Lattice Study of Scalar Mesons and Exotic Hadrons

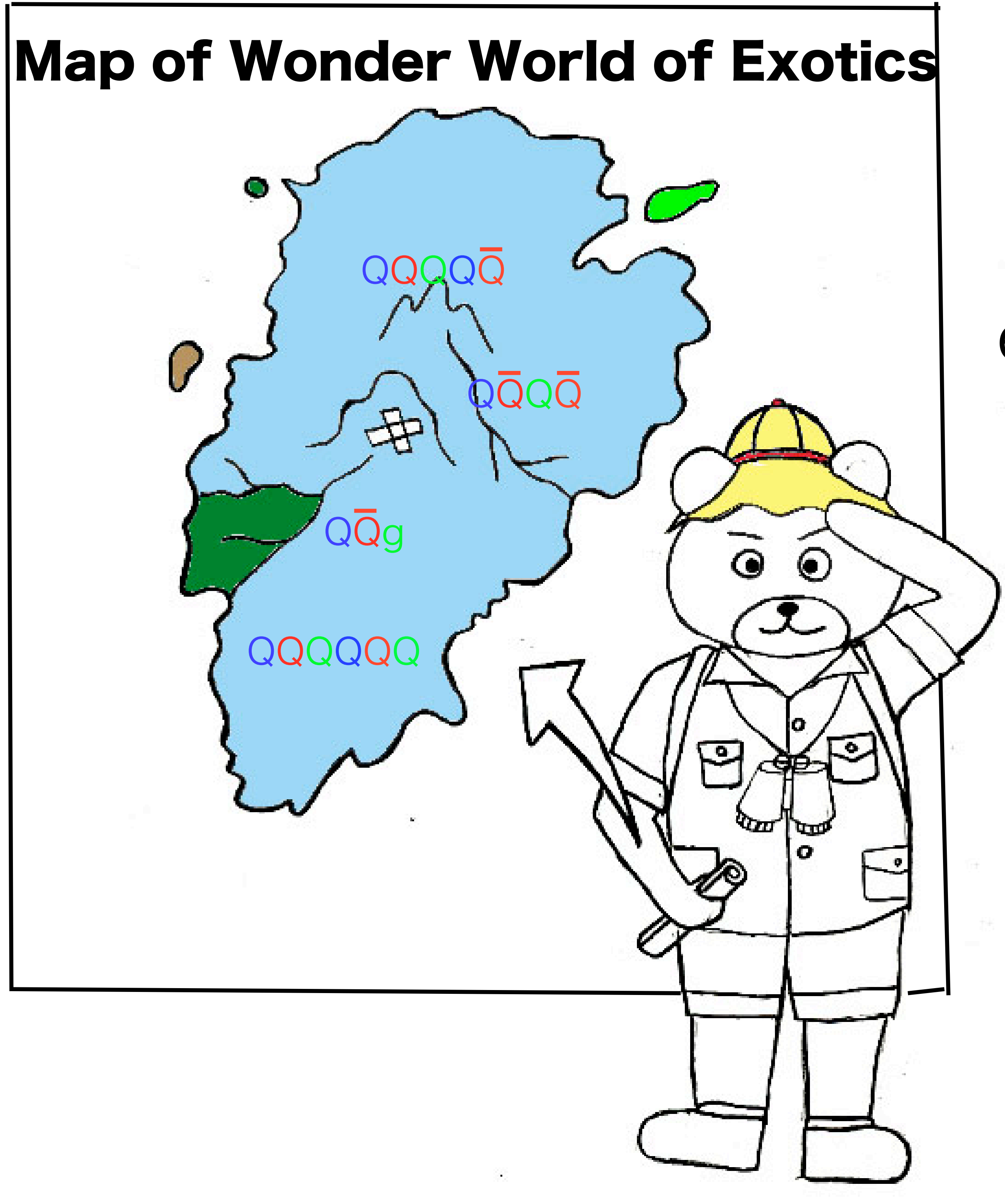
in collaboration with

Scalar Collaboration(Kunihiro, Muroya, A.N., Nonaka, Sekiguchi, Wada)

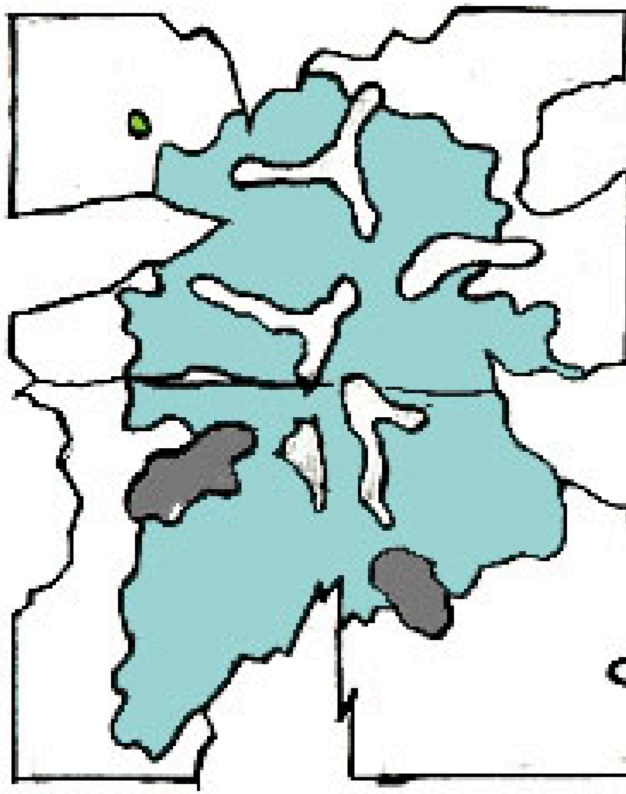
L^3 Collaboration (Nagata, A.N., Muroya)

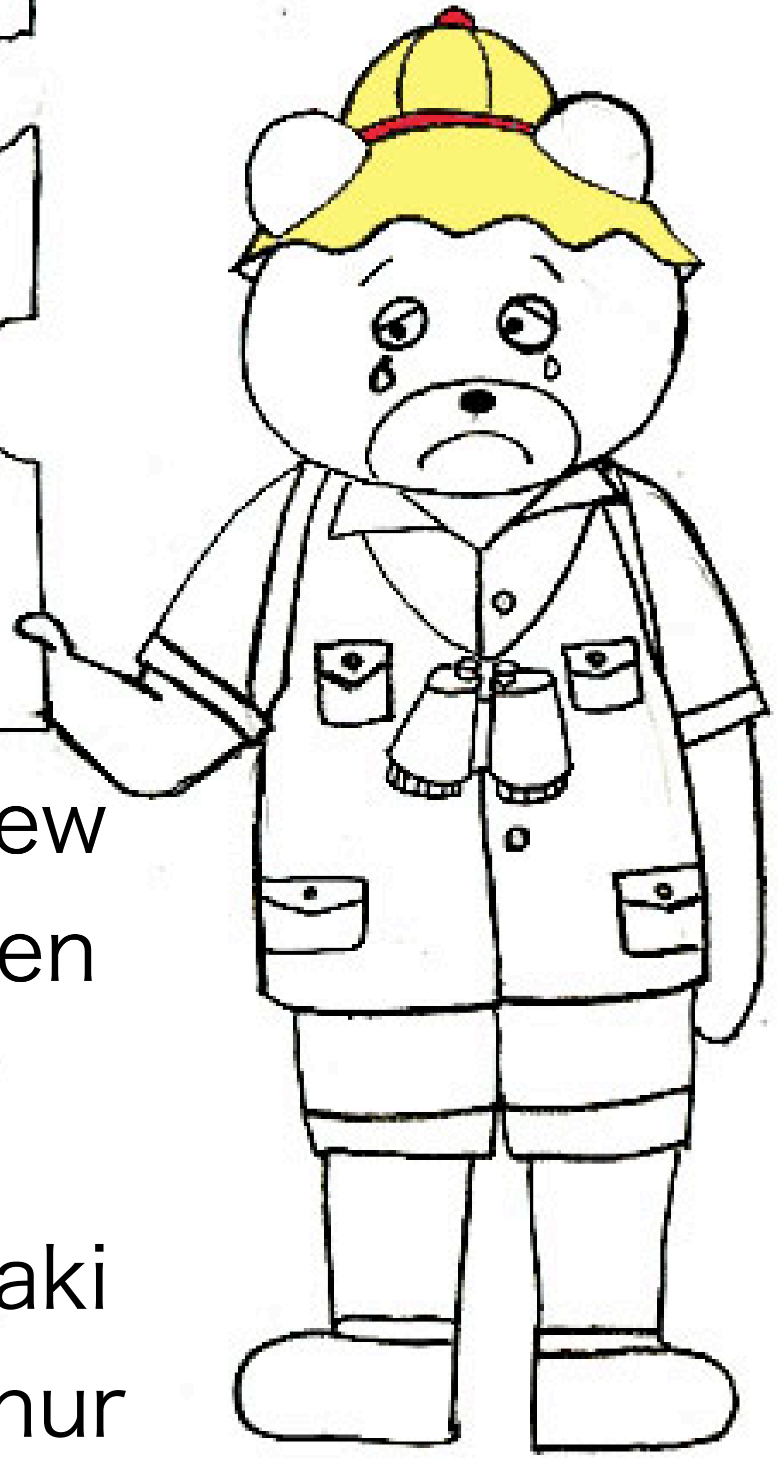
T.Saito and D.Zwanziger

International Symposium on
“Multi-quark Hadrons, four, five and more”
Feb.17-19, 2004, YITP, Kyoto Univ.



Yes, I will explore this wonderful world !





Sorry, I can visit only a few points, which may be even wrong place !

Please go to the talks by → Sasaki Mathur

Lattice QCD Calculation

- Relativistic Formulation
  - Quarks are described by Dirac Fermions
- Not a Model
- Apart from numerical limitations, there is no approximation.
- No bound state Calculation
  - No potential
  - No B-S
  - It measures the mass gap in a given channel.

Euclidean Path Integral

$$Z = \int dU d\bar{\psi} d\psi e^{-\bar{\psi} D \psi - S_G}$$

$$G(x,y) = \frac{1}{Z} \int dU d\bar{\psi} d\psi H(y) H^{\dagger}(x) e^{-\bar{\psi} D \psi - S_G}$$

$$\Rightarrow \quad e^{-m|x-y|}$$

$H(x)$: Hadron Operator. For σ meson $H(x) = \bar{\psi}(x)\psi(x)$

$H^{\dagger}(x)|0>$ State with Quantum Numbers specified by H

You should trust Lattice QCD !

because it is the First Principle Calculation!

You should not trust Lattice QCD !

until the following conditions are satisfied:

- **Enough Statistic**
  - Gauge configurations are generated by Monte Carlo, and there are statistical errors like Experiments.
- **Continuum Limit**
  - Lattice spacing
- **Infinite Volume Limit**
  - Lattice Volume is large enough to include hadron.
- **Chiral Extrapolation**
  - u and d quark masses on the lattice are large, and extrapolated to zero.

Let me start from Story of Scalar Particles in lattice QCD

SCALAR Collaboration
(Super Computer And LAttice Research)

- T. Kunihiro, YITP, Kyoto Univ.
- S. Muroya, Tokuyama Women's Coll.
- A. Nakamura, RIISE, Hiroshima Univ.
- C. Nonaka, Dept. Phys., Duke Univ.
- M. Sekiguchi, Fac. of Eng. Kokushikan Univ.
- H. Wada, Fac. of Sci. and Eng.. Nihon Univ.

◆ and SX5 at RCNP, SR8000 at KEK

Operator for σ Meson

- I=0, scalar

$$\sigma(x) \equiv \sum_{c=1}^{3} \bar{\psi}_c(x)\psi^c(x)$$

$$= \sum_{c=1}^{3}\sum_{\alpha=1}^{4} \frac{\bar{u}_\alpha^c(x)u_\alpha^c(x) + \bar{d}_\alpha^c(x)d_\alpha^c(x)}{\sqrt{2}}$$

c = 1,2,3 ・・・ color

α=1,2,3,4 ・・・ Dirac spin

$$\left[\ \psi \equiv \begin{pmatrix} u \\ d \end{pmatrix}\ \right]$$

Propagator

$$G(y,x) = \left\langle \sigma(y)\sigma(x)^{\dagger} \right\rangle$$

$$= \frac{1}{Z}\int DUD\bar{u}DuD\bar{d}Dd \sum_{a,b=1}^{3} \sum_{\alpha,\beta=1}^{4} \frac{\bar{u}_{\beta}^{b}(y)u_{\beta}^{b}(y) + \bar{d}_{\beta}^{b}(y)d_{\beta}^{b}(y)}{\sqrt{2}}$$

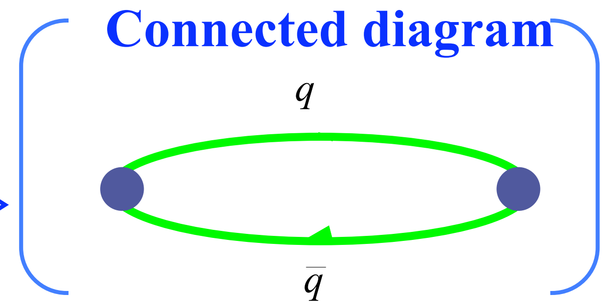
$$\left(\frac{\bar{u}_{\alpha}^{a}(x)u_{\alpha}^{a}(x) + \bar{d}_{\alpha}^{a}(x)d_{\alpha}^{a}(x)}{\sqrt{2}} \right)^{\dagger} e^{-S_g - \bar{u}Wu - \bar{d}Wd}$$

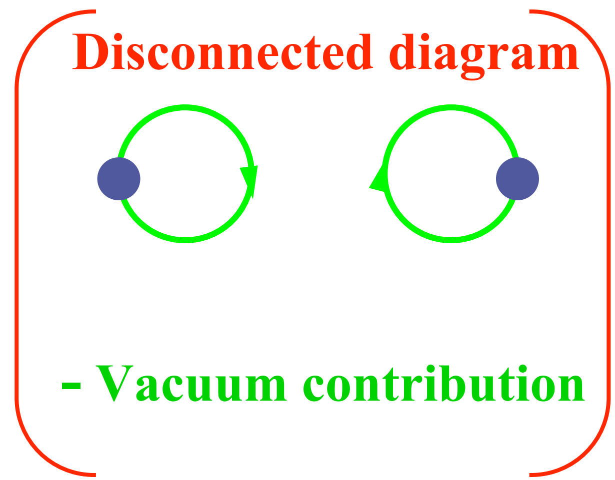
$$= \frac{1}{Z}\int DUD\bar{u}DuD\bar{d}Dd \sum_{a,b=1}^{3} \sum_{\alpha,\beta=1}^{4} \frac{1}{2}\Big[\bar{u}_{\beta}^{b}(y)u_{\beta}^{b}(y)\bar{u}_{\alpha}^{a}(x)u_{\alpha}^{a}(x)$$

$$+ \bar{d}_{\beta}^{b}(y)d_{\beta}^{b}(y)\bar{d}_{\alpha}^{a}(x)d_{\alpha}^{a}(x) + \bar{u}_{\beta}^{b}(y)u_{\beta}^{b}(y)\bar{d}_{\alpha}^{a}(x)d_{\alpha}^{a}(x)$$

$$+ \bar{d}_{\beta}^{b}(y)d_{\beta}^{b}(y)\bar{u}_{\alpha}^{a}(x)u_{\alpha}^{a}(x) \Big] e^{-S_g - \bar{u}Wu - \bar{d}Wd}$$

$$G(x,y) = -\left\langle Tr\left(W^{-1}(x,y) W^{-1}(y,x) \right) \right\rangle + 2\left\langle Tr\left(W^{-1}(y,y) \right) Tr\left(W^{-1}(x,x) \right) \right\rangle - 2\left\langle Tr\left(W^{-1}(y,y) \right) \right\rangle \left\langle Tr\left(W^{-1}(x,x) \right) \right\rangle$$





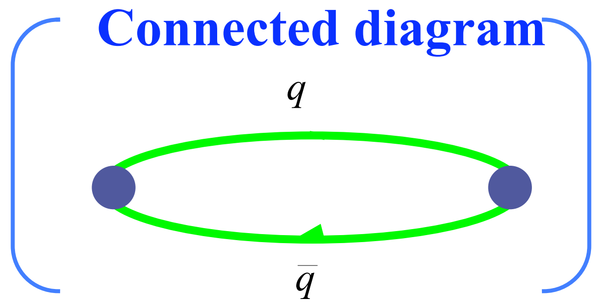
$W^{-1}(x,y)$: Inverse of Fermion Matrix, i.e., Quark Propagators

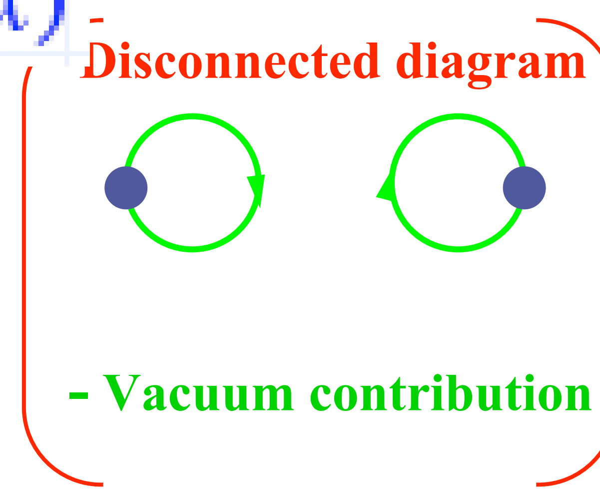
$$G(x,y) = - < Tr W^{-1}(x,y) W^{-1}(y,x) >$$

$$+2 < (\sigma(x) - < \sigma >)(\sigma(y) - < \sigma >) >$$

where

$$\sigma(x) \equiv Tr W^{-1}(x,x) = \overline{\psi}(x)\psi(x)$$





Lattice QCD simulations of σ

- There have been many Lattice Simulations of scalar without the disconnected diagram; "Valence Sigma"

- deTar and Kogut
  - Phy.Rev. D36, (1987) 2828.
  - Screening masses
- Kim and Ohta
  - hep-lat/9609023,hep-lat/9712014
  - KS fermions, β =6.5,
  - a=0.054fm, 48a=2.6fm,

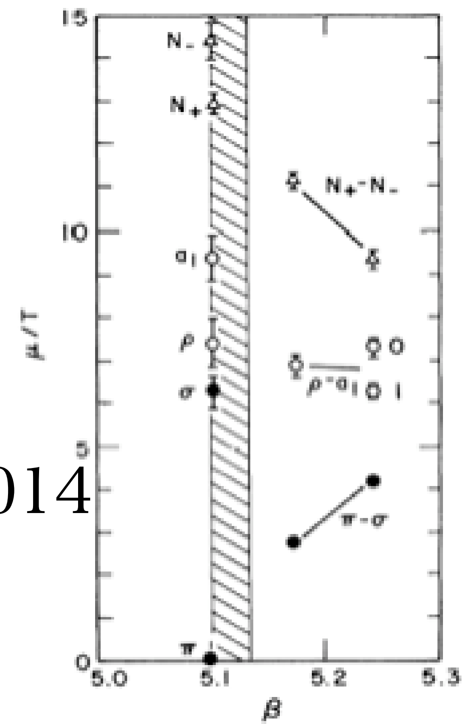
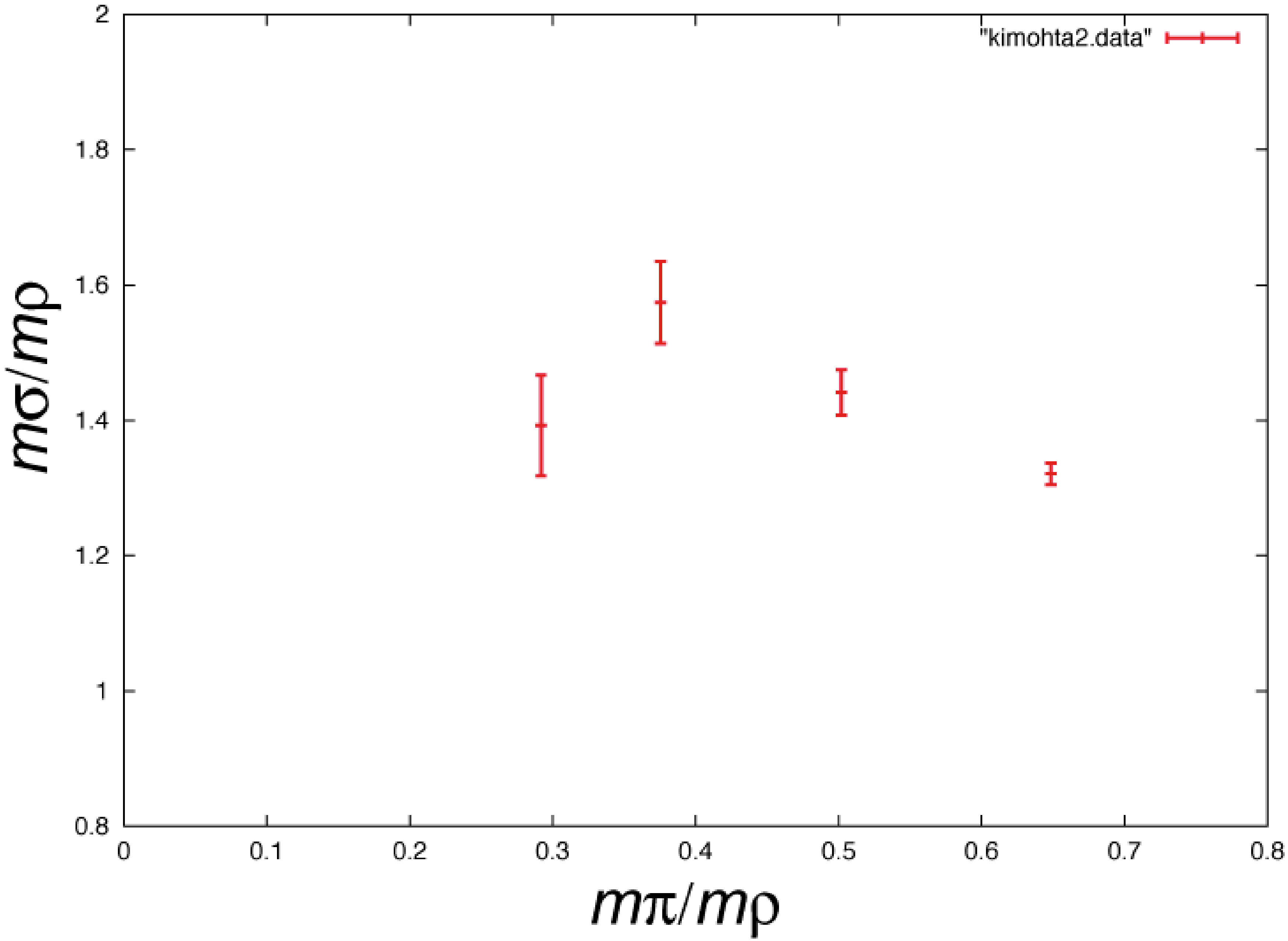


FIG. 10. Screening masses, expressed in units of the temper-ature, as extrapolated to the chiral limit, for the π-, σ-, ρ-, and a₁-meson plasmon modes, and the lowest even-parity (N_+) and



"kimohta2.data"
2
1.8
1.6
1.4
1.2
1
0.8
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
mσ/mρ
mπ/mρ

- W. Lee and D. Weingarten
  - Phys. Rev. **D61** (1999) 014015
  - Quench
  - Mixing of Glue-ball and
- UKQCD C.McNeile and C.Michael
  - Phys. Rev. **D63** (2001) 114503
  - Full QCD
- Alford and Jaffe, Nucl.Phys. B578 (2000)367.
  - Quench
  - $\sigma = qq\overline{qq}$, $E(q\bar{q}q\bar{q}) < E(q\bar{q} + q\bar{q})$

W. Lee and D. Weingarten, Phys. Rev. D61 (1999) 014015

Mixing of $\bar{q}q$ and glueball (I=0, $J^{PC}=0^{++}$)

Quenched approximation
Wilson fermion
Plaquette gauge action

f_0(1710) • • • lightest scalar glueball (73.8 (9.5)%)

f_0(1500) • • • $\bar{s}s$ quarkonium (98.4 (1.4)%)

f_0(1390) • • • $\bar{n}n$ quarkonium (main)

n stands for $u\bar{u} - d\bar{d} / \sqrt{2}$

Lee and Weingarten (cont'd)

$$\begin{pmatrix} m_g & E(\mu_s) & \sqrt{2}rE(\mu_s) \\ E(\mu_s) & m_\sigma(\mu_s) & 0 \\ \sqrt{2}rE(\mu_s) & 0 & m_\sigma(\mu_n) \end{pmatrix} \Rightarrow \begin{pmatrix} f_0(1710) & & \\ & f_0(1500) & \\ & & f_0(1390) \end{pmatrix}$$

Input:

f0(1710)	1697(4)MeV
f0(1500)	1505(9)MeV
f0(1390)	1404(24)MeV
$m_\sigma(\mu_n)$	1470(25)MeV
$r \equiv E(\mu_n)/E(\mu_s)$	1.198(72)

(Only r is given by Lattice.)

Output:

m_g	1622(29) MeV
$m_\sigma(\mu_s)$	1514(11) MeV
$E(\mu_s)$	64(13) MeV

Lattice:

m_g	1654(47) MeV (World Average)
$m_\sigma(\mu_s)$	1322(42) MeV
$E(\mu_s)$	43(31) MeV

C.McNeile and C.Michael (UKQCD), Phys. Rev. D63 (2001) 114503

Mixing of the Iso-singlet scalar (I=0, $J^{PC}=0^{++}$) and Glueball

Mass with Full QCD
<< mass with quench

$M\sigma < M\pi$

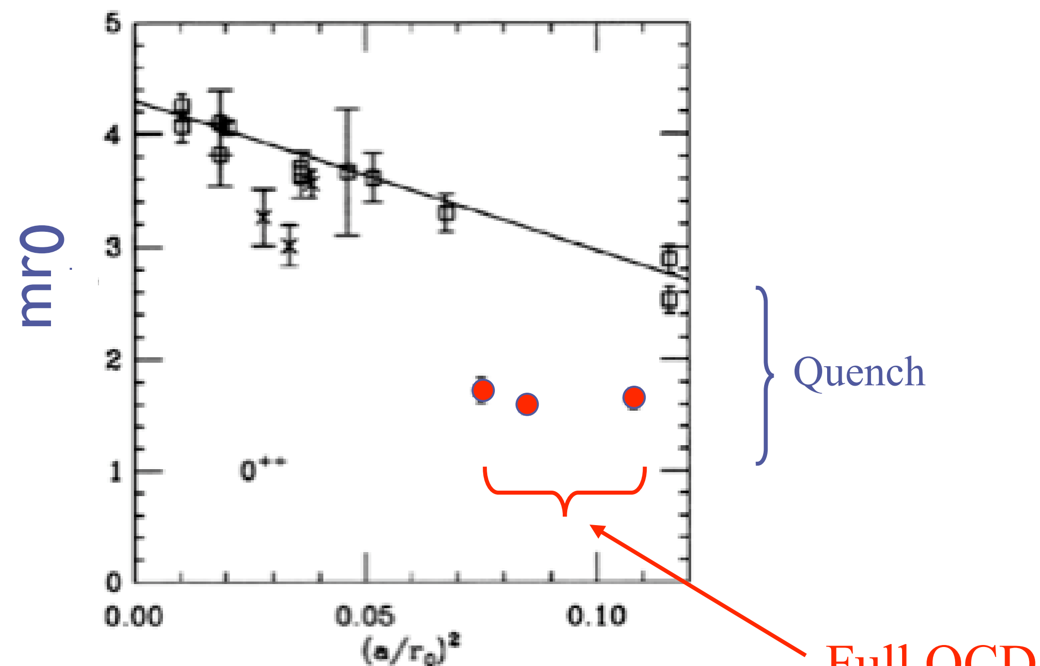


FIG. 5. The scalar mass versus a^2. The quenched results [12,10,13,14] are for the scalar glueball and are shown by boxes. The results from $N_f=2$ flavors of sea quark are from glueballs [15] (crosses from SESAM) and the lightest flavor singlet scalar we find here (circles).

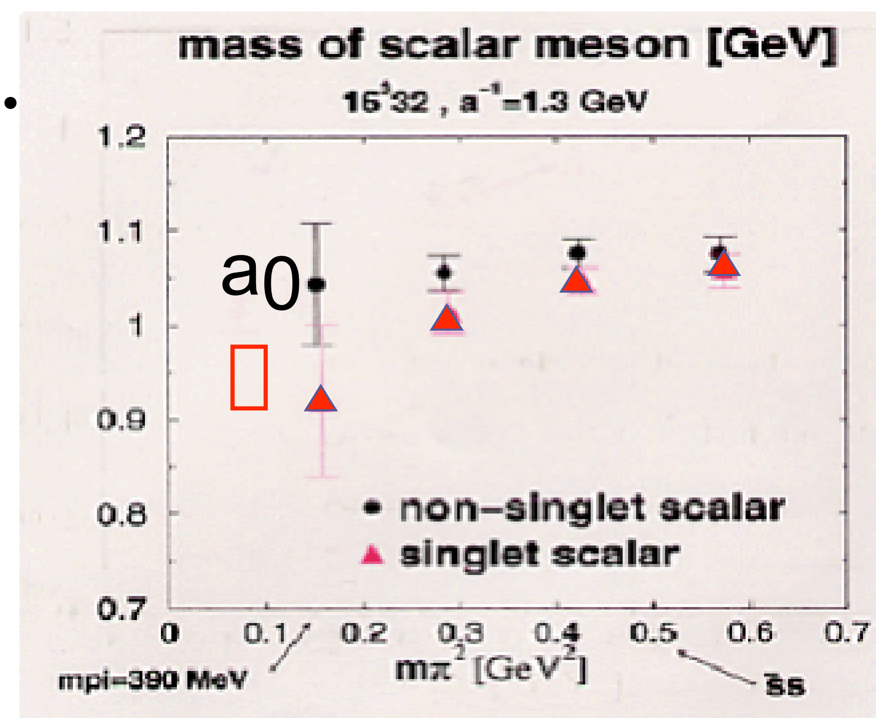
r_0: Sommer factor

Lattice QCD simulations of σ
- current going projects -

- **Riken-Columbia-Brookhaven**
  - Domain Wall Fermions
  - Quench
  - hep-lat/0209132 (Lattice02 Proceedings)
- **Scalar Collaboration**
  - Wilson Fermions
  - Full QCD
  - hep-ph/0310312

Riken-Brookhaven-Columbia

- Domain-wall fermions: Good Chiral nature
- Quench: Check the sickness of the quench calculations by quenched chiral perturbation theory.



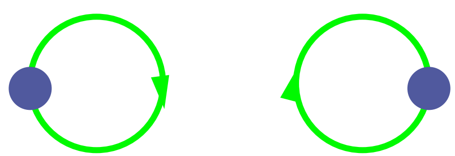
Details of our Calculation (1)

Wilson Fermions (2 flavors)

Plaquette Gauge Action

Full QCD Update by Hybrid Monte Carlo (SX5 at RCNP)

Disconnected Part by Z2 Noise Method (SR8000 at KEK)



Details of our Calculation (2)
- Simulation parameters

Lattice size : $8^3 \times 16$

$\beta = 4.8$

$\kappa = 0.1846,\ 0.1874,\ 0.1891$

(well established by CP-PACS,
a = 0.197(2) fm , κ_c = 0.19286(14)
(CP - PACS, Phys. Rev. D60(1999)114508))

Wilson Fermions & Plaquette gauge action

Number of the Z2 noise = 1000 , 500

Details of our Calculation

[κ = 0.1846]

1470 configurations from 720th trajectory

[κ = 0.1874]

970 configurations from 710th trajectory

[κ = 0.1891]

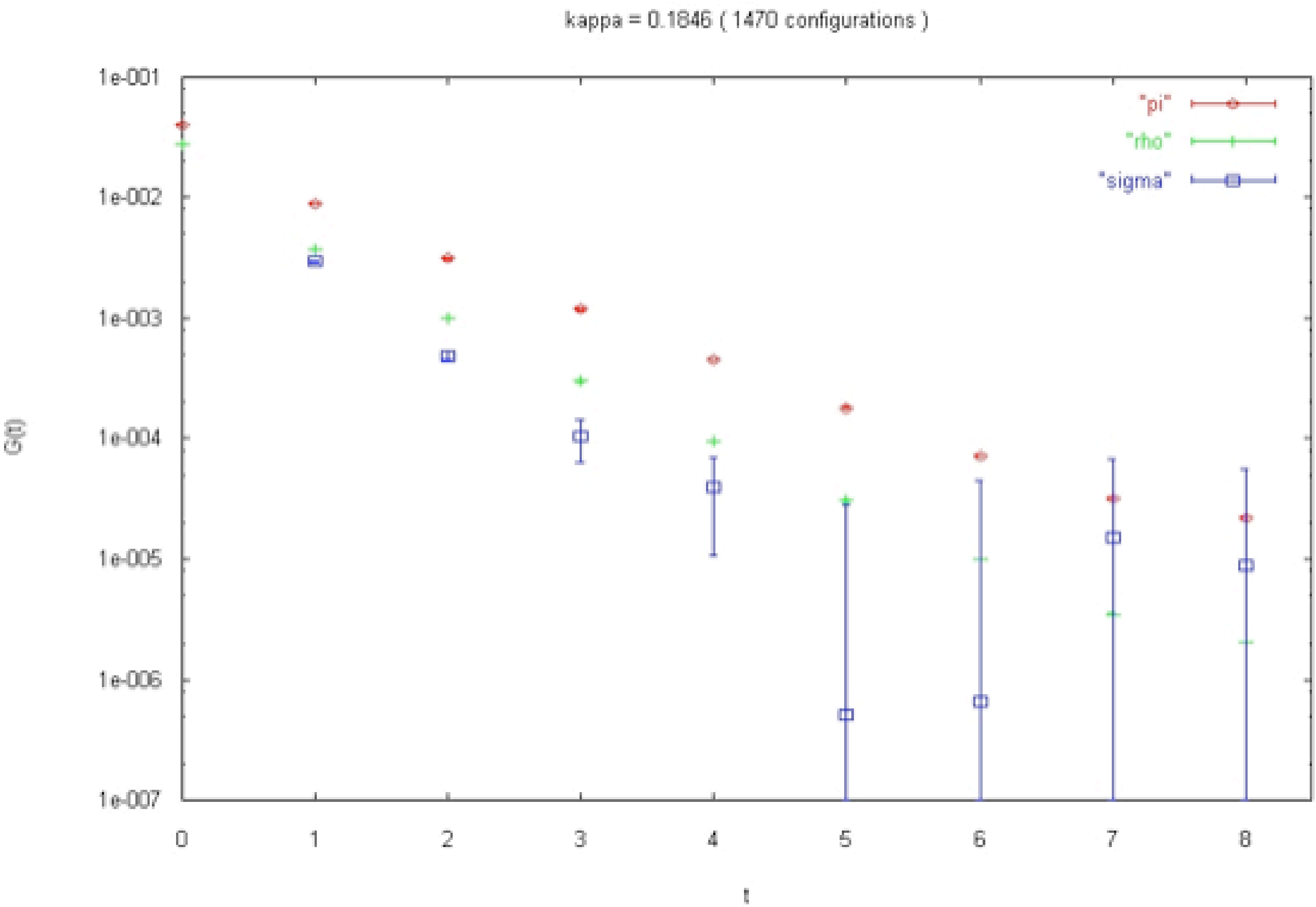
400 configurations from 500th trajectory

(Separation between configurations are 10 trajectories)

Details of our Calculation (5)

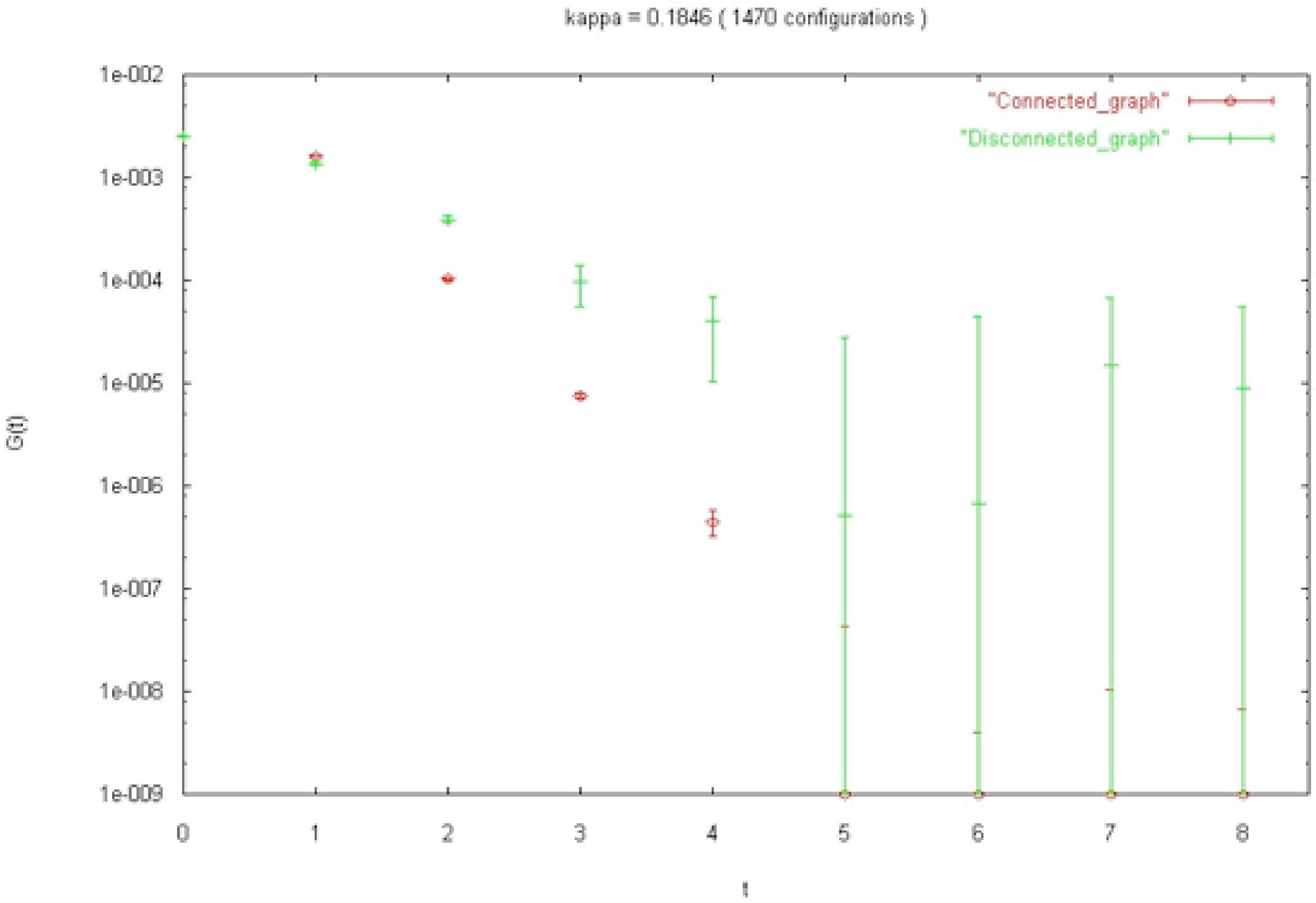
κ		m_π/m_ρ (Our Results)	m_π/m_ρ (CP-PACS)
0.1846		0.825±0.001	0.8291±0.0012
0.1874		0.760±0.002	0.7715±0.0017
0.1891		0.692±0.005	0.7026±0.0032

π , ρ, σ mesons (κ=0.1846)

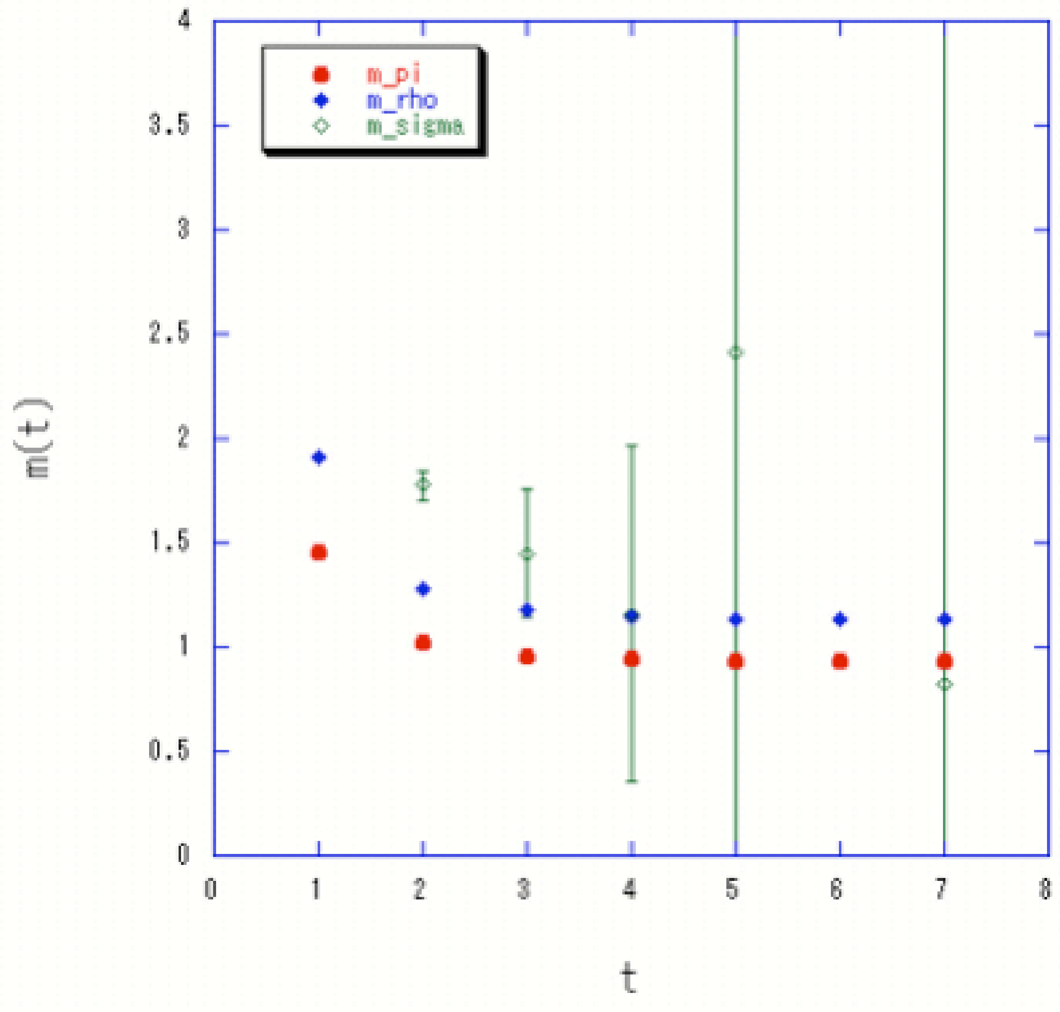


σ meson propagators

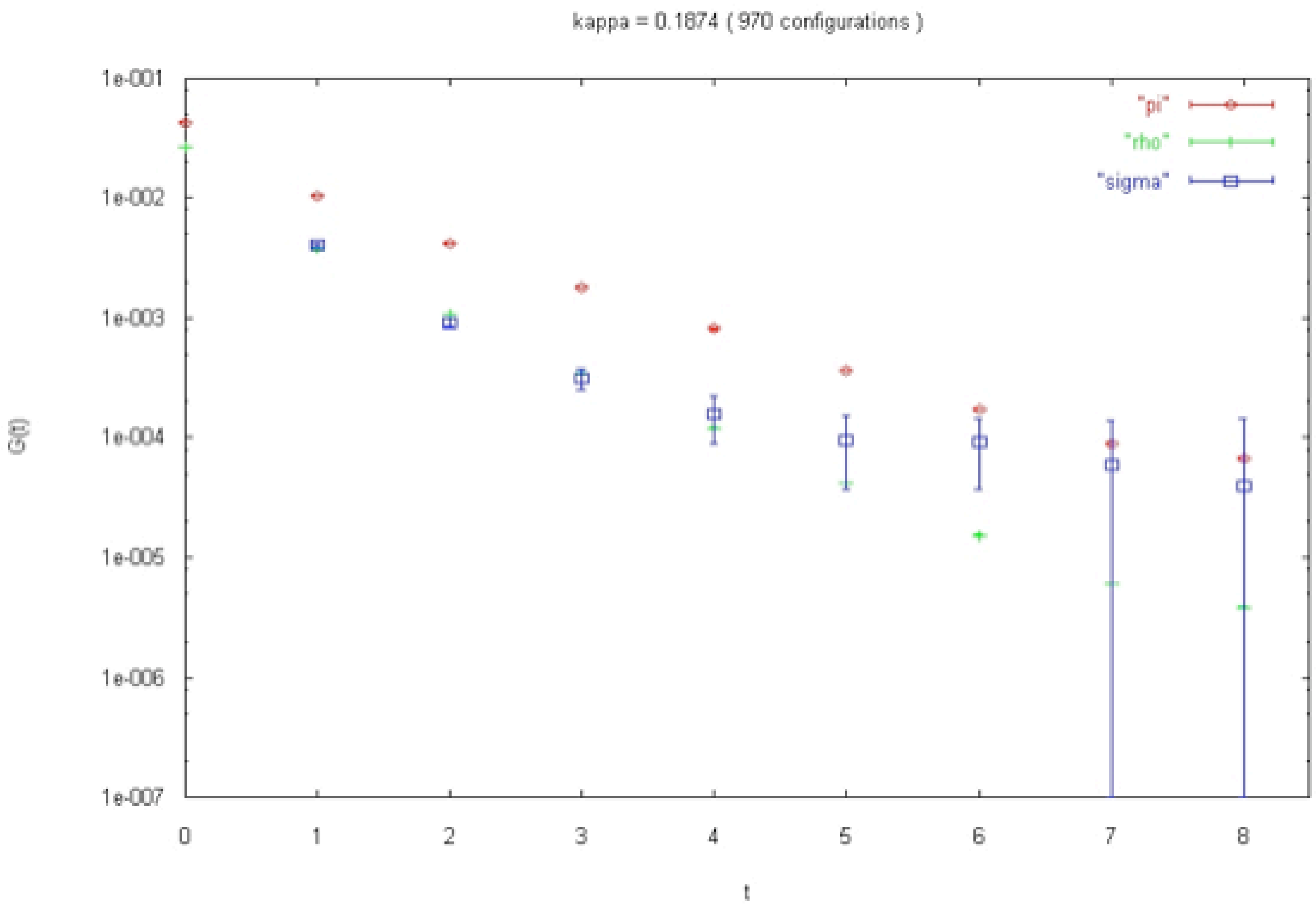
Connected and Disconnected Parts (κ =0.1846)



Effective mass (κ =0.1846)

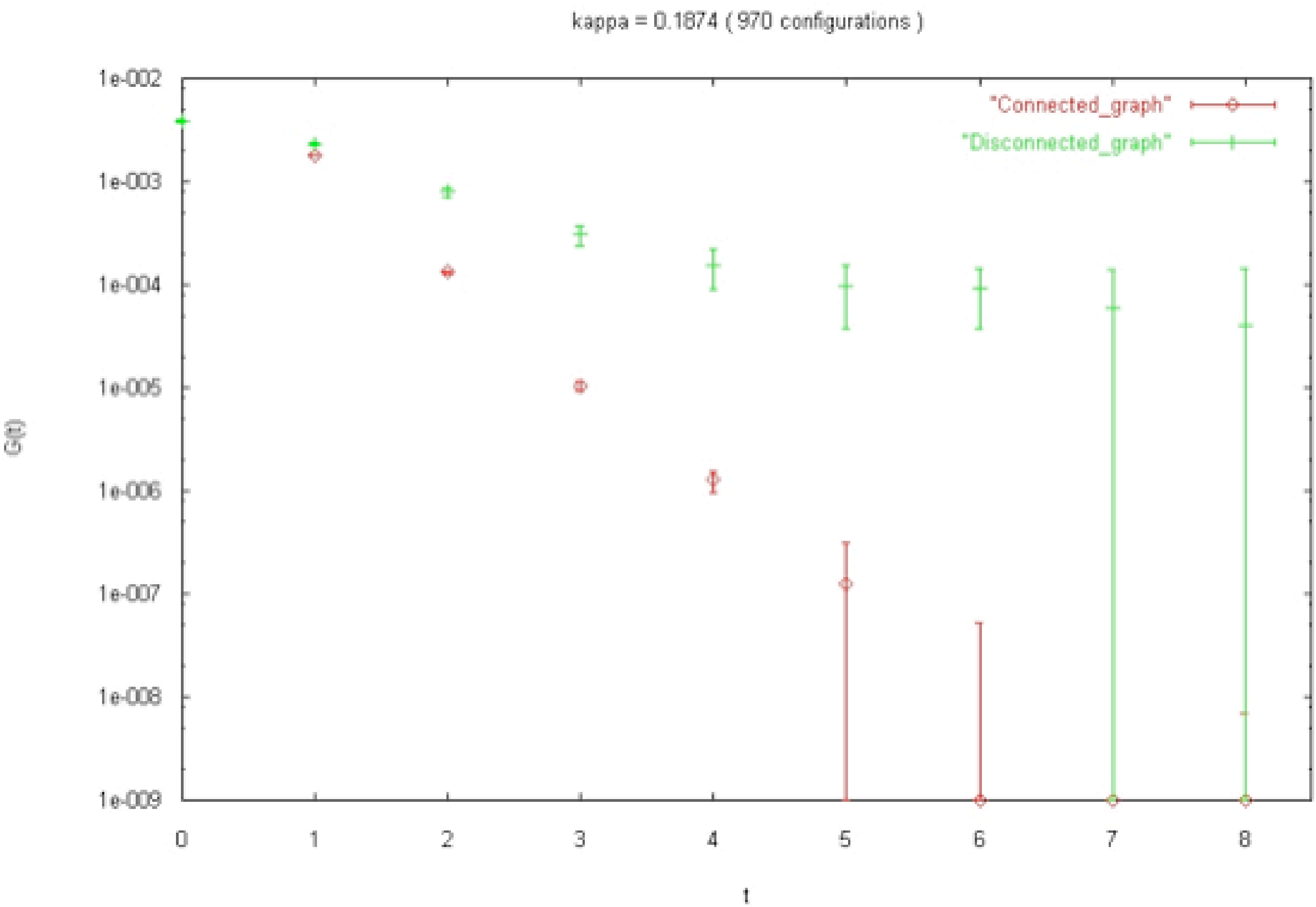


π , ρ , σ mesons (κ =0.1874)

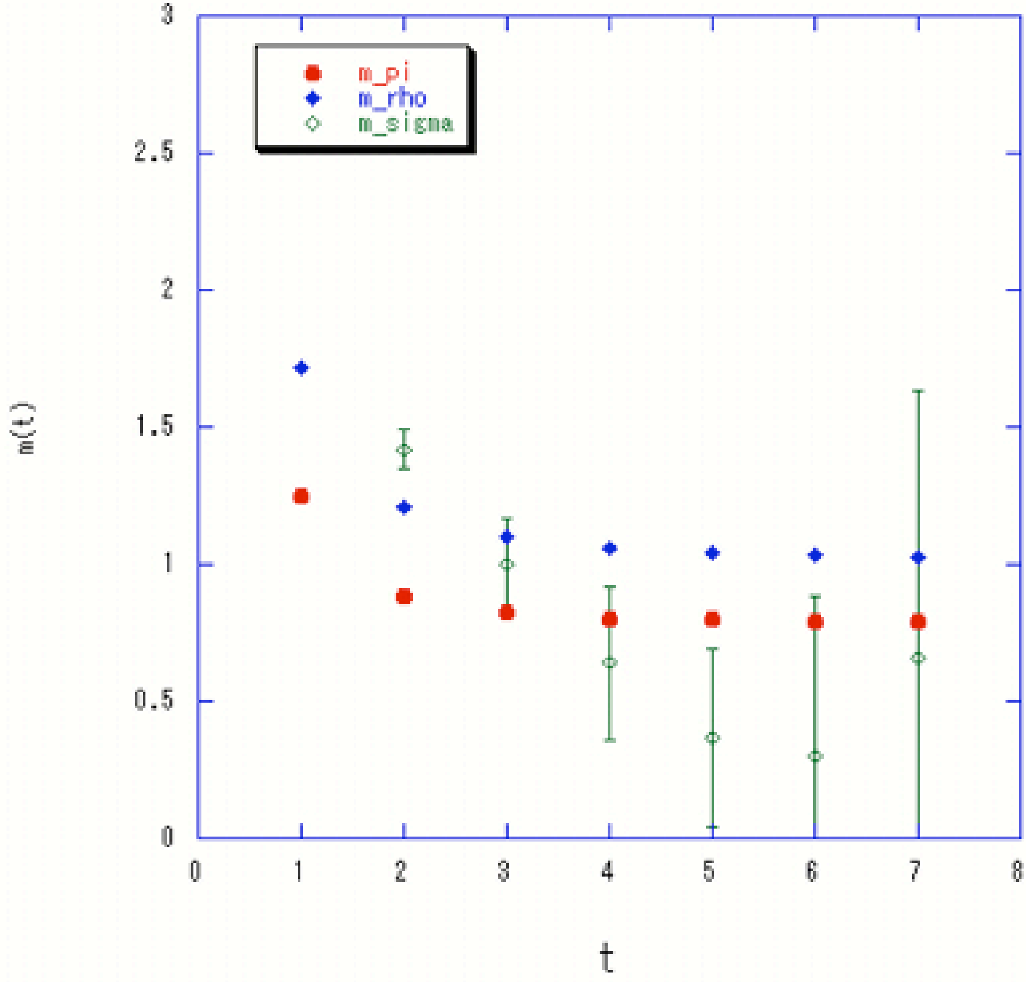


σ meson propagators

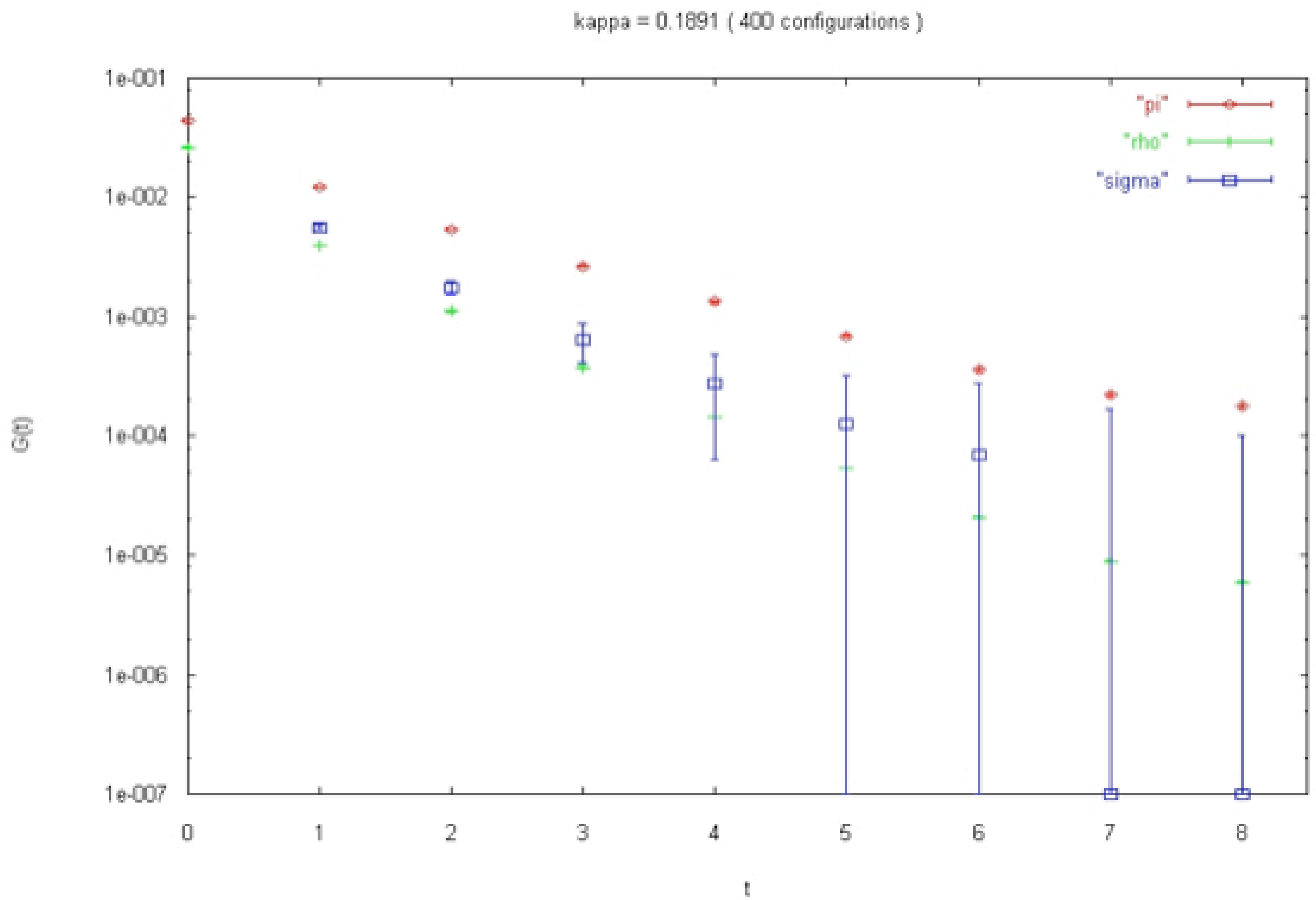
Connected and Disconnected Parts （κ =0.1874 ）



Effective mass (κ =0.1874)

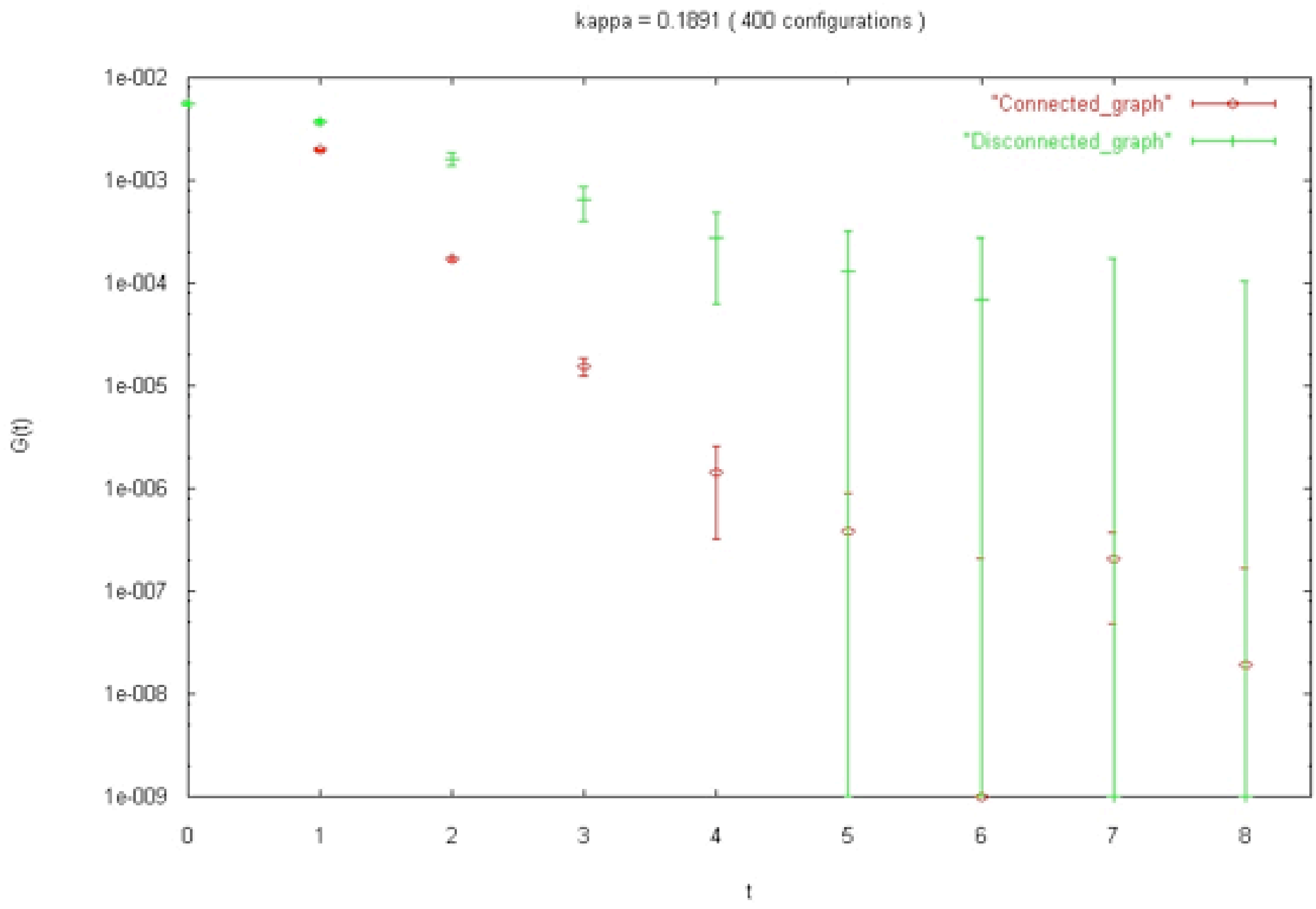


π , ρ , σ mesons (κ =0.1891)

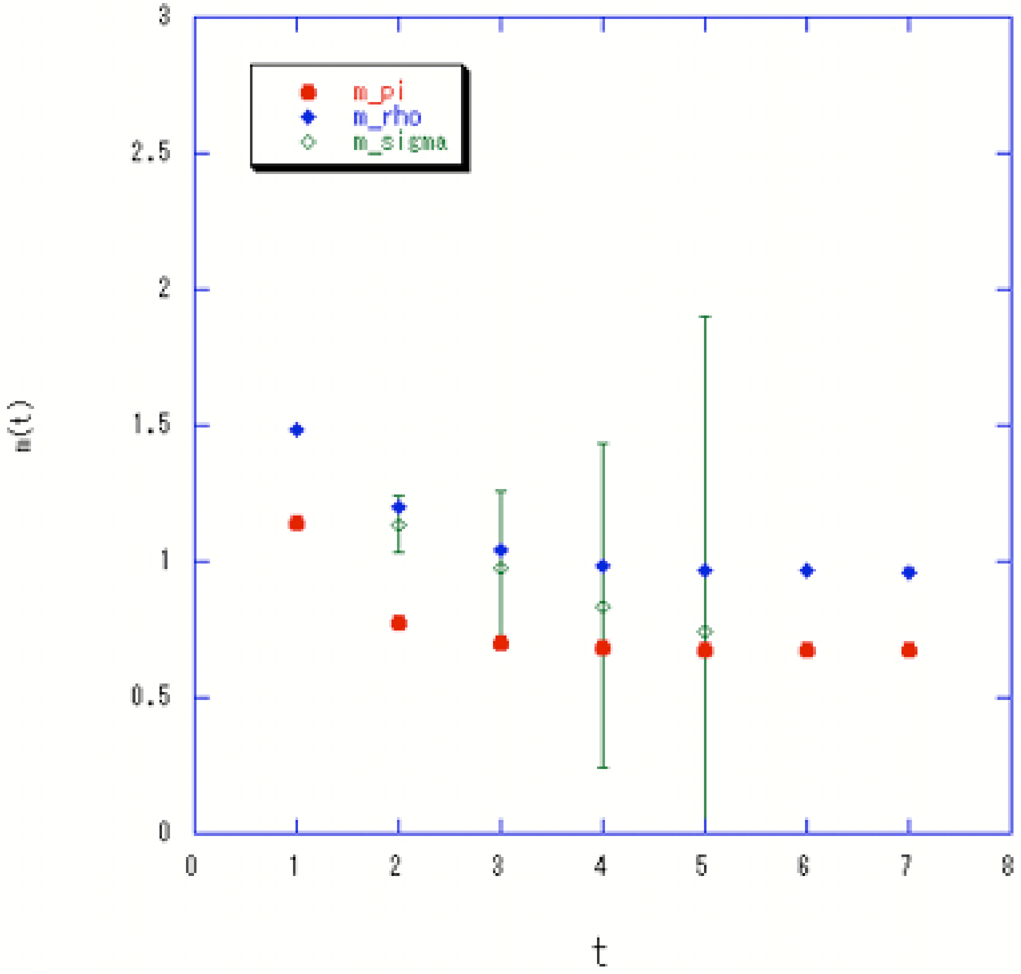


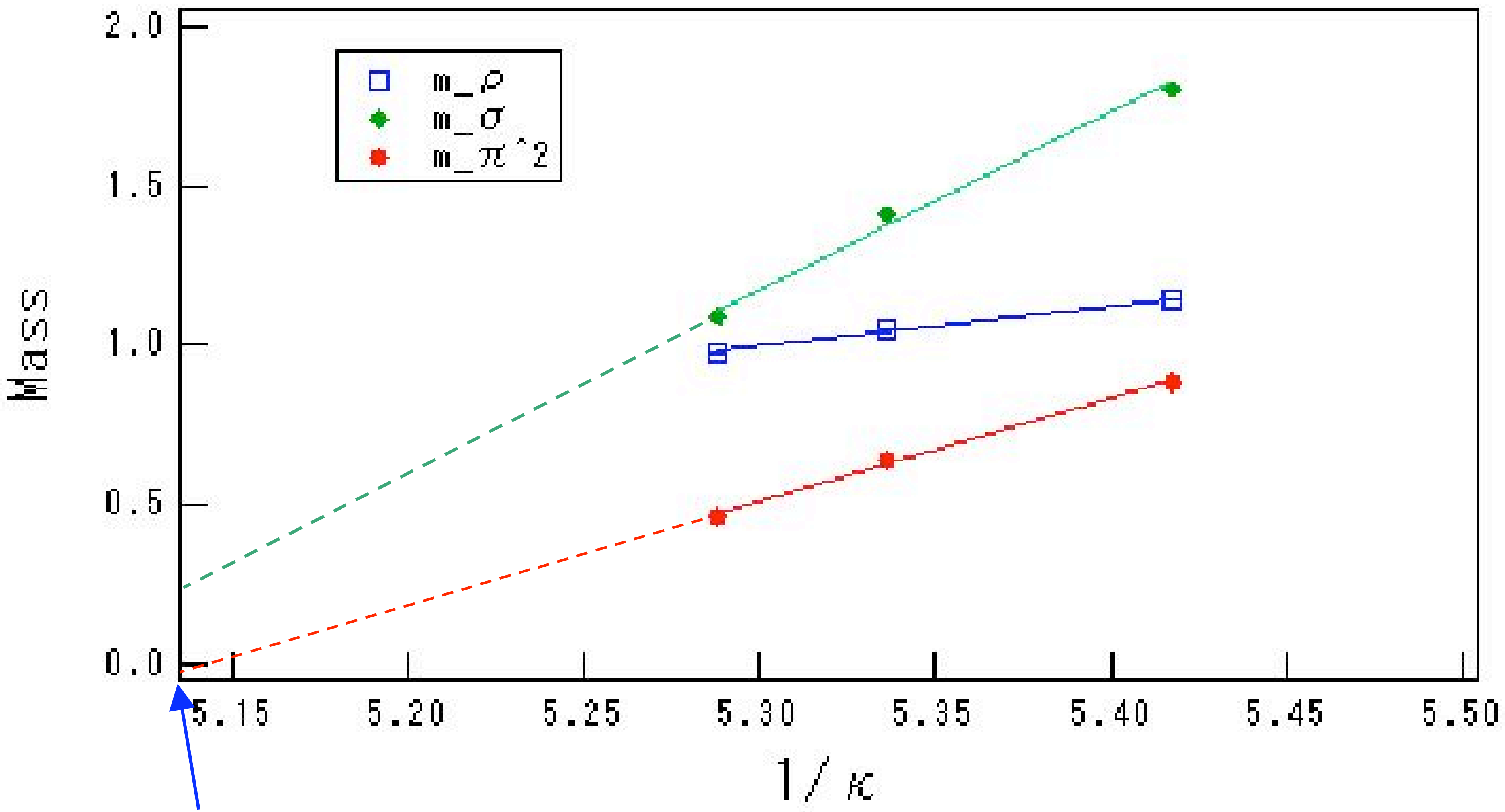
σ meson propagators

Connected and Disconnected Parts (κ =0.1891)



Effective mass (κ =0.1891)



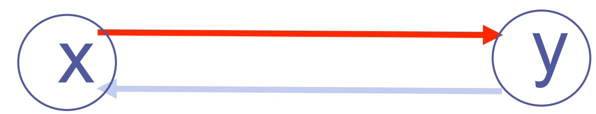


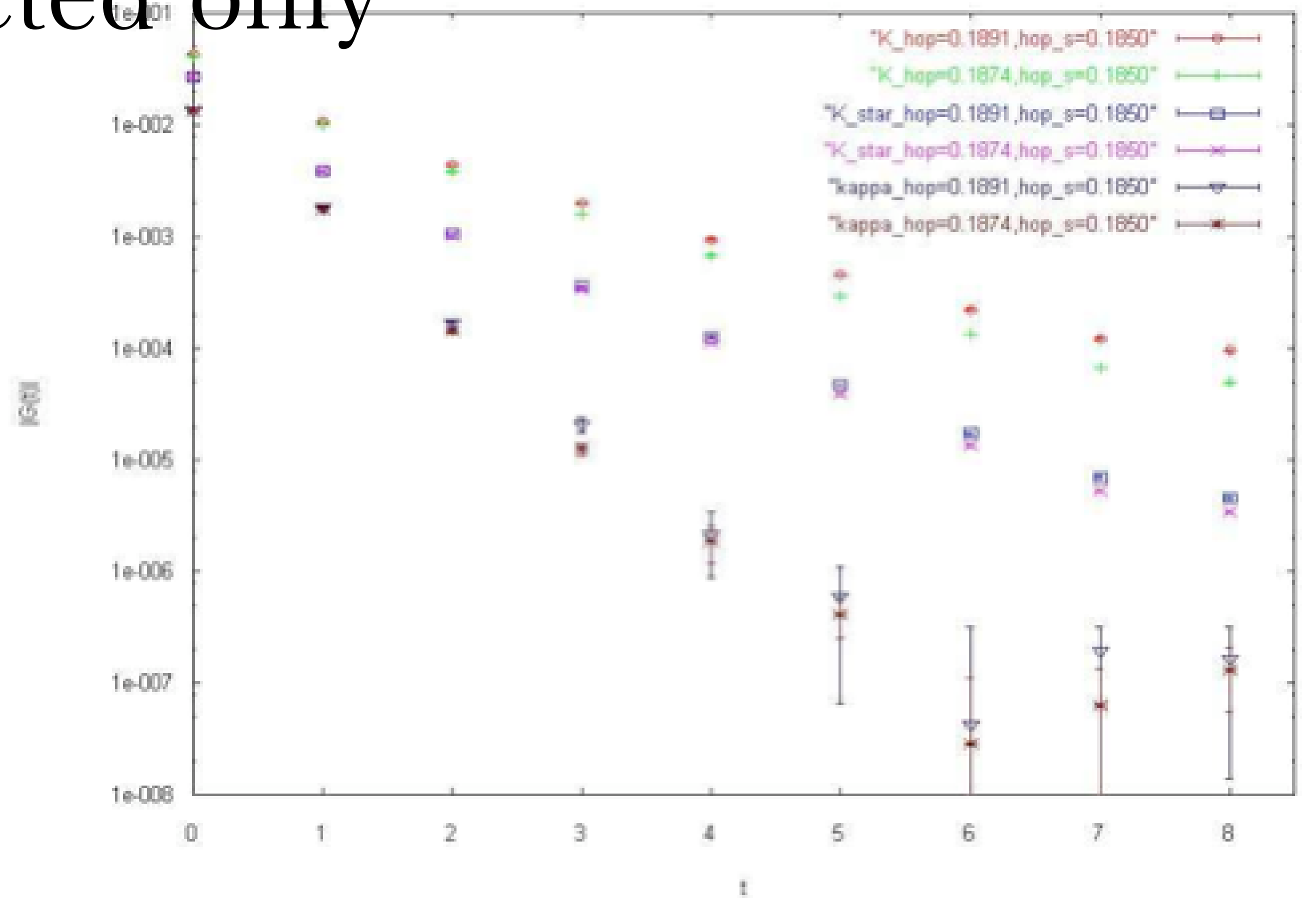
5.1410±0.0747

$\kappa c = 0.1945 \pm 0.0029$

κ meson

- strange scalar
- Connected only



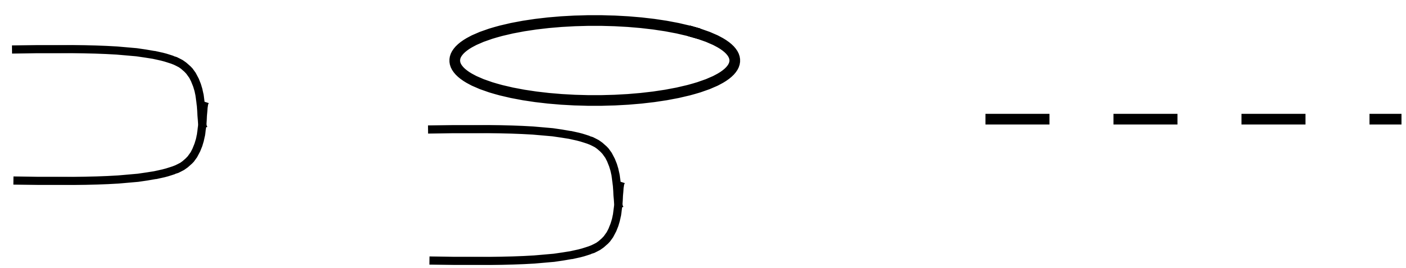


Summary of scalar meson

- Although σ propagators are noisy and we need high statistics, present data suggest that σ appears as a pole of QCD.
- Disconnected diagram dominates at large t.
- $M_\sigma \sim M_\rho$
- Mass of κ is heavier than experimental value.
- Lattice QCD Study of Scalar mesons was crazy before, but is now recognized as a meaningful work. In one year, it becomes matured and produces reliable data on the scalar mesons.

Full QCD simulation is like experiments; the measurement include all states with the prepared quantum number.

$$|0^{++}\rangle = \bar{q}q + \bar{q}q\bar{q}q + G + ...$$



(In quenched approximation, we calculate these states, and diagonalize them.)

qq Force ?

- Di-quarks are interesting object for Color super conductivity and di-quark model.

Static $\bar{q}q$-correlations

- McLerran and Svetisky, Phys.Rev. D24(1981)450.
- Static quarks

$$\left(\frac{1}{i}\frac{\partial}{\partial t} - t^a A_0^a(\vec{x},t)\right)\psi(\vec{x},t) = 0$$

$$\Rightarrow \psi(\vec{x},t) = T\exp\left(i\int_0^t dt' t^a A_0^a(\vec{x},t')\right)\psi(\vec{x},0)$$

$$\sim L(\vec{x})\psi(\vec{x},0)$$

$$L(\vec{x}) \equiv U_t(\vec{x},N_t)U_t(\vec{x},N_t-1)\ldots U_t(\vec{x},1)$$

$TrL(\vec{x})$ **:Polyakov Line**

$\bar{q}$q-state

$$e^{-\beta F_{q\bar{q}}} \sim \sum_{\phi} < \phi | e^{-\beta H} | \phi >$$

$$| \phi > = \psi^a(\vec{x},0)^\dagger (\psi^c)^b(\vec{x},0)^\dagger | Gluons >$$

a,b: Color indices ψ^c : anti-quark

$$e^{-\beta F_{q\bar{q}}} \sim \sum_{a,b,gluons} < Gluons | \psi^a(\vec{x}_1,0)(\psi^c)^b(\vec{x}_2,0)$$

$$\times e^{-\beta H} \psi^a(\vec{x}_1,0)^\dagger (\psi^c)^b(\vec{x}_2,0)^\dagger | Gluons >$$

$$= \sum_{a,b,gluons} < Gluons | e^{-\beta H} \psi^a(\vec{x}_1,\beta) \psi^a(\vec{x}_1,0)^\dagger$$

$$\times (\psi^c)^b(\vec{x}_2,\beta)(\psi^c)^b(\vec{x}_2,0)^\dagger | Gluons >$$

$$= \sum_{a,b,gluons} < Gluons | e^{-\beta H} L(\vec{x}_1)^{aa'} \psi^{a'}(\vec{x}_1, 0)$$

$$\times \psi^{a}(\vec{x}_1, 0)^{\dagger} L(\vec{x}_2)^{\dagger bb'} (\psi^{c})^{b'}(\vec{x}_2, 0)(\psi^{c})^{b}(\vec{x}_2, 0)^{\dagger} | Gluons >$$

$$= \sum_{gluons} < Gluons | e^{-\beta H} Tr L(\vec{x}_1) Tr L(\vec{x}_2) | Gluons >$$

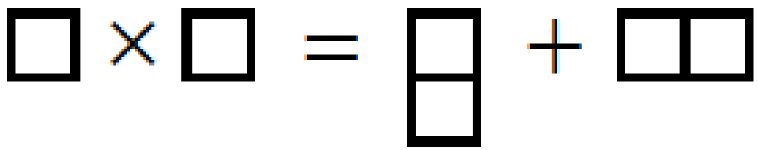
$$\sim < Tr L(\vec{x}_1) Tr L^{\dagger}(\vec{x}_2) >$$ Color averaged

Here we used $[\psi^{a}(\vec{x}, 0), \psi^{b}(\vec{x}', 0)^{\dagger}] = \delta_{a,b}\delta_{\vec{x},\vec{x}'}$

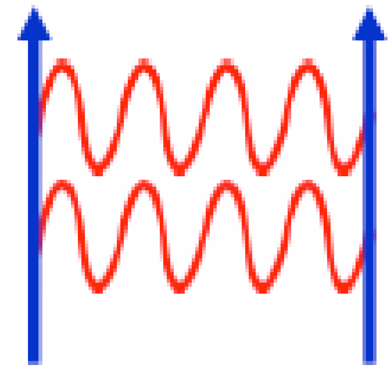
and similar relation for anti-quark fields.

Di-qurak Potential

$3\times 3 = 3^* + 6$



$\varepsilon_{abc}\psi^b\psi^c$



$\psi^b \qquad \psi^c$

$$e^{-\beta F_{qq}} \sim \sum_{\phi} < \phi | e^{-\beta H} | \phi >$$

$$|\phi> = \varepsilon_{abc}\psi^b(\vec{x}_1,0)^\dagger \psi^c(\vec{x}_2,0)^\dagger \,|\, Gluons >$$

Using

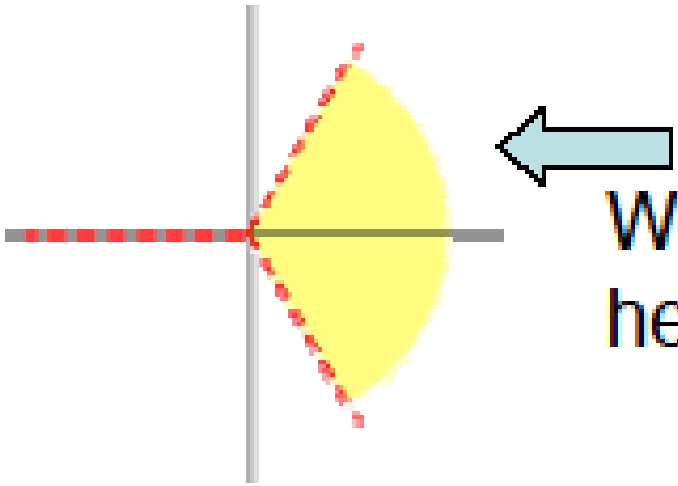
$$\varepsilon_{abc}\varepsilon_{ab'c'} = \delta_{bb'}\delta_{cc'} - \delta_{bc'}\delta_{cb'}$$

$$\Rightarrow \exp(-\frac{V_{ant.sym.}}{T}) = \frac{3}{2} < TrL(R)TrL(0) > - \frac{1}{2} < TrL(R)L(0) >$$

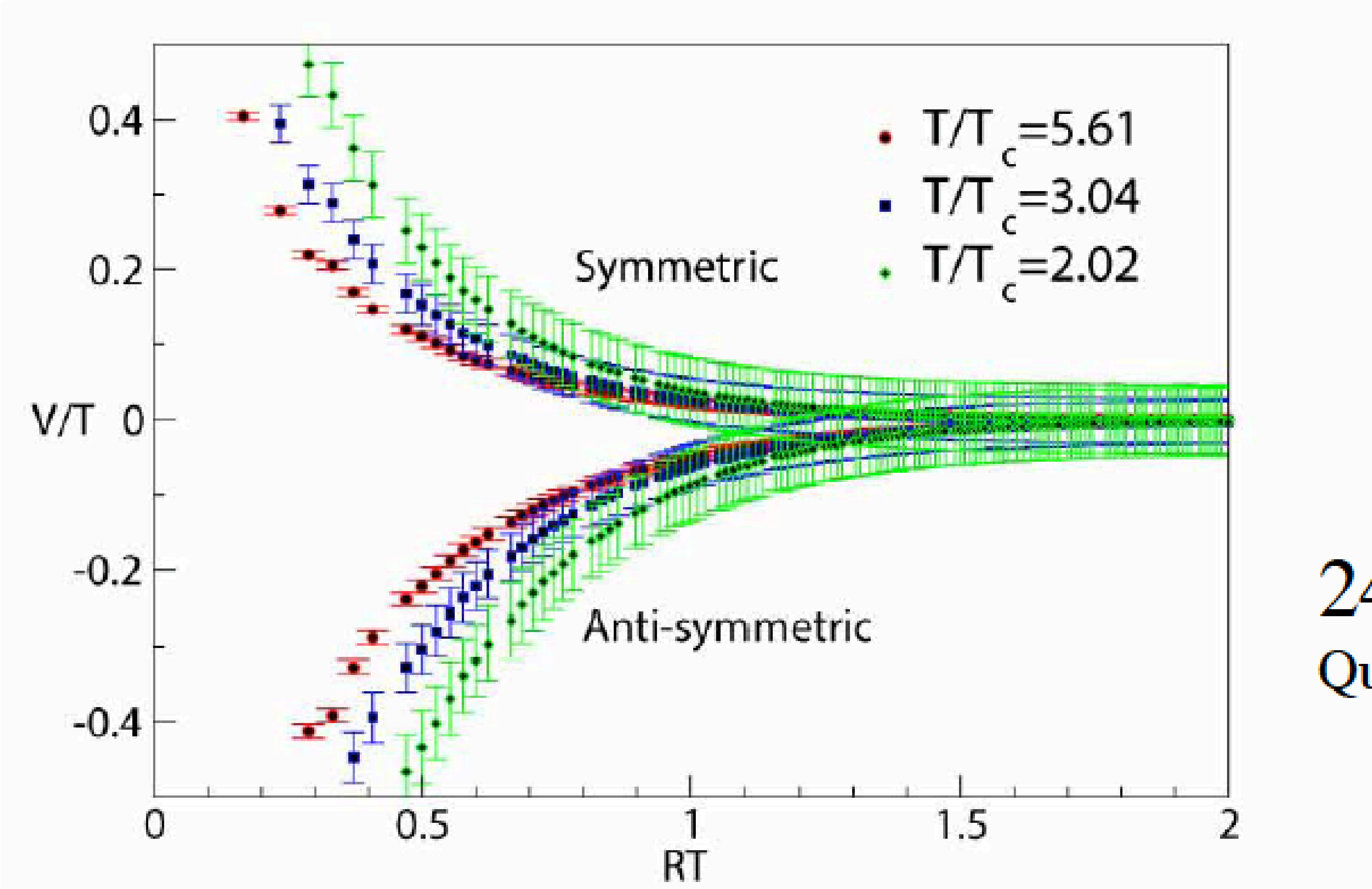
Notice

- In Coulomb Gauge, this is zero due to the residual global symmetry.
- Z3 symmetry in Quench QCD.

$$< Tr(LL) > + < Tr(zLzL) > + < Trz^2Lz^2L >$$
$$= (1 + z^2 + z) < TrLL >= 0$$



We stay here.



$24^3 \times 6$

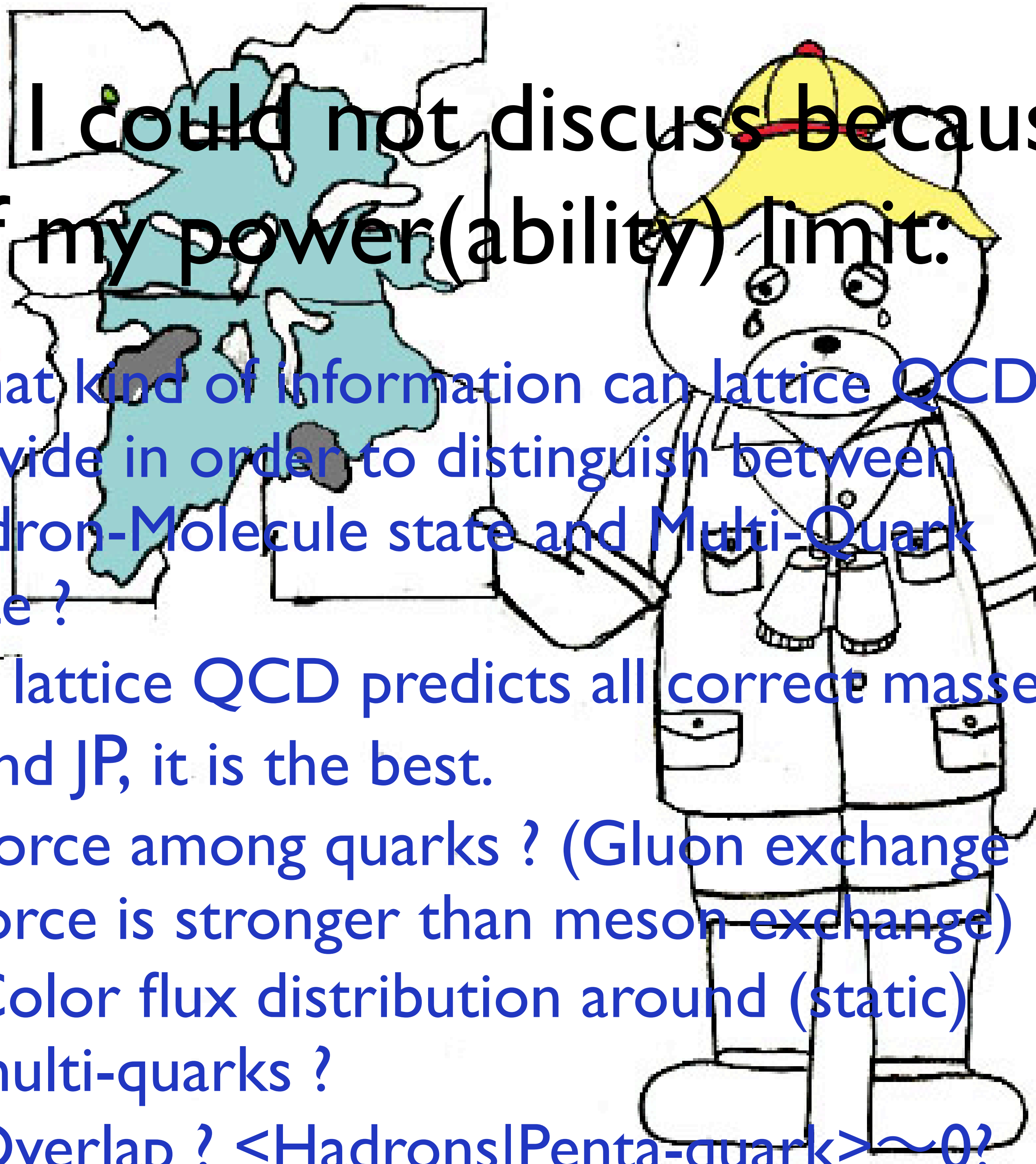
Quench

- These are results at T>0.
- They are interesting.
- But it is more interesting if we calculate such potential or force at T=0. Penta-quarks consist of $(qqq)(\bar{q}q)$ or $(qq)(qq)\bar{q}$?

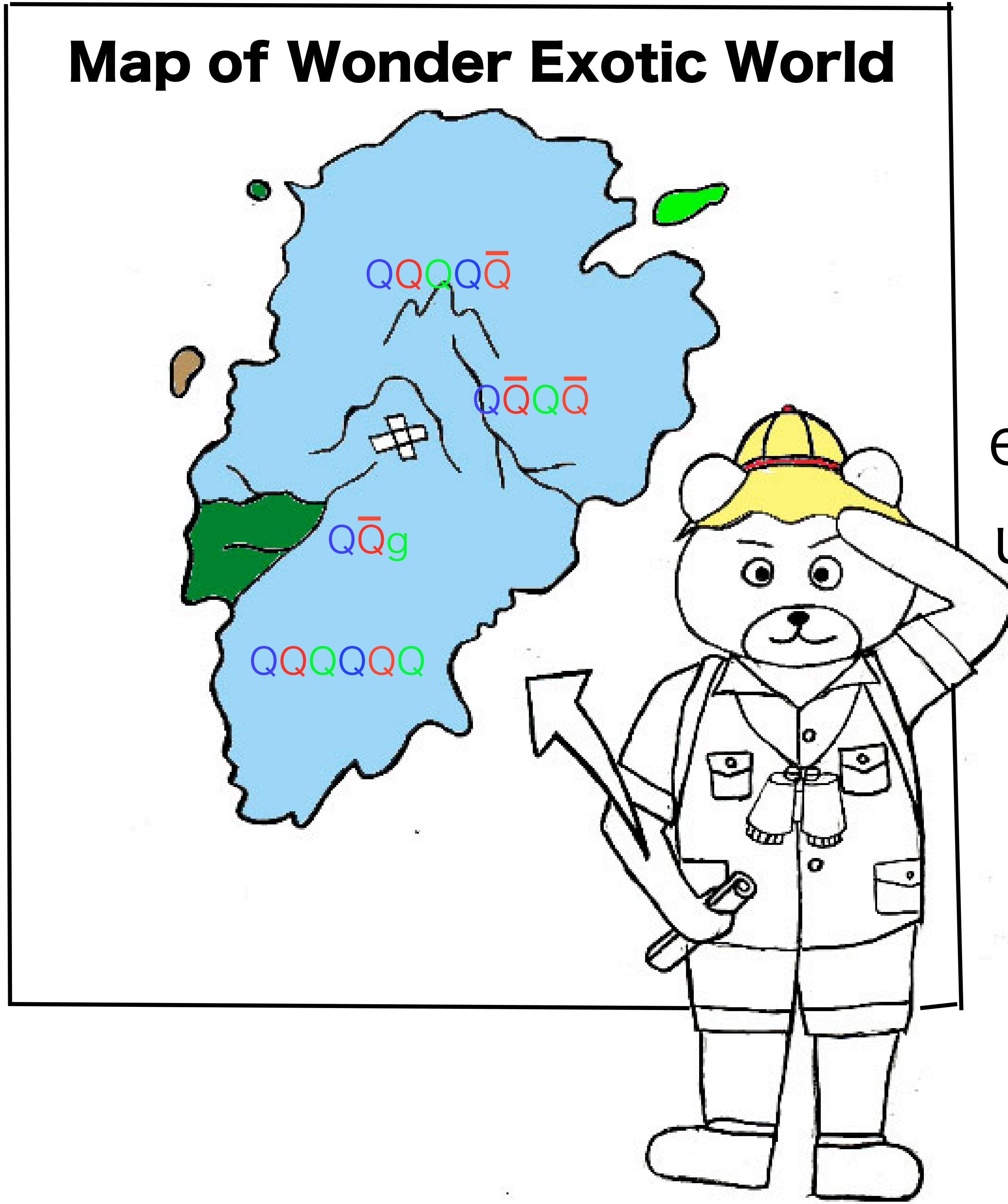
What I could not discuss because of the time:

- Lattice calculations of Penta-quark:
  - Sasaki
  - Csikor-Fodor-Katz-Kovacs, JHEP
  - Kentucky Group
- Di-Baryon (H-Baryon)
  - Wetzorke and Karsch, hep-lat/020829
  - Pochinsky,Negele and Scarlet, hep-lat/9809077
- Hybrid (qqG and/or new type of heavy quark potential)
  - Juge, Kuti and Morningstar,hep-lat/0207004
  - Lacock and Schilling,hep-lat/9809022
  - and many
  -

What I could not discuss because of my power(ability) limit:



- What kind of information can lattice QCD provide in order to distinguish between Hadron-Molecule state and Multi-Quark state ?
  - If lattice QCD predicts all correct masses and JP, it is the best.
  - Force among quarks ? (Gluon exchange force is stronger than meson exchange)
  - Color flux distribution around (static) multi-quarks ?
  - Overlap ? <Hadrons|Penta-quark>~0?



Someday, we will explore and understand this wonderful world !