Scalar Mesons in Radiative ϕ Decays and π-π Scattering

Masayasu Harada (Nagoya Univ.)

based on

- D.Black, M.H. and J.Schechter, Phys. Rev. Lett. **88**, 181603 (2002)
- M.Harada, F.Sannino and J.Schechter, hep-ph/0309206
  (To appear in PRD).

@ Yukawa Institute (February 17, 2004)

1. Introduction

☆ Light Scalar mesons

$a_0(980)$, $f_0(980)$, "$\kappa(900)$" , "$\sigma(560)$"

scalar nonet

◎ **Properties**

- **quark structure ･･･ 2-quark or 4-quark ?**
- **interactions with other mesons**

･･･

⇓

clue for understanding of QCD

◎ Effective Lagrangian

study of the properties of the scalar mesons

- **Radiative decays involving scalar mesons**
- **π - π scattering in large Nc QCD**

Outline

1. Introduction

2. Effective Lagrangian for Scalar Mesons

   - Masses of scalar mesons and thier couplings to pseudoscalar mesons -

3. Radiative Decays Involving Scalar Mesons

4. π - π Scattering in large Nc QCD

5. Summary

2. Effective Lagrangian for Scalar Mesons

- Masses of scalar mesons and their couplings to pseudoscalar mesons -

2.1. Scalar meson nonet field

D.Black, A.H.Fariborz, F.Sannino and J.Schechter, PRD 59, 074026 (1999)

$$N = \begin{pmatrix} (N_T + a_0^0)/\sqrt{2} & a_0^+ & \kappa^+ \\ a_0^- & (N_T - a_0^0)/\sqrt{2} & \kappa_0 \\ \kappa^- & \bar{\kappa}^0 & N_S \end{pmatrix}$$

$$\begin{pmatrix} \sigma \\ f_0 \end{pmatrix} = \begin{pmatrix} \cos\theta_s & -\sin\theta_s \\ \sin\theta_s & \cos\theta_s \end{pmatrix} \begin{pmatrix} N_S \\ N_T \end{pmatrix}$$

θ_s $\cdots$ “scalar mixing angle”

cf: vector meson nonet field

$$V = \begin{pmatrix} (\omega + \rho^0)/\sqrt{2} & \rho^+ & K^{*+} \\ \rho^- & (\omega - \rho^0)/\sqrt{2} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}$$

2.2. Relation to quark structure

- $q\bar{q}$ picture $\cdots$ $\cos\theta_s = 0$: $\theta_s = \pm 90^\circ$

$$N = \begin{pmatrix} (\sigma + a_0^0)/\sqrt{2} & a_0^+ & \kappa^+ \\ a_0^- & (\sigma - a_0^0)/\sqrt{2} & \kappa_0 \\ \kappa^- & \bar{\kappa}^0 & f_0 \end{pmatrix} \sim \begin{pmatrix} \bar{u}u & \bar{u}d & \bar{u}s \\ \bar{d}u & \bar{d}d & \bar{d}s \\ \bar{s}u & \bar{s}d & \bar{s}s \end{pmatrix}$$

- $qq\bar{q}\bar{q}$ picture $\cdots$ $\cos\theta_s = 0$: $\theta_s = 0^\circ,\ 180^\circ$

$$N = \begin{pmatrix} (f_0 + a_0^0)/\sqrt{2} & a_0^+ & \kappa^+ \\ a_0^- & (f_0 - a_0^0)/\sqrt{2} & \kappa_0 \\ \kappa^- & \bar{\kappa}^0 & \sigma \end{pmatrix} \sim \begin{pmatrix} \bar{s}\bar{d}ds & \bar{s}\bar{d}us & \bar{s}\bar{d}ud \\ \bar{s}\bar{u}ds & \bar{s}\bar{u}us & \bar{s}\bar{u}ud \\ \bar{u}\bar{d}ds & \bar{u}\bar{d}us & \bar{u}\bar{d}ud \end{pmatrix}$$

2.3. Mass terms for scalar nonet

D.Black, A.H.Fariborz, F.Sannino and J.Schechter, PRD 59, 074026 (1999)

$$a\,\mathrm{tr}\,[NN] + b\,\mathrm{tr}\,[\mathcal{M}NN] + c\,\mathrm{tr}\,[N]\,\mathrm{tr}\,[N] + d\,\mathrm{tr}\,[\mathcal{M}N]\,\mathrm{tr}\,[N]$$

$$\mathcal{M} = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$$

quark mass

- Determination of θ_s from scalar masses

$M_{a_0} \simeq 980\,\mathrm{MeV}$ $M_{f_0} \simeq 980\,\mathrm{MeV}$

$M_\sigma \simeq 560\,\mathrm{MeV}$ (π-π scattering)

$M_\kappa \simeq 900\,\mathrm{MeV}$ (π-K scattering)

⇓

values of a, b, c, d $\Rightarrow$ $\theta_s = \begin{cases} -20^\circ & \text{(close to } qq\bar{q}\bar{q} \text{ picture)} \\ -90^\circ & \text{(pure } q\bar{q} \text{ picture)} \end{cases}$

2.4. Pseudoscalar meson nonet field

$$P = \begin{pmatrix} (\eta_T + \pi_0^0)/\sqrt{2} & \pi_0^+ & K^+ \\ \pi_0^- & (\eta_T - \pi_0^0)/\sqrt{2} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}$$

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta_p & -\sin\theta_p \\ \sin\theta_p & \cos\theta_p \end{pmatrix} \begin{pmatrix} \eta_T \\ \eta_S \end{pmatrix}$$

$\theta_p \simeq 37^\circ$ $\cdots$ "η-η' mixing angle"

2.5. Interactions among one scalar and two pseudoscalars

◎ **light pseudoscalar mesons (p, *K*, h)**

··· **approximate Nambu-Goldstone bosons**

associated with $SU(3)_L \times SU(3)_R \to SU(3)_V$

⇩

☆ **Pseudoscalar mesons couple to other mesons with derivative interaction**

$$-\mathcal{L}_{NPP} = A\,\varepsilon_{abc}\varepsilon^{def} N_a^d \partial_\mu P_b^e \partial^\mu P_c^f + B\,\mathrm{Tr}\,[N]\,\mathrm{Tr}\,[\partial_\mu P \partial^\mu P]$$

$$+\, C\,\mathrm{Tr}\,[N\partial_\mu P]\,\mathrm{Tr}\,[\partial^\mu P] + D\,\mathrm{Tr}\,[N]\,\mathrm{Tr}\,[\partial_\mu P]\,\mathrm{Tr}\,[\partial^\mu P]$$

☆ **All of *NPP* couplings are expressed by 4 parameters, *A*, *B*, *C*, *D*** (and θ_s)

☆ Determination of *A*, *B*, *C*, *D* and θ_s

D.Black, A.H.Fariborz, F.Sannino and J.Schechter, PRD 59, 074026 (1999)
A.H.Fariborz and J.Schechter, PRD 60, 034002 (1999)

◎ **Fit**

π-K scattering π-π scattering

$\eta' \to \eta\pi\pi$ decay

⇓

$A \simeq 2.5\,\mathrm{GeV}^{-1}$ $B \simeq -2.0\,\mathrm{GeV}^{-1}$

$C \simeq -2.3\,\mathrm{GeV}^{-1}$ $D \simeq -2.3\,\mathrm{GeV}^{-1}$

$\theta_s \simeq -20^\circ \quad \cdots$ (close to $qq\bar{q}\bar{q}$ picture)

3. Radiative Decays Involving Scalar Mesons

•D.Black, M.H. and J.Schechter, Phys. Rev. Lett. **88**, 181603 (2002)

☆ Radiative decays involving light scalar mesons

scalar Æ g + g

vector Æ scalar + g

scalar Æ vector + g

⇑

◎ **Effective Lagrangian**

← SU(3) flavor symmetry

+ vector meson dominance

3.1. Features of our model

◎ SU(3) flavor symmetry

← Effective Lagrangian

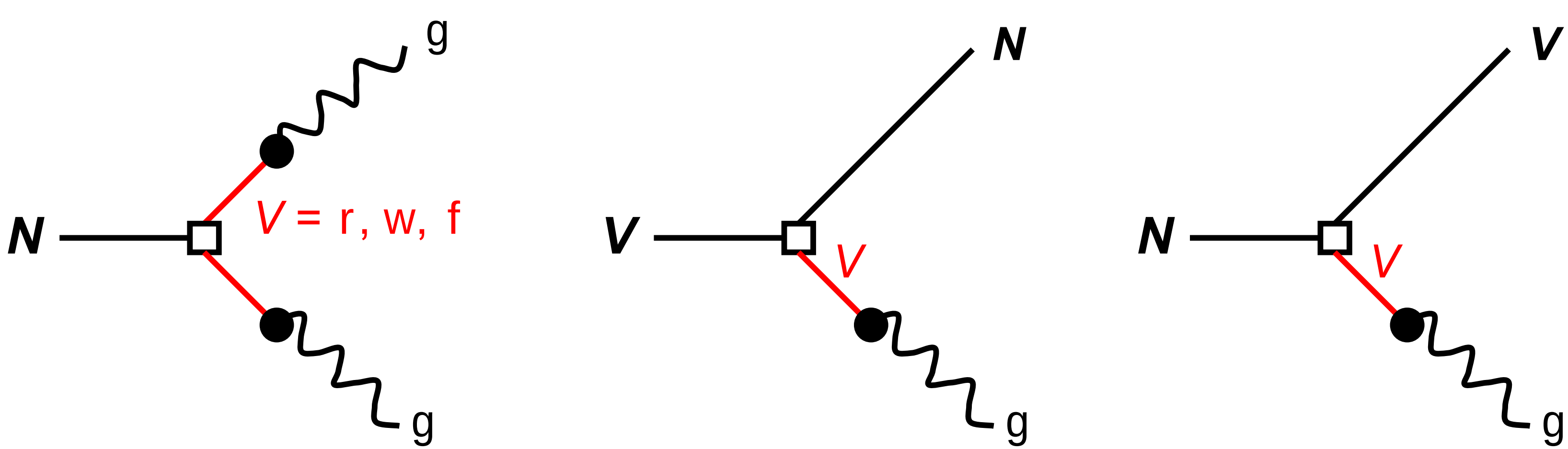
◎ Vector meson dominance (VMD)

- Photon couples to mesons dominantly through vector mesons.

VMD works very well for EM form factor of p.

3.2. Vector Meson Dominance (VMD)
in (*N*Ægg), (*V*Æ*N*g) and (*N*Æ*V*g)

***N* Æ gg** ***V* Æ *N*g** ***N* Æ *V*g**



VMD î *NVV* vertex determines
(*N*Ægg), (*V*Æ*N*g) and (*N*Æ*V*g)

3.3. Effective Lagrangian for *NVV* vertices

$$\mathcal{L}_{NVV} = \beta_A \varepsilon_{abc}\varepsilon^{a'b'c'} [F_{\mu\nu}(V)]^a_{a'} [F^{\mu\nu}(V)]^b_{b'} N^c_{c'}$$

$$+ \beta_B \mathrm{Tr}[N] \mathrm{Tr}[F_{\mu\nu}(V) F^{\mu\nu}(V)]$$

$$+ \beta_C \mathrm{Tr}[N F_{\mu\nu}(V)] \mathrm{Tr}[F^{\mu\nu}(V)]$$

$$+ \beta_D \mathrm{Tr}[N] \mathrm{Tr}[F_{\mu\nu}(V)] \mathrm{Tr}[F^{\mu\nu}(V)]$$

β_D ··· not contribute

☆ **SU(3) flavor symmetry + VMD**

$\Rightarrow$ 3 parameters β_A, β_B and β_C determine

all of $(N \to \gamma\gamma)$, $(V \to N\gamma)$ and $(N \to V\gamma)$.

3.4. Analysis 1 ･･･ processes related to a_0 meson

- Determination of β_A and β_C ･･･ Independent of θ_S

$$\begin{cases} \Gamma(a_0 \to \gamma\gamma) \propto |\beta_A|^2 \\ \Gamma(\phi \to a_0\gamma) \propto |\beta_C - 2\beta_A|^2 \end{cases} \Rightarrow \begin{cases} \beta_A = 0.72 \pm 0.12\,\mathrm{GeV}^{-1} \\ \beta_C = (7.7 \pm 0.5\,,\, -4.8 \pm 0.5)\,\mathrm{GeV}^{-1} \end{cases}$$

- Predicions

$$\left|\frac{4}{3}\beta_A\right|^2 \propto \Gamma(a_0 \to \rho\gamma) = 3.0 \pm 1.0\,\mathrm{keV}$$

$$|2\beta_C|^2 \propto \Gamma(a_0 \to \omega\gamma) = (641 \pm 87\,,\, 251 \pm 54)\,\mathrm{keV}$$

◎ large hierarchy

$$\frac{\beta_C}{\beta_A} \gg 1 \Rightarrow \frac{\Gamma(a_0 \to \omega\gamma)}{\Gamma(a_0 \to \rho\gamma)} \gg 1$$

3.5. Analysis 2 ··· processes related to f_0 meson

- Determination of β_B

$$\Gamma(f_0 \to \gamma\gamma) \propto \left| -\frac{4}{9}\beta_A \left(\sqrt{2}\cos\theta_s + 4\sin\theta_s\right) + \frac{8}{3}\beta_B \left(\sqrt{2}\cos\theta_s + \sin\theta_s\right) \right|^2$$

$$\beta_A = 0.72 \pm 0.12\,\mathrm{GeV}^{-1} \qquad \theta_s \simeq -20^\circ$$

$$\Downarrow$$

$$\beta_B = (-0.62 \pm 0.10\,,\, 0.61 \pm 0.10)\,\mathrm{GeV}^{-1}$$

- Predicions

$$(\beta_A\,,\beta_B\,,\beta_C\,,\theta_s) \;\Rightarrow\; \Gamma(f_0 \to \omega\gamma) = 88 \pm 17\,\mathrm{keV}\,,\cdots$$

$$(\beta_A\,,\beta_B\,,\theta_s) \;\Rightarrow\; \Gamma(f_0 \to \rho\gamma) = 3.3 \pm 2.0\,\mathrm{keV}\,,\cdots$$

◎ **large hierarchy**

$$\frac{\beta_C}{\beta_A}\,,\,\frac{\beta_C}{\beta_B} \gg 1 \quad \Rightarrow \quad \frac{\Gamma(f_0 \to \omega\gamma)}{\Gamma(f_0 \to \rho\gamma)} \gg 1$$

☆ Analysis for $\theta_s \simeq -90^\circ$ (work in progress; preliminary)

(Note: $f_0\pi\pi$ coupling becomes too large to explain $\pi\pi$ scattering amplitude.)

- $\beta_B = (1.1 \pm 0.1\,,\, 0.12 \pm 0.13)\ \mathrm{GeV}^{-1}$

  cf: $\beta_B = (-0.62 \pm 0.10\,,\, 0.61 \pm 0.10)\ \mathrm{GeV}^{-1}$ for $\theta_s \simeq -20^\circ$

- Predicions

$$(\beta_A\,,\,\beta_B\,,\,\beta_C\,,\,\theta_s) \ \Rightarrow\ \Gamma(f_0 \to \omega\gamma) = 86 \pm 16\,\mathrm{keV}\,,\ \cdots$$

cf: $88 \pm 17\,\mathrm{keV}$ for $\theta_s \simeq -20^\circ$

$$(\beta_A\,,\,\beta_B\,,\,\theta_s) \ \Rightarrow\ \Gamma(f_0 \to \rho\gamma) = 3.4 \pm 3.2\,\mathrm{keV}\,,\ \cdots$$

cf: $3.3 \pm 2.0\,\mathrm{keV}$ for $\theta_s \simeq -20^\circ$

◎ **large hierarchy**

$$\frac{\beta_C}{\beta_A}\,,\ \frac{\beta_C}{\beta_B} \gg 1 \quad \Rightarrow \quad \frac{\Gamma(f_0 \to \omega\gamma)}{\Gamma(f_0 \to \rho\gamma)} \gg 1$$

for $\theta_s \simeq -20^\circ$ and $\theta_s \simeq -90^\circ$

☆ Analysis on f→ f_0 g

◎ prediction from present analysis

$$\Gamma(\phi \to f_0 \gamma) = 0.21 \pm 0.03\,\mathrm{keV} \quad \ll \Gamma_{\mathrm{exp}} = 1.51 \pm 0.41\,\mathrm{keV}$$

- *K*-loop effect gives an important contribution

[N.N.Achasov and V.N.Ivanchenko, NPB315, 465 (1989)]

Note : non-derivative $f_0 K \bar{K}$ interaction

◎ New analysis in progress (preliminary)

- Inclusion of K-loop effect through derivative $f_0 K \bar{K}$ interaction together with β_A, β_B and β_C terms

⇒ Interference seems to play an important role.

3.6. Short summary on radiative decays

☆ **Analysis on radiative decays**

(*N*Ægg), (*V*Æ*N*g) and (*N*Æ*V*g)

◎ **Effective Lagrangian**

- **SU(3) flavor symmetry**
- **vector meson dominance**

◎ **Predictions** ・・・ **large hierarchy**

$$\frac{\Gamma(a_0 \to \omega\gamma)}{\Gamma(a_0 \to \rho\gamma)} \gg 1 \qquad \frac{\Gamma(f_0 \to \omega\gamma)}{\Gamma(f_0 \to \rho\gamma)} \gg 1$$

for $\theta_s \simeq -20^\circ$ and $\theta_s \simeq -90^\circ$

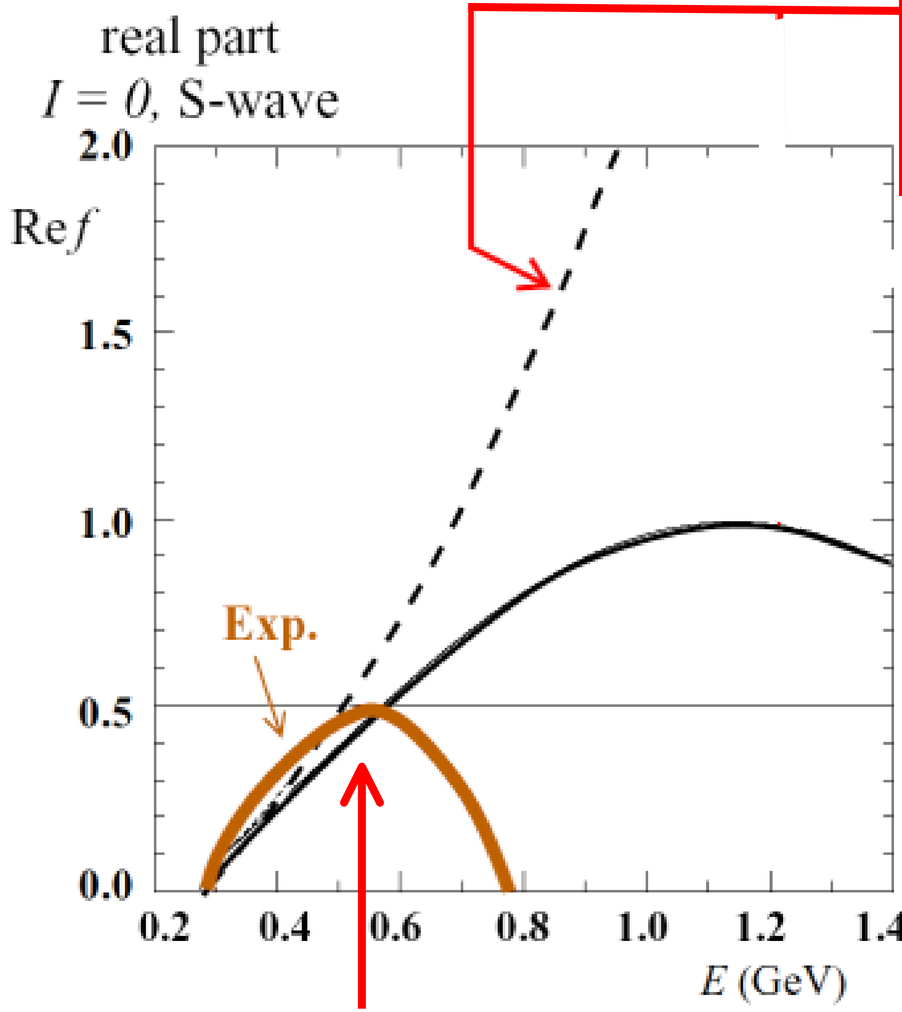
checked by future experiments !

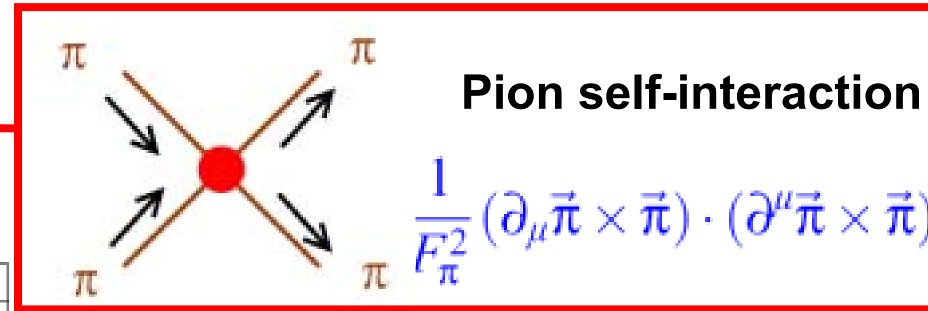
4. $\pi - \pi$ Scattering in Large Nc QCD

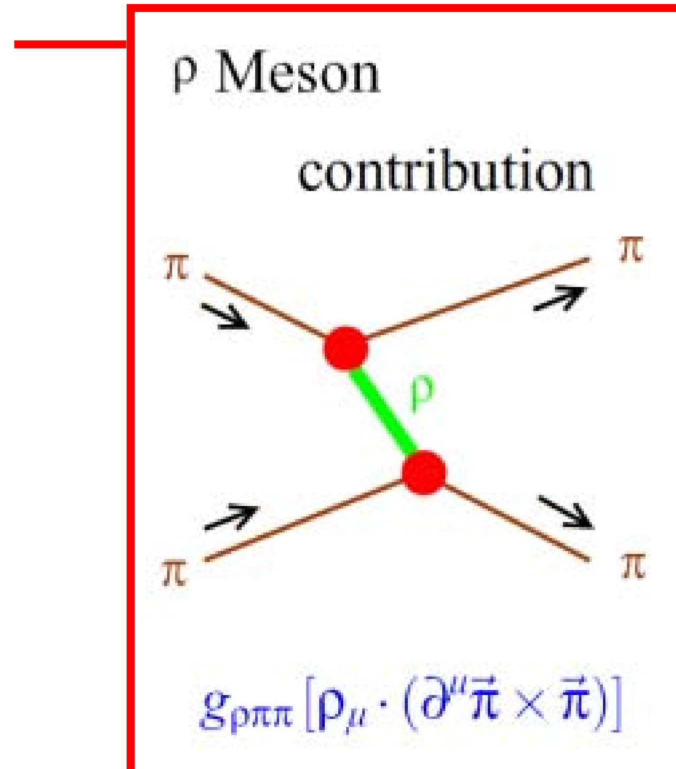
- M.Harada, F.Sannino and J.Schechter, hep-ph/0309206 (To appear in PRD).

4.1. π - π scattering in real QCD

☆ $\pi + \rho$

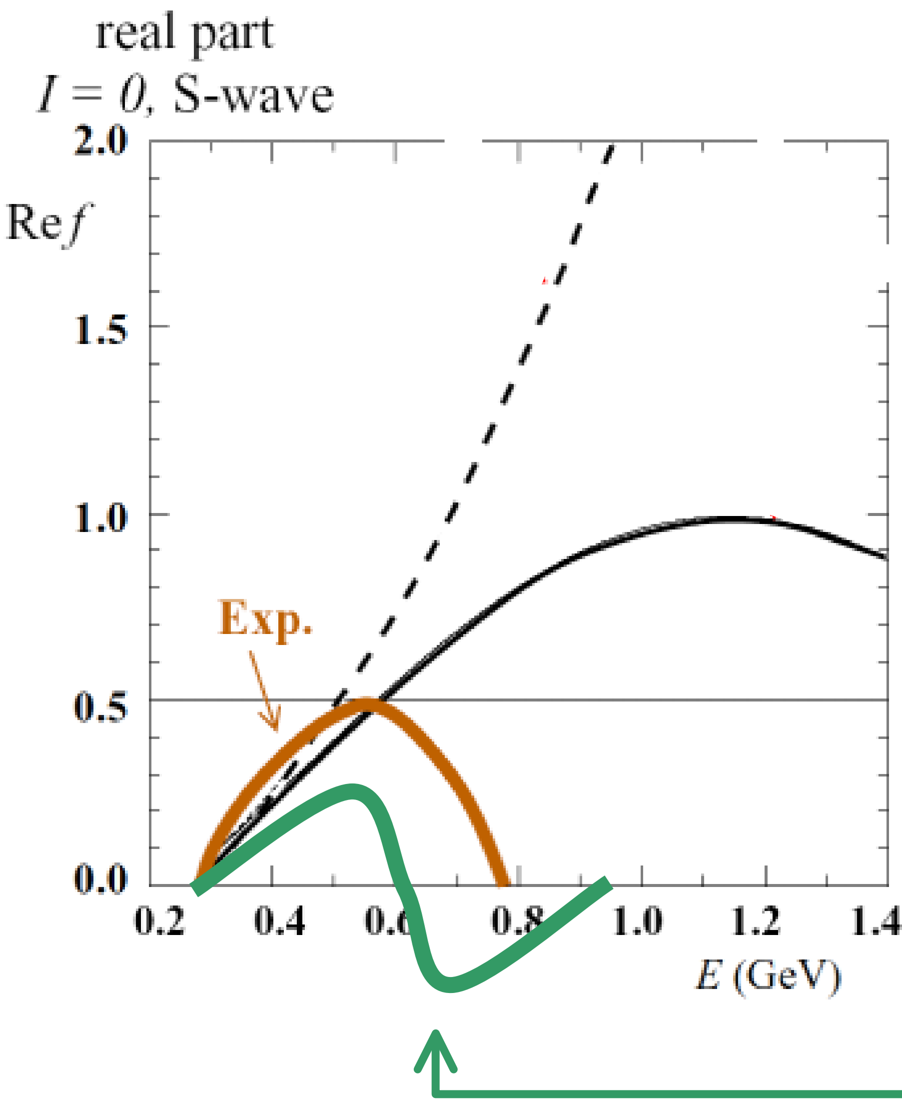






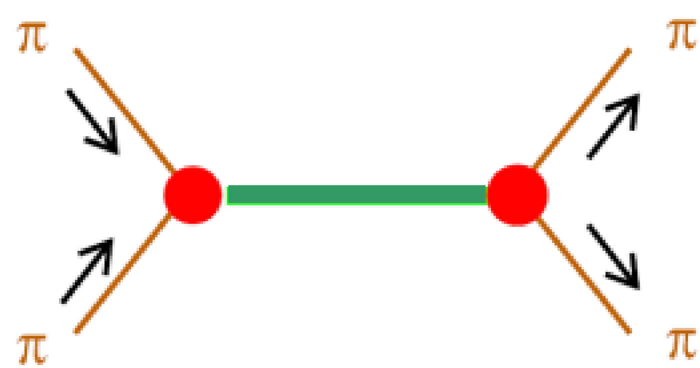
Unitarity violation !

Unitarity ⇒ Resonance !

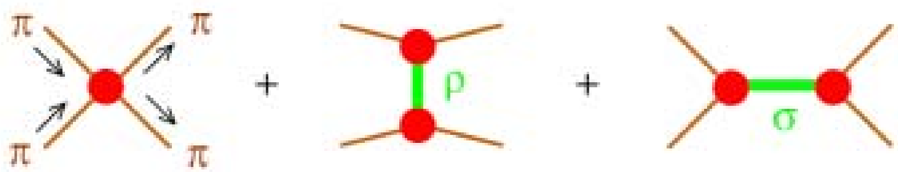


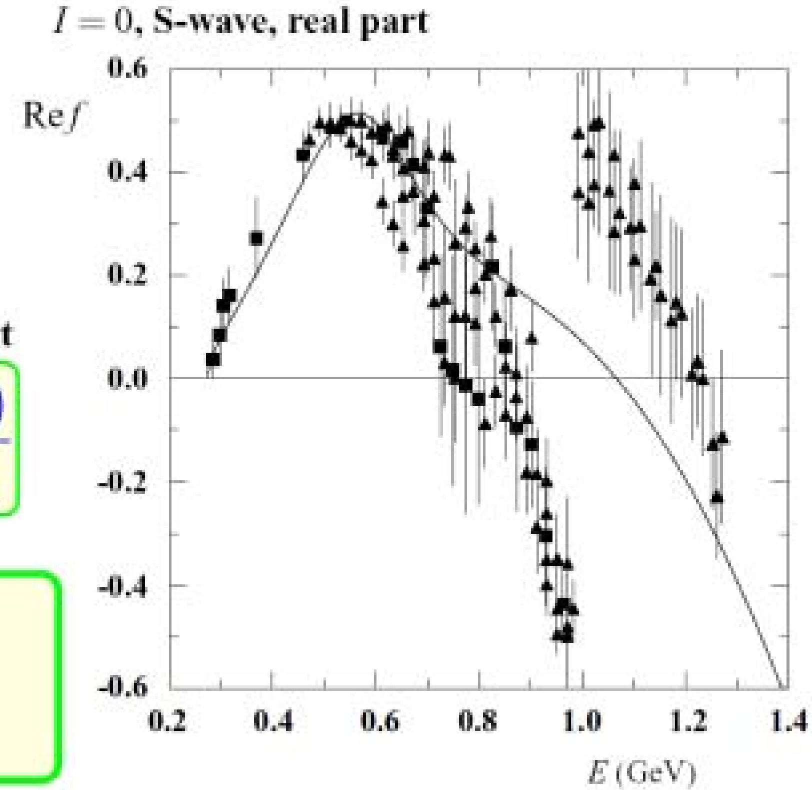
real part of
resonance contribution

$$\frac{(M^2 - E^2)\, MG}{(M^2 - E^2)^2 + M^2 G'^2}$$



☆ $\pi + \rho + \sigma$ contribution





interaction

$$\frac{\gamma_\sigma}{\sqrt{2}} \sigma \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi}$$

→

contribution to the real part

$$\frac{\gamma_\sigma^2}{2} \frac{(M_\sigma^2 - E^2) \cdot (E^2 - 2m_\pi^2)}{(M_\sigma^2 - E^2)^2 + M_\sigma^2 G'^2}$$

$$M_\sigma = 559\,\text{MeV}$$

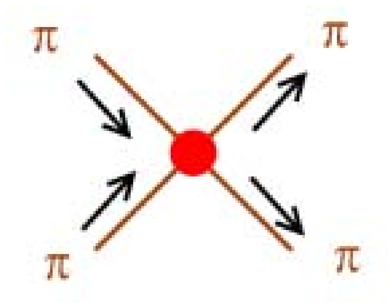
$$G' = 370\,\text{MeV}, \quad \gamma_\sigma = 7.8\,(\text{GeV})^{-1}$$

☆ Lesson from the π-π scattering in real QCD

$M_\sigma \leftarrow$ unitarity violation of $\pi + \rho$ contribution

4.2. π - π scattering in large Nc QCD

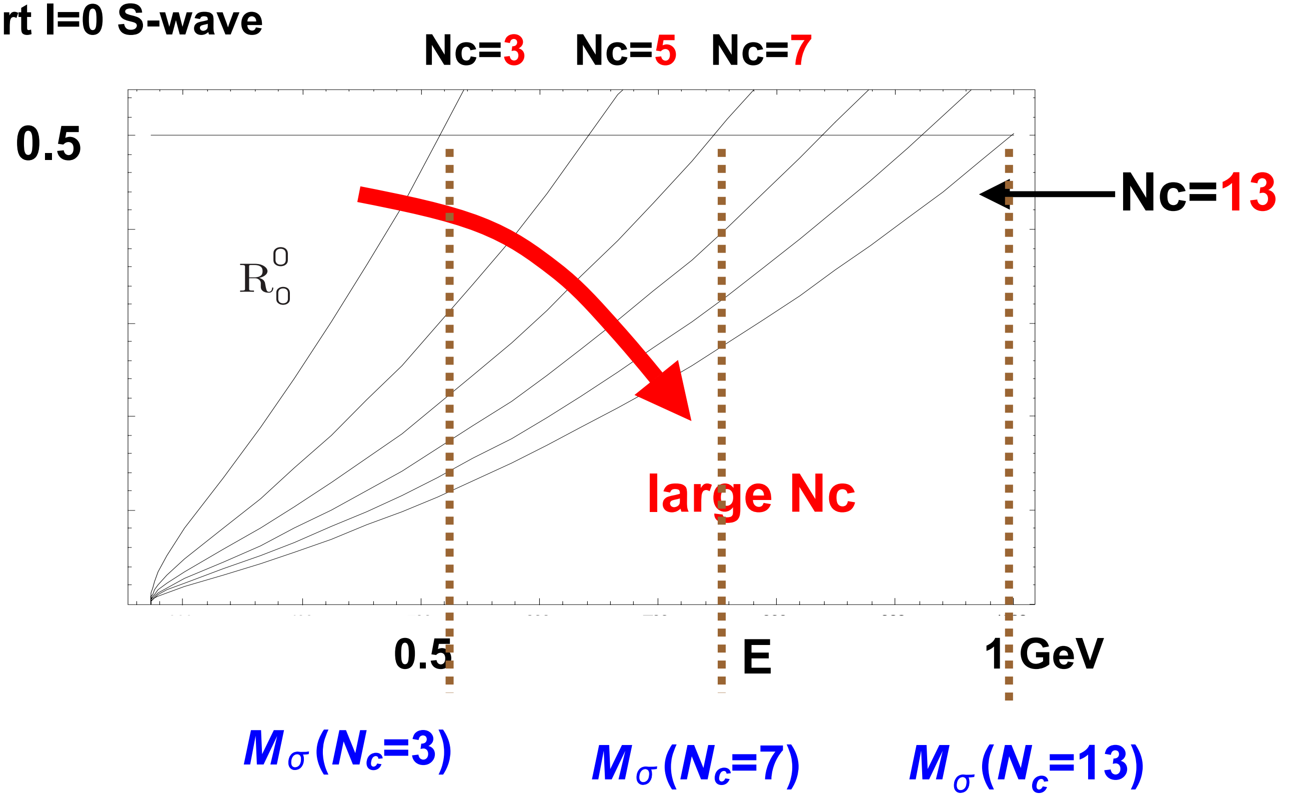
☆ **pion self-interaction**



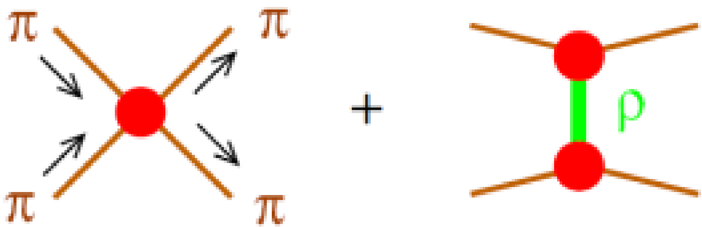
$$\frac{1}{F_\pi^2}(\partial_\mu \vec{\pi} \times \vec{\pi}) \cdot (\partial^\mu \vec{\pi} \times \vec{\pi})$$ ··· smaller for larger Nc

$$(F_\pi)^2 \sim N_c$$

real part I=0 S-wave



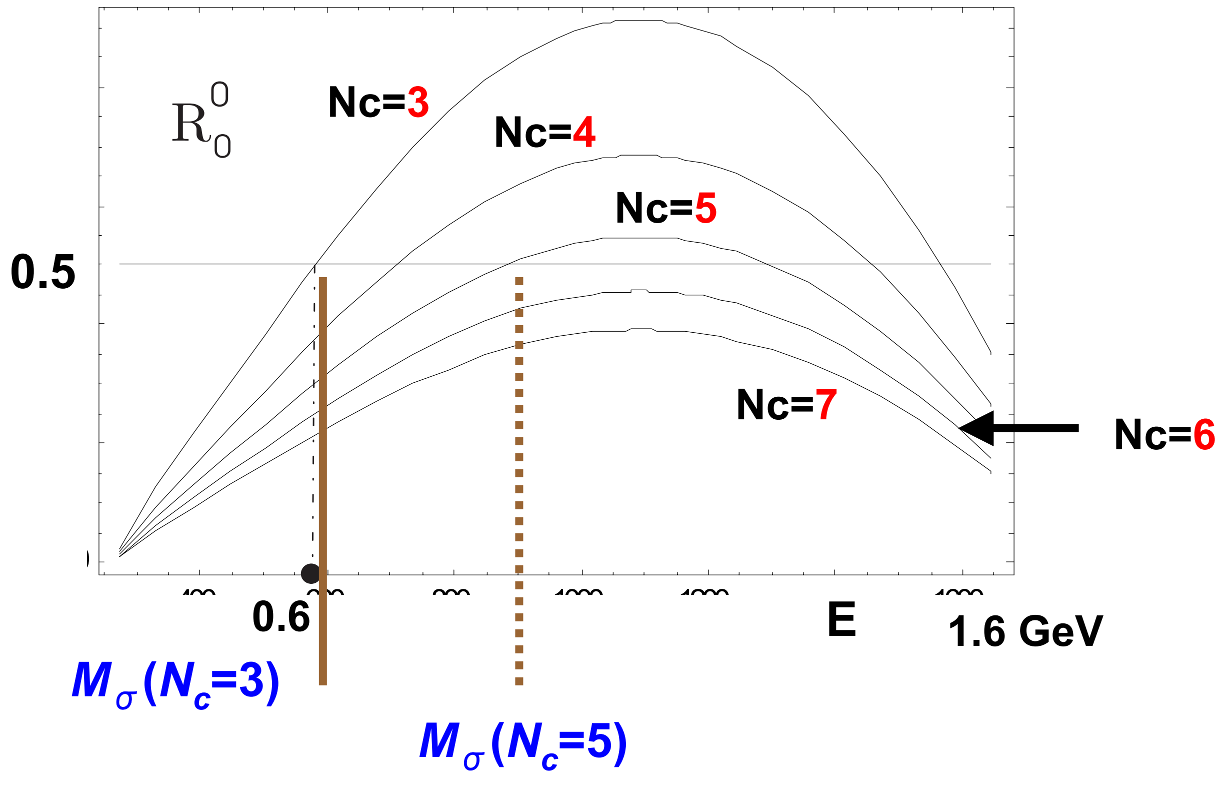
☆ $\pi + \rho$



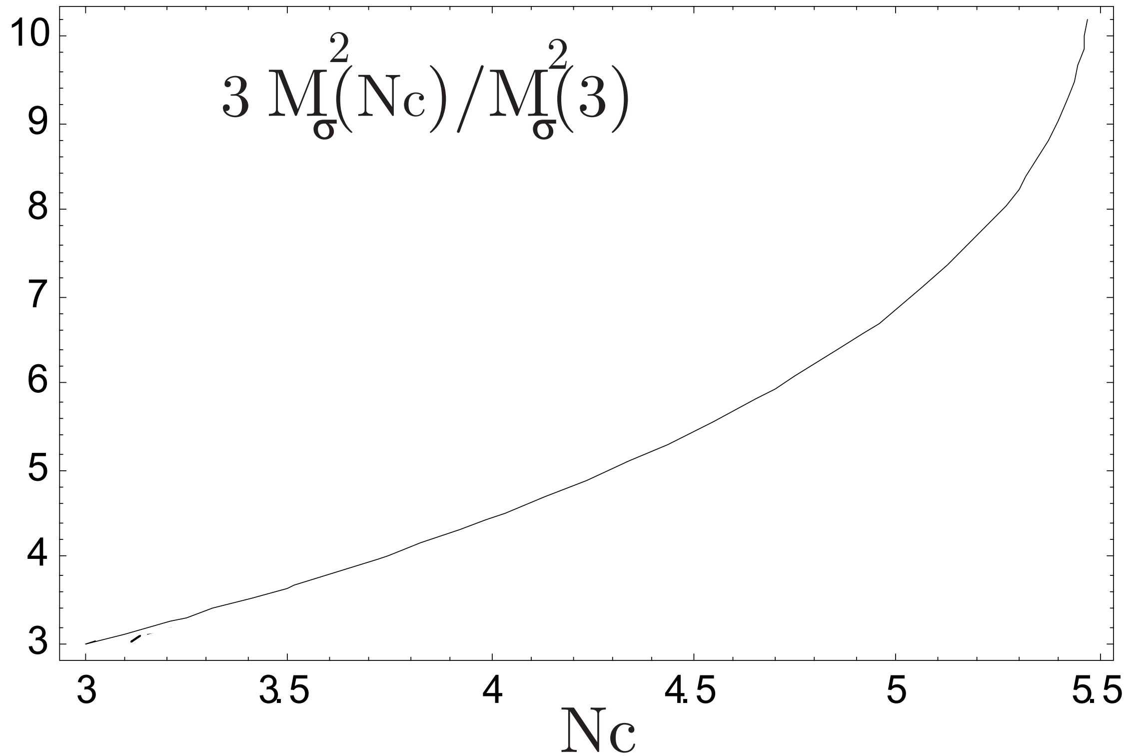
M_ρ ··· independent of N_c

$(g_{\rho\pi\pi})^2 \sim 1/Nc$

real part I=0 S-wave



☆ N_c dependence of M_σ



M_σ becomes larger for larger N_c

◎ **σ is unlikely the 2-quark state.**

⇒ **likely the 4-quark state**

5. Summary

☆ Analysis of the properties of the scalar mesons
using the effective Lagrangian

・ **Radiative decays involving scalar mesons**

◎ vector dominance model predicts large hierarchy

$$\frac{\Gamma(a_0 \to \omega\gamma)}{\Gamma(a_0 \to \rho\gamma)} \gg 1 \qquad \frac{\Gamma(f_0 \to \omega\gamma)}{\Gamma(f_0 \to \rho\gamma)} \gg 1$$

for $\theta_s \simeq -20^\circ$ and $\theta_s \simeq -90^\circ$

checked by future experiments !

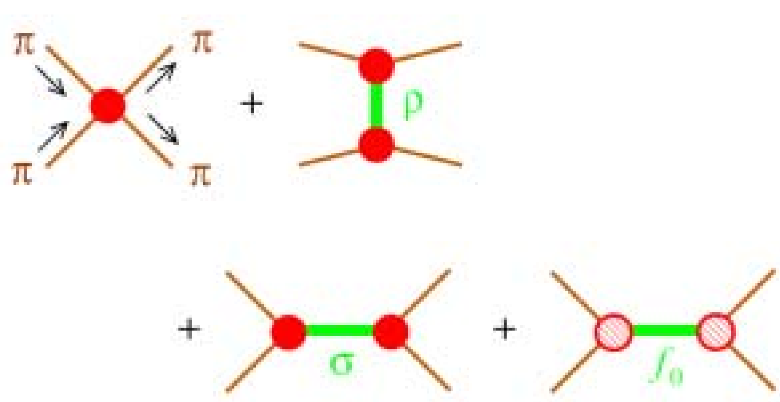
・ π - π **scattering in large Nc QCD**

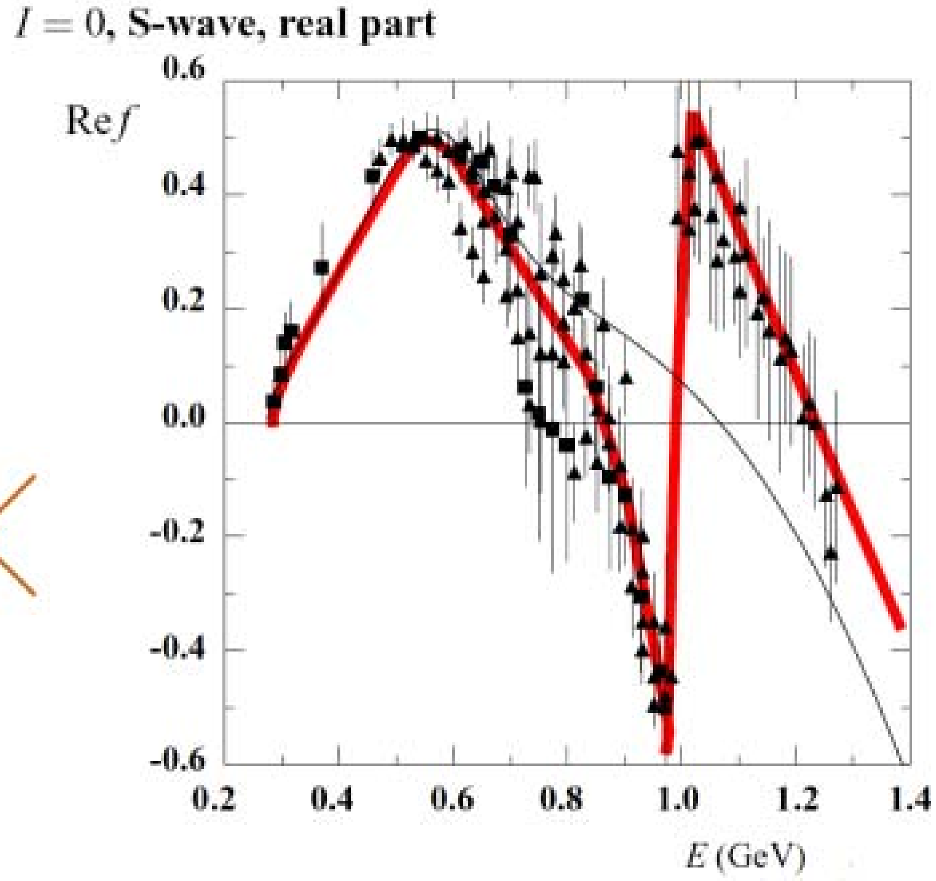
M_σ* becomes larger for larger *N_c

◎ σ **is unlikely the 2-quark state.**

⇒ **likely the 4-quark state**

☆ π + ρ + σ + f_0(980) contribution





$$M_\sigma = 559\,\text{MeV}, \quad G' = 370\,\text{MeV}, \quad \gamma_\sigma = 7.8\,(\text{GeV})^{-1}$$

$$M_{f_0} = 0.99\,\text{GeV}, \quad \Gamma_{f_0} = 0.065\,\text{GeV}$$