-i \frac{G_F}{2} V_{cs} V_{us} a_1 f_K (m_D^2 - m_K^2) f_+^{(\bar{K}D)}(m_K^2)$
$D^+ \rightarrow K^+ \bar{K}^0$	$-i \frac{G_F}{2} V_{cs} V_{us} a_1 f_K (m_D^2 - m_K^2) f_+^{(\bar{K}D)}(m_K^2)$
$D_s^+ \rightarrow \pi^+ K^0$	$-i \frac{G_F}{\sqrt{2}} V_{cs} V_{us} a_1 f_\pi (m_{D_s}^2 - m_K^2) f_+^{(KD_s)}(m_\pi^2)$
$D_s^+ \rightarrow \pi^0 K^+$	$-i \frac{G_F}{2} V_{cs} V_{us} a_2 f_\pi (m_{D_s}^2 - m_K^2) f_+^{(KD_s)}(m_\pi^2)$

### §9. Nonfactorizable amplitudes

$$\begin{aligned}
 M_{NF} \\
 \left\{ \text{Diagram 1} \right\}_{NF} &\approx \sum \left\{ \text{Diagram 2} \right. \\
 &\quad + \\
 &\quad \left. \text{Diagram 3} \right\} M_S^{(n)} \\
 &\quad + \\
 &\quad \left. \text{Diagram 4} \right\} M_S^{(l)} \\
 &\quad + \\
 &\quad \left. \text{Diagram 5} \right\} M_{ETC}
 \end{aligned}$$

V. S. Mathur and L. K. Pandit,  
 in *Advances in Particle Physics*,  
 edited by R. L. Cool and  
 R. E. Marshak (Interscience  
 Publishers, 1968), vol.2, p. 383.



- Important players in the non-factorizable amplitudes for two body decays of charm mesons

	ETC term ( $f_\pi \sim f_K$ )	Surface term			
		$[qq][\bar{q}\bar{q}]$	$(qq)(\bar{q}\bar{q})$	G.B.	Hyb
$D^+ \rightarrow \pi^+ \bar{K}^0$	$\sim (*) \cdot e^{i\delta_{3/2}}$	$\sim 0$	$E_{\pi\bar{K}}^*$	—	—
$D^0 \rightarrow \pi^+ K^-$	$\sim e^{i\delta_{1/2}}$	$\hat{\kappa}^*$	$E_{\pi\bar{K}}^*, C_{\bar{K}}^*$	—	$\kappa_H$
$D^0 \rightarrow \pi^0 \bar{K}^0$	$\sim \sqrt{\frac{1}{2}} e^{i\delta_{1/2}}$	$\hat{\delta}^{s*}$	$C_\pi^{s*}$	—	$\kappa_H$
$D_s^+ \rightarrow K^+ \bar{K}^0$	$-e^{i\delta_1}$	$\hat{\delta}^{s*}$	$C_\pi^{s*}$	—	$\delta_H$
$D^0 \rightarrow K^0 \bar{K}^0$	$\sim 0$	—	—	$S^*$	$\sigma_H, \sigma_H^s$
$D^0 \rightarrow K^+ K^-$	$\sim -e^{i\delta_0}$	$\hat{\sigma}^{s*}, \hat{\delta}^{s*}$	$C_\pi^{s*}, C_\pi^{s*}$	$S^*$	$\sigma_H^s$
$D^+ \rightarrow K^+ \bar{K}^0$	$\sim -e^{i\delta_1}$	$\hat{\delta}^{s*}$	$C_\pi^{s*}$	—	$\delta_H$
$D^+ \rightarrow \pi^+ \pi^0$	—	—	$E_{\pi\pi}^*$	—	—
$D^0 \rightarrow \pi^+ \pi^-$	$e^{i\delta_0}$	$\hat{\sigma}^*$	$E_{\pi\pi}^*, C^*$	$S^*$	$\sigma_H$
$D^0 \rightarrow \pi^0 \pi^0$	$\sqrt{\frac{1}{2}} e^{i\delta_0(\pi\pi)}$	$\hat{\sigma}^*$	$E_{\pi\pi}^*, C^*$	$S^*$	$\sigma_H$
$D_s^+ \rightarrow \pi^+ K^0$	$\sim e^{i\delta_{1/2}}$	$\hat{\kappa}^*$	$E_{\pi K}^*, C_K^*$	—	$\kappa_H$
$D_s^+ \rightarrow \pi^0 K^+$	$\sim \sqrt{\frac{1}{2}} e^{i\delta_{1/2}}$	$\hat{\kappa}^*$	$E_{\pi K}^*, C_K^*$	—	$\kappa_H$

$$(*) = (1 - \frac{f_\pi}{f_K})$$

§10. A possible solution

Taking the following values of parameters involved,

- Wilson coefficients:

$$a_1 = 0.825, \quad a_2 = -0.159$$

$$(a_1^{\text{BSW}} = 1.09, \quad a_2^{\text{BSW}} = -0.09)$$

- Form factors:

$$F^{(\bar{K}D)}(0) = 0.74 \pm 0.03, \quad (\text{PDG96})$$

$$\left| \frac{F^{(\pi D)}(0)}{F^{(\bar{K}D)}(0)} \right| = 1.00 \pm 0.13, \quad (\text{E687})$$

$$= 0.99 \pm 0.08. \quad (\text{CLEO})$$

- Asymptotic matrix element:

$$\langle \pi^+ | \tilde{H}_w | D_s^+ \rangle = 0.05501 \times 1.166 \times 10^{-5} \text{ (GeV}^2\text{)}$$

$$k_a^* = 0.0771, \quad k_s^* = -0.0217, \quad f_g = 0.0387,$$

$$k_H = -0.0144, \quad Z = 1.56$$

- Phases:

– Relative phase between factorized and non-factorizable amplitudes;

$$\delta = -19.9^\circ$$

– Strong phases;

$$\delta_0(\pi\pi) = \delta_0(K\bar{K}) = 57.0^\circ, \quad \delta_1(K\bar{K}) = 58.7^\circ,$$

$$\delta_{1/2}(\pi K) = 84.4^\circ, \quad \delta_{3/2}(\pi K) = -26.7^\circ$$

- Masses and widths of non- $(q\bar{q})$  mesons:

$$m_{\hat{\sigma}^*} = 1.514 \text{ GeV}, \quad m_{E_{\pi\pi}^*} = 2.164 \text{ GeV},$$

$$m_{\sigma_H} = 2.012 \text{ GeV};$$

$$\Delta m_s = 0.12 \text{ GeV}, \quad \Delta m_c = 1.3 \text{ GeV} :$$

$$\Gamma_{\square\square} = 0.198 \text{ GeV}, \quad \Gamma_{\square\square} = 0.256 \text{ GeV},$$

$$\Gamma_H = 0.0456 \text{ GeV}$$

- Glue-rich scalar meson –  $S^*$ :

$$m_{S^*} = 1.71 \text{ GeV}, \quad \Gamma_{S^*} = 0.125 \text{ GeV}$$



Branching ratios (%) for  $D \rightarrow PP$  decays

(PDG'03)

Decays	(1)	(2)	(3)	(4)	(5)	$\mathcal{B}_{\text{exp}}$ (PDG'03)
$D^+ \rightarrow \bar{K}^0 \pi^+$	3.27	1.06	1.06	2.72	2.72	$2.71 \pm 0.20$
$D^0 \rightarrow K^- \pi^+$	2.41	8.41	8.41	3.37	3.83	$3.83 \pm 0.09$
$D^0 \rightarrow \bar{K}^0 \pi^0$	0.00	3.85	3.85	2.21	2.31	$2.30 \pm 0.22$
$D_s^+ \rightarrow \bar{K}^0 K^+$	0.20	5.82	5.82	1.70	3.50	$3.6 \pm 1.1$
$D^0 \rightarrow \pi^- \pi^+$	0.15	0.63	0.42	0.21	0.14	$0.143 \pm 0.007$
$D^0 \rightarrow \pi^0 \pi^0$	0.00	0.19	0.11	0.07	0.09	$0.084 \pm 0.022$
$D^+ \rightarrow \pi^0 \pi^+$	0.12	0.12	0.12	0.23	0.23	$0.25 \pm 0.07$
$D^0 \rightarrow K^- K^+$	0.19	0.48	0.72	0.45	0.42	$0.412 \pm 0.014$
$D^0 \rightarrow \bar{K}^0 K^0$	0.00	0.00	0.03	0.03	0.04	$0.071 \pm 0.019$
$D^+ \rightarrow \bar{K}^0 K^+$	0.47	1.19	1.19	0.70	0.52	$0.57 \pm 0.06$
$D_s^+ \rightarrow \pi^+ K^0$	0.17	0.24	0.24	0.09	0.05	$< 0.8$
$D_s^+ \rightarrow \pi^0 K^+$	0.00	0.07	0.07	0.05	0.06	—

 (1)  $M_{\text{FA}}$ 

 (2)  $M_{\text{FA}} \oplus M_{\text{ETC}}$ 

 (3)  $M_{\text{FA}} \oplus M_{\text{ETC}} \oplus M_S^{(S^*)}$ 

 (4)  $M_{\text{FA}} \oplus M_{\text{ETC}} \oplus M_S^{(S^*)} \oplus M_S^{\{qq\bar{q}\bar{q}\}}$ 

 (5)  $M_{\text{FA}} \oplus M_{\text{ETC}} \oplus M_S^{(S^*)} \oplus M_S^{\{qq\bar{q}\bar{q}\}} \oplus M_S^{\{q\bar{q}g\}}$



## §11. Summary on the **Part II**

Hadronic two-body decays of charm mesons assuming that their amplitude is given by a sum of factorizable and non-factorizable ones.

- Color suppression and helicity suppression in the factorized amplitudes.
- Large contribution of multi-hadron intermediate states to decays into non-exotic final states
- Important role of four-quark mesons
  - Masses of four-quark mesons are somewhat higher than the ones predicted by Jaffe.
  - Important role of  $\sigma^{s*}$  in  $D^0 \rightarrow K^+K^-$  in a possible solution to the long standing puzzle

$$\frac{\Gamma(D^0 \rightarrow K^+K^-)}{\Gamma(D^0 \rightarrow K^+K^-)} \sim 3$$

- Hybrid scalar mesons can play a role in annihilation decays  
( $m_{\pi_H} \sim 2.01$  GeV and  $\Gamma_{\pi_H} \sim 50$  MeV).

"Hadronic weak interactions are intimately related to hadron spectroscopy."

It is awaited that existence of  
four-quark mesons is confirmed.

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