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The Potentials of the pure SU(3) representations

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Using a new lattice spacing in the spatial directions of .19 fm, I propose to compute potentials and string tensions between static sources of a variety of representations by measuring Wilson loops. Simulations have been done on an anisotropic lattice ($16^3 * 24$) using a tadpole improved action. A good scaling behavior is obtained for this new lattice based on new measurements. The lattice spatial size is 3.08 fm which is large enough to not have finite volume effect error. Results have been compared with other lattice calculations and approximate Casimir scaling is confirmed.

1 Introduction

Measuring the potential energy of a pair of heavy sources and investigating the theory of confinement in QCD is one of the problems that has been studied by lattice gauge theorists. Recent lattice studies [1], [2] confirm the existence of the linear potential for the intermediate distances for the fundamental and higher representations in SU(3). Based on these measurements, quarks are confined in all representations of SU(3). Although lattice calculations are very accurate but like all numerical measurements, they have some statistical and systematic errors. One of the errors of the lattice calculations is finite volume effect. In this paper, I give results of a new lattice spacing and by comparing it with previous measurements show that for one of the old lattices, the lattice spatial size is not big enough and, therefore, the finite volume effect gets important so that we have to put aside that measurement.

In reference [1], I have reported results of simulations on three anisotropic lattices: $10^3 \times 24$, $18^3 \times 24$ and $16^3 \times 24$ at β equal to 1.7, 2.4 and 3.1 with aspect ratios of 5, 3, and 1.5 respectively. Good scaling behaviors have been obtained for the coupling constants 1.7 and 2.4 which are corresponding to lattice spacing of .45 fm and .25 fm. But results for the $16^3 \times 24$ lattice with the coupling constant 3.1 (lattice spacing of .11 fm) were not scaled well with other two lattices, especially for higher representations. Therefore, another coupling constant with the same lattice dimensions has been tried to study the scaling window. With this new lattice spacing which is .19 fm, a good scaling behavior is obtained. In this paper, I show that the lattice spatial size of the finer lattice is significantly smaller than others such that we encounter finite volume effects error. Hence, we can throw out the finer lattice measurements and have an estimation of the lattice size without this error.

Like previous measurements, with this new coupling, potentials are linear at intermediate distance and they are qualitatively in agreement with Casimir scaling. I still do not see screening and change of the slope of the potential for higher representations. Probably because Wilson loops do not couple well to screened states.

2 Calculations

For simulations, Wilson loops have been measured to calculate potentials between static sources. The potential $V(r)$, as a function of r which is the spatial separation of the quark, may be found by looking for the area law fall-off for large t from Wilson loops: $W(r,t) \simeq \exp[-V(r)t]$. $W(r,t)$ is Wilson loop as a function of r and t, where t is the propagation time. Wilson loops for higher representations may be found from Wilson loop of the fundamental representation [1]. To calculate the string tension, potentials obtained from Wilson loops have been fitted to a linear plus a Coulombic form: $V(r) \simeq -A/r + Kr + C$ where K is the string tension.

Simulations have been done on an $16^3 \times 24$ anisotropic lattice with $a_s/a_t = 2$, where a_s and a_t are the spatial and temporal spacing, respectively. The improved action used for the calculations has the form [3]:

$$S = \beta\{\frac{5}{3}\frac{\Omega_{sp}}{\xi u_s^4} + \frac{4}{3}\frac{\xi\Omega_{tp}}{u_s^2 u_t^2} - \frac{1}{12}\frac{\Omega_{sr}}{\xi u_s^6} - \frac{1}{12}\frac{\xi\Omega_{str}}{u_s^4 u_s^2}\} \quad (1)$$

where $\beta = 6/g^2$, g is the QCD coupling, and ξ is the aspect ratio ($\xi = a_s/a_t$ at tree level in perturbation theory). Ω_{sp} and Ω_{tp} include the sum over spatial and temporal plaquettes; Ω_{sr} and Ω_{str} include the sum over 2×1 spatial rectangular and short temporal rectangular (one temporal and two spatial links), respectively. For $a_t \ll a_s$ the discretization error of this action is $O(a_s^4, a_t^2, a_t a_s^2)$. The coefficients are determined using tree level perturbation theory and tadpole improvement [4]. (The spatial mean link, u_s is given by $\langle\frac{1}{3}ReTrP_{ss'}\rangle^{\frac{1}{4}}$, where $P_{ss'}$ denotes the spatial plaquette. When $a_t \ll a_s$, u_t, the temporal mean link can be fixed to $u_t = 1$, since its value in perturbation theory differs by unity by $O(\frac{a_t^2}{a_s^2})$.)

To minimize the excited state contamination in correlation functions, spatial links are smeared. In the smearing procedure, each spatial link is replaced by itself plus a sum of its four neighboring spatial staples times a smearing factor λ [5]. Projection back to SU(3) after smearing or averaging over different paths in Wilson loops has been done, since I want to use these links for higher representations and they should be in SU(3). I have not done thermal averaging since it takes timelike links out of SU(3). (Thermal averaging is the replacement of a timelike link by its average with fixed neighbors, which is normally useful to increase statistics.)

I have used MILC Code as a platform for the simulations on Origin2000 supercomputer at NCSA and the super computer of the university of Indiana(single node jobs).

3 Results and Discussions

Figure 1 shows a typical plot for representation 6 for $R = 2$. The confidence level of the fit, Q, is calculated by measuring the covariance matrix evaluated by the jack knife method. Fitting is done at large T's to get $V(R)$ from Wilson loop.

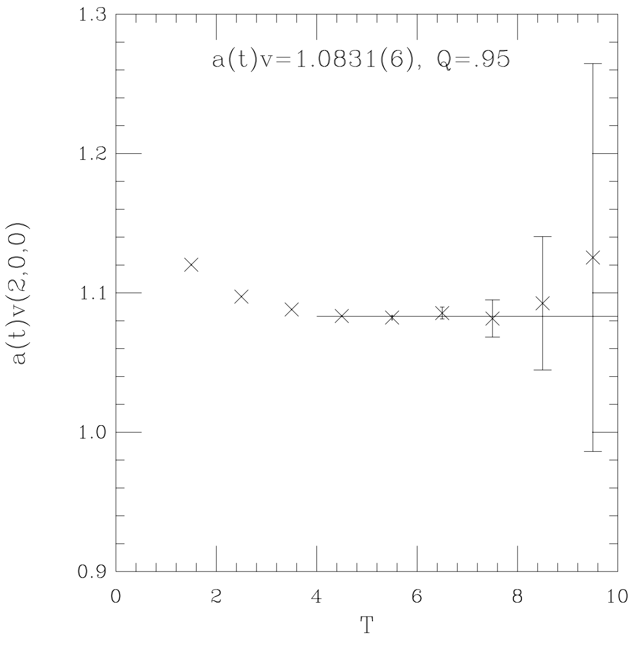

Figure 1. Potential for representation 6. The fit range is from $T = 4$ to $T = 10$ and is shown by the solid line. Q is the confidence level of the fit.

Figure 2 shows potentials versus r for the fundamental, 6, 8, 10, $15a$, $15s$ and 27 representations. The data have been fitted to a Coulombic plus linear term. As seen from the plot, at intermediate distances potentials are linear for all representations which means that quarks are confined. String tensions are in rough agreement with Casimir scaling. Like previous measurements [1], no sign of screening or change of the slope of potentials to the slope of the fundamental representation is observed. One of the main reason could be overestimation of potentials for higher representations specially for large R. This is because for higher representations, Wilson loops become very small (even for small R) so that errors get bigger than Wilson loops. Therefore, there are not reliable values for Wilson loops at large T. As an example, the potential for representation 27 and $R = 3$ is plotted in figure 3. The fit range is from $T = 2$ to $T = 4$. Comparing this figure with figure 1, there is an indication that the potential may be overestimated, even though a systematic error has been applied to the amount of the potential by comparing it with the potential of other distances that large T's have been used in the fitting. I recall that the potential can be measured from Wilson loops for large T.

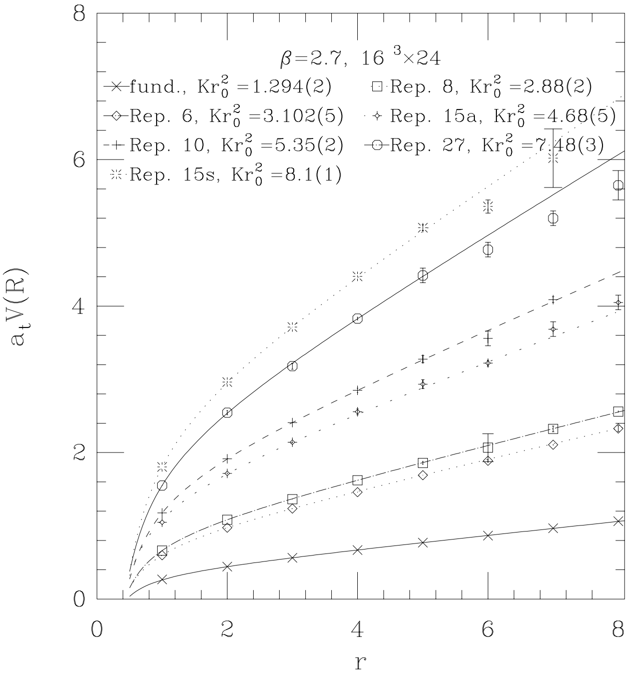

Figure 2. Potentials for the fundamental, 6, 8, 10, 15a, 15s and 27 representations. The fits are based on 12800 measurements. Rough agreement with Casimir scaling is observed in the intermediate distances, but no color screening for representations 8, 10 or 27 and no change of the slope is seen for other representations with zero triality.

To study the scaling behavior, the potential between the static sources are found in terms of hadronic scale, r_0, ($[r^2 dV/dr]_{r=r_0} = 1.65$), where V is the potential between quarks in the fundamental representation. In Figure 4, the potential between two sources in the fundamental representation in term of r_0 is plotted for different lattice measurements. Good scaling behavior is observed for the fundamental representation. This behavior is seen for all higher representations for all lattices including the new one with $\beta = 2.7$ except for the $16^3 \times 24$ lattice with $\beta = 3.1$. Figures 5 and 6 show potentials for representations 10 and $15s$. The lattice spatial size for the finer lattice is about

1.58 fm which is significantly smaller than other lattices where lattice spatial sizes are 3.98 fm, 4.15 fm and 3.04 fm for $10^3 \times 24$, $18^3 \times 24$ and $16^3 \times 24$ with the lattice spacing .45 fm, .25 fm and .19 fm, respectively. Although in lattice calculations we are always looking for finer lattices to get the continuum where real physics exists, but in this case dimensions of the finer lattice (with .11 fm lattice spacing) are not big enough so that the lattice spatial size is about one third of other coarser lattices. Hence, finite volume effect error gets important for the lattice with the smaller size and that is why it does not scale well. So this new measurements confirm that we have a good reason to throw out results of the finer lattice and have a rough estimation of the scaling window and the lattice size that does not encounter the finite size effects error.

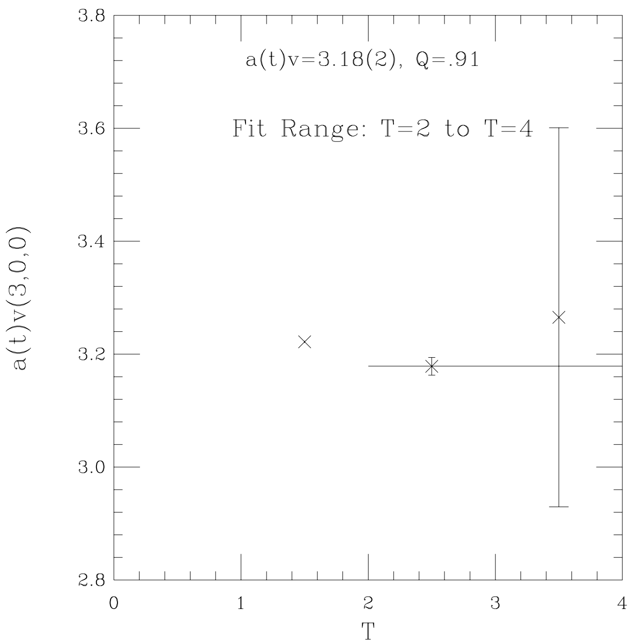

Figure 3. Potential for representation 27 for $R = 3$. The fit area is from $T = 2$ to $T = 4$. An overestimation of the potential may have been occurred because of using small T's in the fitting.

Table 1. Parameters of the potentials as a function of representation. K is the string tension, A is the Coulombic coefficient term, C is the Casimir number and "f" stands for the fundamental representation. Errors shown are statistical only.

Repn.	*K*	$\frac{K}{K_f}$	$\frac{A}{A_f}$	$\frac{C}{C_f}$
3	.1838(4)	-	-	-
8	.40(2)	2.2(1)	1.89(2)	2.25)
6	.4406(8)	2.40(1)	2.24(1)	2.5
15*a*	.664(6)	3.61(3)	3.3(1)	4.
10	.760(4)	4.14(2)	4.02(3)	4.5
27	1.06(2)	5.7(1)	5.11(6)	6
15*s*	1.15(2)	6.3(1)	6.40(4)	7.

Table 2. String tensions in terms of r_0 for different coupling constants, lattice sizes, and the best estimate.

Rep.	$Kr_0^2(\beta = 1.7)$ $10^3 \times 24$	$Kr_0^2(\beta = 2.4)$ $18^3 \times 24$	$Kr_0^2(\beta = 2.7)$ $16^3 \times 24$	Best estimate
3	1.25(8)	1.32(1)	1.294(2)	1.295(2)(36))
8	2.60(1)	2.60(3)	2.88(2)	2.651(9)(170)
6	2.9(2)	3.00(3)	3.102(5)	3.099(5)(157)
15a	4.4(2)	4.6(1)	4.68(5)	4.65(4)(18)
10	4.9(3)	5.4(2)	5.35(2)	5.35(2)(32)
27	5.9(5)	6.62(6)	7.48(3)	7.30(3)(100)
15s	7.1(5)	7.6(2)	8.1(1)	7.97(9)(67)

Like previous measurements for this new lattice, potentials are linear at intermediate distances. The coefficient of the linear term, string tension, is shown in table 1. Ratios of the string tension, Coulombic coefficient, and Casimir number of each representation to the corresponding values in the fundamental representation are shown in the second, third and fourth column, respectively. As seen from the table, rough agreement between measured ratios and casimir number exists.

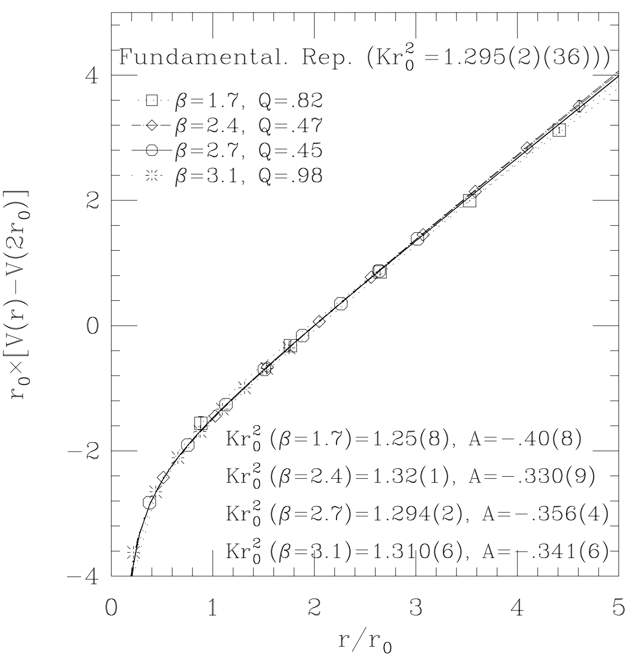

Figure 4. The static quark potential $V(R)$ in terms of hadronic scale r_0 for the fundamental representation. A is the coefficient of the Coulombic term for each lattice measurement. Kr_0^2 is the best estimate for the dimensionless string tension among lattices that do not encounter finite volume effect error.

In table 2, dimensionless string tensions of different representations found by lattices which show good scaling behavior are given. In last column, best estimate of the string tension for each representation is presented. The best estimate is obtained by the weighted average of the three lattice measurements. The errors on the string tensions are the statistical error (from the weighted average), and the systematic error of discretization (determined by the standard deviation of the results over the 3 couplings).

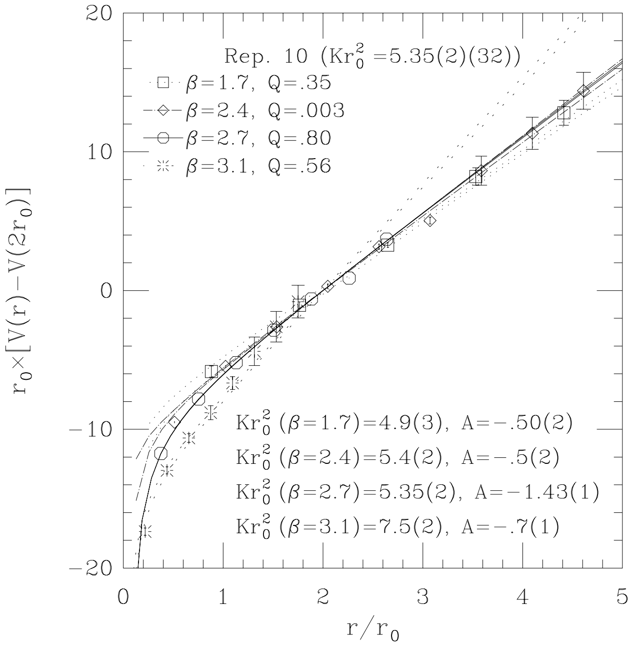

Figure 5. The static quark potential $V(R)$ in terms of hadronic scale r_0 for representation 10. Results from three lattices including the new one, scales well. Results for $\beta = 3.1$ do not scale since the lattice spatial size is significantly smaller than others and finite size affects measurements.

4 Conclusion

Finite volume effects error has been studied by looking at four lattice measurements. The lattice spatial size of the finer lattice is significantly smaller than other lattices (about one third). This explains why this lattice does not show a good scaling behavior. It seems that in this case, the lattice spatial volume is not big enough to study physics and therefore we can exclude this measurements from our calculations. Approximate Casimir scaling is observed for the new measurement and once again results confirm that quarks are confined in all representations of $SU(3)$.

5 Acknowledgement

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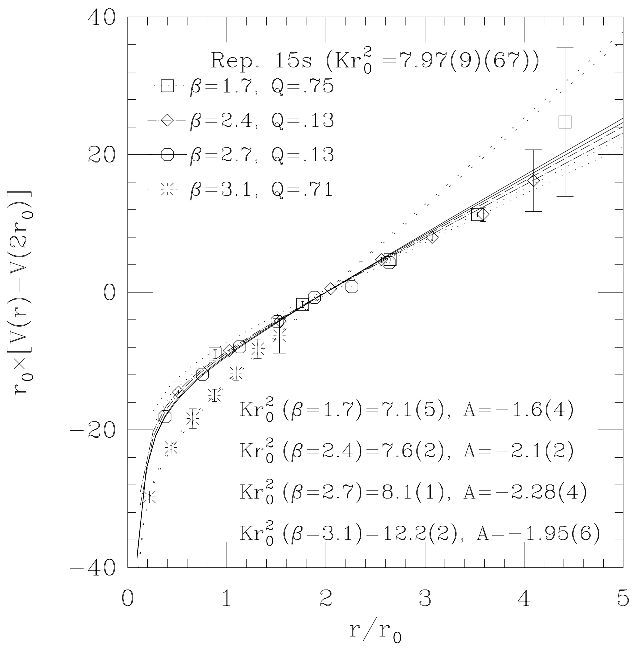

Figure 6. Same as figure 5 but for representation 15s.

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