

Physical Nature of Lobachevsky Parallel Lines and a New Inertial Frame Transformation

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Processes of the particle and light beams propagation synchronized by Huygens principle have been found as the physical nature for the V postulate denial and for the Lobachevsky parallel lines definition in the velocity space. The physical nature of Lobachevsky parallel lines reveals a new way to solve the main difficulty in relativity – the problem of definition of the moments of time for different space points. The first obvious consequences from the established physical correspondence, including simultaneity, proper time and inertia system coordinate transformation, are presented in this paper.

1 Introduction

In modern high energy physics the Lobachevsky velocity space is widely used for investigation of particle scattering processes. In spite of that the physical nature of Lobachevsky parallel lines (LPL) is still absent. As the existence of LPL is based on denial of the Euclidean V postulate, a physical foundation for its violation is also absent. At the present time LPL have a geometrical interpretation only either as infinite lines on a pseudospherical surface, or as hordes on an Euclidean circle [1].

Exposition of the physical nature of LPL and obtaining the first obvious consequences from processes corresponding to LPL are results of further development of main ideas of [2] and presented in this paper.

It is reasonable to sketch some general remarks on our approach. Let us consider light propagation on the basis of the Huygens principle and of the light beams independence law. So, light phenomena of diffraction and interference are omitted. Also, let us accept the constant light velocity principle. Let us imply that the reader is familiar with Lobachevsky geometry [1, 3].

There is a special remark for Huygens principle: a time moment of emitting a secondary light sphere (semisphere) from any point reached by a light front can be taken as the initial (zero) moment of time counting for that point.

Let us use the same plane light fronts as used to explain the light reflection and refraction phenomena.

2 The physical nature of Lobachevsky parallel lines

Let us have two inertial reference frames K and K_s , and one (K_s) is moving relatively to the other with some velocity V . Both systems may be associated somehow with corresponding particles. As usual, all space axes of the both frames are parallel, and the motion goes along the X -axis of the K . Let us assume that when the origins O and O_s of both frames coincide, then a plane light front (side beam directed from down to up in some plane, for instance in XY) hits the point O under a parallel angle to the X -axis (see Fig. 1a):

$$\cos \theta_L \equiv \cos \Pi(\rho/k) = th(\rho/k) = V/c \equiv \beta, \quad (k = c) \quad (1)$$

and a light sphere (semisphere to the falling front) starts to spread out from the O (here β is a velocity V in units of c , ρ/k is a rapidity in units of $k = c$, $\Pi(\rho/k) \equiv \theta_L$ is a parallel angle, k is Lobachevsky constant, c is a light velocity). The second equality $\beta = th(\rho/c)$ in (1) is known

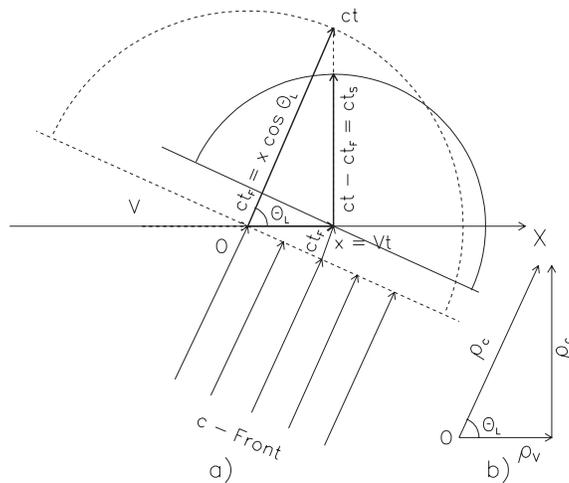


Figure 1. a) synchronization of light rays (ct and ct_s) propagation and the K_s -motion Vt by the Huygens principle due to the side light beam; b) Lobachevsky parallel lines in the velocity space plane corresponding to synchronous motions of ct , ct_s and Vt (in the Euclidean plane).

from the Beltrami model [1] and used in physics to define particle rapidity:

$$\rho/c = 1/2 \ln((1 + \beta)/(1 - \beta)). \quad (2)$$

The first equality in (1) can be rewritten in the form:

$$\theta_L \equiv \Pi(\rho/k) = 2 \operatorname{arctg} e^{-\rho/c}, \quad (3)$$

known as Lobachevsky function. It is seen from (1) that for any rapidity (or/and for any velocity) there is a definite angle θ_L . For negative argument of the Lobachevsky function the parallel angle is $\pi - \theta_L$ [1]. So, this case corresponds to $(\cos(\pi - \theta_L) = -\cos \theta_L = -V/c)$ the same velocity, but in the opposite direction.

Let us consider an event $(x = Vt, t)$ in the K . Then the side beam hits a given x -point in the moment of time t_F (see Fig. 1a):

$$ct_F = x \cos \theta_L = Vt \cos \theta_L = ct \cos^2 \theta_L, \quad (4)$$

i.e., later than it hits the origin O , and a new light sphere starts to spread out from a given x -point. By a given moment of time t a new sphere spreads up to the distance (or radius) ct_s :

$$ct_s = ct - ct_F = ct - x \cos \theta_L = ct - xV/c, \quad t_s = t - xV/c^2, \quad (5)$$

and for $x = Vt$:

$$ct_s = ct - ct \cos^2 \theta_L = ct \sin^2 \theta_L = ct(1 - V^2/c^2), \quad (6)$$

where ct is the light sphere radius from origin O , so that $ct_s < ct$. It is obvious that the origin O_s of the K_s moves along X at the distances Vt .

Let us choose two light rays from these two spheres: one is ct from O at the angle θ_L to the X -axis on some plane, the other is ct_s from O_s (from the given x) perpendicular to the X -axis on the same plane (see Fig. 1a). These three segments ct , Vt and ct_s form some kind of

a rectangular triangle. But sides ct and ct_s have no common (intersection) point for any moment of time t , so they are parallel in any chosen Euclidean plane. As a rapidity for c is an infinity (see (2)), then this obtained triangle transforms into LPL, or more precisely, into parallel lines in one side on the Lobachevsky plane in the velocity space (see Fig. 1b).

Thus, the LPL in a velocity space corresponds to the light rays ct and ct_s emitted from different points and different times and synchronized by Huygens principle with particle motion Vt . The physical reason of absence of an intersection point is the time delay t_F (see (4)). This time is obviously a physical foundation for denial of the Vth postulate.

To find out light rays corresponding to LPL in another side, one should use analogous consideration with a side beam directed to another side (from up to down) in the same plane (see Fig. 2a and Fig. 2b). To find out light rays corresponding to the LPL for negative argument of Lobachevsky function (for $V < 0$ in the both sides), one should use side beams directed opposite to X -axis, i.e. from the right to the left (the previous ones for $V > 0$ were directed from the left to the right), see Fig. 2c and Fig. 2d. The complete picture in the Euclidean plane corresponding to the LPL on a plane in the velocity space is presented at Fig. 3.

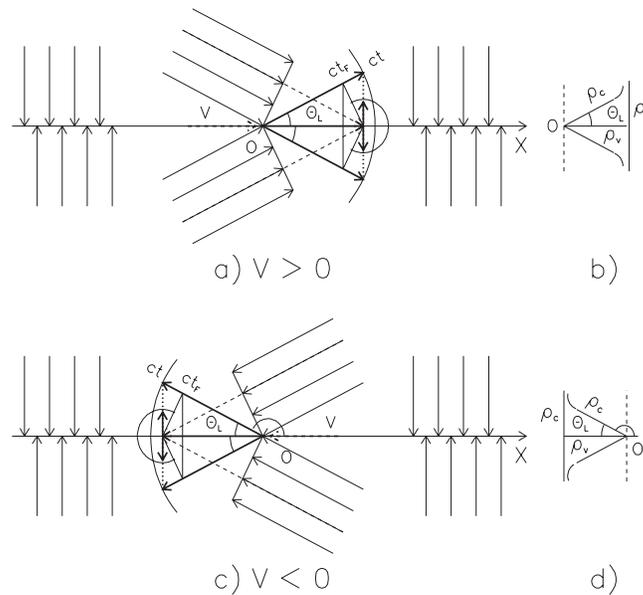


Figure 2. a) two side light beams (for $V > 0$) give two pairs of light rays ct and ct_s for both sides of the plane (up and down), synchronous with a K_s -motion Vt ; b) parallel lines in both sides on Lobachevsky plane, corresponding to synchronous motions in a); c) and d) are the same one as in a) and b), but for $V < 0$.

Thus, a moving reference frame ($V > 0$ or $V < 0$) is associated with definite side light beams. For fixed frame $V = 0$ and the fixed frame is associated with straight beams as in this case $\theta_L = \pi/2$ (see (1) and Fig. 2). The physical nature of Lobachevsky parallel lines reveals a new way to solve the main difficulty in relativity – the problem of definition of moments of time for different space points.

3 x and t – coordinate transformation and light ether concept

Let us continue the previous consideration of two inertia systems K and K_s ($V > 0$). Let us assume that a straight beam hits X -axis at the same moment of time when a side beam hits

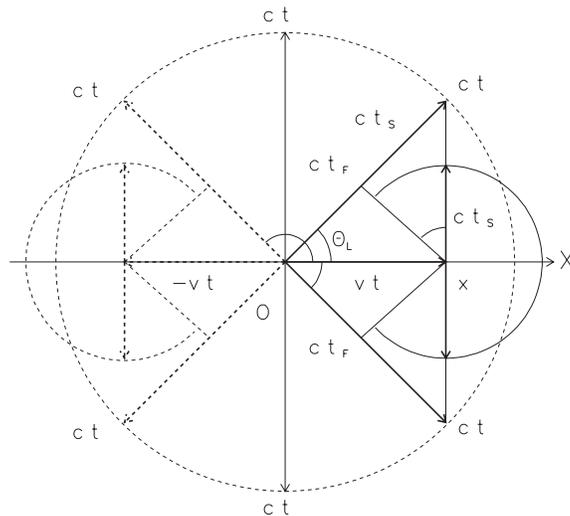


Figure 3. A summary diagram to illustrate of Huygens synchronization of corresponding light rays and two motions ($V > 0$ and $V < 0$) in K -frame.

a point where the both origins coincide. Then all x -points (including O) are “fired” simultaneously due to the straight beam, and this moment of time is usually taken as the initial one for the K frame. With respect to the side beam the initial moment of time for any x -point is shifted by the delay time t_F (see (4)). The time t_s in a given x -point (in K) by a given moment of time t (in K) is defined by (5). Thus, due to synchronization of K and K_s systems any x point has two times: t and t_s . As the velocity of K_s is known then t_s depends only on a chosen event.

Let us a measure time moment t in the fixed frame through the distance of light ray ct emitted from the point O at the parallel angle to X -axis in some plane. Then for any event (x, t) the delay time ct_F is just a projection of the given x on the chosen light ray ct (see Figs. 1–4).

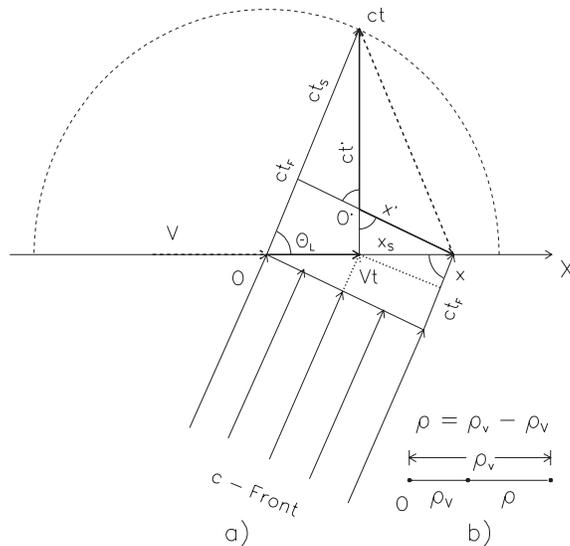


Figure 4. a) an illustration for the inertial frame x and t coordinate transformation (including Lorentz transformation); b) a velocity space diagram corresponding to x and t shifts (by the moment of time t a given x coordinate is assumed as x -position of a particle, moving with a velocity $v = x/t$ in K -frame).

It is obvious that the K_s origin displacement $Vt = ct \cos \theta_L$ is just a projection of light ray ct on the X -axis. So, a given x at a given time t has a value x_s relative to the origin O_s :

$$x_s = x - Vt = x - ct \cos \theta_L. \tag{7}$$

For any event $(x = Vt, t)$ a relative coordinate is $x_s = 0$. It means that time t_s (see (5) and (6)) is a proper time of K_s – the time “measured” by means of the “moving clock”. An observer in K sees light sphere with radius ct and at the same time t a moving observer sees another light sphere with radius ct_s . Thus, for any event (x, t) in K the corresponding coordinates for K_s one can find as simple shifts (see (5) and (7)). To find out the values of shifts, one should produce the mentioned above symmetrical projections.

Let us use the established symmetry to find out Lorentz coordinates x' and t' for a moving frame. To obtain them, one should find the crossing point O' of two perpendiculars producing the mentioned projections (see Fig. 4). Then the length of the side from O' up to the x corresponds to x' :

$$x' = (x - ct \cos \theta_L) / \sin \theta_L = (x - Vt) / \sqrt{1 - V^2/c^2}, \quad x_s = x' \sin \theta_L, \quad (8)$$

and the length of the side from O' up to the ct corresponds to ct' :

$$ct' = (ct - x \cos \theta_L) / \sin \theta_L = (ct - xV/c) / \sqrt{1 - V^2/c^2}, \quad ct_s = ct' \sin \theta_L. \quad (9)$$

It is seen from (8) and (9) that primed and shifted coordinates are related as corresponding projections. But the point O' , which is always accepted as the origin of the moving frame, does not coincide in space with O_s . It is also seen that $O'x'$ line is not parallel to the X -axis. So, it seems obvious that primed values cannot be accepted as coordinates of a moving frame.

Now, let us look at the length of a side between the given points x and ct (dashed line in Fig. 4). It can be obviously written through the primed and unprimed values:

$$l^2 = c^2 t^2 + x^2 - 2ctx \cos \theta_L = c^2 t'^2 + x'^2 + 2ct'x' \cos \theta_L, \quad (10)$$

or as a sum of two terms, either as $l^2 = s_1^2 + s_2^2$ (for that one should add $\pm x^2$ to the left part of (10) and $\pm x'^2$ to the right part of it), or as $l^2 = -s_1^2 + s_3^2$ (for that one should add $\pm c^2 t^2$ to the left part of (10) and $\pm c^2 t'^2$ to the right part of it), where:

$$s_1^2 = c^2 t^2 - x^2 = c^2 t'^2 - x'^2 = \gamma^2 (c^2 t_s^2 - x_s^2), \quad \gamma = 1 / \sin \theta_L = 1 / \sqrt{1 - V^2/c^2}, \quad (11)$$

$$s_2^2 = 2x(x - ct \cos \theta_L) = 2x'(x' \pm ct' \cos \theta_L),$$

$$s_3^2 = 2ct(ct - x \cos \theta_L) = 2ct'(ct' \pm x' \cos \theta_L). \quad (12)$$

Term s_1^2 is known as an invariant interval. It is seen that it is only a part of full length l^2 and that this part is a result of cancelling of two equal values, either s_2^2 or s_3^2 , in the possible expressions for l^2 . Terms s_2^2 and s_3^2 may differ by sign: (+) corresponds to the case when the point O' is inside and (–) when it is outside of the angle θ_L . For an event $(x = Vt, t)$ term s_2^2 is equal to zero (as $x' = 0$) and $s_3^2 = 2s_1^2$, so $s_1^2 \equiv l^2$. Just for this case the Lorentz coordinate transformation are usually proved in manuals (see e.g. [4]).

By using second formulas in (8), (9) one can find from (3):

$$x = (x_s + ct_s \cos \theta_L) / \sin^2 \theta_L = (x_s + Vt_s) / (1 - V^2/c^2), \quad (13)$$

$$ct = (ct_s + x_s \cos \theta_L) / \sin^2 \theta_L = (ct_s + Vx_s/c) / (1 - V^2/c^2), \quad (14)$$

just the reverse transformation from the moving frame to the fixed one. To be sure of that, one has to solve a system of (5) and (7) with respect to x and ct . To make a geometrical meaning of the latter formulas more clear, it is useful to insert the factor $1/\sin \theta_L$ into brackets (then terms in brackets are lengths of perpendiculars corresponding to the above-mentioned projection symmetry).

As seen from (5), (7) and (13), (14) that the straight and reverse transformations are different: back formulas could not be taken by changing V on $-V$. It means that one knows either that

the same way. The reverse transformation is obvious:

$$y = y_s / \sin \theta_L = y_s / \sqrt{1 - V^2/c^2}, \quad z = z_s / \sin \theta_L = z_s / \sqrt{1 - V^2/c^2}. \quad (18)$$

For the non-invariant interval (see (11)) by using the latter formulas one can get:

$$c^2 t^2 - x^2 - y^2 - z^2 = \gamma^2 (c^2 t_s^2 - x_s^2 - y_s^2 - z_s^2). \quad (19)$$

Thus, the obtained coordinate transformations lead to the reduced interval (i.e. to the light sphere with the reduced radius) but they do not contradict to the velocity summation law. Since the energy-momentum transformation is a direct consequence of the velocity summation law, then the Lorentz energy-momentum transformation is valid in this approach [5]. Also in [5] relativistic effects considered in detail, and the four elements complex fraction invariant and a new wave equation in framework of this approach were proposed.

5 Conclusion

- A complete correspondence has been established between Lobachevsky parallel lines in the velocity space and the processes of particle and light beams propagation in the ordinary space, synchronized by Huygens principle.
- The time delay in the emission of two light rays has been found as the physical reason for their intersection point absence and for the V postulate denial.
- New contents of the simultaneity conception, common time and proper time have been formulated.
- New inertial frames coordinate transformations (as shifts) have been obtained.
- It has been shown, that relativistic effects happen due to the coordinate and time reference point shifts. Changing the way of measuring the space or time interval lengths, one can find the way when these values are the same in fixed and moving frames.
- It has been shown that Lorentz energy-momentum transformation is a direct consequence of the relativistic velocity summation law.
- It has also been shown that Lobachevsky function is a tool to express the constant light velocity principle.
- The four elements of the complex fractional invariant and a possible wave equation have been proposed.

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