QCD Corrections to ZZ Boson Production at Hadron Colliders

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The effect of strong interaction on the Z pair production at hadron colliders are studied through a calculation of the QCD corrections including both virtual and real gluon emission. The calculation is a combination of analytical and computional methods. We find that the corrections are positive and of order of 20-30% to the total cross section.

1 Introduction:

Electro weak vector boson pair production is an important topics at future high-energy hadron colliders [1]. Cross sections for the process $p\overline{p} \to ZZ + X$ arising from the lowest order become quite sizable at the LHC (14 TeV) energy.

The production of the $p\overline{p} \to WW, WZ, ZZ$ has been calculated long times ago and the first order QCD corrections have only been estimated using soft gluon approximation [2]. In this work we present a next-to-leading logarithm (NLL) calculation of hadronic production of Z pair production. At the parton level this involves computing the contribution for real $2 \to 3$ emission processes as well as the loop corrections to the $2 \to 2$ process. The calculated corrections contain divergences, represented in the dimensional regularization procedure by poles $\mathrm{O}(1/\epsilon)$ and $\mathrm{O}(1/\epsilon^2)$, where and $\epsilon = (4-n)/2$ is the number of space-time dimension. The terms with double pole are eliminated when real and virtual corrections are combined and the remaining single divergences will be 'absorbed' into the quark-momentum distribution functions.

2 Lowest order results

To obtain the lowest order cross-section for $p\overline{p} \to ZZ + X$, one has first calculate the matrix element for the process where the Dirac fermions annihilate to produce a boson pair.The algebra was evaluated using the algebra manipulation program REDUCE, The square amplitude for the process becomes

$$[M_{Born}]^2 = \frac{g_v^4 + g_a^4 + 6g_v^2 g_a^2}{e^4} \left[\frac{t}{u} + \frac{u}{t} + \frac{4m_z^2 s}{tu} - m_z^4\left(\frac{1}{t^2} + \frac{1}{u^2}\right)\right] \tag{1}$$

All the kinematics invariants are defined by

$$s = (k_1 + k_2)^2, \quad t = (k_1 - p_1)^2, \quad u = (k_1 - p_2)^2, \quad s + t + u = 2m_z^2 \tag{2}$$

The differential cross section, for the sub-process is given by

$$\frac{d\sigma}{d\Omega} = \frac{g_v^4 + g_a^4 + 6g_v^2 g_a^2}{e^4} \left[\frac{t}{u} + \frac{u}{t} + \frac{4m_z^2 s}{tu} - m_z^4\left(\frac{1}{t^2} + \frac{1}{u^2}\right)\right] \tag{3}$$

Where the differential cross section included the spin average, color average, and identical particle factors.

As a check to the calculation, the differential cross-section for $q\overline{q} \to ZZ$ is exactly same as the differential cross-section for sub-process of two photon production, when we substitute $\mathrm{m}_z = 0$.

The leading-logarithm (LL) cross-section is obtained by convoluting the sub-process cross-section

$$\sigma(p\overline{p} \to ZZ) = \sum_{ij} \int d\sigma^{Born} \left[G(x_1, M^2)G(x_2, M^2) + x_1 \to x_2\right] dx_1 dx_2 \tag{4}$$

3 Virtual Gluon Corrections

To calculate the first order virtual gluon correction to the sub-process $q\overline{q} \to, ZZ$, we have to evaluate all the Feynman diagrams where the gluon is emitted and absorbed again by the quark.

Each of these diagrams contains a loop, the resulting integral over the gluon momentum leads to ultraviolet and infrared divergences. These divergences occur due to the behavior of the integrands at high and low virtual momentum (high and low energy behavior of the loop integrals). In order to work out of these diagrams, we may either introduce cutoffs or work in a space with dimension more than four. Using the continuous dimension method, all the poles arising from the calculation become poles of single and double orders.

It is clear that there are in fact three types of virtual diagrams called self-energy, vertex and box diagrams. We shall calculate each of them separately as follows:

Using Feynman parameterization of multiple denominators, with the use of an n-dimensional Minkowski space integrals [3], and combining with the lowest order amplitude, we get the following correction due to the self-energy diagrams:

$$\sum_{self} = (4\pi)^{\epsilon} \left[\frac{-1}{2\epsilon} - \frac{1}{2} + \frac{1}{2M_0}\left[f(\ln(-t)) + t \to u\right]\right] \quad (5)$$

Where $f = 8m_z^4 tu - 2m_z^4 u^2 - 4m_z^4 t^2 u - 4m_z^2 tu^2$. and M_0 is the lowest order matrix element for the sub-process.

For the vertex correction, we have to write the loop integral, which is more complicated than the self-energy case. Using Feynman parameterization of multiple denominators, using of an n-dimensional Minkowski space integrals , with a suitable change of variables and taking into the account the crossed diagrams, combining with the lowest order amplitude, we get the following correction due to the vertex diagrams:

$$\sum_{vertex} = (4\pi)^{\epsilon} \left[\frac{1}{\epsilon}(1 + \frac{8}{M_0}) + 1 - \frac{1}{M_0}[f(\ln(m_z^2 - t) + (t \to u))\right] \quad (6)$$

And finally for the box diagram we have to work out a four-denominator integral. The calculation is lengthier than the previous cases. we made use of the symbolic manipulation program to calculate the trace of the numerator interfering with the lowest order. . A double pole term appear as a result of the infrared divergences as well as mass singularities in the amplitude. Taking into account the crossed diagrams, the final form of the correction from the box diagram has the form:

$$\sum_{box} = (4\pi)^{\epsilon} [-\frac{1}{\epsilon^2} + \frac{1}{\epsilon}(\ln(s)) - 2 + \frac{8}{M_0} + f(m_z, u, t) + (t \to u)] \quad (7)$$

Where $f(m_z, u, t)$ is the rest finite terms. We have remind that we have only concentrated on terms that contain a singularity. In the limit m_z=0, the results for all virtual diagrams agree with the results in Ref. [4].

4 Real Gluon Corrections

In order to calculate the corrections from the real gluon emission, we have to calculate the cross-section corresponding to the Feynman diagrams, where real qluon has emitted from quarks.As it is clear, these diagrams contain no loop integrals, but there is an integral over the final 3-body phase space. We again face divergences, generated in the continuous dimension method as poles in the contribution to the cross section. These divergences arise from two limits: First, when the gluon is emitted in a direction parallel to either the quark or anti-quark, this is called collinear limit, and second, when the gluon is emitted withy a very small energy, we have what is called soft gluon emission.

Where the summation is over the polarization states of the boson, the gluon and the quark. To calculate the traces we made use the symbolic manipulation program (REDUCE). The gluon has treated in n-dimension, the boson in 4-dimension. Working the matrix element out we get:

$$[M]^2 = \frac{2g^2(g_v^4 + g_a^4 + 6g_v^2 g_a^2)}{k \cdot k_1 \, k \cdot k_2} [R + (n-4)F] \quad (8)$$

Where

$$R = \left(\frac{t^2 + u^2}{TU}\right) + 4m_z^2 S(\frac{1}{tU}) - m_z^4 \left(\frac{t}{TuU} + \frac{u}{UtT}\right) \quad (9)$$

$$F = \frac{(t^2 - u^2)}{TU} + 2 - \frac{t}{U} - \frac{u}{T} \quad (10)$$

$$T = (p_2 - k_2)^2, \, U = (p_2 - k_1)^2, \, S = (p_1 + p_2)^2 \quad (11)$$

Finally for the case of soft gluon emission, the gluon energy is small so that the soft gluon matrix element is proportional to the purely elastic cross-section.

The integral has two divergences, with the gluon being soft (k→ 0), and with quark being mass less. We will generate both infrared and mass singularities, working in the continuous dimension, double pole in the soft correction appears. Integrating over the gluon energy and using some gamma function properties, the final result for the correction can be written in the following form:

$$\sum_{soft} = (4\pi)^{\epsilon} \left[\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\left(\ln(4E^2\epsilon^2\right) + \frac{1}{2}(\ln(4E^2\epsilon^2)^2 + \frac{\pi^2}{4}\right] \quad (12)$$

5 Discussion

In order to obtain results which are free from divergences, we add up the corrections due to virtual and real gluon, the various double poles cancel, and left-over a single pole, corresponds to a mass singularity, which is absorbed into the structure function beyond the leading order. Since the appearance of the mass singularities in higher order correction have universal structure, process in depended they can factorized according to a fundamental property of factorization theorem which tells that the bare cross-section can be factorized into universal function containing the singularities which can be removed by subtraction procedure and a well defined short distance cross-section, which is free from mass singularities.

The numerical values of the total cross section for LL and NLL using MRST [5] structur function for different collider energy are listed in table (1) below:

Table 1. Total cross-section (LL and NLL) for ZZ production as function of C.M. energy

$\sqrt{s}$ TeV	σ_{LL} (pb)	σ_{NLL} (pb)
2	0.91	1.22
10	5.01	6.20
14	12.10	16.11
20	20.12	27.21

6 References

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