



# Polarized parton distributions: present status and prospects

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We review the present status of the measurements of polarized structure functions, and of the corresponding extraction of polarized parton distribution functions in the context of perturbative QCD. We illustrate how present and future experiments are expected to improve our knowledge of the structure of polarized hadrons, with special attention to the capabilities of neutrino factories.

## 1 Present status

Most of our present knowledge of polarized parton distribution functions originates from inclusive, neutral-current deep-inelastic scattering experiments. This class of experiments has a number of intrinsic limitations. It is easy to show, for example, that only the C–even combination  $\Delta q^+ = \Delta q + \Delta \bar{q}$  of polarized quark plus polarized antiquark distribution function is accessible. Correspondingly, it is impossible to separate quarks and antiquarks densities.

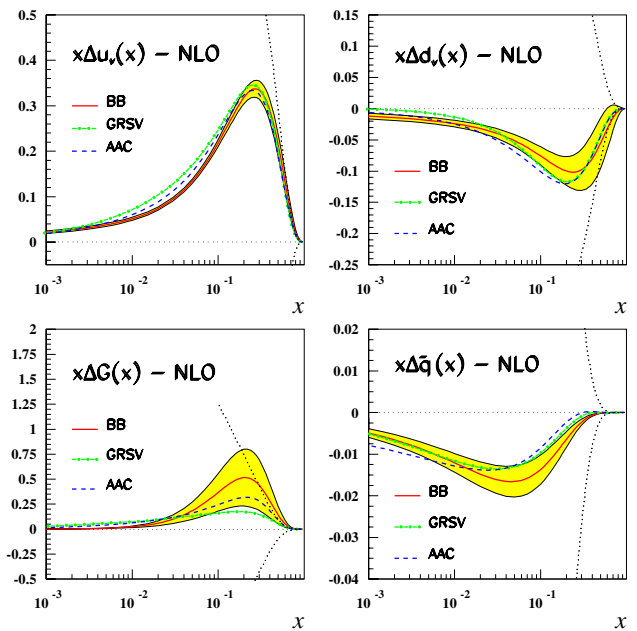
These experiments are also characterized by a weak sensitivity to the polarized strange quark density  $\Delta s$ . In general, it proves very difficult to perform an efficient determination of individual flavor densities.

The polarized gluon distribution function  $\Delta g$  and the flavor singlet combination of quark polarized distributions  $\Delta \Sigma^+ = \Delta u^+ + \Delta d^+ + \Delta s^+$  are of great phenomenological interest. In polarized deep-inelastic scattering experiments,  $\Delta g$  can only be reconstructed through a measurement of scaling violations (similarly to what happens in the case of unpolarized parton densities); we will see that this determination is affected by large uncertainties. The situation is even worse for  $\Delta \Sigma^+$ , since only its first moment can be directly accessed, and even in this case the determination is somehow unsatisfactory, since it is based on the knowledge of non-singlet first moments from other experiments, like hyperon decay rates, and relies on theoretical assumptions.

It should be stressed that indirect measurements based on scaling violations are only possible in the perturbative regime, which in the case of QCD corresponds to relatively large values of the squared momentum transfer  $Q^2$ . This means that a large amount of experimental information, corresponding to  $Q^2$  less than about 1 GeV<sup>2</sup>, is useless for this purpose. It should also be noted that, due to experimental difficulties, the kinematic coverage of polarized deep-inelastic scattering experiments is much less extensive than in the unpolarized case. This means in particular that scaling violation measurements are difficult, because of the limited lever arm in the  $Q^2$  evolution, and that the small- $x$  region is still largely unexplored.

A full next-to-leading order analysis of polarized DIS data

in the context of perturbative QCD has been performed by many different groups; a comparison among the different assumptions and techniques adopted is beyond the scope of this talk. An accurate and up-to-date analysis of polarized parton densities, as well as a complete list of references, can be found in ref. [1]. The results are summarized in fig. 1, which is taken from ref. [1]. The different flavor



**Figure 1.** The polarized parton distributions of ref. [1]. The shaded areas correspond to 1- $\sigma$  errors.

densities in ref. [1] have been obtained assuming a flavor-symmetric sea,  $\Delta \bar{u} = \Delta \bar{d} = \Delta \bar{s} = \Delta s$ . In fig. 1, the results of refs. [2] and [3] are also shown, together with a band representing the 1- $\sigma$  error from the fitting procedure.

A central issue in the subject of physics of polarized nucleons is to explain the unexpected smallness of the axial charge

$$a_0(Q^2) = \int_0^1 dx \left( \Delta \Sigma^+ - \frac{n_f \alpha_s}{2\pi} \Delta g \right) \quad (1)$$

(this expression of the singlet axial charges holds in a factorization scheme, such as the so-called AB scheme, where the first moment of  $\Delta\Sigma^+$  is scale independent. In the  $\overline{\text{MS}}$  scheme one finds  $a_0(Q^2) = \int_0^1 dx \Delta\Sigma^+$ .) Present data indicate that  $a_0$  is compatible with zero, but values as large as  $a_0(10 \text{ GeV}^2) = 0.3$  are not excluded. A possible theoretical interpretation is to assume that a cancellation between a large (scale-independent, or AB-scheme)  $\Delta\Sigma^+$  and a large  $\Delta g$  takes place. In this case,  $|\Delta u^+|, |\Delta d^+| \gg |\Delta s^+|$  (in the AB-scheme), as expected in the quark model. Alternatively, one may assume that  $\Delta g$  is indeed small, and that  $\Delta s^+$  is large and negative. This might be explained by invoking non-perturbative, instanton-like vacuum configuration. In this case,  $\Delta s = \Delta\bar{s}$ . Alternatively, one can imagine a scenario in which  $\Delta s^+$  is large, but  $\Delta s$  is significantly different from  $\Delta\bar{s}$ . Future experiments will aim at improving the accuracy in the determination of the axial charge and of the polarized gluon distribution. Furthermore, it will be important to understand whether  $\Delta s$  is large, compared to the light flavor distributions, and how different it is from  $\Delta\bar{s}$ .

The first moments of polarized parton densities as obtained in ref. [4] are shown in table 1,

**Table 1.** Fits to neutral-current polarized DIS data.

	generic fit	$\eta_g = 0$ fit
$\eta_\Sigma$	$0.38 \pm 0.03$	$0.31 \pm 0.01$
$\eta_g$	$0.79 \pm 0.19$	0
$\eta_3$	$1.110 \pm 0.043$	$1.039 \pm 0.029$
$\eta_8$	0.579	0.579
$\eta_u$	0.777	0.719
$\eta_d$	-0.333	-0.321
$\eta_s$	-0.067	-0.090
$a_0$	$0.183 \pm 0.030$	$0.284 \pm 0.012$

where  $\eta_\Sigma, \eta_3$  and  $\eta_8$  are the first moments of  $\Delta u^+ + \Delta d^+ + \Delta s^+$ ,  $\Delta u^+ - \Delta d^+$  and  $\Delta u^+ + \Delta d^+ - 2\Delta s^+$  respectively (all of them are scale-independent quantities in the AB scheme), and  $\eta_g$  is the first moment of the polarized gluon distribution at  $Q^2 = 1 \text{ GeV}^2$ . The value of  $a_0$  at  $Q^2 = 10 \text{ GeV}^2$  is also given. The second column corresponds to a fit with  $\eta_g$  forced to zero. This value, although disfavoured by the data, can only be excluded at about two standard deviations.

In the rows labelled  $\eta_u, \eta_d, \eta_s$  in table 1 we show the values of the first moments of  $\Delta q^+$  at  $Q^2 = 1 \text{ GeV}^2$  for  $q = u, d, s$ . It can be seen that  $|\eta_s| \ll |\eta_u|, |\eta_d|$ .

As mentioned earlier, the polarized gluon distribution  $\Delta g(x, Q^2)$  has historically attracted a special attention, because of its special features. In particular, a  $\Delta g$  with a

large and positive first moment is one of the invoked explanations of the unexpected smallness of the singlet axial charge [5,6], since the quantity  $\alpha_s \eta_g$  does not vanish at large values of  $Q^2$ . Unfortunately, its determination through scaling violations is difficult. This can be easily understood by considering the  $Q^2$  evolution of the Mellin moments

$$\Delta g(N, Q^2) = \int_0^1 dx x^{N-1} \Delta g(x, Q^2). \quad (2)$$

Moments corresponding to small values of  $N \ll 1$  are mainly determined by the region of very small values of the Bjorken variable  $x$ . This kinematic region is very poorly known: only a few data points for the structure function  $g_1(x, Q^2)$  have been measured for  $x$  between  $5 \cdot 10^{-3}$  and  $\sim 10^{-2}$  by the SMC collaboration, with relatively large errors, while experiments at SLAC only access  $x$  larger than  $\sim 10^{-2}$ .

On the other hand, moments with  $N \gg 1$  are determined by the large  $x$  region, where the effects of evolution are less important. As a result, an analysis of scaling violations can only determine moments with  $N \sim 1$  with a reasonable accuracy. The detailed  $x$  dependence of  $\Delta g$  therefore remains largely unknown, as can be seen from fig. 1.

Given the large uncertainties involved, different determinations of the first moment of  $\Delta g$  are in good agreement with each other. In table 2, we have collected the determinations of the first moment of  $\Delta g$  obtained by some of the most recent analyses.

**Table 2.** Determinations of  $\Delta g$  in different analyses, as indicated in the first column. The value of the axial charge is also shown.

	$Q^2$ (GeV <sup>2</sup> )	$\Delta g$	$n_f \alpha \Delta g / (2\pi)$	$a_0$
ABFR [4]	1	0.79	0.182	0.199
BB 1 [1]	4	1.026	0.194	0.140
BB 2 [1]	4	0.931	0.177	0.153
E155 [7]	5	1.6	0.29	0.23
AAC 1 [2]	1	0.532	0.122	0.05
AAC 2 [2]	1	0.533	0.122	0.24

As already mentioned, these values are obtained by a fitting procedure of measured values of the structure function  $g_1(x, Q^2)$  for different targets, performed with the expression of  $g_1$  given by the parton model to next-to-leading order in QCD. Thus, these results are affected both by the experimental errors, taken into account by the fitting procedure, and by theoretical uncertainties.

An obvious source of theoretical uncertainty is the use of perturbation theory at a fixed order. The truncation of the

perturbative expansion of coefficient functions to order  $\alpha_s$  induces an uncertainty which is reflected by the dependence of the results on the choice of the renormalization scale  $\mu_R$ . Truncation of the perturbative expansion of the evolution kernels to order  $\alpha_s^2$ , on the other hand, corresponds to a sensitivity of the result on a factorization scale  $\mu_F$ , unrelated to  $\mu_R$ . Both scales are usually chosen around the value of  $Q^2$  for each data point, to minimize the impact of large logarithms in the perturbative coefficients. Varying  $\mu_R^2$  and  $\mu_F^2$  independently around  $Q^2$  (e.g. by a factor 2) is a way of estimating the size of the neglected higher orders. This turns out to be the most important source of uncertainty on interesting quantities.

It turns out that the assumed  $x$  dependence of the parton densities affects sizeably the extrapolation towards the unmeasured small- $x$  region, and consequently the estimate of physically relevant quantities, such as the first moment of  $\Delta g$ . The corresponding uncertainty may be estimated by performing different fits with different functional forms, as in refs. [8,9].

Other sources of theoretical uncertainty, such as the assumption of exact  $SU(3)_{flavor}$  symmetry for the first moments of non-singlet combinations, the value of  $\alpha_s(m_Z)$  (when fixed), and the positions of heavy quark thresholds, all produce very small effects. The impact of target mass corrections has also been explored [10], and found to be small.

Since some of the data points included in these analyses are at values of  $Q^2$  close to 1 GeV<sup>2</sup>, higher twist terms, suppressed by powers of  $\Lambda^2/Q^2$ , may give sizeable contributions. The corresponding uncertainty is difficult to estimate. It has been checked in ref. [8] that, repeating the whole analysis without low- $Q^2$  data (data between 1 GeV<sup>2</sup> and 2 GeV<sup>2</sup>) has a negligible effect, compared to the uncertainties induced by scale variations.

All theoretical uncertainties on first moments of polarized parton densities, as obtained in refs. [8,9], are collected in table 3.

**Table 3.** Uncertainties in the first moments of polarized parton distributions. The value of  $\eta_g$  is given at  $Q^2 = 1$  GeV<sup>2</sup>.

	$\eta_3$	$\Delta\Sigma$	$\eta_g$	$a_0(\infty)$	$\alpha_s(m_Z)$
fitting	$\pm 0.05$	$\pm 0.05$	$\pm 0.2$	$\pm 0.07$	$\pm 0.001$
$\alpha_s$ & $a_8$	$\pm 0.03$	$\pm 0.01$	$\pm 0.1$	$\pm 0.02$	$\pm 0.000$
thresholds	$\pm 0.02$	$\pm 0.05$	$\pm 0.1$	$\pm 0.01$	$\pm 0.003$
higher orders	$\pm 0.03$	$\pm 0.04$	$\pm 0.3$	$^{+0.15}_{-0.07}$	$^{+0.007}_{-0.004}$
higher twists	$\pm 0.03$	-	-	-	$\pm 0.004$
theoretical	$\pm 0.07$	$\pm 0.08$	$\pm 0.4$	$^{+0.17}_{-0.010}$	$^{+0.009}_{-0.006}$

We conclude this section by noting that the value of

$$\begin{aligned} \eta_3 &= \int_0^1 dx [\Delta u^+(x, Q^2) - \Delta d^+(x, Q^2)] \\ &= 1.110 \pm 0.043(\text{exp}) \pm 0.070(\text{th}) \end{aligned} \quad (3)$$

is in fact a test of the Bjorken sum rule. Comparing this value with the value of the triplet axial charge measured in beta decays

$$|g_A/g_V| = 1.2670 \pm 0.0035 \quad (4)$$

we can conclude that the Bjorken sum rule is tested at the level of  $\sim 10\%$  accuracy.

## 2 Direct determination of $\Delta g$

The results summarized in the previous section lead naturally to consider the possibility of measuring the polarized gluon distribution directly, rather than indirectly from scaling violations. Such a direct measurement can only be performed by observing processes that receive leading contributions by gluon-initiated parton processes. The natural candidates are

- production of heavy quark-antiquark pairs;
- jets with large transverse momentum;
- single-photon production.

Charm-anticharm pairs with polarized beams [11] can be observed in photoproduction (e.g. at polarized HERA), electroproduction (especially interesting for the COMPASS experiment) or hadroproduction (RHIC). For example, the leading order subprocesses relevant for hadroproduction are

$$q\bar{q} \rightarrow c\bar{c} \quad (5)$$

$$gg \rightarrow c\bar{c}, \quad (6)$$

and gluon-gluon fusion dominates at RHIC energies. In all cases, the quantity of interest is the asymmetry

$$A_c = \frac{\sigma_c^{++} - \sigma_c^{+-}}{\sigma_c^{++} + \sigma_c^{+-}}, \quad (7)$$

where  $\sigma_c^{\pm\pm}$  are the cross sections for production of  $c\bar{c}$  pairs with parallelly or antiparallely polarized particles in the initial state. The extraction of  $\Delta g$  from the asymmetry  $A_c$  is in principle feasible, provided that quark contamination from higher order corrections is under control. The characteristic energy scale of this process is the charm mass,  $m_c \sim 1.5$  GeV. Is this large enough for a reliable perturbative expansion? Explicit calculations in the unpolarized

case show that NLO QCD correction are reasonably small for photoproduction, but rather large in hadroproduction. In the polarized case, NLO corrections have been computed for photoproduction [12] and hadroproduction [13]. The experiment COMPASS expects to be able to measure  $\Delta g/g$  with an accuracy of about 10 % at  $x \sim 0.1 - 0.2$  and  $Q^2 \sim m_c^2$ , provided all the design performances of the experiment are achieved.

Two-jet production receives a sizeable contribution from photon-gluon fusion. Potentially, this is a good candidate for a direct measurement of  $\Delta g$ . However, at fixed-target energies jet identification is difficult, because of the large angular spread of hadronic jets, and because of low multiplicities. In order to overcome this difficulty, it has been suggested to observe inclusive production of pairs of hadrons of a definite species (e.g. pions) with opposite azimuth and large transverse momentum. The asymmetry

$$\Delta\sigma_{\pi\pi} = \sum_{f,i,j} \Delta f \otimes \Delta\hat{\sigma}_{ij} \otimes D_{\pi}^i \otimes D_{\pi}^j \quad (8)$$

$$\sigma_{\pi\pi} = \sum_{f,i,j} f \otimes \hat{\sigma}_{ij} \otimes D_{\pi}^i \otimes D_{\pi}^j \quad (9)$$

allows in principle an extraction of the polarized gluon distribution. Also in this case, however, one needs some information about quark contamination and fragmentation functions. A detailed study of these uncertainties would be useful (single-inclusive pion production at NLO has been studied in ref. [14].)

The expected sensitivity of different experiments to  $\Delta g$  by this kind of measurements is shown in fig. 2, together with the gluon distribution as obtained in ref. [15] (The fits of ref. [15] are now obsolete, since they do not include recent sets of data, but they just serve as a comparison between the spread among present determinations and the sensitivity of future experiments.) The HERMES Collaboration has already published [16] a measurement of  $\Delta g$  based on high- $p_T$  hadron pair production; the result is shown in fig. 3.

The possibility of extracting  $\Delta g$  from the observation of isolated photons in polarized  $pp$  collision in the energy range of RHIC was studied in ref. [17]; the conclusion was reached that an accurate determination of  $\Delta g$  in the kinematic range  $0.04 \leq x \leq 0.25$ ,  $10 \text{ GeV} \leq p_T \leq 30 \text{ GeV}$  should be possible.

### 3 Flavor decomposition

Unpolarized densities for quarks of a definite flavor are usually extracted from deep-inelastic scattering data by studying processes mediated by charged weak currents, which involve leptons of different charges in the initial and

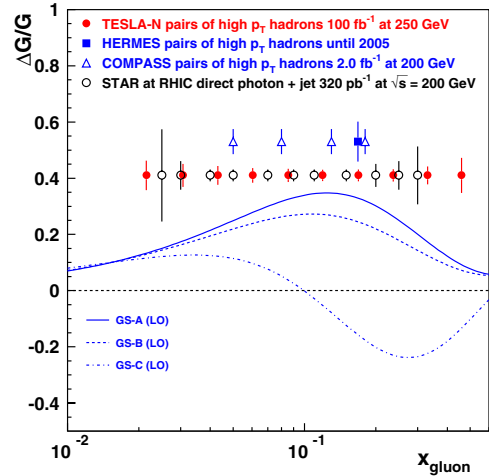


Figure 2. Sensitivity of different experiments to the first moment of  $\Delta g$  through direct measurements.

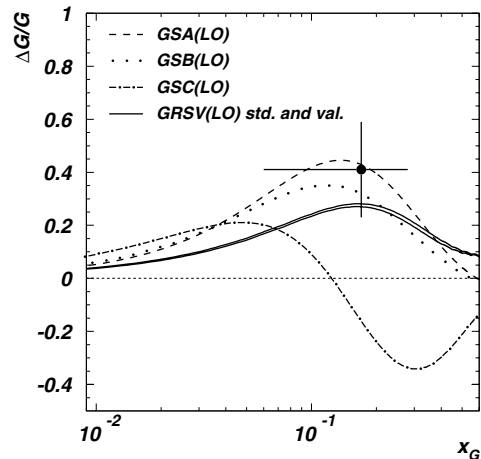
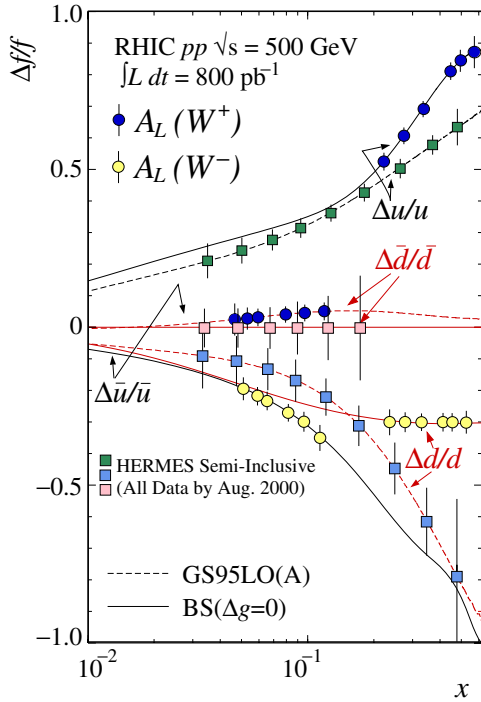


Figure 3. Measurement of  $\Delta g/g$  from high- $p_T$  hadron pair production by the HERMES Collaboration.

final states. So far, it has not been possible to adopt the same procedure in the case of polarized targets and beams. This is easy to understand: using neutrino beams, one would need an exceedingly large polarized target. On the other hand, charged current experiments with an electron or a muon in the initial state are only sensitive to charged current at very high energies. Thus, in the absence of intense neutrino beams (to be discussed later in this section), one has to rely upon different strategies in order to obtain information about individual polarized flavor densities.

There are essentially two possibilities: the first one is the study of semi-inclusive processes, in which the nature of the observed hadron may provide information on the underlying parton process. Measurements of this kind have been performed by the SMC [18] and HERMES [19] Collaborations.

The second possibility, pursued for example at RHIC, is the measurement of asymmetries in  $W$  production in polarized proton-proton collisions. The sensitivity of both kinds of measurements is summarized in fig. 4.



**Figure 4.** Sensitivity of RHIC and HERMES to polarized distributions of individual flavors.

An interesting opportunity is offered by the so-called ‘neutrino factories’, that is, intense neutrino beams arising from the decays of muons along straight sections of the accumulator of a muon storage ring (see ref. [20] for details.) This would allow an accurate decomposition of the partonic content of the nucleon in terms of individual (spin-averaged and spin-dependent) flavor densities.

In ref. [4], an accurate analysis was performed, aimed at estimating the impact of charged-current DIS data from a neutrino factory on our knowledge of the proton spin structure. It was assumed that both  $\nu$  and  $\bar{\nu}$  beams will be available, and therefore that the four structure functions  $g_1(W^\pm)$ ,  $g_5(W^\pm)$  will be measured (at leading twist, charged-current polarized DIS can be described in terms of the two independent structure functions  $g_1$  and  $g_5$ . For the precise definitions of these quantities, see e.g. ref. [4].) Under this assumption, suitable combinations of structure functions for different targets below and above the charm threshold provide a complete separation of all four flavors and antiflavors.

The expected errors on the measurements of  $g_1$  and  $g_5$  (for proton and deuteron targets) in a region of the  $x, Q^2$  plane compatible with the features of the device, were estimated;

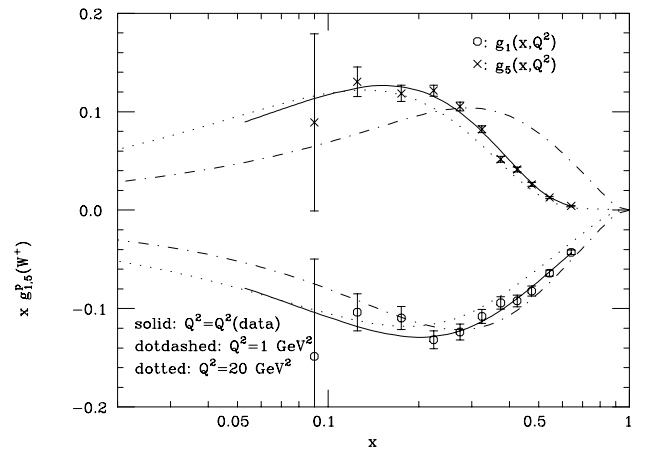
with the parameter choice of ref. [4], this corresponds to about  $0.01 \leq x \leq 0.7$  and  $1 \text{ GeV}^2 \leq Q^2 \leq 100 \text{ GeV}^2$ . The neutrino beam was assumed to be generated by a muon beam with energy  $E_\mu = 50 \text{ GeV}$  and number of muon decays per year  $N_\mu = 10^{20}$ .

Then, sets of ‘fake’ charged current data for  $g_1(W^\pm)$  and  $g_5(W^\pm)$  were produced. The C-even combinations  $\Delta q^+$  were taken from neutral current data fits (namely, those of table 1), while C-odd distributions were built according to three different assumptions:

1. generic fit of table 1 and  $\Delta\bar{s} = 0$ ;
2.  $\eta_g = 0$  fit of table 1 and  $\Delta\bar{s} = \Delta s$
3.  $\eta_g = 0$  fit of table 1 and  $\Delta\bar{s} = 0$ .

In all three cases, we have fixed  $\Delta\bar{u} = \Delta\bar{d} = 0$ , consistently with the indications of semi-inclusive DIS data, and with positivity constraints.

The whole set of (real and fake) data were then fitted by the usual NLO procedure. Figure 5, which shows the result for  $g_1(W^\pm)$  and  $g_5(W^\pm)$  for a proton target, gives an idea of the quality of these fits.



**Figure 5.** Fit to the structure functions  $g_1^p(W^+)$ ,  $g_5^p(W^+)$  for  $Q^2$  between 4 and 8  $\text{GeV}^2$ .

The results for first moments of quark and gluon distributions are displayed in table 4, where the rows labelled  $\eta_u$ ,  $\eta_d$  and  $\eta_s$  now give the best-fit values and errors on the first moments of  $\Delta q^-$  for  $u, d, s$ .

Comparing these values with those of our original fits (first and second columns), we see that the precision on the singlet quark first moment is significantly improved by the charged-current data: the error on the first moment of  $\Delta\Sigma^+$

is now of a few percent in comparison to about 10% with neutral-current DIS. With this accuracy, the scenario in which the singlet axial charge receive a large negative contribution from the first moment of  $\Delta g$  can be experimentally distinguished from other scenarios, at the level of several standard deviations. This is also reflected by the improvement in the precision on the gluon first moment.

The determination of the singlet axial charge is improved by an amount comparable to the improvement in the determination of the singlet quark first moment. It will be possible to establish at the level of a few percent whether the axial charge differs from zero or not. The determination of the isotriplet axial charge is also significantly improved: the improvement is comparable to that on the singlet quark. This would allow an extremely precise test of the Bjorken sum rule, and an accurate determination of the strong coupling. Finally, the octet C-even combination is now also determined with an uncertainty of a few percent. Therefore, the strange C-even component can be measured with less than 10% accuracy. This direct determination of the octet axial charge can be compared to the value obtained from baryon decays, and therefore provides a test of different models of SU(3) violation.

The first moments of the C-odd  $u$  and  $d$  distributions can be measured with an accuracy of a few percent, just sufficient to establish whether the up and down antiquark distributions, which are constrained by positivity to be quite small, differ from zero, and whether they are equal to each other or not. Furthermore, the strange C-odd component can be determined at a level of about 10%, which is enough to distinguish between the two cases  $\Delta s^- \sim 0$  and  $\Delta s^- \sim \Delta s^+$ .

In summary, we can conclude that a neutrino factory with the assumed parameters would have a strong impact on the study of polarized parton distributions. With such a facility the accuracy in the determination of the first moments of C-even distributions can be improved by about an order of magnitude with respect to the current uncertainties. The first moments of the C-odd distributions  $\Delta u^-$ ,  $\Delta d^-$  can be determined at the level of few percent, while  $\Delta s^-$  could only be measured at the 10% level. This would be sufficient to test for intrinsic strangeness.

The determination of the shapes of the distributions is severely limited by the uncertainty on the shape of the gluon distribution. It turns out that  $\Delta u(x)$  and  $\Delta d(x)$  can only be measured with a precision around 15-20%, while no significant shape information can be obtained for  $\Delta s(x)$ .

## 4 Conclusions

To summarize, we have reviewed existing analyses of polarized neutral-current deep-inelastic scattering data. We have seen that they allow an indirect determination of the polarized gluon distribution  $\Delta g$  through scaling violation,

but not quark-antiquark or flavor separation. Within this class of experiments, the first moment of  $\Delta g$  is determined with  $\sim 50\%$  accuracy, while its shape in  $x$  remains quite unconstrained. A rather accurate test of the Bjorken sum rule is also provided by these data.

There are interesting prospects for a direct measurement of  $\Delta g$ , based on charm production, hadron pair production at high transverse momentum and isolated photon asymmetries.

Polarized distributions of individual flavors and antiflavors are at present almost completely unknown; they can be extracted from the measurement of asymmetries for semi-inclusive processes, and from  $W$  production asymmetries in polarized  $pp$  collisions. However, the most promising progress in this respect is expected from neutrino factories, which will provide data on charged-current deep inelastic scattering structure functions accurate enough to perform flavor separation with good precision.

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**Table 4.** Fits to neutral-current data and fake charged-current data obtained according to the three different assumptions described in the text.

	1.	2.	3.
$\eta_\Sigma$	$0.39 \pm 0.01$	$0.321 \pm 0.006$	$0.324 \pm 0.008$
$\eta_g$	$0.86 \pm 0.10$	$0.20 \pm 0.06$	$0.24 \pm 0.08$
$\eta_3$	$1.097 \pm 0.006$	$1.052 \pm 0.013$	$1.066 \pm 0.014$
$\eta_8$	$0.557 \pm 0.011$	$0.572 \pm 0.013$	$0.580 \pm 0.012$
$\eta_u$	$0.764 \pm 0.006$	$0.722 \pm 0.010$	$0.728 \pm 0.009$
$\eta_d$	$-0.320 \pm 0.008$	$-0.320 \pm 0.009$	$-0.325 \pm 0.009$
$\eta_s$	$-0.075 \pm 0.008$	$-0.007 \pm 0.007$	$-0.106 \pm 0.008$
$a_0$	$0.183 \pm 0.013$	$0.255 \pm 0.006$	$0.250 \pm 0.007$