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MUON $g-2$

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ABSTRACT

The Muon $g-2$ collaboration has measured the anomalous magnetic g value of the positive muon to within a relative uncertainty of 0.7 parts per million. The result, $a_{\mu^+}(\text{expt}) = 11\,659\,204(7)(5) \times 10^{-10}$, is in good agreement with the preceding data on a_{μ^+} and a_{μ^-} and has about twice smaller uncertainty. The measurement tests standard model theory, which at the level of the present experimental uncertainty involves quantum electrodynamics, quantum chromodynamics, and electroweak interaction in significant ways. The analysis of the data on the anomalous magnetic g value of the negative muon is well underway.

1 Introduction

The anomalous g values, $a = (g-2)/2$, of leptons arise from quantum mechanical effects. Their precise measurement has historically played an important role in the development of particle theory. The anomalous magnetic g value of the electron, a_e, has been measured to within about four parts per billion (ppb) [2], and is thus among the most accurately known quantities in physics. Its value is described in terms of standard model (SM) field interactions, with nearly all of the measured value contributed by QED processes involving virtual photons, electrons, and positrons. Heavier particles contribute to a_e only at the level of the present experimental uncertainty.

The anomalous magnetic g value of the muon, a_μ, is more sensitive than a_e to processes involving particles more massive than the electron, typically by a factor $(m_\mu/m_e)^2 \sim 4 \cdot 10^4$. A series of three experiments [3] at CERN measured a_μ to within 7 parts per million (ppm), which is predominantly statistical. The CERN generation of experiments thus tested electron-muon universality and established the existence of a hadronic contribution to a_μ with a relative size of $\sim$ 59 ppm. Electroweak processes are expected to contribute at the level of 1.3 ppm, as are many speculative extensions of the SM.

The muon $g-2$ experiment at Brookhaven National Laboratory (BNL) is conceptually similar to the last CERN experiment, and has determined a_{μ^+} of the positive muon with an uncertainty of 0.7 ppm from a sample of about $4 \cdot 10^9$ decay positrons collected in the year 2000. The analysis of a_{μ^-} of the negative muon from a similarly sized sample of decay electrons collected in 2001 is well underway.

2 Experiment

The measurements at the Brookhaven Alternating Gradient Synchrotron used a secondary pion beamline. For most of the data taking periods, longitudinally polarized muons of about 3.1 GeV from forward decays were momentum-selected and injected into a 14.2 m diameter storage ring magnet [4] through a field-free inflector [5] region in the magnet yoke. A pulsed magnetic kicker [6] located at approximately one quarter turn from the inflector region produced a 10 mrad deflection which placed the muons onto stored orbits. Pulsed electrostatic quadrupoles [7] provided vertical focusing. The magnetic dipole field of about 1.45 T was measured with an NMR system [8] relative to the free proton NMR frequency ω_p over most of the 9 cm diameter circular storage aperture. Twenty-four electromagnetic calorimeters [9] read

Figure 1: Top view of the $g-2$ apparatus. The beam of longitudinally polarized muons enters the superferric storage ring magnet through a superconducting inflector magnet located at 9 o'clock and circulates clockwise after being placed onto stored orbit with three pulsed kickers modules in the 12 o'clock region. Twenty-four lead scintillating-fiber calorimeters on the inner, open side of the C-shaped ring magnet are used to measure muon decay positrons (electrons). The central platform supports the power supplies for the four electrostatic quadrupoles and the kicker modules.

out by 400 MHz custom waveform digitizers (WFD) were used on the open, inner side of the C-shaped ring magnet to measure the decay positrons and electrons. Muon decay violates parity, which in the laboratory frame results in a modulation of the number of positrons (electrons),

$$N(t) = N_0(E) \exp\left(\frac{-t}{\gamma\tau}\right) \left[1 + A(E) \sin\left(\omega_a t + \phi(E)\right)\right], \tag{1}$$

above an energy threshold E. Here, N_0 is a normalization, $\gamma\tau \sim 64\,\mu$s is the dilated muon lifetime, $A \sim 0.4$ is an asymmetry factor, ϕ is a phase, and ω_a is the angular difference frequency of muon spin precession and momentum rotation.

The muon anomalous magnetic g value is evaluated from the ratio of the measured frequencies, $R = \omega_a/\omega_p$, according to:

$$a_\mu = \frac{R}{\lambda - R}, \tag{2}$$

in which $\lambda = \mu_\mu/\mu_p$ is the ratio the muon and proton magnetic moments. The value with smallest stated uncertainty, $\lambda = \mu_\mu/\mu_p = 3.183\,345\,39(10)$ [10], results from measurements of the microwave spectrum of ground state muonium [11] and theory [12].

3 Data Analysis

The proton NMR frequency ω_p and the muon spin precession frequency ω_a were analyzed independently by several groups within the collaboration. The values of $R = \omega_a/\omega_p$ and a_μ were evaluated only after each of the frequency analyses had been finalized; at no earlier stage were the absolute values of both frequencies known to any of the collaborators.

3.1 The magnetic field

During the data collection period from January to March 2000, a field trolley with 17 NMR probes was used 2-3 times per week, 22 times in total, to measure the field throughout the muon storage region. Figure 2a shows the field value measured in the storage ring with the center trolley probe versus the azimuthal angle. The field is seen to be uniform to within about ± 50 ppm of its average value over the full azimuthal range, in particular also in the region near $350°$ where the inflector magnet is located. Since the field averaged over azimuth is uniform to within 1.5 ppm over the storage aperture (Fig. 2b), the field integral encountered by the (analyzed) muons is rather insensitive to the precise location and profile of the beam.

The probes inside the field trolley were calibrated with respect to each other during the data collection period using dedicated measurements in which a single NMR probe was plunged into the storage vacuum. This so-called plunging probe, as well as a subset of the trolley probes, were calibrated *in situ* with respect to a standard probe [15].

The 22 measurements with the field trolley were used to relate the readings of 370 NMR fixed probes in the outer top and bottom walls of the storage vacuum chamber to the field values in the beam region, so that the fixed NMR probe readings could be used to interpolate the field when the field trolley was 'parked' in the storage vacuum just outside the beam region and muons circulated in the storage ring.

For the data collection between January and March 2000, the field frequency ω_p weighted by the muon distribution was found to be,

$$\omega_p/(2\pi) = 61\,791\,595(15)\,\text{Hz} \quad (0.2\,\text{ppm}), \tag{3}$$

where the uncertainty has a leading contribution from the calibration of the trolley probes and is thus predominantly systematic. The result was confirmed by a second, largely independent analysis, which made use of additional calibration data, a different selection of fixed NMR probes, and a different method to relate the trolley

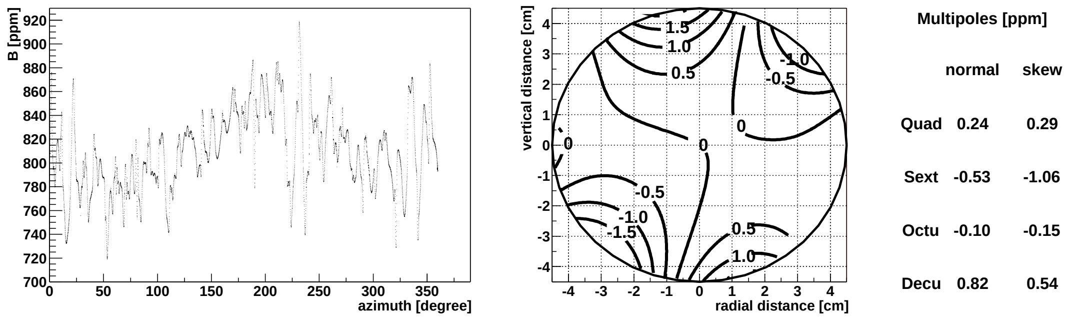

Figure 2: The NMR frequency measured with the center trolley probe relative to a 61.74 MHz reference versus the azimuthal position in the storage ring (left), and (right) a 2-dimensional multipole expansion of the azimuthal average of the field measured with 15 trolley probes with respect to the central field value of 1.451 275 T. The multipole amplitudes are given at the storage ring aperture, which has a 4.5 cm radius as indicated by the circle.

and fixed probe readings. The result from the data collection on the negative muon in the year 2001 is expected to have further improved uncertainty.

3.2 The muon spin precession frequency

About $4 \cdot 10^9$ reconstructed positrons with energies greater than 2 GeV and times between 50 μs and 600 μs following the beam injection were available for analysis from the data collection between January and March 2000. Figure 3a shows their time spectrum after corrections for the bunched time structure of the beam and for overlapping calorimeter pulses, so called pile-up [13], had been applied.

The main characteristics of the spectrum are muon decay and spin precession (Eq. 1), however, additional effects need to be considered as illustrated by the Fourier spectrum in Fig. 3b. These effects include detector gain and time instability, muon losses, and oscillations of the beam as a whole, so-called coherent betatron oscillations (CBO). The latter are caused by the injection of the beam through the relatively narrow 18(w) $\times$ 57(h) mm^2 aperture of the 1.7 m long inflector channel into the 90 mm diameter aperture of the storage region. Their frequencies are determined by the focusing index of the storage ring, and have been observed directly with fiber harp monitors that were plunged into the beam region for this purpose. Since the calorimeter acceptances vary with the muon decay position in the storage ring and with the momentum of the decay positron, CBO cause modulation of the time and energy spectra of the observed positrons.

Numerically most important to the determination of ω_a from the data col-

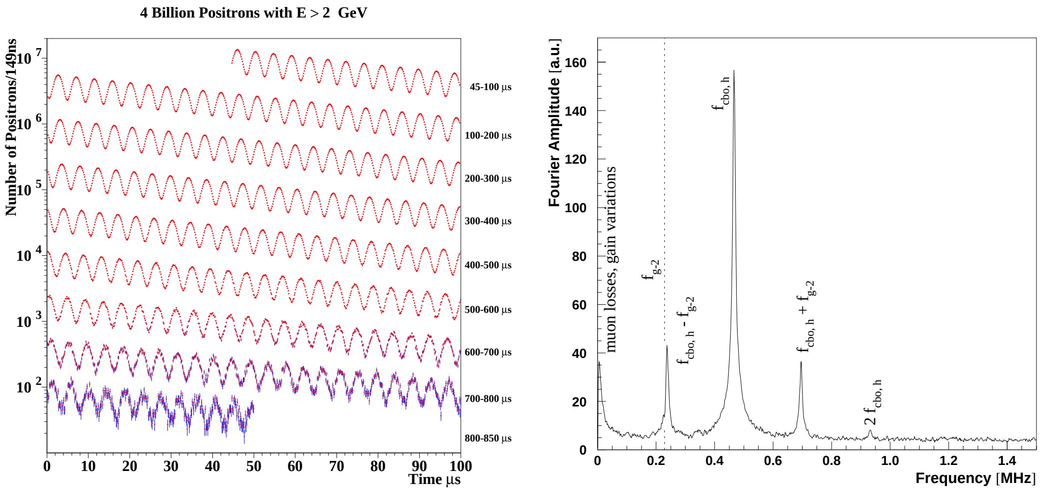

Figure 3: The time spectrum for $4 \cdot 10^9$ positrons with energies greater than 2 GeV collected from January to March 2000, after corrections for pile-up and for the bunched time structure of the injected beam (left) were made, and (right) the Fourier transform of the time spectrum, in which muon decay and spin precession (cf. Eq. 2) has been suppressed to emphasize other effects.

lected between January and March 2000 were the CBO in the horizontal plane, whose frequency was numerically close to twice the frequency ω_a. When the modulations of the asymmetry and phase with frequency $\omega_{\mathrm{cbo,h}} \simeq 2 \times \omega_a$ were not accounted for in the function fitted to the data, artificial shifts of up to 4 ppm in the frequency values ω_a determined from individual calorimeter spectra were observed. In the joined calorimeter spectrum, such shifts are largely canceled because of the circular symmetry of the experiment design.

Several approaches were pursued in the analysis of ω_a. In one approach, the time spectra from individual positron calorimeters was fitted in narrow energy intervals using a fit function as in Eq. 1 extended by the aforementioned number, asymmetry, and phase modulations. Other approaches made use of the cancellation in the joined calorimeter spectra and either fitted for the residual of the leading effects, or accounted for their neglect in a contribution to the systematic uncertainty. The results were found to agree, on ω_a to within the expected 0.5 ppm statistical variation resulting from the slightly different selection and treatment of the data in the respective analyses. The combined result was found to be,

$$\omega_a/(2\pi) = 229\,074\,11(14)(7)\,\mathrm{Hz} \quad (0.7\,\mathrm{ppm}), \tag{4}$$

in which the first uncertainty is statistical and the second is systematic. The above

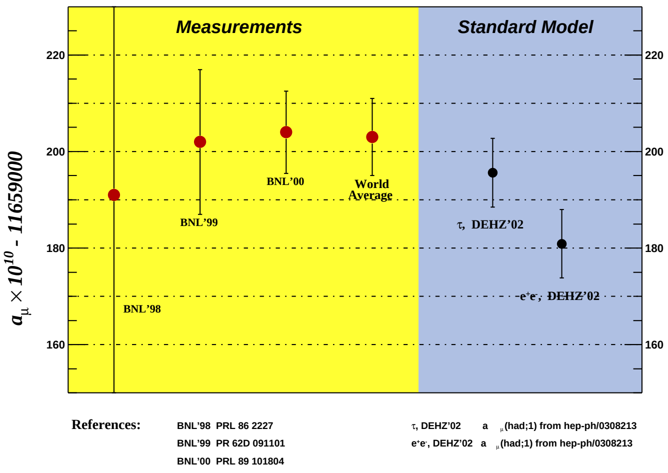

Figure 4: Recent measuremens of a_μ and standard model evaluations using the estimates in Ref. [25] of the lowest order contribution from hadronic vacuum polarization.

frequency includes a correction of $+0.76(3)$ ppm for the net contribution to the muon spin precession and momentum rotation caused by vertical beam oscillations and, for muons with $\gamma \neq 29.3$, by horizontal electric fields [14]. The systematic uncertainty has a leading contribution of 0.2 ppm caused by CBO. In the year 2001, an event sample of comparable size on the negative muon was collected. The storage ring was operated with two different values of the focusing index, which is expected to reduce the leading systematic uncertainty.

4 Results and Discussion

The value of a_μ was evaluated after the analyses of ω_p and ω_a had been finalized,

$$a_{\mu^+} = 11\,659\,204(7)(5) \ \times \ 10^{-10} \quad (0.7\,\text{ppm}), \tag{5}$$

where the first uncertainty is statistical and the second systematic. The result agrees well with the preceding measurements [3, 13, 16] and drives the present world average,

$$a_\mu(\text{exp}) = 11\,659\,203(8) \ \times \ 10^{-10} \quad (0.7\,\text{ppm}), \tag{6}$$

in which the uncertainty accounts for known correlations between the systematic uncertainties in the measurements. Figure 4 shows our recent measurements of a_{μ^+}, together with two SM evaluations discussed below.

In the SM, the value of a_μ receives contributions from QED, hadronic, and electroweak processes, $a_\mu(\text{SM}) = a_\mu(\text{QED}) + a_\mu(\text{had}) + a_\mu(\text{weak})$. The QED and weak contributions can, unlike the hadronic contribution, be evaluated perturbatively, $a_\mu(\text{QED}) = 11\,658\,470.57(29) \times 10^{-10}$ [18] and $a_\mu(\text{weak}) = 15.4(2) \times 10^{-10}$ [19]. The hadronic contribution is, in lowest order, related by dispersion theory to the hadron production cross sections measured in e^+e^- collisions and, under additional assumptions, to hadronic τ-decay. Clearly, the hadronic contribution has a long history of values as new data appeared and analyses were refined.

At the time of the PIC-2003 conference, a recent and detailed evaluation was the one by Davier and co-workers, which – unlike preceding analyses – incorporated the low-energy e^+e^- annihilation cross section into hadrons by the CMD-2 collaboration [20] , e^+e^- measurements [21] with improved accuracy in the 2–5 GeV energy region from BES, preliminary results from the final ALEPH analysis [22] of hadronic τ-decay at LEP1, and additional data [23] from CLEO. Significant discrepancies between the e^+e^- and τ data were found.

The CMD-2 collaboration has since released a reanalysis of their cross section measurements [24] and Davier and co-workers have provided updated estimates for the contribution to $a_\mu(\text{SM})$ from lowest order hadronic vacuum polarization, $a_\mu(\text{had}, 1) = 696(7) \times 10^{-10}$ from e^+e^- data and $a_\mu(\text{had}, 1) = 711(6) \times 10^{-10}$ from τ data [25]. The authors refrain from averaging the values, noting that significant discrepancies remain in the underlying data in the center-of-mass region between 0.85 and 1.0 GeV. Radiative return measurements at the e^+e^- factories may reach the required precision to shed light on the situation, as might lattice calculations. Higher order contributions to $a_\mu(\text{had})$ include higher order hadronic vacuum polarization [26] and hadronic light-by-light scattering [27].

Open questions thus concern the SM value of a_μ, in particular its hadronic contribution, and the experimental value of a_{μ^-} at sub-ppm precision. Stay tuned!

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References

1. G.W. Bennett[2], B. Bousquet[9], H.N. Brown[2], G. Bunce[2], R.M. Carey[1], P. Cushman[9], G.T. Danby[2], P.T. Debevec[7], M. Deile[11], H. Deng[11],

W. Deninger[7], S.K. Dhawan[11], V.P. Druzhinin[3], L. Duong[9], E. Efstathiadis[1], F.J.M. Farley[11], G.V. Fedotovich[3], S. Giron[9], F.E. Gray[7], D. Grigoriev[3], M. Grosse-Perdekamp[11], A. Grossmann[6], M.F. Hare[1], D.W. Hertzog[7], X. Huang[1], V.W. Hughes[11,†], M. Iwasaki[10], K. Jungmann[5], D. Kawall[11], B.I. Khazin[3], J. Kindem[9], F. Krienen[1], I. Kronkvist[9], A. Lam[1], R. Larsen[2], Y.Y. Lee[2], I. Logashenko[1,3], R. McNabb[9], W. Meng[2], J. Mi[2], J.P. Miller[1], W.M. Morse[2], D. Nikas[2], C.J.G. Onderwater[7], Y. Orlov[4], C.S. Özben[2], J.M. Paley[1], Q. Peng[1], C.C. Polly[7], J. Pretz[11], R. Prigl[2], G. zu Putlitz[6], T. Qian[9], S.I. Redin[3,11], O. Rind[1], B.L. Roberts[1], N. Ryskulov[3], P. Shagin[9], Y.K. Semertzidis[2], Yu.M. Shatunov[3], E.P. Sichtermann[11], E. Solodov[3], M. Sossong[7], A. Steinmetz[11], L.R. Sulak[1], A. Trofimov[1], D. Urner[7], P. von Walter[6], D. Warburton[2], and A. Yamamoto[8].

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2. R.S. Van Dyck *et al*, Phys. Rev. Lett. **59**, 26 (1987).

3. G. Charpak *et al*, Phys. Lett. **1**, 16 (1962); J. Bailey *et al*, Nuovo Cimento **A9**, 369 (1972); J. Bailey *et al*, Nucl. Phys. **B510**, 1 (1979).

4. G. Danby *et al*, Nucl. Instrum. Meth. **A457**, 151 (2002).

5. A. Yamamoto *et al*, Nucl. Instrum. Meth. **A491**, 23 (2002).

6. E. Efstathiadis *et al*, Nucl. Instrum. Meth. **A496**, 8 (2003).

7. Y.K. Semertzidis *et al*, Nucl. Instrum. Meth. **A503**, 458 (2003).

8. R. Prigl *et al*, Nucl. Instrum. Meth. **A374**, 118 (1996).

9. S. Sedykh *et al*, Nucl. Instrum. Meth. **A455**, 346 (2000).

10. K. Hagiwara *et al*, Phys. Rev. **D66**, 010001 (2002).

11. W. Liu *et al*, Phys. Rev. Lett. **82**, 711 (1999).

12. T. Kinoshita, hep-ph/9808351 (1998); T. Kinoshita and M. Nio in: Frontier Tests of Electrodynamics and Physics of the Vacuum (eds. E. Zavattini, D. Bakalov, and C. Rizzo), 151 (Heton Press, Sofia, 1998).

13. H.N. Brown *et al*, Phys. Rev. Lett. **86**, 2227 (2001).

14. F. Farley and E. Picasso in: Quantum Electrodynamics (ed. T. Kinoshita), 479 (World Scientific, Singapore, 1990).

15. X. Fei *et al*, Nucl. Instrum. Meth. **A394**, 349 (1997).

16. H.N. Brown *et al*, Phys. Rev. Lett. **D62**, 091101 (2000); R.M. Carey *et al*, Phys. Rev. Lett. **82**, 1632 (1999).

17. M. Davier *et al*, Eur. Phys. J. **C27**, 497 (2003).

18. P.J. Mohr and B.N. Taylor, Rev. Mod. Phys. **72**, 351 (2000).

19. A. Czarnecki *et al*, Phys. Rev. **D67**,073006 (2003); and references therein.

20. R.R. Akhmetshin *et al*, Phys. Lett. **B527**, 161 (2002).

21. J.Z. Bai *et al*, Phys. Rev. Lett. **84**, 594 (2000); J.Z. Bai *et al*, Phys. Rev. Lett. **88**, 101802 (2002).

22. ALEPH Collaboration, ALEPH 2000-030 CONF 2002-019 (2002).

23. S. Anderson *et al*, Phys. Rev. **D61**, 112002 (2000); K.W. Edwards *et al*, Phys. Rev. **D61**, 072003 (2000).

24. R.R. Akhmetshin *et al*, hep-ex/0308008 (2003).

25. M. Davier *et al*, hep-ph/0308213 (2003).

26. B. Krause, Phys. Lett. **B390**, 392 (1997); R. Alemany *et al*, Eur. Phys. J. **C2**, 123 (1998).

27. M. Hayakawa and T. Kinoshita, hep-ph/0112102 (2001); J. Bijnens *et al*, Nucl. Phys. **B626**, 410 (2002); M. Knecht and A. Nyffeler, Phys. Rev. **D65**, 073034 (2002); M. Knecht *et al*, Phys. Rev. Lett. **88**, 071802 (2002); I. Blokland *et al*, Phys. Rev. Lett. **88**, 071803 (2002).