HEAVY QUARKONIUM HYPERFINE STRUCTURE AND LEPTONIC DECAYS

D. Ebert[a], R. N. Faustov[a,b], V. O. Galkin[a,b]
[a]*Institut für Physik, Humboldt–Universität zu Berlin, Berlin, Germany*
[b]*Russian Academy of Sciences, Scientific Council for Cybernetics, Moscow, Russia*

ABSTRACT

The relation between the hyperfine structure and leptonic decay rates of heavy quarkonia is considered with the account of relativistic and radiative corrections. The calculated decay rates agree well with the available experimental data, while the predicted $\eta_c(2S)$ mass is significantly smaller than the value measured recently by the Belle and BaBar Collaborations.

Recent observation of the charmonium 2^1S_0 state $\eta_c(2S)$ by the Belle and BaBar Collaborations yielded the following values for its mass:

$$M(\eta_c(2S)) = \begin{cases} 3.654(14)\ \text{GeV (see Ref. [1])} \\ 3.622(12)\ \text{GeV (see Ref. [2])} \\ 3.632(6)\ \ \ \text{GeV (see Ref. [3])} \end{cases} . \tag{1}$$

These values are significantly larger than most predictions of the constituent quark models [4, 5] and the previous (unconfirmed) experimental value [6] $M[\eta_c(2S)] = 3.594(5)$ GeV. The resulting $2S$ hyperfine splitting (HFS) would be about 2–4 times smaller than the $1S$ HFS in charmonium which is quite unexpected [5, 7]. The HFS is closely connected with leptonic decay rates and vector decay constants of heavy quarkonia. This connection for charmonium was discussed at length in the recent paper [7]. Here we extend this discussion in order to include the relativistic corrections for the vector constants. We considered these corrections for heavy-light

(B and D) mesons in our recent paper [8]. The vector decay constant f_V is defined by $\left\langle 0|\bar{Q}\gamma^\mu Q|V(\mathbf{K},\varepsilon)\right\rangle = f_V M\varepsilon^\mu$, where $\mathbf{K}$ is the quarkonium momentum, ε and M are the polarization vector and mass of the quarkonium. The relativistic expression for f_V can be obtained from Eq. (3) of Ref. [8] by putting $m_q = m_Q = m$

$$f_V = \sqrt{\frac{12}{M}} \int \frac{d^3p}{(2\pi)^3} \left\{1 - \frac{\epsilon(p) - m}{3\epsilon(p)}\right\} \Phi_V(p), \tag{2}$$

where $\epsilon(p) = \sqrt{\mathbf{p}^2 + m^2}$ and $\Phi_V(p)$ is the vector quarkonium wave function in the momentum space. In the nonrelativistic limit $p^2/m^2 \to 0$ this expression reduces to the well-known formula $f_V^{\rm NR} = \sqrt{12/M}\,|\Psi_V(0)|$, where $\Psi_V(0)$ is the wave function at the origin $r = 0$. The leptonic decay rate for zero lepton mass is given by

$$\Gamma_0(V \to e^+e^-) = \frac{4\pi\alpha^2 e_Q^2}{3M} f_V^2, \tag{3}$$

where α is the QED fine structure constant and e_Q is the quark charge in units of the elementary electric charge. In the nonrelativistic limit we obtain the widely-used relation $\Gamma_0^{\rm NR} = (16\pi\alpha^2 e_Q^2/M^2)\,|\Psi_V(0)|^2$. The one-loop QCD corrections modify Eq. (3) in the following way [9] $\Gamma = \Gamma_0\left(1 - \frac{16}{3\pi}\alpha_s\right)$, where we take the QCD coupling constant α_s according to PDG [10] equal to $\alpha_s(m_c) = 0.26$, $\alpha_s(m_b) = 0.18$.

Table 1: $1S$ *and* $2S$ *heavy quarkonium masses (in GeV) and HFS* ΔM *(in MeV).*

States	1^1S_0	1^3S_1	$\Delta M_{1S}^{\rm HFS}$	2^1S_0	2^3S_1	$\Delta M_{2S}^{\rm HFS}$
$c\bar{c}$	$\eta_c(1S)$	$J/\psi(1S)$		$\eta_c(2S)$	$\psi(2S)$	
theory [4]	2.979	3.096	117	3.588	3.686	98
exp. [10]	2.9797(15)	3.09687(4)	117(1)		3.68596(9)	
exp. [1]	2.979(2)		117(1)	3.654(14)		32(14)
exp. [2]	2.979(2)		117(1)	3.622(12)		64(12)
exp. [3]				3.632(6)		54(6)
exp. [6]				3.594(5)		92(5)
$b\bar{b}$	$\eta_b(1S)$	$\Upsilon(1S)$		$\eta_b(2S)$	$\Upsilon(2S)$	
theory [4]	9.400	9.460	60	9.993	10.023	30
exp. [10]		9.46030(26)			10.02326(31)	

The heavy quarkonium mass spectra including all relativistic v^2/c^2 and one-loop radiative corrections were calculated in Ref. [4]. The masses M and HFS $\Delta M_{nS}^{\rm HFS} \equiv M(n^3S_1) - M(n^1S_0)$ of nS states ($n = 1, 2$) are presented in Table 1. Inserting into Eq. (2) the same wave functions as used in Ref. [4] we can calculate the leptonic decay rates of vector quarkonia. The results of these calculations in comparison with experimental data are shown in Table 2. Relativistic corrections considerably reduce

Table 2: *Vector decay constants and leptonic decay rates of vector quarkonia.*

Decay modes	f_V MeV	$\Gamma^{\rm NR}$ keV	Γ keV	$\Gamma^{\rm exp}$ keV
$J/\psi(1S) \to e^+e^-$	551	6.7	5.4	5.26(37)
$\psi(2S) \to e^+e^-$	401	3.2	2.4	2.19(15)
$\Upsilon(1S) \to e^+e^-$	839	1.4	1.3	1.32(5)
$\Upsilon(2S) \to e^+e^-$	562	0.6	0.5	0.52(3)

the calculated decay rates and bring them in good agreement with experimental values. Thus we observe the overall selfconsistency between predictions for HFS and leptonic decay rates of heavy quarkonia. It means that the new Belle and BaBar data [1, 2, 3] for the $\eta_c(2S)$ mass cannot be easily explained in the framework of relativistic constituent quark models and, if confirmed, require some novel ideas and approaches (scale choice [7] for α_s, state mixing [11], etc.). This work was supported in part by the *Deutsche Forschungsgemeinschaft* under contract Eb 139/2-2.

References

1. Belle Collaboration, S.-K. Choi *et al.*, Phys. Rev. Lett. **89**, 102001 (2002).

2. Belle Collaboration, K. Abe *et al.*, Phys. Rev. Lett. **89**, 142001 (2002).

3. BaBar Colaboration, G. Wagner, hep-ex/0305083.

4. D. Ebert, R. N. Faustov, and V. O. Galkin, Phys. Rev. D **62**, 034014 (2000); **67**, 014027 (2003).

5. W. Buchmüller, J. Y. Ng, and S.-H. H. Tye, Phys. Rev. D **24**, 3003 (1981); E. Eichten, *et al.*, Phys. Rev. D **21**, 203 (1980); Phys. Rev. D **49**, 5845 (1994).

6. Crystal Ball Collaboration, C. Edwards *et al.*, Phys. Rev. Lett. **48**, 70 (1982).

7. A. M. Badalian and B. L. G. Bakker, Phys. Rev. D **67**, 071901(R) (2003).

8. D. Ebert, R. N. Faustov, and V. O. Galkin, Mod. Phys. Lett. A **17**, 803 (2002).

9. R. Barbieri, *et al.*, Phys. Lett. B **57**, 455 (1975).

10. Particle Data Group, K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002).

11. E. Eichten *et al.*, Phys. Rev. D **17**, 3090 (1978); A. Martin and J. M. Richard, Phys. Lett. B **115**, 323 (1982).