

# Chiral dynamics for exotic resonances

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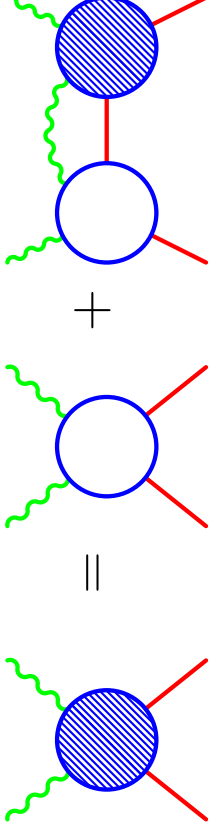
- ✓ Introduction and motivation
- ✓ Resonances and coupled-channel dynamics
- ✓ Predictions of chiral  $SU(3)$  symmetry
- ✓ Summary

# Resonances and coupled-channel dynamics

- ✓ **Dynamic generation of resonances:** some good candidates
  - $\Lambda(1405)$  resonance as  $\bar{K}N$  quasi-bound state
    - Dalitz, Wyld, Rajasekaran, Weise, Siegel, ...
  - $N(1535)$  resonance as  $K\Sigma$  quasi-bound state
    - Dalitz, Wyld, Weise, Kaiser, Oset, ...
  - $\Lambda(1520)$  resonance as  $\bar{K}_\mu N$  quasi-bound state
    - Ball, Frazer, Aaron, Amado, ...
  - $f_0(980)$  resonance as  $K\bar{K}$  quasi-bound state
    - Van Beveren, Weinstein, Isgur, Janssen, Oller, Oset, Pelaez, ...
- ✓ **To be or not to be:** which resonances are generated by coupled channels ?
- if one member of a large- $N_c$  multiplet is generated, so should be the full multiplet
- ✓ **Conjecture:**

excited baryon and meson resonances generated by coupled-channel dynamics

# Coupled-channel Bethe-Salpeter equation



- Scattering amplitude:

- Effective interaction kernel  $V$ :

$$[1 - K \cdot G]^{-1} \cdot K \quad \leftarrow \text{on-shell} \rightarrow \quad [1 - V \cdot G]^{-1} \cdot V$$

defined by:

$$K = V + (1 - V \cdot G) \cdot V_L + V_R \cdot (1 - G \cdot V) + (1 - V \cdot G) \cdot V_{LR} \cdot (1 - G \cdot V) - V_R \cdot \frac{1}{1 - G \cdot V_{LR}} \cdot G \cdot V_L.$$

note:  $V_L$  and  $V_R$  vanish for on-shell kinematics

- **Strategy:** chiral and large- $N_c$  expansion of effective interaction  $V$
- **No dependence on choice of fields**

# Solution of Bethe-Salpeter scattering equation

- Decompose effective interaction  $V$ :

$$V = \sum_{J,P} V^{(J,P)}(\sqrt{s}) \mathcal{Y}^{(J,P)}(\bar{q}, q; w)$$

- Unique covariant projection operators:

$$\mathcal{Y}^{(J,P)}(\bar{q}, q; w) \quad \text{with} \quad w^2 = s$$

- preserve total angular momentum  $J$  and parity  $P$
- regularity in  $q$  (initial meson) and  $\bar{q}$  (final meson)
- defined for any off-shell kinematics

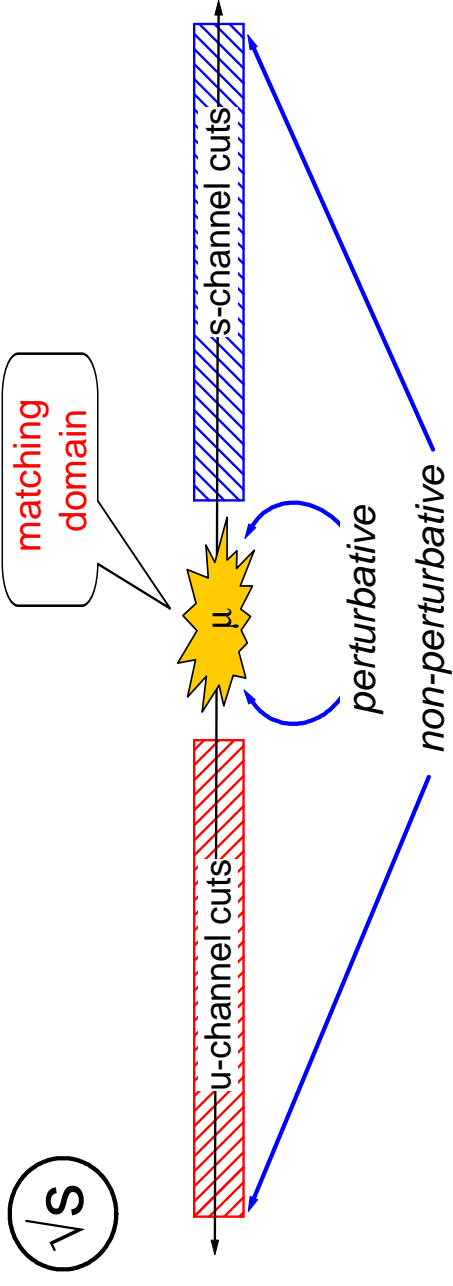
$$\left( \mathcal{Y}^{(J,P)} \cdot G \cdot \mathcal{Y}^{(J',P')} \right) (\bar{q}, q; w) \stackrel{!}{=} \delta_{J,J'} \delta_{P,P'} J^{(J,P)}(\sqrt{s}) \mathcal{Y}^{(J,P)}(\bar{q}, q; w)$$

→ divergent loop-functions  $J^{(J,P)}(\sqrt{s})$

- Scattering amplitude  $T$ : algebraic matrix equation

$$T = \sum_{J,P} \left( 1 - V^{(J,P)} J^{(J,P)} \right)^{-1} V^{(J,P)} \mathcal{Y}^{(J,P)} + \dots$$

# Gluing of $s$ - and $u$ -channel unitarized amplitudes



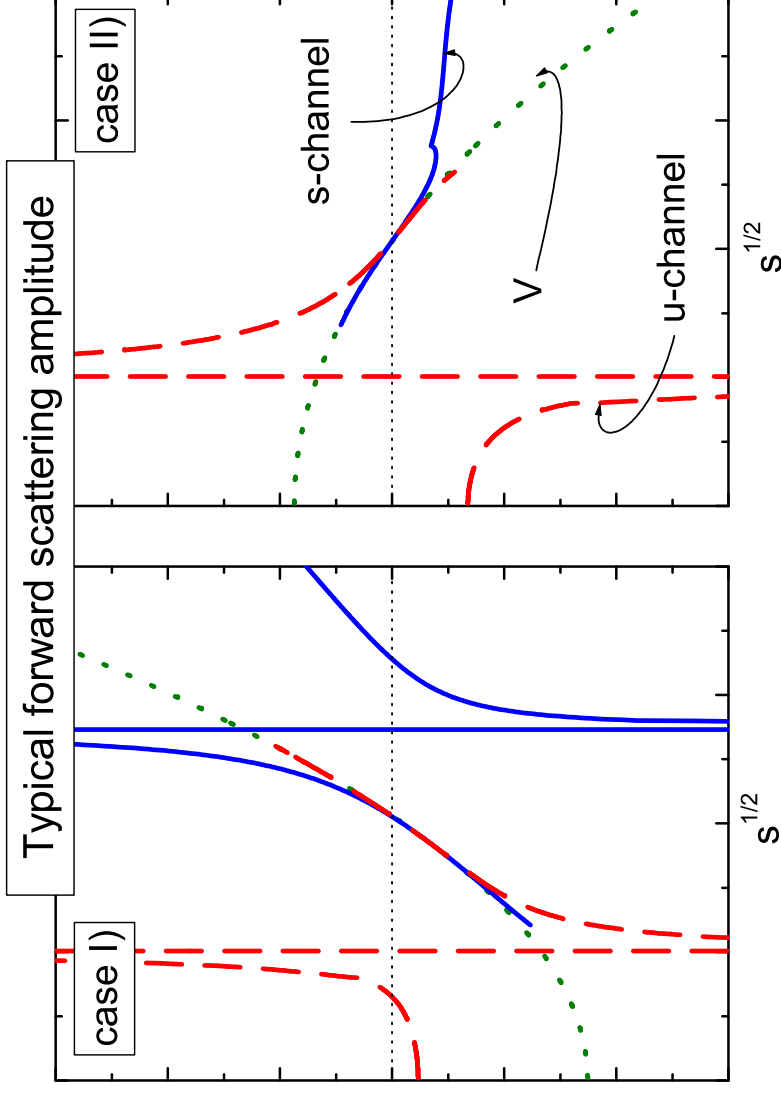
- Approximated crossing symmetry:

$$\begin{aligned}
 & \text{[Feynman diagrams: } s\text{-channel exchange, } u\text{-channel exchange, and } t\text{-channel exchange}] + \dots \implies T_{u\text{-chan.}}(\mu) \sim T_{s\text{-chan.}}(\mu) \iff \\
 & T = \begin{cases} T_{s\text{-chan.}} & \text{if } \sqrt{s} > \mu \\ T_{u\text{-chan.}} & \text{if } \sqrt{s} < \mu \end{cases} + \dots
 \end{aligned}$$

- Renormalization condition:  $T^{(J,P)}(\sqrt{s} = \mu) = V^{(J,P)}(\sqrt{s} = \mu)$

- Optimal matching point: e.g., for  $\pi H$ -scattering  $\rightarrow \mu = m_H$

# Crossing-symmetric scattering amplitudes



## ✓ Gluing of s- and u-channel unitarized amplitudes:

- requires good matching properties
- by construction: exact crossing symmetry in physical region
- approximate crossing symmetry at subthreshold energies

# Chiral SU(3) interaction terms and large- $N_c$ QCD

- Large- $N_c$  ground states:
  - Goldstone boson octet  $\Phi_{[8]} = (\pi, K, \bar{K}, \eta)$
  - Vector meson nonet  $\Phi_{[9]}^\mu = (\rho^\mu, K^\mu, \bar{K}^\mu, \omega^\mu, \phi^\mu)$
  - Baryon octet  $B_{[8]} = (N, \Sigma, \Lambda, \Xi)$
  - Baryon decuplet  $B_{[10]} = (\Delta, \Sigma^*, \Xi^*, \Omega)$
- Systematic approximation strategy:

expand in *powers* of the **small** current quark masses, momenta and  $1/N_c$

$$\frac{m_{\text{quark}}}{\Lambda_{\chi SB}} \ll 1, \quad \frac{1}{N_c} \ll 1$$

- heavy fields:  $M_{[8,9,10]} \sim \Lambda_{\chi SB}$  but  $M_{[10]} - M_{[8]} \sim \frac{1}{N_c}$
- light Goldstone bosons:  $m_{[8]} \sim m_{\text{quark}}^{1/2}$

# Leading-order chiral interaction

- **Leading-order interaction**

use covariant derivative  $D_\mu$  (local chiral SU(3) rotations) in kinetic term:

e.g.  $\text{Tr}(\bar{B}_{[8]} i \gamma_\mu D^\mu B_{[8]})$  for baryon octet

- **Weinberg–Tomazawa term for meson-baryon interaction**

$$\mathcal{L}_{\text{WT}} = \frac{i}{8 f^2} \text{tr} \bar{B}_{[8]} \gamma_\mu \left[ \Phi_{[8]}, (\partial^\mu \Phi_{[8]}) \right]_- , B_{[8]} \Big]_- + \frac{3i}{8 f^2} \text{tr} g_{\alpha\beta} \left( \bar{B}_{[10]}^\alpha \gamma^\mu B_{[10]}^\beta \right) \cdot \left[ \Phi_{[8]}, (\partial_\mu \Phi_{[8]}) \right]_-$$

$$\begin{array}{c}
 \text{Diagram: } \text{red wavy line} \text{ and } \text{green arrow} \text{ meet at a blue square vertex} \\
 = \frac{C_{\alpha,\beta}}{4 f^2} (k_\alpha + k_\beta) \quad \longleftarrow \text{linear in meson 4-momentum}
 \end{array}$$

- **vector meson t-channel exchange**

$$\begin{array}{c}
 \text{Diagram: } \text{red wavy line} \text{ and } \text{green arrow} \text{ meet at a blue square vertex, which is connected to another blue square vertex, which then splits into a red wavy line and a green arrow} \\
 \sum_{\text{vector mesons}} \quad \Longrightarrow \quad \mathcal{K} C_{\alpha,\beta} (k_\alpha + k_\beta)
 \end{array}$$

- **Pion-decay constant**  $f \iff f_\pi/f \simeq 1.07 \pm 0.12$       $f_\pi \simeq 92.42 \pm 0.33 \text{ MeV}$



# Parity-flip reactions in s-wave

- meson-baryon

$$0^- + \frac{1^+}{2} \rightarrow \frac{1^-}{2} \rightarrow 0^- + \frac{1^+}{2}$$

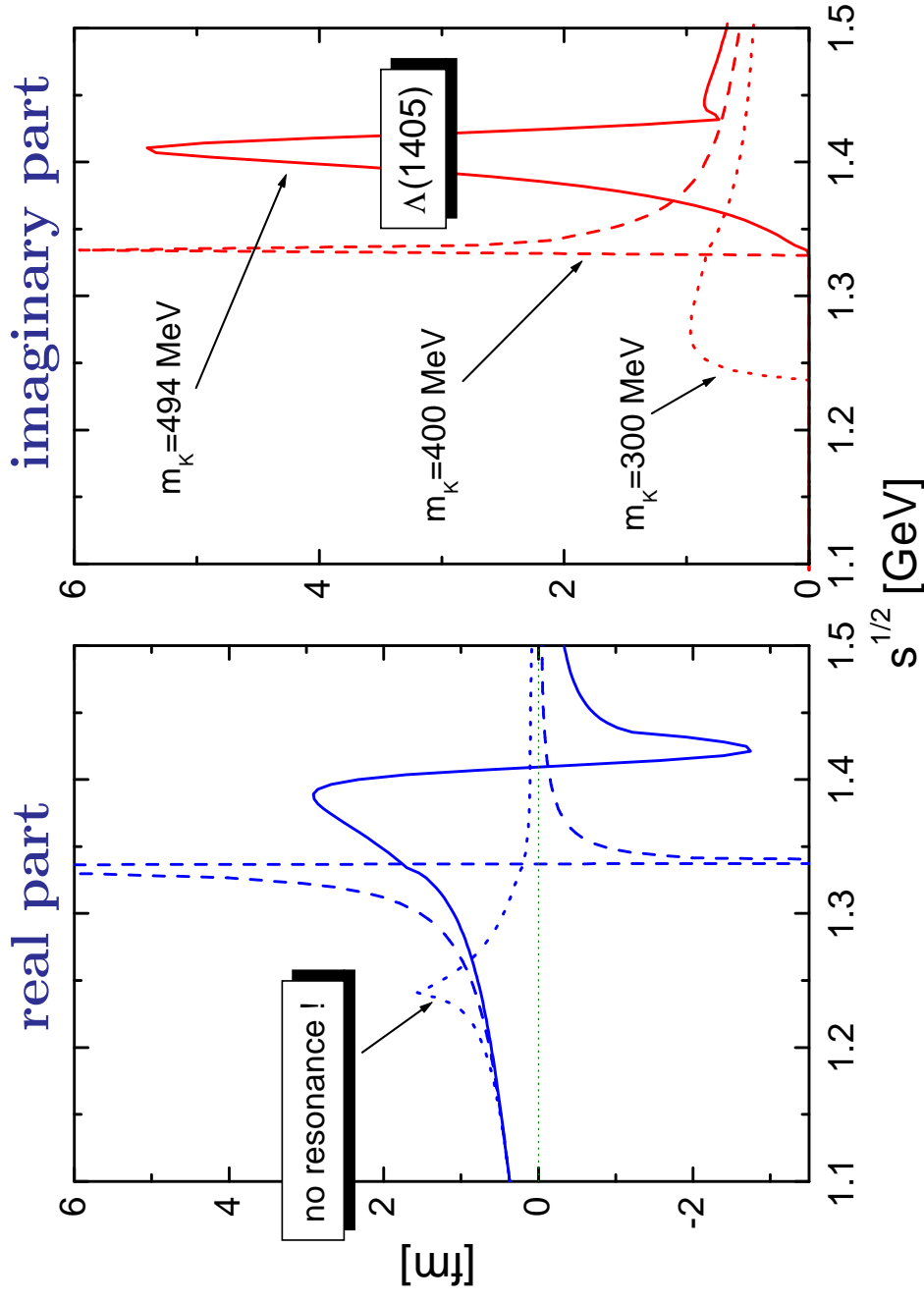
$$0^- + \frac{3^+}{2} \rightarrow \frac{3^-}{2} \rightarrow 0^- + \frac{3^+}{2}$$

- meson-meson

$$0^- + 0^- \rightarrow 0^+ \rightarrow 0^- + 0^-$$

$$0^- + 1^- \rightarrow 1^+ \rightarrow 0^- + 1^-$$

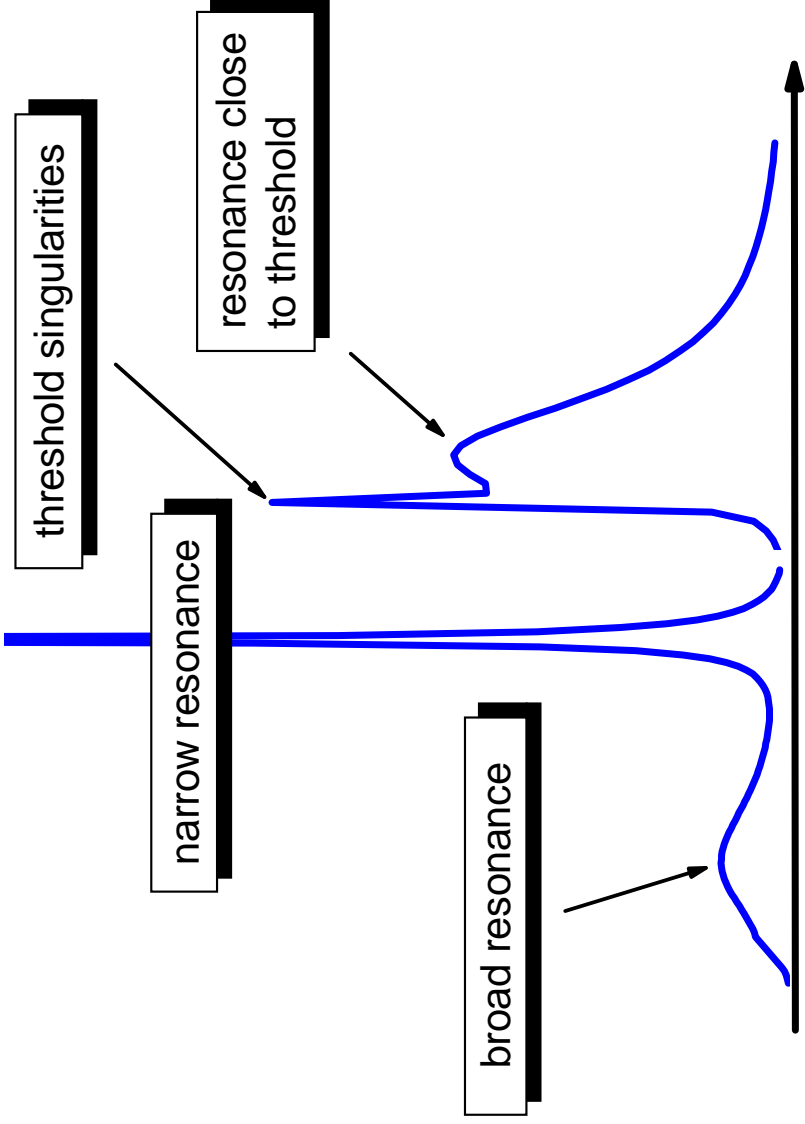
# Antikaon-Nucleon S-wave amplitude ( $Q^1$ )



Found. Phys. **31** (2001)

- **Parameter free result** : use  $f_\pi = 90$  MeV !
- **Λ(1405) resonance dynamically generated** : disappears at  $m_K = 300$  MeV !
- **Strong dependence on current quark masses**: see lattice QCD

# Speed plots for multichannel scattering

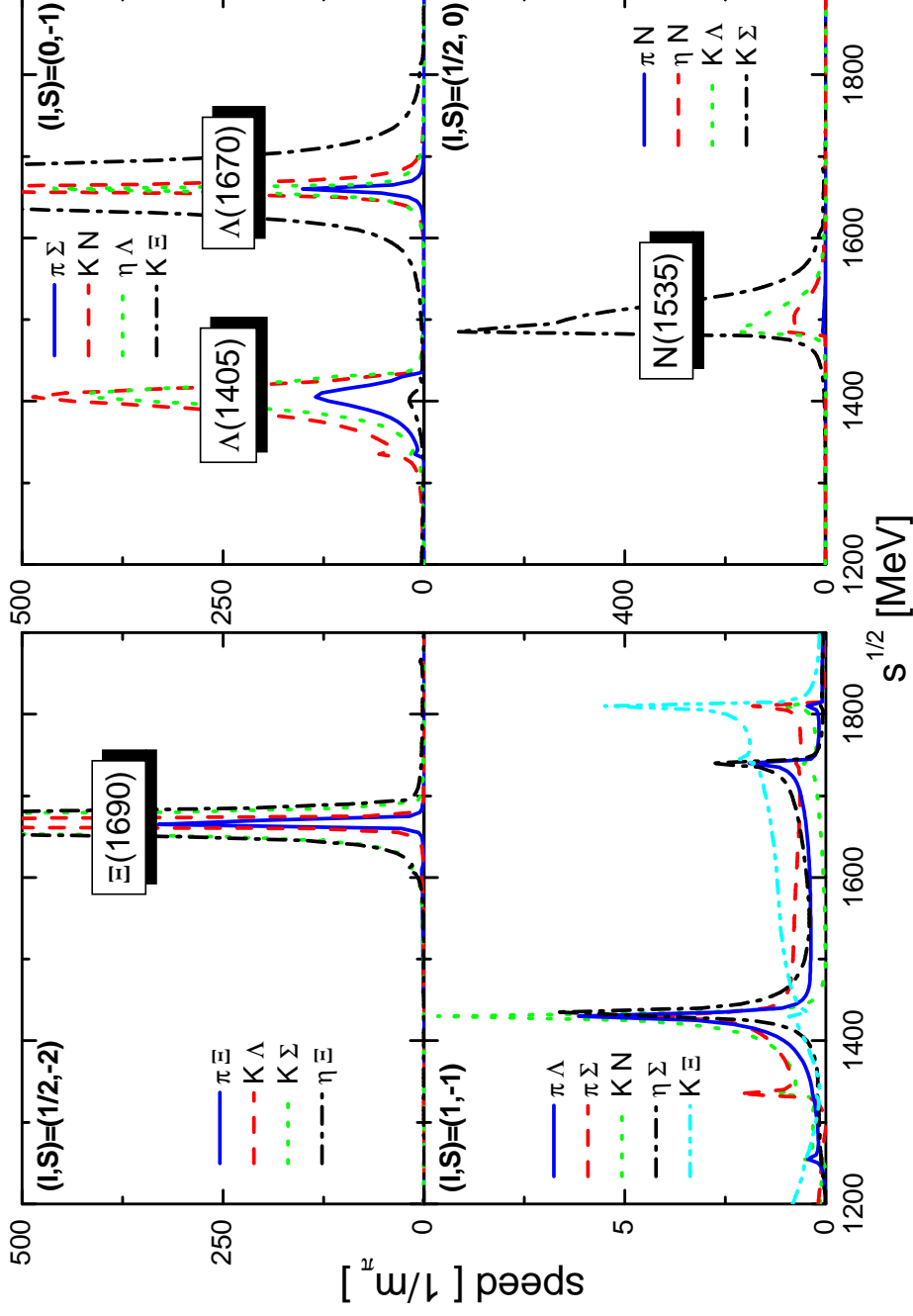


- generalization of time-delay for closed/open channels  $\frac{d\delta(E)}{dE}$
- resonance position  $\leftarrow$  local maximum of the Speed
- for s-wave resonances - cusp effects at thresholds

- analytic continuation of the S-matrix:  $S_{ab} = \delta_{ab} + 2i T_{ab}$

$$\text{Speed}_{ab}^{(I,S)}(\sqrt{s}) = \left| \sum_c \left[ \frac{d}{d\sqrt{s}} S_{ac}^{(I,S)}(\sqrt{s}) \right] \left( S_{cb}^{(I,S)}(\sqrt{s}) \right)^* \right|$$

# $J^P = \frac{1}{2}^-$ baryon resonances ( $\mathbf{Q}^1$ )



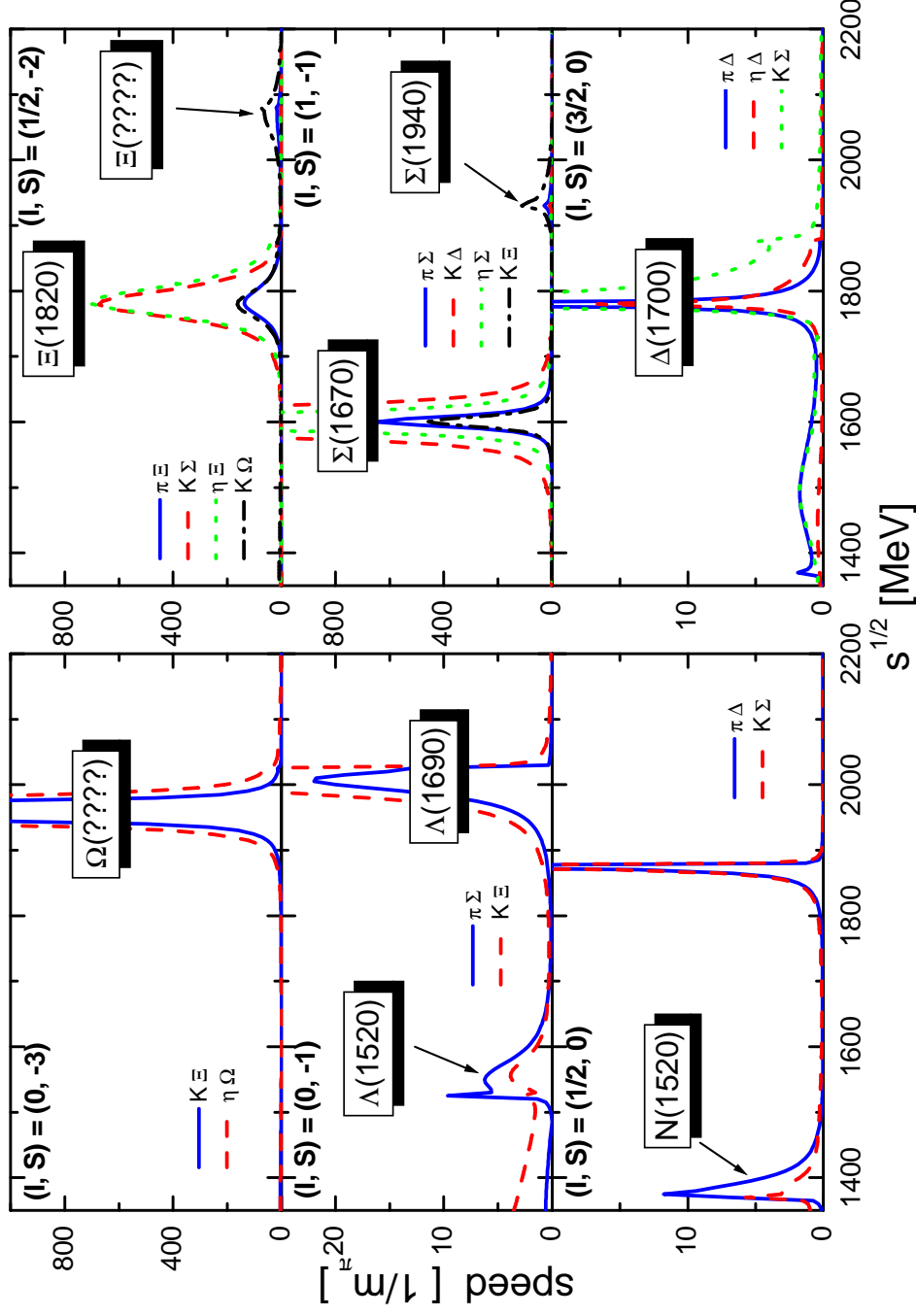
- no adjustable parameter:  
 $f = 90$  MeV
- two octets and one singlet
- strong signals:  $N(1535)$ ,  $\Lambda(1405)$ ,  $\Lambda(1670)$  and  $\Xi(1690)$
- weak signals:  $N(1650)$ ,  $\Xi(1620)$  and  $\Sigma(1620)$ ,  $\Sigma(1750)$

Chiral SU(3) symmetry predicts:  $\mathbf{8} \otimes \mathbf{8} = \mathbf{27} \oplus \mathbf{10} \oplus \mathbf{10} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{1}$

## SU(3) limit

- ✓ **”heavy” SU(3) limit:**  $m_\pi = m_\eta = m_K = 495 \text{ MeV}$ 
  - resonances turn into bound states
- ✓ **”light” SU(3) limit:**  $m_\pi = m_\eta = m_K = 139 \text{ MeV}$ 
  - resonances are gone
- ✓ **Predictions that can be tested with lattice QCD:**
  - requires unquenched QCD
  - no quark-hadron duality here
  - constituent-quark model does not predict such behavior

# $J^P = \frac{3}{2}^-$ baryon resonances ( $\mathbf{Q}^1$ )

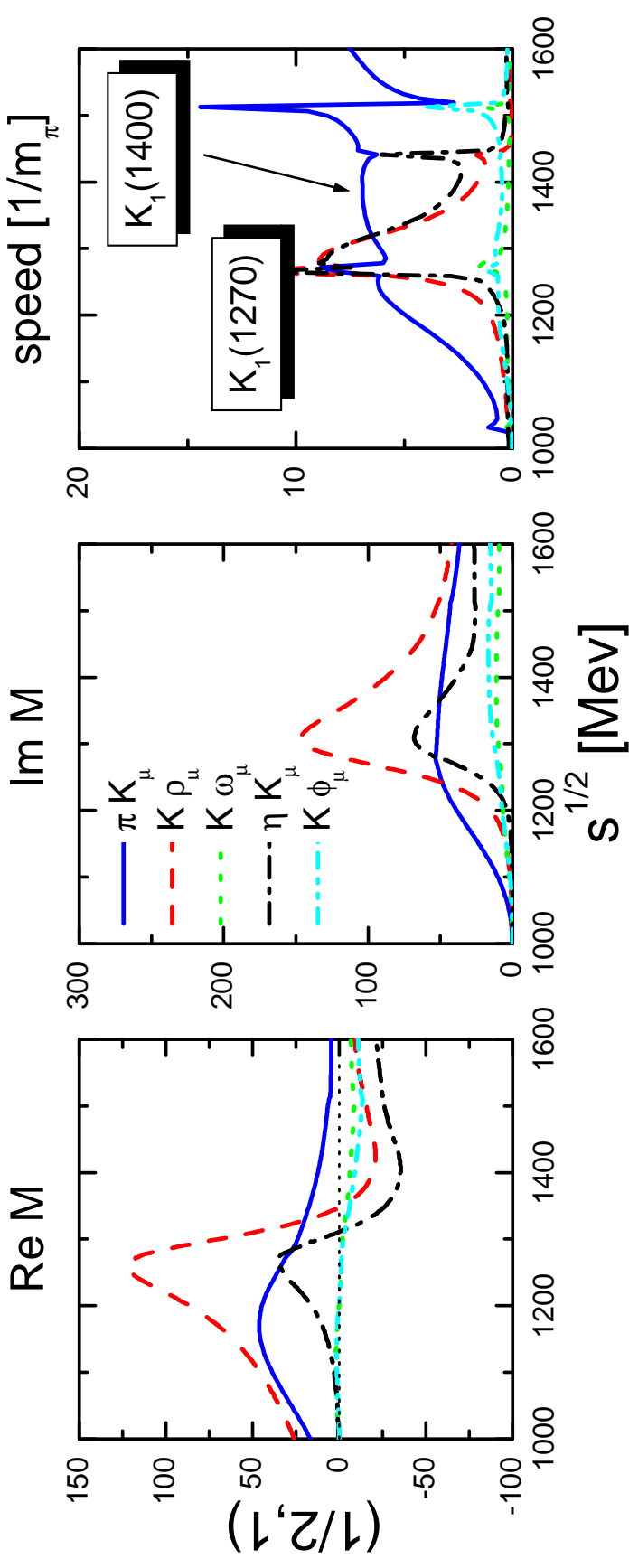


- no adjustable parameter:  
 $f = 90$  MeV
- decuplet and octet
- bound state in  $(0, -3)$ -sector
- 27-plet state in  $(0, -1)$ -sector
- $\Lambda(1520)$ – $\Lambda(1690)$  singlet-octet ??

Chiral SU(3) symmetry predicts:  $8 \otimes 10 = 35 \oplus 27 \oplus 10 \oplus 8$

# $J^P = 1^+$ meson resonances ( $Q^1$ )

- channel  $(I, S) = (1/2, 1)$ : two octet states

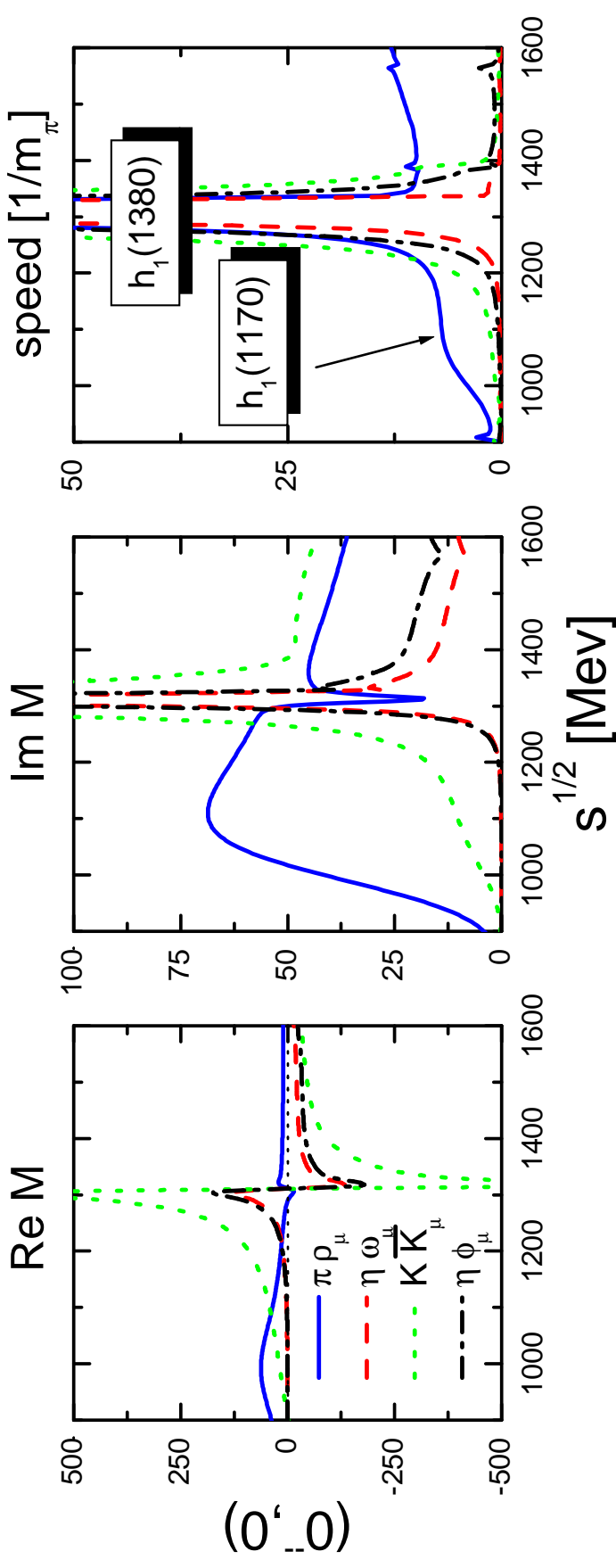


- experimental pattern:

$K_1(1270)$ : dominant decay into  $K \rho_\mu$      $\Gamma \simeq 90$  MeV  
 $K_1(1440)$ : dominant decay into  $K_\mu \pi$      $\Gamma \simeq 175$  MeV

# $J^P = 1^+$ meson resonances ( $Q^1$ )

- channel  $(I^G, S) = (0^-, 0)$ : octet and singlet states



- experimental pattern:

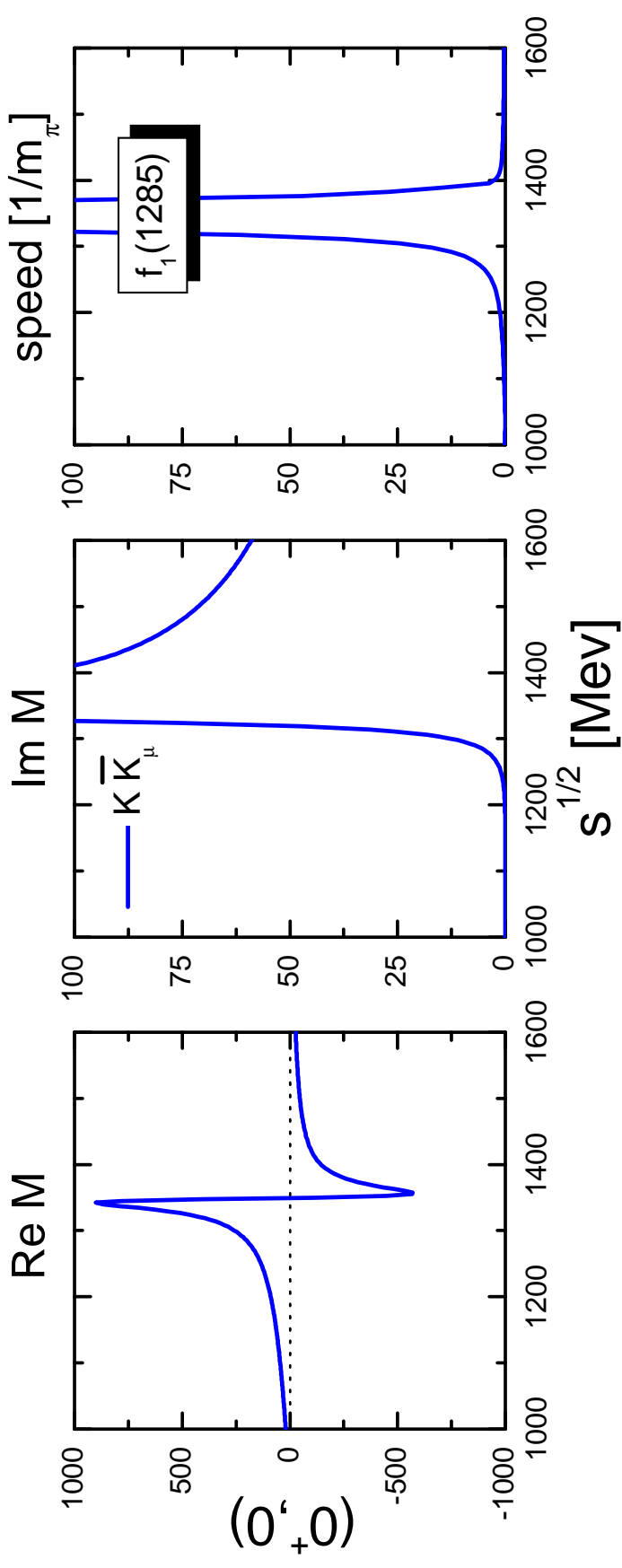
$h_1(1170)$ : seen only through  $\pi \rho_\mu$  channel  $\Gamma \simeq 360$  MeV

$h_1(1380)$ : dominant decay into  $K_\mu \bar{K}$  and  $K \bar{K}_\mu$   $\Gamma \simeq 80$  MeV



# $J^P = 1^+$ meson resonances ( $Q^1$ )

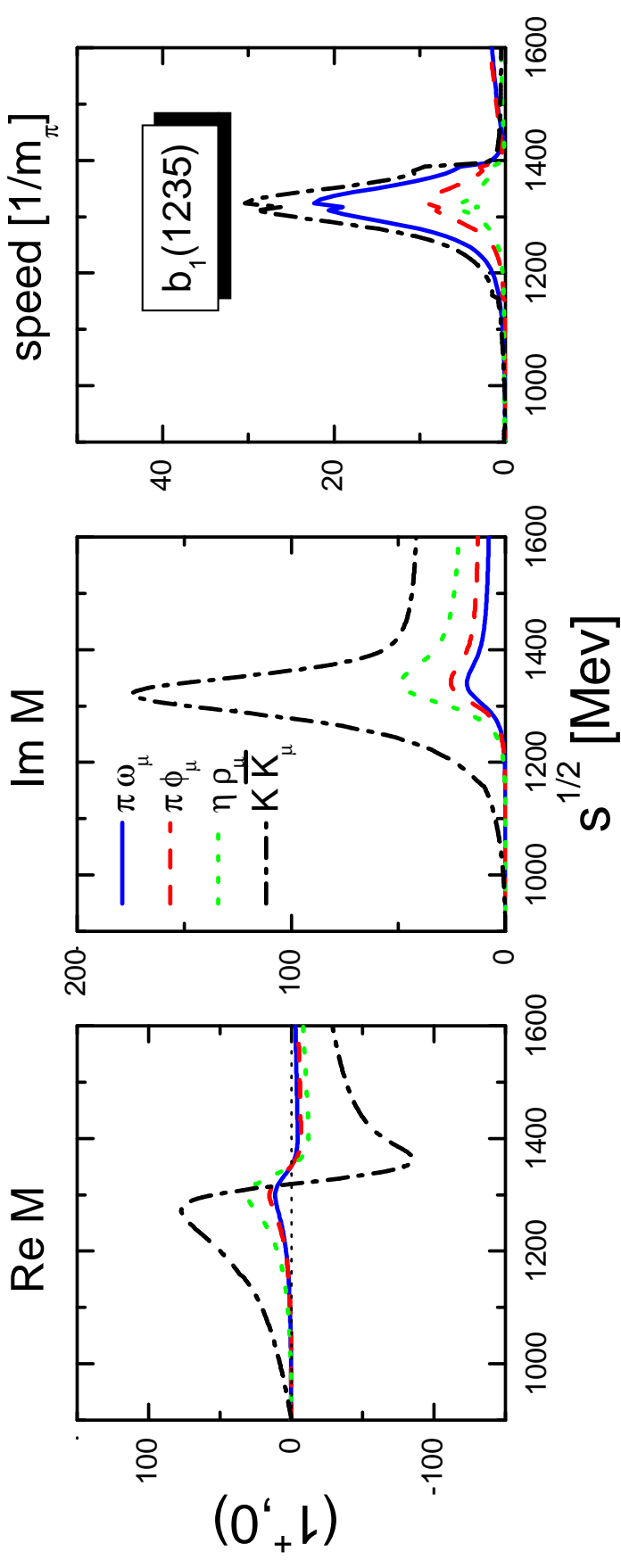
- channel  $(0^+, 0)$ : octet state



- experimental width:  $\Gamma \simeq 20$  MeV

# $J^P = 1^+$ meson resonances ( $Q^1$ )

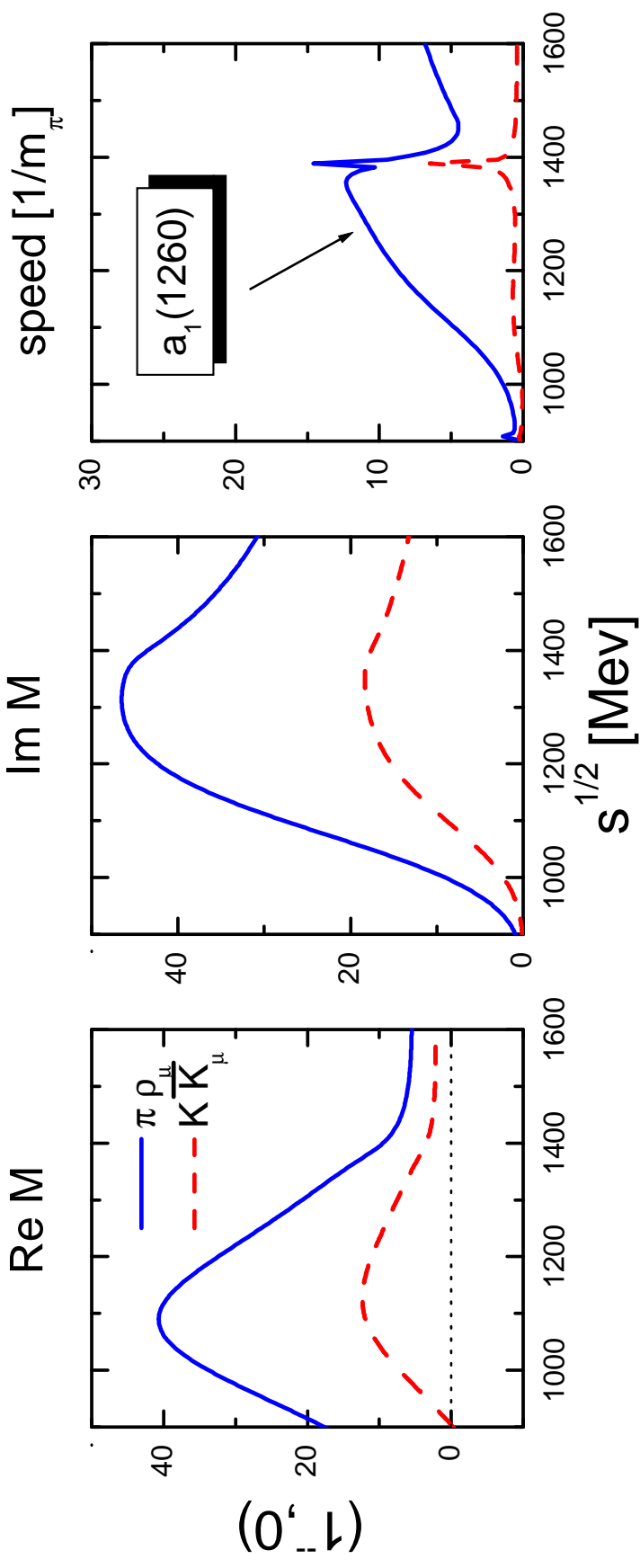
- channel  $(I^G, S) = (1^+, 0)$ : octet state



- experimental width dominated by  $\pi \omega_\mu$  channel:  $\Gamma \simeq 140$  MeV
- couples strongly to  $K_\mu \bar{K}$  and  $K \bar{K}_\mu$  states
- strangeness channels crucial for generation of resonances

# $J^P = 1^+ 1^+$ meson resonances ( $Q^1$ )

- channel  $(1^-, 0)$ : octet state



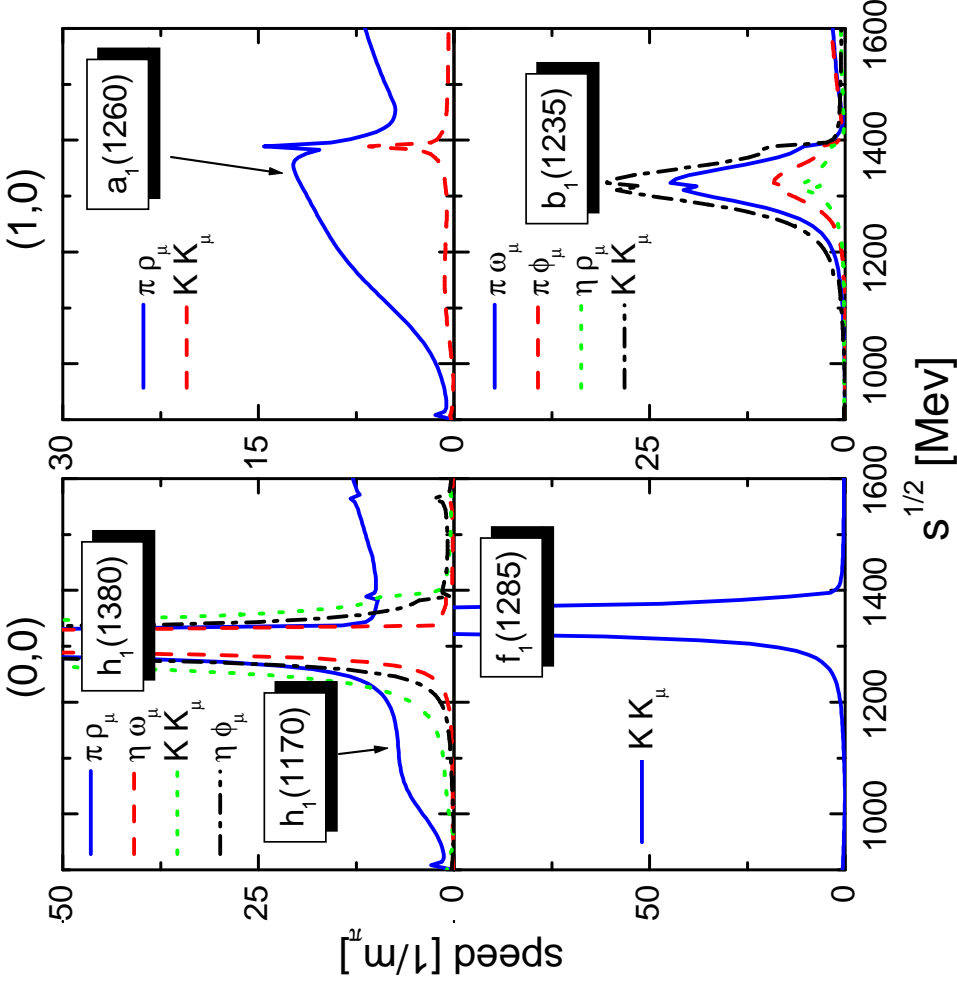
- couples strongly to  $\pi \rho_\mu$  state

# $J^P = 1^+$ meson resonances ( $Q^1$ )

- Chiral SU(3) symmetry:

2 octets and singlet

$$8 \otimes 8 = 27 \oplus \bar{10} \oplus 10 \oplus 8 \oplus 8 \oplus 1$$



- Predictions:

- $h_1(1380)$ ,  $f_1(1285)$  and  $b_1(1235)$  couples strongly to  $K \bar{K}_\mu$ -channel
- $h_1(1380) \leftrightarrow (I^G = 0^+)$  state

# Charmed meson and baryon resonances

- **Heavy-light mesons:**  $(c \bar{q}_i) - \text{SU}(3)$  anti-triplet [3]
  - $0^-$  mesons  $H = (D_0(1867), D_+(1867), D_s(1969))$
  - $1^-$  mesons  $H^\mu = (D_0^\mu(2008), D_+^\mu(2008), D_s^\mu(2110))$
- **Heavy-light baryons:**  $(c q_i q_j) - \text{SU}(3)$  anti-triplet [3] or sextet [6]
  - $\frac{1}{2}^+$   $[\bar{3}]$ -baryons  $B_{[\bar{3}]} = (\Xi_c(2470), \Lambda_c(2284))$
  - $\frac{1}{2}^+$   $[6]$ -baryons  $B_{[6]} = (\Sigma_c(2453), \Xi'_c(2580), \Omega_c(2704))$

- **Chiral SU(3) symmetry predicts:**

$$\bar{3} \otimes 8 = \bar{3} \oplus 6 \oplus \bar{15}$$

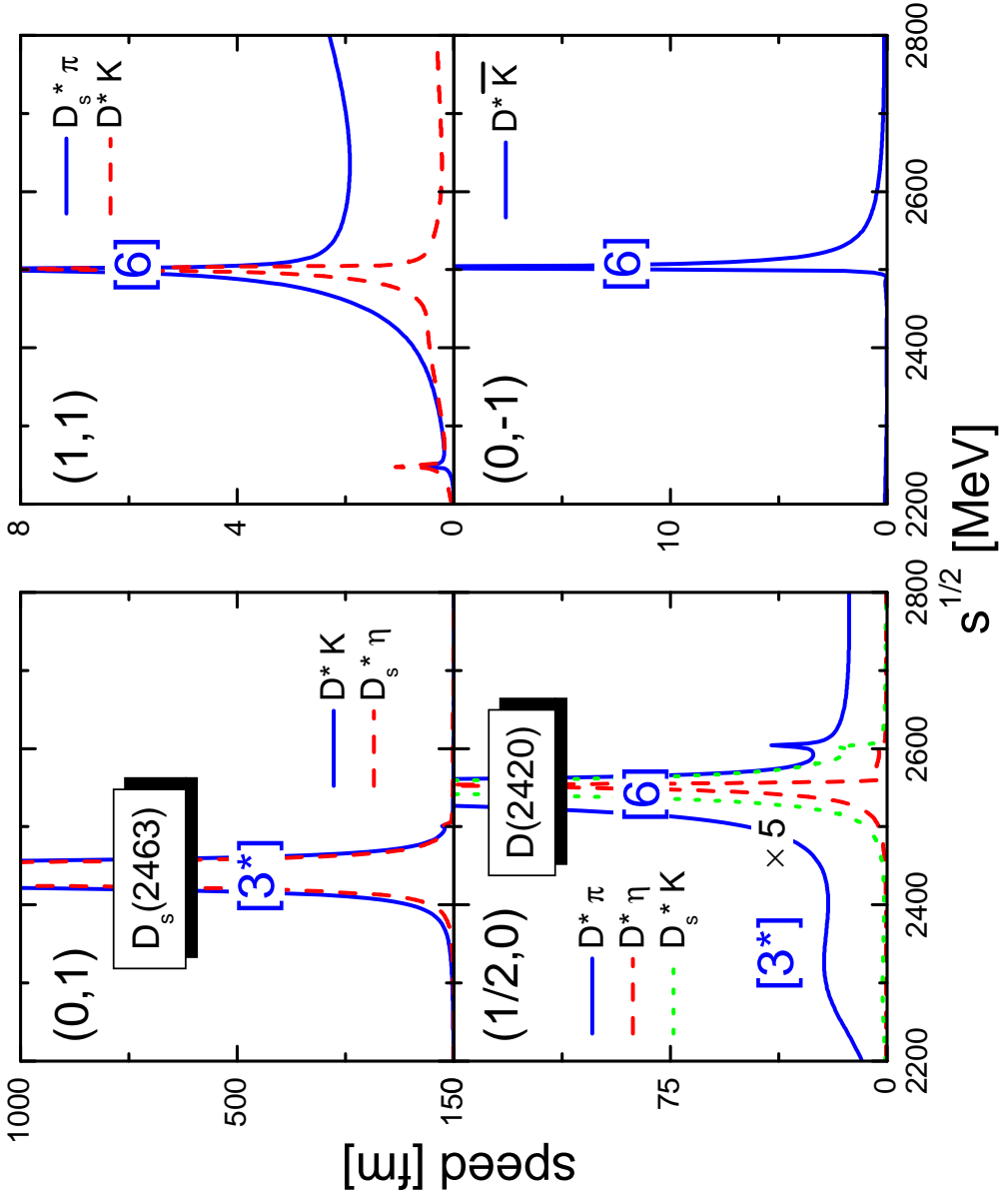
- attraction in  $[\bar{3}]$  and  $[6]$  but repulsion in  $[\bar{15}]$
- $6 \otimes 8 = \bar{3} \oplus 6 \oplus \bar{15} \oplus 24$
- attraction in  $[\bar{3}]$ ,  $[6]$ , and  $[\bar{15}]$  but repulsion in  $[24]$



# $J^P = 1^+$ charmed meson resonances ( $\mathbf{Q}^1$ )

- Chiral SU(3) predicts: anti-triplet and sextet states

$$\bar{\mathbf{3}} \otimes \mathbf{8} = \bar{\mathbf{3}} \oplus \mathbf{6} \oplus \bar{\mathbf{15}}$$



- bound  $[\bar{\mathbf{3}}]$  in  $(0, 1)$  at 2440 MeV [CLEO at 2463 MeV]

- $M(1^+) - M(0^+) \simeq 140$  MeV

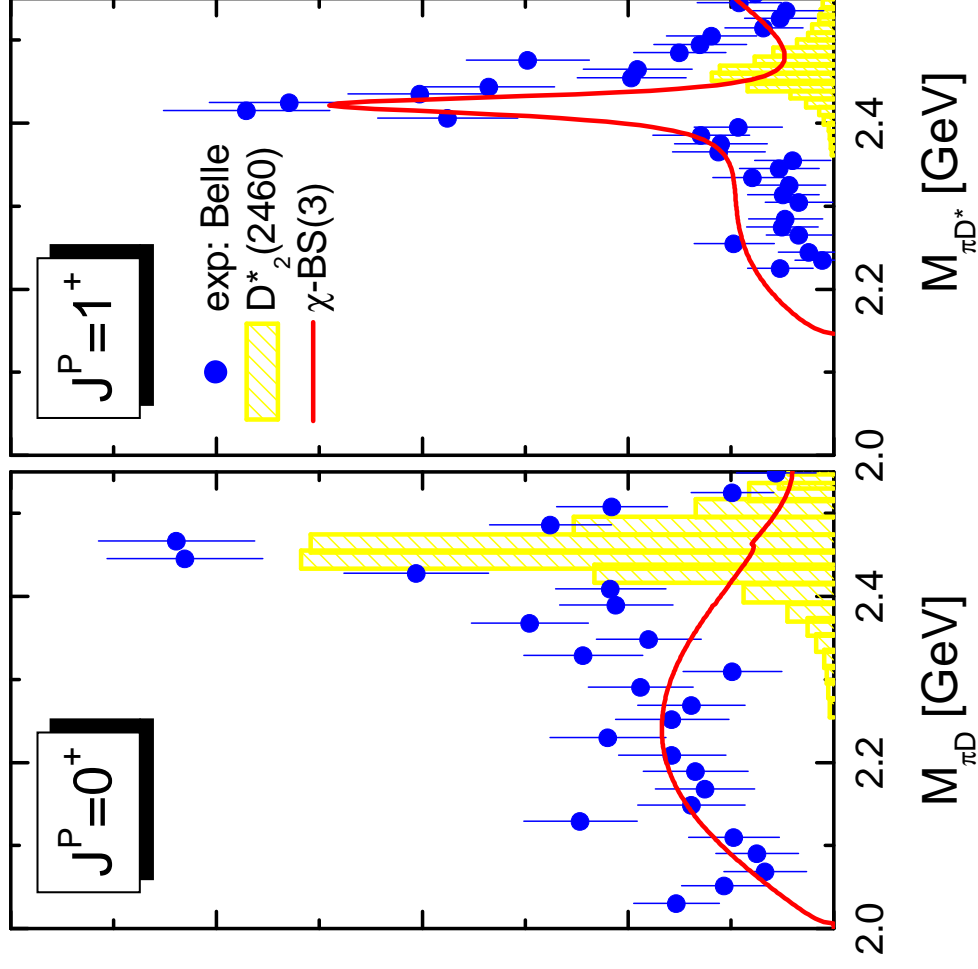
- broad  $[\bar{\mathbf{3}}]$  in  $(\frac{1}{2}, 0)$  at 2325 MeV

- narrow [6] in  $(\frac{1}{2}, 0)$  at 2552 MeV [identify with D(2420)]

- cusp structures in  $(0, -1)$  and  $(1, 1)$

# Charmed meson resonance spectrum ( $Q^2$ )

✓ Chiral corrections: 3 parameters tuned to data



✓ "hidden"  $0^+$ -resonance  $(I, S) = (\frac{1}{2}, 0)$

- narrow [6]-state at 2389 MeV
- couples weakly to  $\pi D(1867)$  channel

✓ Is the  $D(2420)$  a sextet state ?

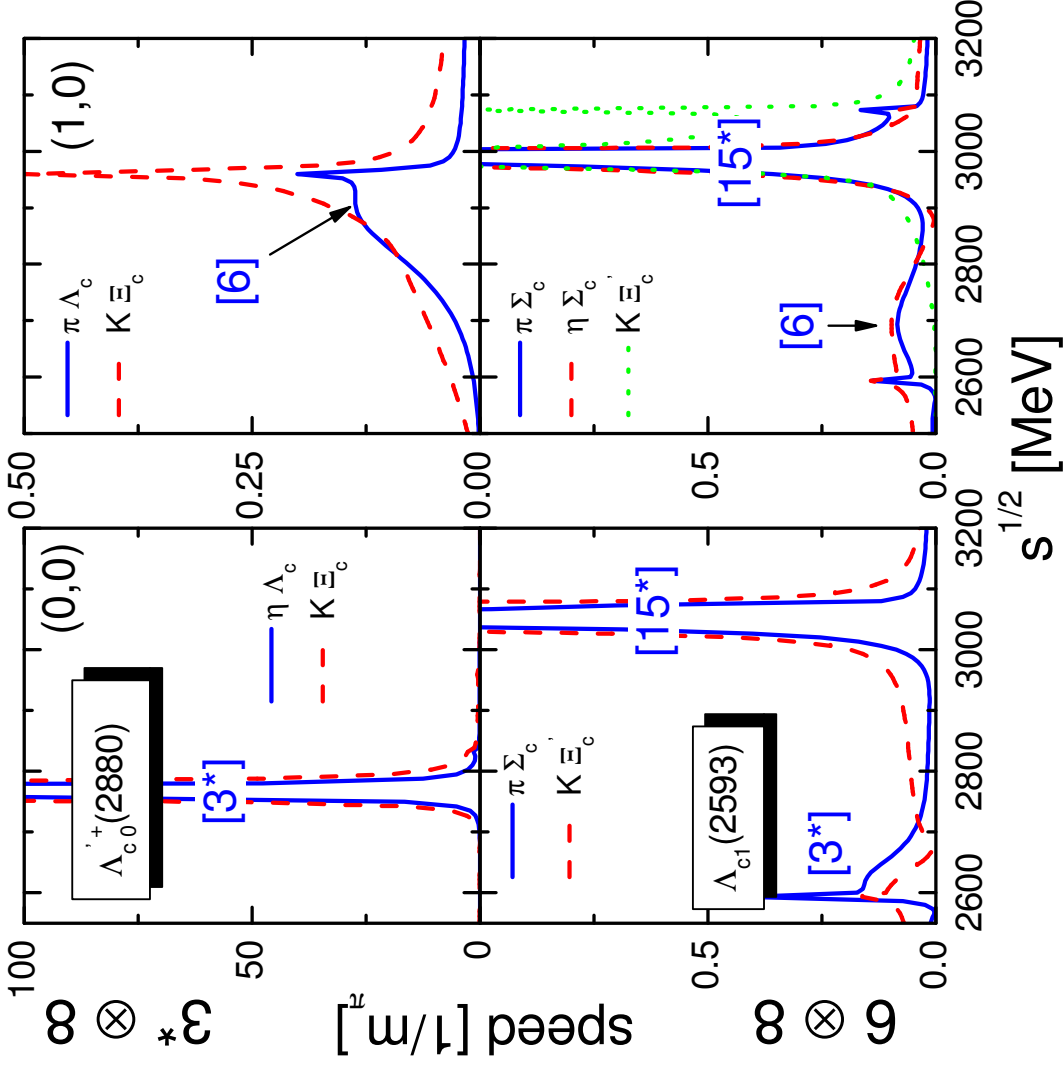
- where is the heavy-quark partner of the  $2^+ D(2460)$  ?

✓ charmed mesons with negative strangeness

- $\bar{K}$  bound at  $D(1867)$  and  $D_\mu(2008)$
- predict: 2352 MeV ( $J^P = 0^+$ ) and 2416 MeV ( $J^P = 1^+$ )



# $J^P = \frac{1}{2}^-$ charmed baryon resonances



✓ Chiral SU(3) dynamics predicts:

$$\bar{3} \otimes 8 = \bar{3} \oplus 6 \oplus \bar{15}$$

$$6 \otimes 8 = \bar{3} \oplus 6 \oplus \bar{15} \oplus 24$$

✓ bound  $[3]$  in  $(0,0)_{[3]}$  at 2767 MeV

• identify with CLEO's  $\Lambda_c(2880)$

✓ broad  $[3]$  in  $(0,0)_{[6]}$  at 2650 MeV

• identify with CLEO's  $\Lambda_c(2593)$

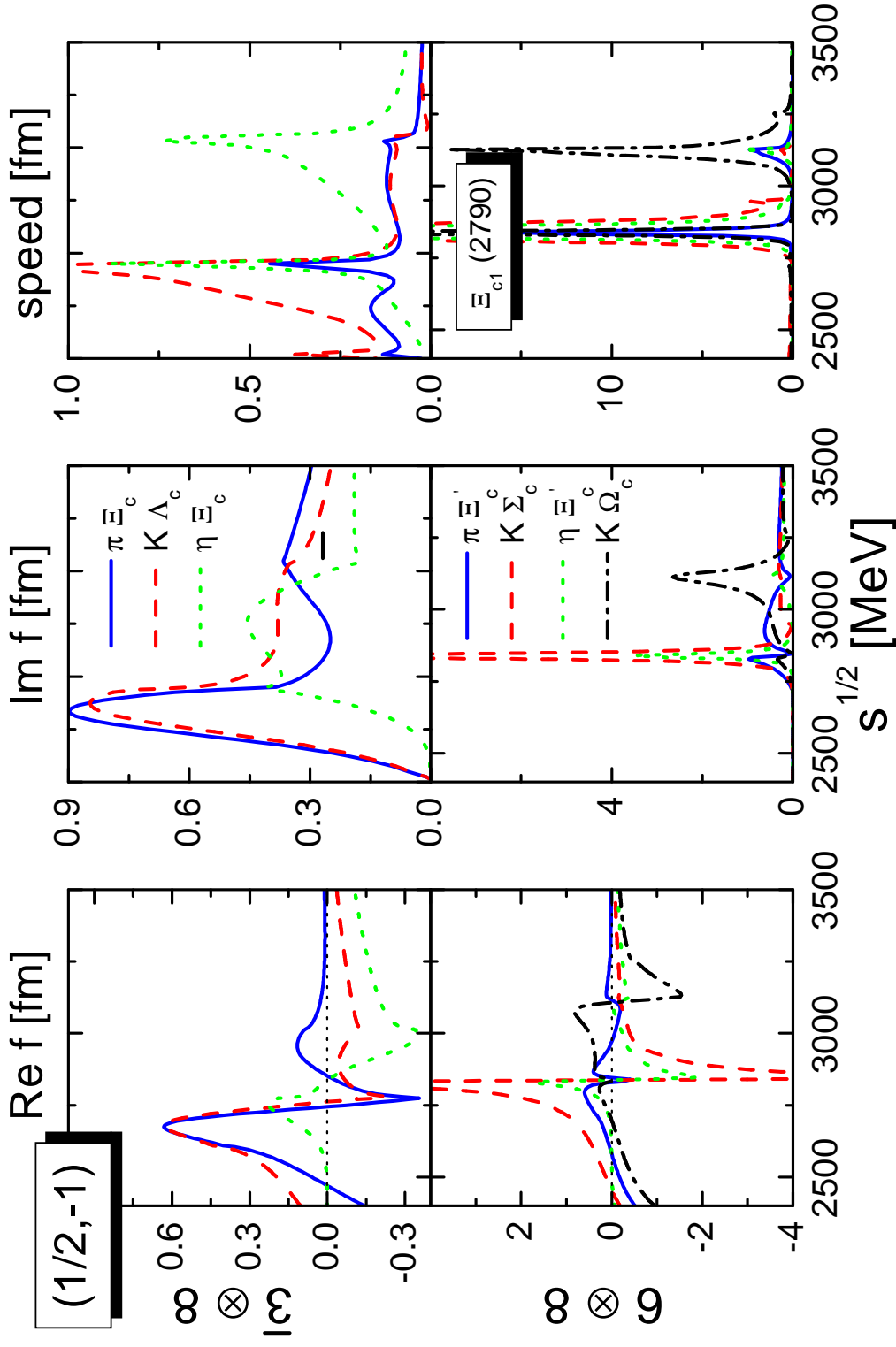
✓ mixing of  $(0,0)_{[3]}$  and  $(0,0)_{[6]}$  sectors

• level-level repulsion  $\rightarrow$  good!

✓ predict exotic resonances

• states with  $(\frac{1}{2}, +1)$ ,  $(\frac{3}{2}, -1)$  and  $(1, -2)$   
systems of  $K @ \Sigma_c$ ,  $\bar{K} @ \Sigma_c$  and  $\bar{K} @ \Xi_c'$

$J^P = \frac{1}{2}^-$  charmed baryon resonances



✓ recover CLEO's  $\Xi_c(2790)$  - complex coupled channel dynamics!

# Summary

- ✓ meson and baryon resonances that do not belong to the large- $N_c$  ground states are dynamically generated by coupled-channel dynamics
  - degrees of freedom: large- $N_c$  meson and baryon ground-state fields
  - crossing symmetry constrains renormalization scheme
  - independence on choice of chiral coordinates by covariant on-shell reduction
  - at leading order: parameter-free prediction for baryon and meson resonances in light and heavy-light quark sectors
  - prediction of new multiplets