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Hadronic processes and electromagnetic corrections

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The inclusion of electromagnetism in a low energy effective theory is worth further study in view of the present high precision experiments (muon $g-2$, $\pi_0 \to \gamma\gamma$, τ decays, etc.). In particular in many applications of chiral perturbation theory, one has to purify physical matrix elements from electromagnetic effects. The theoretical problems that I want to point out here are following: the splitting of a pure QCD and a pure electromagnetic part in a hadronic process is model dependent: is it possible to parametrise in a clear way this splitting? What kind of information (scale dependence, gauge dependence,..) is actually included in the parameters of the low energy effective theory? I will attempt to answer these questions introducing a possible convention to perform the splitting between strong and electromagnetic parts in some examples.
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1 Introduction

The low energy effective theory of the Standard Model in the hadron sector is the Chiral Perturbation Theory (ChPT). The chiral Lagrangian has been enlarged in order to include also electromagnetic effects, (f. i. in the meson sector [1,2]). In order to illustrate the object of the present work let us consider the example of the decay of $\eta \to 3\pi$ in the framework of QCD [3]. The amplitude for this decay is proportional to $1/Q^2$, where Q^2 denotes a ratio of quark masses in pure QCD. One may evaluate Q^2 from the meson mass ratio

$$Q^2 = \frac{m_K^2}{m_\pi^2} \frac{m_K^2 - m_\pi^2}{(m_{K^0}^2 - m_{K^+}^2)_{\rm QCD}} (1 + O(m_{quark}^2)),$$

and predict the width. In this manner, the mass difference of the kaons in pure QCD shows up. In order to determine this difference, one has to properly subtract the contributions from electromagnetic interactions to the kaon masses [4]. However, due to ultraviolet divergences generated by photon loops, the splitting of the Hamiltonian of QCD+γ into a strong and an electromagnetic piece is ambiguous. The calculation of $(M_{K^+}^2 - M_{K^0}^2)_{\rm QCD}$ in the effective theory must therefore reflect this ambiguity as well. An analogous problem occurs whenever one wants to extract hadronic quantities from matrix elements which are contaminated with electromagnetic contributions. One is confronted with two separate issues here. The first one is a proper definition of strong and electromagnetic contributions in a given theory. The second, separate point concerns the construction of the corresponding effective low-energy Lagrangian (see also Ref. [5]). The aim of this discussion is i) to investigate the problem of electromagnetic corrections in QCD+γ, in the sense that the generating functional of Green functions of scalar, vector and axial vector currents is extended to include radiative corrections at order α, and ii) to construct the relevant effective theory at low energies, taking into account the ambiguities mentioned. The Lagrangian built by Urech [1] so is worth a deeper study.

The problem is a complex one and I refer to our recent work [6] for a complete discussion of some relevant examples and technical details. In this work I will present an overview of our work [6].

2 Parametrisation of the splitting

In order to see how the splitting of strong and electromagnetic contributions works let me consider as an example the linear sigma model (LσM). Without electromagnetism the Lagrangian of the model has an $O(4)$ symmetry spontaneously broken to $O(3)$. The corresponding effective theory at low energies may be analysed with the Lagrangian used in ChPT, with low-energy constants that are fixed in terms of the couplings of the LσM [7,8]. Thus in this example the LσM acts as the strong high energy part of the theory. I couple the four real scalar fields ϕ^A in the LσM to external vector and axial vector fields and incorporate electromagnetic interactions, $\mathcal{L}_\sigma = \mathcal{L}_0 + \mathcal{L}_{\rm ct}$,

$$\begin{aligned}\mathcal{L}_0 \;=\; & \frac{1}{2}\,(d_\mu\phi)^T d^\mu\phi + \frac{m^2}{2}\,\phi^T\phi - \frac{g}{4}(\phi^T\phi)^2 + c\phi^0 \\ & + \frac{\delta m^2}{2}\,(Q\cdot\phi)^T(Q\cdot\phi) - \frac{\delta g}{2}\,(\phi^T\phi) \\ & \times (Q\cdot\phi)^T(Q\cdot\phi) - \frac{1}{4}\,F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}\,(\partial_\mu A^\mu)^2 \,, \qquad (1)\end{aligned}$$

The details of the notation and definitions can be found in ref. [6]. What is important to note here is that in our metric the spontaneously broken phase occurs at $m^2 > 0$. Since the electromagnetic interactions break isospin symmetry, we have explicitly introduced the isospin breaking terms $\sim \delta m^2, \delta g$ from the very beginning. The counterterms are collected in $\mathcal{L}_{ct}$, see ref. [6]. The symmetry breaking parameter c is considered to be of non-electromagnetic origin – it provides the Goldstone bosons with a mass also at $e = 0$. In order to render the formulae more compact and make the counting more evident, I will use also the following notation for the couplings δm^2 and δg: $\delta g = e^2 g c_g$, $\delta m^2 = e^2 m^2 c_m$. The new couplings c_g and c_m are assumed to be independent of e at this order, $c_{g,m} \simeq O(p^0)$ and $e^2, c \simeq O(p^2)$. At tree level the masses of the pions, the sigma and the vacuum expectation value are

$$m^2_{\pi^0} \;=\; \frac{c}{v_0} \;\;,\; m^2_{\pi^+} = m^2_{\pi^0} - \delta m^2 + \delta g v_0^2 \;\;, \tag{2}$$

$$m^2_\sigma \;=\; 2m^2 + 3m^2_{\pi^0} \;\;,\; v_0 = \frac{m}{\sqrt{g}} + \frac{c}{2m^2} + O(p^4) \;\;.$$

I omit here the issue of the splitting in the vector currents for simplicity. Among the others the discussion of this issue is important in order to understand the gauge dependence of the splitting in the effective theory. I refer to [6] for this discussion.

2.1 The splitting procedure and the matching scale μ_1

In order to illustrate the splitting procedure I will consider the effect of the splitting only on some (running, physical) masses of the model and also some strong coupling as g and c. Strong effects are computed at one loop and all the effects of order e^4 are neglected. For any kind of mass X (and also the strong couplings g and c) it is possible to write $X = \bar{X} + e^2 X_1$, where $\bar{X}$ is the pure strong part of the mass. We want to define the pure strong contribution as that which is obtainable in a theory with $e = 0$. This definition is consistent at one loop if $\frac{d}{d\mu} X\Big|_{e=0} = \frac{d}{d\mu}\bar{X}$. This equation defines the dependence of $\bar{X}$ on the renormalization scale μ. This relation shows also that one has to fix a boundary condition in order to fix $\bar{X}$. A natural condition consists in choosing that at a scale μ_1 one has $\bar{X}(\mu;\mu_1)\big|_{\mu=\mu_1} \equiv X(\mu_1)$. The pure e.m. contribution comes then by the difference $X_1(\mu;\mu_1) = X(\mu) - \bar{X}(\mu;\mu_1)$.

To make things more explicit let us see the couplings m and g of the Lagrangian in eq. 1. The matching equations are

$$g(\mu) \;=\; \bar{g}(\mu;\mu_1)\left\{1 + c_g \frac{e^2\bar{g}}{2\pi^2} \ln\frac{\mu}{\mu_1}\right\}, c = \bar{c}\,,$$

$$m^2(\mu) \;=\; \bar{m}^2(\mu;\mu_1)\left\{1 + (c_g + c_m)\frac{e^2\bar{g}}{4\pi^2} \ln\frac{\mu}{\mu_1}\right\} \;\;. \tag{3}$$

In the following I denote with a barred quantity an expression evaluated at $e = 0$, with $(g, m) \to (\bar{g}, \bar{m})$.

Another example is provided by the physical pion masses ($M_{\pi^{0,+}}$) at one loop. To determine the physical pion masses, one evaluates the pole positions in the Fourier transform of the two-point functions $\langle 0|T\phi^i(x)\phi^i(0)|0\rangle, i = 1, 3$. In the following I will consider only the neutral pion mass for simplicity,

$$M^2_{\pi^0} \;=\; m^2_{\pi^0}\left\{1 + \frac{g}{m^2_\sigma}\left(V_0 + 2L_{\pi^+} - L_{\pi^0}\right)\right\} + O(e^4, p^6) \;\;. \tag{4}$$

where

$$V_0 \;=\; (3+2y)L_\sigma - \frac{m^2_\sigma}{48\pi^2}(3+7y) \;\;;\;\; y = \frac{m^2_{\pi^0}}{m^2_\sigma};$$

$$L_X \;=\; \frac{m^2_X}{16\pi^2}\left\{\ln\frac{m^2_X}{\mu^2} - 1\right\} \;\;.$$

Starting from these equations one then expresses the parameters g, m, c through the isospin symmetric couplings $\bar{g}, \bar{m}$ and $\bar{c}$ by use of Eq. (3). Next, we observe that the dependence on the electric charge in Eq. (3) is an effect of order $\hbar$. Therefore, to the accuracy considered here, the splitting (3) must be applied to the tree-level expressions only,

$$v_0 \;=\; \bar{v}_0\left\{1 - C\ln\mu^2/\mu_1^2\right\} + O(p^4) \;\;,$$

$$m^2_{\pi^0} \;=\; \bar{m}^2_\pi\left\{1 + C\ln\mu^2/\mu_1^2\right\} + O(p^6) \;\;, \tag{5}$$

where

$$C \;=\; (c_g - c_m)\frac{e^2\bar{g}}{16\pi^2} \;\;,\; \bar{m}^2_\pi = \frac{\bar{c}}{\bar{v}_0} \;\;. \tag{6}$$

The μ_1 dependence is $\mu_1 \frac{d}{d\mu_1}(\bar{m}^2_\pi, \bar{v}_0) = 2C(\bar{m}^2_\pi, -\bar{v}_0)$. The strong part of $M^2_{\pi^0}$ is the same for the neutral and for the charged pion mass, $\bar{M}^2_\pi \doteq \bar{M}^2_{\pi^0} = \bar{M}^2_{\pi^+}$,

$$\bar{M}^2_\pi \;=\; \bar{m}^2_\pi\left\{1 + \frac{\bar{g}}{\bar{m}^2_\sigma}(\bar{V}_0 + \bar{L}_\pi)\right\} + O(p^6) \;\;. \tag{7}$$

The electromagnetic corrections are given by $e^2 M^{2,1}_{\pi^0} = M^2_{\pi^{0,+}} - \bar{M}^2_\pi$. For the neutral pion mass they are

$$e^2 M^{2,1}_{\pi^0} \;=\; \frac{\bar{m}^2_\pi\bar{g}}{16\pi^2\bar{m}^2}\left(m^2_{\pi^+}\ln\frac{m^2_{\pi^+}}{\mu^2} - m^2_{\pi^0}\ln\frac{m^2_{\pi^0}}{\mu^2}\right) + \bar{m}^2_\pi C\left(\ln\frac{\mu^2}{\mu_1^2} - 1\right) + O(e^4, p^6) \;\;. \tag{8}$$

A similar expression holds for the charged pion mass.

The quantity $\bar{M}_\pi$ denotes the isospin symmetric part of the pion mass. It coincides neither with the neutral nor with the charged pion mass, and is independent of the running scale μ. It depends, however, on the scale μ_1 where the matching has been performed, $\mu_1 \frac{d}{d\mu_1} \bar{M}_\pi^2 = 2C\bar{m}_\pi^2 + O(e^4, p^6)$. As C is of order e^2, this scale dependence of the isospin symmetric part is of order p^4. The electromagnetic part $e^2 M_{\pi^0}^{2,1}$ has the same scale dependence, up to a sign, as a result of which the total mass is independent of μ_1.

3 Splitting in the effective theory

At low energy the LσM with the inclusion of electromagnetism can be analyzed with the low energy effective theory of Gasser and Leutwyler, [7], enlarged by Urech, [1]. The matching condition states that the Green functions in the effective theory must coincide with those in the original theory at momenta much smaller than the σ-mass. At the end, one evaluates Green functions in the limit where the charge matrices become space-time independent. Now the linear sigma model with space-time dependent spurion fields has the same symmetry as the theory that underlies the construction of the effective Lagrangian performed by Urech [1]. Thus I will determine particular light energy constants (LECs) by comparing physical quantities calculated in the underlying and in the effective theory.

3.1 Matching pion masses

I first consider the purely strong part in the pion mass, displayed in Eq. (7). For the low-energy expansion one finds that

$$\bar{M}_\pi^2 = \bar{M}^2 \left[1 - \frac{1}{32\pi^2} \frac{\bar{M}^2}{\bar{F}^2} \left(\frac{16\pi^2}{\bar{g}} - 11 \ln \frac{2\bar{m}^2}{\mu^2} + \frac{22}{3} - \ln \frac{\bar{M}^2}{\mu^2} \right) \right] + O(p^6) \ , \tag{9}$$

where the complete expression for $\bar{F}^2$ and $\bar{M}^2$ are reported in [6]. The quantity $\bar{F}$ denotes the pion decay constant in the chiral limit, evaluated in the framework of the linear sigma model at order $\hbar$, see Refs. [7,8], from where the expression for $\bar{F}$ is taken. I have used the fact that $\bar{M}^2$ is linear in c [7,8] – this fixes the structure of the expansion uniquely.

I may now compare Eq. (9) with the expansion of the pion mass in the effective theory at $e = 0$. I find for the parameters in the effective theory

$$M^2 = 2\hat{m}B = \bar{M}^2 \ , \quad F^2 = \bar{F}^2 \ , \quad l_7 = 0 \ ,$$

$$l_3^r(\mu_{\rm eff}) = -\frac{1}{64\pi^2} \left(\frac{16\pi^2}{\bar{g}} - 11 \ln \frac{2\bar{m}^2}{\mu^2} + \frac{22}{3} + \ln \frac{\mu^2}{\mu_{\rm eff}^2} \right) . \tag{10}$$

Note that M^2, l_7 and F^2 are independent of the scales μ and $\mu_{\rm eff}$ of the underlying and of the effective theory. On the other hand, the pion decay constant and the mass parameter M^2 depend on the matching scale μ_1. At one loop,

$$\frac{\mu_1}{F^2} \frac{d}{d\mu_1} F^2 = -2 \frac{\mu_1}{M^2} \frac{d}{d\mu_1} M^2 = \frac{e^2 \bar{g}(c_m - c_g)}{4\pi^2} \ . \tag{11}$$

The last term in this equation is proportional to the charged pion (mass)^2 in the chiral limit, see below. Using the DGMLY sum rule [11] gives

$$F(\mu_1 = 500\,{\rm MeV}) - F(\mu_1 = 1\,{\rm GeV}) = 0.1\,{\rm MeV} \ . \tag{12}$$

The uncertainty related to μ_1 so is of the order of the PDG error [12].

One can also determine the linear combinations $\mathcal{K}_{\pi^0}^r$, of the electromagnetic couplings k_i^r that occur in the expansion of the neutral pion mass in the effective theory, see [6]. One finds also that whereas the coupling $\mathcal{K}_{\pi^0}^r$, is independent of the scale μ, it depends on the matching scale μ_1. Finally, I display the neutral pion mass in the linear sigma model, properly expanded in powers of momenta up to $O(p^4)$, and electromagnetic corrections disentangled, $M_{\pi^0}^2 = \bar{M}_\pi^2 + e^2 M_{\pi^0}^{2,1} + O(e^4)$,

$$\bar{M}_\pi^2 = M^2 \left\{ 1 + \frac{2M^2}{F^2} \left(l_3^r + \frac{1}{64} \ln \frac{M^2}{\mu_{\rm eff}^2} \right) \right\}$$

$$e^2 M_{\pi^0}^{2,1} = \frac{M^2}{16\pi^2 F^2} \left\{ M_{\pi^+}^2 \ln \frac{M_{\pi^+}^2}{\mu_{\rm eff}^2} - M^2 \ln \frac{M^2}{\mu_{\rm eff}^2} \right\} + e^2 M^2 \mathcal{K}_{\pi^0}^r \ . \tag{13}$$

3.2 A comparison to other approaches within the model

The splitting of pure e.m. and strong effects has been considered also in other papers, see f.i. [9,10]. What we want to discuss here is the approach presented here versus the ones used previously. To this aim I write the result (4) for the neutral pion mass in the form $M_{\pi^0}^2 = f_0 + e^2 f_1 + O(e^4, p^6)$, where

$$f_0 = m_{\pi^0}^2 \left\{ 1 + \frac{g}{m_\sigma^2} (V_0 + L_{\pi^0}) \right\} \ , \tag{14}$$

and $e^2 f_1$ is the rest. Since the physical mass is scale independent, one has $\mu \frac{df_0}{d\mu} = -e^2 \mu \frac{df_1}{d\mu}$. Consider now the splitting of electromagnetic and strong effects. In the language of Ref. [9,10], f_0 ($e^2 f_1$) is the *strong* (*electromagnetic*) part of the physical mass. Both, the strong and the electromagnetic parts of the mass are μ-dependent in this case. One may again work out the low-energy representation of $M^2_{\pi^0}$ and identify the low-energy constants in this language. For the strong part, one finds the expressions displayed in Eqs. (9)-(10), with $(\bar{g}, \bar{m}^2) \to (g, m^2)$, whereas the electromagnetic LECs are collected in $\mathcal{K}^r_{\pi^0} = \frac{(c_m - c_g)g}{16\pi^2}\left(1 - \ln \mu^2_{\rm eff}/\mu^2\right)$. Here, the μ dependence of $\mathcal{K}^r_{\pi^0}$ shows up. This scale dependence of the electromagnetic part is canceled by the corresponding scale dependence of the strong part.

In our framework, the *strong* part is given by

$$\bar{M}^2_\pi = f_0\bigg|_{g=\bar{g}, m=\bar{m}, c=\bar{c}} , \tag{15}$$

where the couplings $\bar{g}, \bar{m}$ run with the strong part alone, see the discussion in earlier sections. The difference $M^2_{\pi^0} - \bar{M}^2_\pi$ is called *electromagnetic correction* in this article. Both, the strong and the electromagnetic part are μ-independent. We note that the μ dependence of $\mathcal{K}^r_{\pi^0}$ is the same as the μ_1 dependence in our procedure for the splitting. One can show that such a correspondence exists for all quantities that are μ-independent. On the other hand, it does not hold anymore e.g. in the case of the charged form factor, whose matrix elements are μ dependent.

4 Conclusions

In this work I have summarized some of the main results of ref. [6]. In this work it is outlined a method to split consistently e.m. effects in Quantum Field Theory. The splitting that is proposed is done order by order in the loop expansion. The strong part of a quantity depends only on couplings defined in a theory with $e = 0$ (up to the desired perturbative order in e) and it has no running proportional to the electromagnetic coupling e (still, up to the perturbative order in e which is considered). In order to proceed correctly in the construction of an effective theory it is important to characterize the relevant scales of the problem: μ (the renormalization scale of the underlying theory), $\mu_{\rm eff}$ (the renormalization scale of the effective theory) and μ_1 (the scale at which the strong part of a quantity is defined). The splitting ambiguities are parametrised by the scale μ_1. The uncertainty related to μ_1 can be of numerical relevance as it is shown in eq. 12. In fact the error induced on F by μ_1 is of the order of the PDG error [12]. Another advantage of the splitting which is proposed here, is that in the effective Lagrangian the parameters in the strong sector are expressed through the ones of the underlying theory in its strong sector. This makes the matching between the underlying and the effective theory more transparent. Finally the LECs of the effective theory also contain all information about scale and gauge dependence of the Green functions in the underlying theory with electromagnetism.

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