

# Axially Symmetric Black Hole Skyrmions

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It has been known that a  $B = 2$  skyrmion is axially symmetric. We consider the Skyrme model coupled to gravity and obtain static axially symmetric black hole solutions numerically. The black hole skyrmion no longer has integer baryonic charge but has fractional charge outside the horizon as in the spherically symmetric case. Therefore, the solution represents a black hole partially swallowing a deuteron.

## 1 Introduction

It has been shown that the no-hair conjecture for black holes [1] is violated when some non-linear matter fields are considered. The first counter example was provided by Luckock and Moss [2] who found the Schwarzschild black hole with Skyrme hair. The presence of the horizon in the core of skyrmion unwinds the skyrmion, leaving fractional baryon charge outside the horizon. The full Einstein–Skyrme system was solved later by Droz *et al.* to obtain spherically symmetric black holes with Skyrme hair [3]. Other counterexamples include static spherically symmetric black holes in the Einstein–Yang–Mills (EYM) [4], the Einstein–Yang–Mills–Dilaton (EYMD) [5, 6] and the Einstein–Yang–Mills–Higgs (EYMH) theory [7]. More interestingly, it has been also shown that these Einstein–Yang–Mills theories have static axially symmetric black hole solutions [8, 9].

Motivated by the axially symmetric hairy black holes in Refs. [8, 9], we shall study the Einstein–Skyrme model with axial symmetry. It has been shown that a  $B = 2$  skyrmion is axially symmetric and represents a deuteron [10]. Our model, therefore, provides a convenient framework to study the interactions between a deuteron and a black hole. By examining the baryon number of the solution, the absorption of the deuteron by the black hole is observed as in the spherically symmetric case. We expect our solutions to be stable as skyrmions are topologically stable objects.

## 2 The model

The Skyrme model is an effective theory of QCD based on pion fields alone [11]. At low energy, the symmetry of the strong interaction is broken spontaneously and hence the Skyrme Lagrangian retains the chiral symmetry. The Skyrme model coupled to gravity is defined by

$$\mathcal{L} = \mathcal{L}_S + \mathcal{L}_G, \quad (1)$$

where

$$\mathcal{L}_S = \frac{f_\pi^2}{16} g^{\mu\nu} \text{tr}(U^{-1} \partial_\mu U U^{-1} \partial_\nu U) + \frac{1}{32a^2} g^{\mu\rho} g^{\nu\sigma} \text{tr}([U^{-1} \partial_\mu U, U^{-1} \partial_\nu U][U^{-1} \partial_\rho U, U^{-1} \partial_\sigma U]),$$

$$\mathcal{L}_G = \frac{1}{16\pi G} R.$$

Let us introduce an ansatz for the metric given in Ref. [8]

$$ds^2 = -f dt^2 + \frac{m}{f} (dr^2 + r^2 d\theta^2) + \frac{l}{f} r^2 \sin^2 \theta d\varphi^2, \quad (2)$$

where  $f = f(r, \theta)$ ,  $m = m(r, \theta)$ , and  $l = l(r, \theta)$ .

The axially symmetric Skyrme field can be parameterized by

$$U = \cos F(r, \theta) + i\vec{\tau} \cdot \vec{n}_R \sin F(r, \theta), \quad (3)$$

with  $\vec{n}_R = (\sin \Theta \cos n\varphi, \sin \Theta \sin n\varphi, \cos \Theta)$ . In terms of  $F$  and  $\Theta$ , the Lagrangian (1) has the form

$$\mathcal{L}_S = \mathcal{L}_S^{(1)} + \mathcal{L}_S^{(2)}, \quad (4)$$

where

$$\begin{aligned} \mathcal{L}_S^{(1)} &= -\frac{f_\pi^2 f}{8m} \left\{ (\partial_r F)^2 + \frac{1}{r^2} (\partial_\theta F)^2 + \left[ (\partial_r \Theta)^2 + \frac{1}{r^2} (\partial_\theta \Theta)^2 \right] \sin^2 F + \frac{n^2}{r^2 \sin^2 \theta} \frac{m}{l} \sin^2 \Theta \sin^2 F \right\}, \\ \mathcal{L}_S^{(2)} &= -\frac{1}{2a^2 r^2} \left( \frac{f}{m} \right)^2 \left\{ (\partial_r F \partial_\theta \Theta - \partial_\theta F \partial_r \Theta)^2 + \frac{n^2}{\sin^2 \theta} \frac{m}{l} \left[ (\partial_r F)^2 + \frac{1}{r^2} (\partial_\theta F)^2 \right] \sin^2 \Theta \right\} \sin^2 F \\ &\quad - \frac{1}{2a^2 r^2} \left( \frac{f}{m} \right)^2 \left\{ \frac{n^2}{\sin^2 \theta} \frac{m}{l} \left[ (\partial_r \Theta)^2 + \frac{1}{r^2} (\partial_\theta \Theta)^2 \right] \sin^2 F \sin^2 \Theta \right\} \sin^2 F. \end{aligned}$$

Since we are interested in  $B = 2$ , we shall take the winding number  $n = 2$ .

The baryon current in curved space-time is obtained by taking the space-time covariant derivative  $\nabla_\mu$ ,

$$b^\mu = \frac{1}{24\pi^2} \epsilon^{\mu\nu\rho\sigma} \text{tr}(U^{-1} \nabla_\nu U U^{-1} \nabla_\rho U U^{-1} \nabla_\sigma U). \quad (5)$$

The baryon number then is given by integrating  $b^0$  over the hypersurface  $t = 0$ ,

$$\begin{aligned} B &= \int dr d\theta d\varphi \sqrt{g^{(3)}} b^0 = -\frac{1}{\pi} \int dr d\theta (\partial_r F \partial_\theta \Theta - \partial_\theta F \partial_r \Theta) \sin \Theta (1 - \cos 2F) \\ &= -\frac{1}{\pi} \int dF \wedge d\Theta \sin \Theta (1 - \cos 2F) = \left[ \frac{1}{2\pi} (2F - \sin 2F) \cos \Theta \right]_{F_0, \Theta_0}^{F_1, \Theta_1}, \end{aligned}$$

where  $(F_0, \Theta_0)$  and  $(F_1, \Theta_1)$  are the values at the inner and outer boundary, respectively. In flat space-time [10], we have

$$(F_0, \Theta_0) = (\pi, 0) \quad \text{and} \quad (F_1, \Theta_1) = (0, \pi), \quad (6)$$

which gives  $B = 2$ . In the presence of a black hole, the integration should be performed from the horizon to infinity, which changes the values of  $F_0$  and allows the  $B$  to be fractional.

The energy density of the skyrmion outside the horizon can be obtained by the zero-zero component of the stress-energy tensor  $-T_0^0$ ,

$$\begin{aligned} \epsilon &= -T_0^0 \\ &= \frac{f_\pi^2 f}{8m} \left[ (\partial_r F)^2 + \frac{1}{r^2} (\partial_\theta F)^2 + \left\{ (\partial_r \Theta)^2 + \frac{1}{r^2} (\partial_\theta \Theta)^2 \right\} \sin^2 F + \frac{n^2}{r^2 \sin^2 \theta} \frac{m}{l} \sin^2 F \sin^2 \Theta \right] \\ &\quad + \frac{1}{2a^2 r^2} \frac{f^3}{m^2} \left[ (\partial_r F \partial_\theta \Theta)^2 + \frac{n^2}{\sin^2 \theta} \frac{m}{l} \left\{ (\partial_r F)^2 + \frac{1}{r^2} (\partial_\theta F)^2 \right\} \sin^2 \Theta \right] \\ &\quad + \frac{n^2}{\sin^2 \theta} \frac{m}{l} \left\{ (\partial_r \Theta)^2 + \frac{1}{r^2} (\partial_\theta \Theta)^2 \right\} \sin^2 F \sin^2 \Theta \sin^2 F \end{aligned} \quad (7)$$

### 3 Boundary conditions

Let us consider the boundary conditions for the chiral fields and metric functions with help of Ref. [8]. At the horizon  $r = r_h$ , the time-time component of the metric satisfies

$$g_{tt} = -f(r_h, \theta) = 0. \quad (8)$$

Regularity of the metric at the horizon requires

$$m(r_h, \theta) = l(r_h, \theta) = 0. \quad (9)$$

The boundary conditions for  $F(r, \theta)$  and  $\Theta(r, \theta)$  at the horizon are obtained by expanding them at the horizon and inserting into the field equations derived from  $\delta\mathcal{L}_S/\delta F = 0$  and  $\delta\mathcal{L}_S/\delta\Theta = 0$  respectively,

$$\partial_r F(r_h, \theta) = \partial_r \Theta(r_h, \theta) = 0. \quad (10)$$

The condition that the space-time is asymptotically flat requires

$$f(\infty, \theta) = m(\infty, \theta) = l(\infty, \theta) = 1. \quad (11)$$

The boundary conditions for  $F$  and  $\Theta$  at infinity remain the same as in flat space-time

$$F(\infty, \theta) = 0, \quad \partial_r \Theta(\infty, \theta) = 0. \quad (12)$$

For the solution to be axially symmetric, we have

$$\partial_\theta f(r, 0) = \partial_\theta m(r, 0) = \partial_\theta l(r, 0) = 0, \quad (13)$$

$$\partial_\theta f\left(r, \frac{\pi}{2}\right) = \partial_\theta m\left(r, \frac{\pi}{2}\right) = \partial_\theta l\left(r, \frac{\pi}{2}\right) = 0. \quad (14)$$

Likewise for  $F$ ,

$$\partial_\theta F(r, 0) = \partial_\theta F\left(r, \frac{\pi}{2}\right) = 0. \quad (15)$$

Regularity on the axis and axisymmetry impose the boundary conditions on  $\Theta$  as

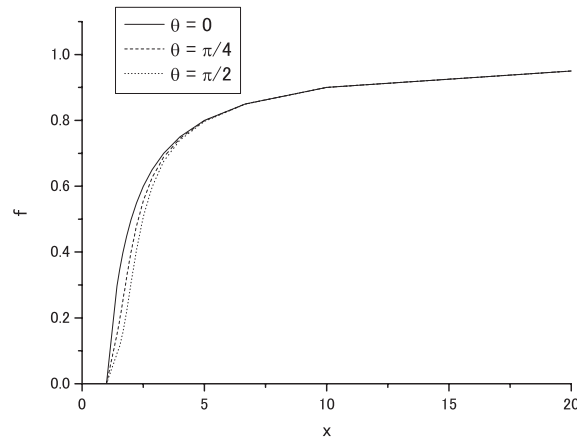
$$\Theta(r, 0) = 0, \quad \Theta\left(r, \frac{\pi}{2}\right) = \frac{\pi}{2}. \quad (16)$$

### 4 Numerical results and discussions

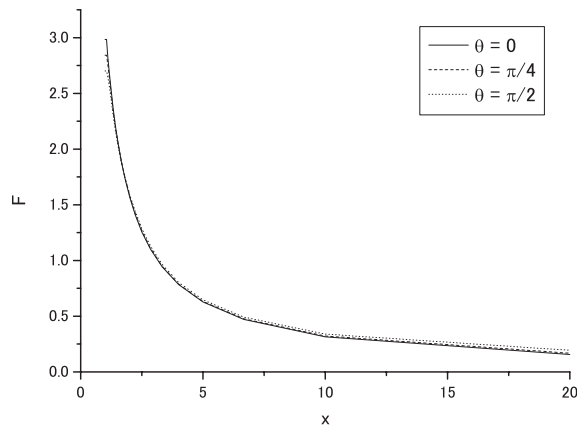
For the purpose of numerical computation, we shall introduce a dimensionless radial coordinate  $x$  and coupling constant  $\alpha$ ,

$$x = af_\pi r, \quad \alpha = \pi G f_\pi^2. \quad (17)$$

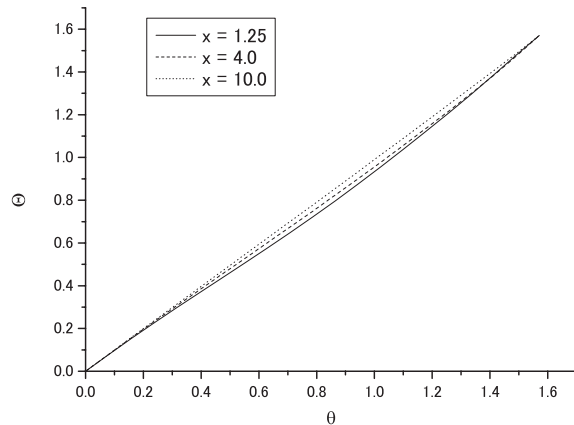
Then, in this system, free parameters are only  $x_h$  and  $\alpha$ . Fig. 1 shows dependence of the metric function  $f$  on  $\theta$  with  $\alpha = 1.0$ . For smaller values of  $\alpha$ , the results are slightly lower than that of  $\alpha = 1.0$ . Other metric functions  $l$  and  $m$  exhibit similar behavior as  $f$ . The Skyrme functions  $F$  and  $\Theta$  are shown in Figs. 2, 3.  $\alpha$  dependence as well as  $\theta$  dependence of  $F$  is rather small.  $\Theta$  is less distorted from  $\Theta = \theta$  for smaller  $\alpha$ . We show the energy density of the Skyrme fields ( $\epsilon = -T_0^0$ ) in Figs. 4, 5 with  $\alpha = 0.01, 1.0$ . As can be seen, the density becomes dumbbell in shape with the highest along  $z$ -axis while in flat space-time it is toroidal. As  $\alpha$  becomes small, it approaches to a more spherical shape. It is interesting to see how the presence of the black hole



**Figure 1.** The metric function  $f$  as a function of  $x$  with  $\alpha = 1.0$  and  $x_h = 1.0$ .



**Figure 2.** The Skyrme function  $F$  as a function of  $x$  with  $\alpha = 1.0$  and  $x_h = 1.0$ .



**Figure 3.** The Skyrme function  $\Theta$  as a function of  $\theta$  with  $\alpha = 1.0$  and  $x_h = 1.0$ .

affects the shape of the skyrmion. The domain of existence of the solutions in the parameter space is shown in Fig. 6. For  $\alpha \gtrsim 2.0$ , there exists no solution since the chiral fields become too massive for the black hole to support outside the horizon. It is also observed that the black hole has a finite minimum size unlike the spherically symmetric case. Hence one cannot recover regular solutions as the limit of zero horizon size. Fig. 7 shows the dependence of the baryon number on  $\alpha$  and  $x_h$ . It is observed that the baryon number gets more absorbed by the black hole in increase of the size of the black hole and the coupling constant.

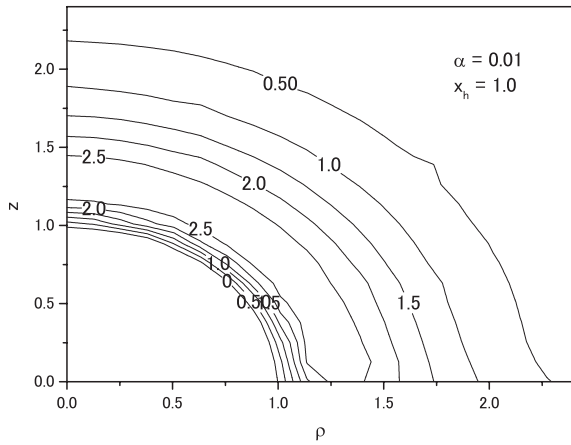
We suspect that our solutions should be stable since the skyrmions are highly stable objects. This statement may be verified by applying the catastrophe theory of hairy black holes proposed in Ref. [12].

Finally, recent studies of theories with large extra dimensions indicate that a true Planck scale is of order a TeV and the production rate of black holes massive than the Planck scale become quite large [13–15]. It will be interesting to extend our model to higher dimensions since it bring us an interesting possibility that the deuteron black holes could be produced in the LHC by collision of two protons.

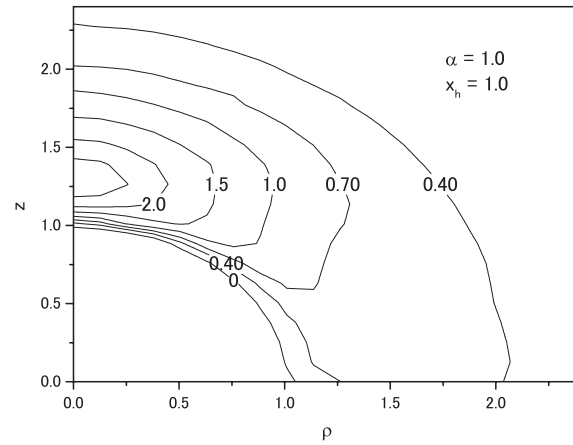
Inclusion of gauge fields will be also interesting to study electrically charged deuteron black holes [16].

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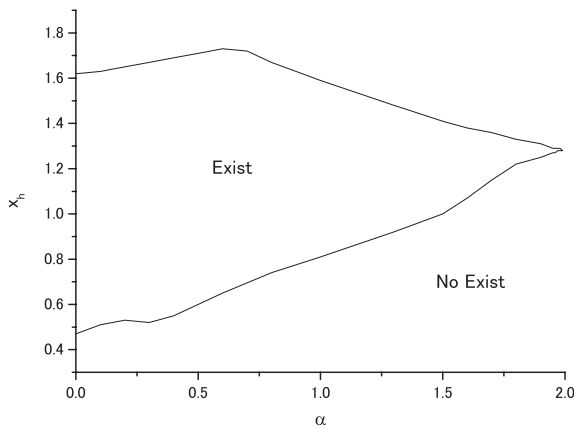
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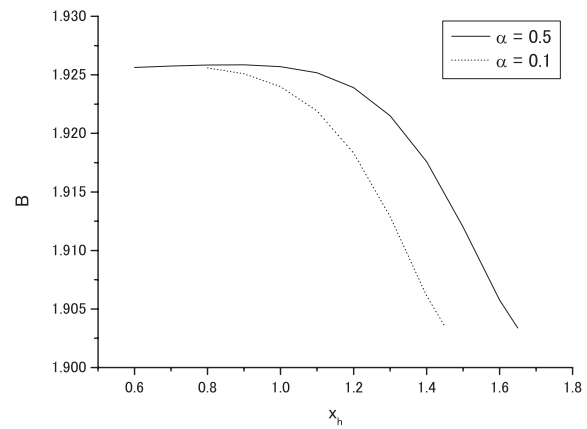
**Figure 4.** A contour plot of the energy density  $\epsilon$  in cylindrical coordinates  $\rho$  and  $z$  with  $x_h = 1.0$ .



**Figure 5.** A contour plot of the energy density  $\epsilon$  in cylindrical coordinates  $\rho$  and  $z$  with  $x_h = 1.0$ .



**Figure 6.** The domain of existence of the solution. For  $\alpha \gtrsim 2.0$ , there exists no non-trivial solution.



**Figure 7.** The dependence of the baryon number on the size of the horizon.

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