Flavour and CP Violation in the Lepton Sector 
and New Physics

Stéphane Lavignac
Theory Division, CERN
CH-1211 Genève 23, Switzerland

We give a pedagogical review of flavour and CP violation in the lepton sector, with a particular emphasis on new physics – and in particular supersymmetric – contributions to flavour and CP violating observables involving leptons.

1 Introduction

In the quark sector, the only source of flavour and CP violation, in the Standard Model, is the CKM matrix. A number of observables, mainly in the $K$ and $B$ meson sectors, allow to constrain the mixing angles and the phase of this matrix and to check the consistency of the CKM picture. If there is new physics beyond the Standard Model, new sources of flavour and CP violation are generally present. Their contributions to flavour and CP violating processes may lead to observable deviations from the Standard Model predictions. A well-known example of this is the explanation of the possible discrepancy between $S_{J/ΨK_S}$ and $S_{ΦK_S}$ by supersymmetric loop contributions to $B → ΦK_S$ [1], while $B → J/ΨK_S$ is dominated by the Standard Model tree-level contribution.

The situation in the lepton sector is very different. The only experimental evidence for flavour violation comes from neutrino oscillations. These imply the existence of a non-trivial lepton mixing matrix, the so-called PMNS (Pontecorvo-Maki-Nakagawa-Sakata) [2] matrix $U$, which is the analogue of the CKM matrix for leptons. This non-trivial mixing matrix induces in turn lepton flavour violating (LFV) processes like $µ → eγ$, $µ → 3e$ or $K^0_L → µe$, and, if it contains nonzero CP-violating phases, dipole electric moments for charged leptons. Due to the smallness of the neutrino masses, however, the corresponding observables are negligibly small and unaccessible to experiments. For example, the branching ratio for $µ → eγ$ is suppressed by $(m_µ/M_W)^4$ [3]:

$$BR(µ → eγ) = \frac{3α}{32π} \left| \sum_i U_{µi}^* U_{ei} \frac{m_ν^2}{M_W^2} \right|^2.$$  (1)

For $m_ν < 1$eV, one obtains $BR(µ → eγ) < 10^{-48}$, well below the present experimental upper limit. As for charged lepton electric dipole moments (EDMs), in the absence of CP-violating phases in the PMNS matrix, they are induced by $δ_{CKM}$ beyond the 3-loop level [4], which gives $d_e < 10^{-38}$ e.cm [5], well below the present experimental limit. If the PMNS matrix contains

\textsuperscript{1}Permanent address: Service de Physique Théorique, CEA-Saclay, F-91191 Gif-sur-Yvette Cédex, France.
CP-violating phases, again the EDMs of charged leptons arise at the multiloop level and are unobservably small.

It follows from these considerations that the observation of any flavour violating process in the lepton sector other than neutrino oscillations, or the measurement of charged lepton EDMs, would be a direct signature of new physics\(^2\). This is a strong difference with the quark sector, in which new physics contributions come in addition to the Standard Model ones.

We have summarized in Table 1 the current upper limits on some LFV processes and on the charged lepton EDMs, as well as the expected improvement in the experimental sensitivity. “SM prediction” refers to the prediction of the Standard Model with Dirac neutrinos. There are many other observables of interest, such as \(K_L^0 \rightarrow \mu e\), \(K^+ \rightarrow \pi^+ \mu^- e^+\), the rates of \(\mu - e\) conversion in nuclei, or the CP asymmetries in LFV decays of taus and muons.

2 CP violation in neutrino oscillations

Let us first write the standard parametrization of the PMNS matrix:

\[
U = \begin{pmatrix}
  c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\
  -c_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta} & c_{23}c_{12} - s_{13}s_{23}s_{12}e^{i\delta} & -s_{23}s_{12} - s_{13}c_{23}s_{12}e^{i\delta} \\
  s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta} & -s_{23}s_{12} + s_{13}c_{23}s_{12}e^{i\delta} & c_{23}c_{12}
\end{pmatrix} \times P , \tag{2}
\]

where \(P = 1\) for Dirac neutrinos, and \(P = \text{Diag}(1,e^{i\Phi_2},e^{i(\Phi_3+\delta)})\) for Majorana neutrinos. This reflects the fact that, in the case of Dirac neutrinos, the PMNS matrix can be parametrized by 3 angles and 1 CP-violating phase \(\delta\), exactly like the CKM matrix. In the case of Majorana neutrinos, the PMNS matrix contains 2 additional CP-violating phases \(\Phi_2\) and \(\Phi_3\) [6, 7] which play a rôle in neutrinoless double beta decay [7] (for recent discussions, see e.g. Refs. [8]).

The only source of information we have so far on the PMNS matrix are neutrino oscillation experiments, which also constrain the squared mass differences \(\Delta m^2_{ij} \equiv m^2_{\nu_i} - m^2_{\nu_j}\) \(\theta_{23}\) and \(\Delta m^2_{32}\) (resp. \(\theta_{12}\) and \(\Delta m^2_{21}\)) are associated with oscillations of atmospheric (resp. solar) neutrinos; both have been found to be large, and possibly maximal for \(\theta_{23}\). The third angle \(\theta_{13}\) has not been measured yet, but is constrained to be smaller than the Cabibbo angle by nuclear reactor experiments [9]. A recent 3-neutrino fit [10] of all available oscillation data [11] gives the following allowed ranges of parameters at the 1σ (3σ) confidence level\(^3\):

\[
\begin{align*}
(1.5) & \quad 2.2 < \Delta m^2_{32}/10^{-3} \text{ eV}^2 < 3.0 (3.3) , \\
(0.45) & \quad 0.75 < \tan^2 \theta_{23} < 1.3 (2.3) , \\
(5.4) & \quad 6.7 < \Delta m^2_{21}/10^{-5} \text{ eV}^2 < 7.7 (10) \quad \text{and} \quad (14) < \Delta m^2_{32}/10^{-5} \text{ eV}^2 < (19) , \\
(0.29) & \quad 0.39 < \tan^2 \theta_{12} < 0.51 (0.82) , \\
\sin^2 \theta_{13} & < 0.02 (0.052) .
\end{align*} \tag{3-7}
\]

\(^2\)New physics could also play a subdominant rôle in neutrino oscillations.

\(^3\)Since then, the SNO collaboration has published new neutral current data which strongly disfavour the high \(\Delta m^2_{21}\) region [12].
We immediately see that the Majorana phases $\Phi$ where the last term is CP-odd and takes a minus sign for $\beta$ of $(\alpha, \beta, \gamma)$ is the mass squared differences $\Delta m^2_{ij}$ and on the ratio $L/E$, where $E$ is the neutrino energy and $L$ the distance travelled by the neutrino between the production and detection points. For any two distinct neutrino flavours $\alpha$ and $\beta$:

$$P_{\nu_{\alpha} \to \nu_{\beta}}(\nu_{\alpha} \to \nu_{\beta}) = -4 \sum_{i<j} \Re \left( U_{\alpha i} U_{\beta i}^\ast U_{\alpha j} U_{\beta j}^\ast \right) \sin^2 \left( \frac{\Delta m^2_{ij} L}{4E} \right)$$

$$+ 2J \left[ \sin \left( \frac{\Delta m^2_{21} L}{4E} \right) + \sin \left( \frac{\Delta m^2_{31} L}{4E} \right) - \sin \left( \frac{\Delta m^2_{32} L}{4E} \right) \right],$$

where the last term is CP-odd and takes a minus sign for $\nu_{\alpha} \to \nu_{\beta}$ and $(\alpha, \beta, \gamma)$ (with $\gamma \neq \alpha, \beta$) an even permutation of $(e, \mu, \tau)$, and a plus sign for $\overline{\nu}_{\alpha} \to \overline{\nu}_{\beta}$ and $(\alpha, \beta, \gamma)$ an even permutation of $(e, \mu, \tau)$. $J$ is the Jarlskog invariant [14] associated with the PMNS matrix:

$$J \equiv \Im \left( U_{e2} U_{\mu2}^\ast U_{\tau2}^\ast U_{\mu3} U_{\tau3} \right) = \frac{1}{8} \cos \theta_{13} \sin 2\theta_{23} \sin \theta_{13} \sin 2\theta_{12} \sin \delta .$$

We immediately see that the Majorana phases $\Phi_2$ and $\Phi_3$ drop from the oscillation formulae, so that CP violation in oscillations probes only the "CKM-like" phase $\delta$ [6, 7]. Using the hierarchy $\Delta m^2_{21} \ll \Delta m^2_{32}$, one can write:

$$P_{\nu_{\alpha} \to \nu_{\beta}} - P_{\overline{\nu}_{\alpha} \to \overline{\nu}_{\beta}} \simeq \pm 8J \left( \frac{\Delta m^2_{21} L}{2E} \right) \sin^2 \left( \frac{\Delta m^2_{31} L}{4E} \right),$$

Table 1: Current experimental limits on some flavour and CP violating observables in the lepton sector.

<table>
<thead>
<tr>
<th>observable</th>
<th>SM prediction</th>
<th>present experimental limit</th>
<th>future expected limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$BR(\mu \to e\gamma)$</td>
<td>$&lt; 10^{-48}$</td>
<td>$1.2 \times 10^{-11}$ [MEGA]</td>
<td>$10^{-14}$ (PSI)</td>
</tr>
<tr>
<td>$BR(\tau \to \mu\gamma)$</td>
<td>$&lt; 10^{-48}$</td>
<td>$5.0 \times 10^{-7}$ [Belle]</td>
<td>$10^{-15}$ ($\nu$ factories)</td>
</tr>
<tr>
<td>$BR(\tau \to e\gamma)$</td>
<td>$&lt; 10^{-48}$</td>
<td>$2.7 \times 10^{-6}$ [CLEO]</td>
<td>$10^{-8}$ ($B$ factories)</td>
</tr>
<tr>
<td>$BR(\mu \to eee)$</td>
<td>$&lt; 10^{-50}$</td>
<td>$1.0 \times 10^{-12}$ [SINDRUM]</td>
<td>$10^{-16}$ ($\nu$ factories)</td>
</tr>
<tr>
<td>$BR(\tau \to \mu\mu\mu)$</td>
<td>$&lt; 10^{-51}$</td>
<td>$8.7 \times 10^{-7}$ [Belle]</td>
<td>?</td>
</tr>
<tr>
<td>$d_e$ (e.cm)</td>
<td>$&lt; 10^{-38}$</td>
<td>$1.6 \times 10^{-27}$ (Regan 02)</td>
<td>$10^{-32}$ (nucl-ex/0109014)</td>
</tr>
<tr>
<td>$d_\mu$ (e.cm)</td>
<td>$&lt; 10^{-35}$</td>
<td>$d_\mu = (3.7 \pm 3.4) \times 10^{-18}$ (Bailey 78)</td>
<td>$10^{-24}$ (BNL)</td>
</tr>
<tr>
<td>$d_\tau$ (e.cm)</td>
<td>$&lt; 10^{-34}$</td>
<td>$-2.2 &lt; \Re(d_\tau) &lt; 4.5 \times 10^{-17}$</td>
<td>$5 \times 10^{-26}$ ($\nu$ factories)</td>
</tr>
</tbody>
</table>

The mixing pattern in the lepton sector is very different from the quark sector, which has only small mixing angles, but the mass hierarchy is much less pronounced in the neutrino sector than in the up and down quark sectors.

CP is violated in neutrino oscillations if oscillation probabilities are different for neutrinos and antineutrinos of the same flavour, i.e. $P_{\nu_{\alpha} \to \nu_{\beta}} \neq P_{\overline{\nu}_{\alpha} \to \overline{\nu}_{\beta}}$ for any two lepton flavours $\alpha$ and $\beta$ [13]. The oscillation probabilities in vacuum depend on the entries of the PMNS matrix $U_{\alpha i}$ on the mass squared differences $\Delta m^2_{ij}$ and on the ratio $L/E$, where $E$ is the neutrino energy and $L$ the distance travelled by the neutrino between the production and detection points. For any two distinct neutrino flavours $\alpha$ and $\beta$:

$$P_{\nu_{\alpha} \to \nu_{\beta}}(\nu_{\alpha} \to \nu_{\beta}) = -4 \sum_{i<j} \Re \left( U_{\alpha i} U_{\beta i}^\ast U_{\alpha j} U_{\beta j}^\ast \right) \sin^2 \left( \frac{\Delta m^2_{ij} L}{4E} \right)$$

$$+ 2J \left[ \sin \left( \frac{\Delta m^2_{21} L}{4E} \right) + \sin \left( \frac{\Delta m^2_{31} L}{4E} \right) - \sin \left( \frac{\Delta m^2_{32} L}{4E} \right) \right],$$

where the last term is CP-odd and takes a minus sign for $\nu_{\alpha} \to \nu_{\beta}$ and $(\alpha, \beta, \gamma)$ (with $\gamma \neq \alpha, \beta$) an even permutation of $(e, \mu, \tau)$, and a plus sign for $\overline{\nu}_{\alpha} \to \overline{\nu}_{\beta}$ and $(\alpha, \beta, \gamma)$ an even permutation of $(e, \mu, \tau)$. $J$ is the Jarlskog invariant [14] associated with the PMNS matrix:

$$J \equiv \Im \left( U_{e2} U_{\mu2}^\ast U_{\tau2}^\ast U_{\mu3} U_{\tau3} \right) = \frac{1}{8} \cos \theta_{13} \sin 2\theta_{23} \sin \theta_{13} \sin 2\theta_{12} \sin \delta .$$

We immediately see that the Majorana phases $\Phi_2$ and $\Phi_3$ drop from the oscillation formulae, so that CP violation in oscillations probes only the "CKM-like" phase $\delta$ [6, 7]. Using the hierarchy $\Delta m^2_{21} \ll \Delta m^2_{32}$, one can write:

$$P_{\nu_{\alpha} \to \nu_{\beta}} - P_{\overline{\nu}_{\alpha} \to \overline{\nu}_{\beta}} \simeq \pm 8J \left( \frac{\Delta m^2_{21} L}{2E} \right) \sin^2 \left( \frac{\Delta m^2_{31} L}{4E} \right),$$

393
with a minus sign for the “golden channel” \( \nu_e \rightarrow \nu_\mu \). Like in the quark sector, CP violation effects are proportional to the Jarlskog invariant \( J \), which is itself proportional to \( \sin \delta \) and \( \sin \theta_{13} \). We also see that the CP asymmetry in neutrino oscillations is proportional to \( \Delta m_{21}^2 L / 2E \). The conditions for observing CP violation effects in oscillations are therefore the following: (i) \( \Delta m_{21}^2 \) and \( \theta_{12} \) should be large, which we know is the case since the KamLAND experiment has identified the so-called LMA (Large Mixing Angle) solution, characterized by both a large \( \theta_{12} \) and a high \( \Delta m_{21}^2 \), as the origin of the solar neutrino deficit observed on the Earth; (ii) \( \theta_{13} \) should not be too small; (iv) the CP-violating phase \( \delta \) should be large; and (v) the baseline should be long enough so that subdominant oscillations, which are governed by the solar squared mass difference, can develop.

The best experimental conditions for observing CP violation in oscillations would be provided by a neutrino beam produced from muon decays at a neutrino factory, and a very long baseline (typically 3000 km or 7000 km). For such large distances, matter effects [15] – which induce an apparent CP asymmetry, due to the fact that matter effects are different for neutrinos and antineutrinos – must be taken into account. Another, less ambitious option is to use neutrino superbeams (which could be produced by JHF in Japan, NuMI in Fermilab or the SPL at CERN). These beams are characterized by a lower energy and therefore allow for CP violation searches on shorter baselines (resp. 130, 300 and 730 km). The SPL superbeam could be used in combination with a “beta beam”. For a review on these projects, see e.g. Ref. [16].

### 3 Lepton flavour violating processes

As already mentioned in the introduction, LFV processes are unobservable in the Standard Model with Dirac neutrinos, and more generally if the only source of flavour violation at low energy is the PMNS matrix. On the other hand, most extensions of the Standard Model include new sources of flavour violation (and of CP violation) which, depending on the case, may be related to the mechanism responsible for neutrino masses or not. Let us mention, as examples of such extensions, models in which Majorana neutrino masses are generated radiatively or through couplings to an \( SU(2)_L \) Higgs triplet, supersymmetric extensions of the Standard Model with or without a seesaw mechanism, and supersymmetric Grand Unified Theories (GUTs). In the following, we discuss lepton flavour violation in models with radiative generation of neutrino masses and in supersymmetric extensions of the Standard Model.

#### 3.1 Models with radiative generation of neutrino masses

In this subsection, we consider models in which neutrino masses are generated at the quantum level by non-standard interactions of the neutrinos. These interactions necessarily violate lepton flavour and therefore induce LFV processes, generally at much larger a rate than the PMNS matrix itself, although they may still be too weak to be observable.

The prototype of models with radiative generation of neutrino masses, the Zee model [17], is now excluded by solar neutrino experiments. We could consider more sophisticated models that pass all experimental data; however this would only complicate our discussion of LFV processes, and we prefer to stick to the Zee model. This model has two identical Higgs doublets
$H$, $H'$ and a charged Higgs $SU(2)_L$ singlet $h^+$, which couples to two lepton doublets with antisymmetrized $SU(2)_L$ indices:

$$f_{\alpha\beta} \left( \nu^T_{\alpha} C e_{L\beta} - e^c_{L\alpha} C \nu_{L\beta} \right) h^+ + \text{h.c.},$$

where $C$ is the charge conjugation matrix, and the couplings $f_{\alpha\beta} = -f_{\beta\alpha}$ are antisymmetric, hence flavour violating. Neutrinos are strictly massless at the tree level, but (Majorana) neutrino masses are induced by the $f_{\alpha\beta}$ couplings at the one-loop level. The one-loop neutrino mass matrix has the following structure:

$$M_\nu = m \begin{pmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{pmatrix},$$

where $a = f_{e\mu}(\frac{m_{\mu}}{m_{\tau}})^2$, $b = f_{e\tau}$ and $c = f_{\mu\tau}$, and the mass scale $m$ depends on the physical charged Higgs boson masses and mixing angles. This structure implies a very large solar mixing angle, $\tan^2 \theta_{12} \approx 0.98$, which is now excluded by solar neutrino data (see Eq. (6)). As mentioned above, the $f_{\alpha\beta}$ couplings induce LFV processes at the one-loop level; in particular the branching ratio for $\mu \to e\gamma$ is found to be [18]:

$$\text{BR} \left( \mu \to e\gamma \right) = \alpha \frac{f_{e\tau} f_{\mu\tau}^2}{48\pi (\sqrt{M^2 G_F})^2},$$

where $\sqrt{M}$ is a function of the physical charged Higgs boson masses, and $G_F$ is the Fermi constant. For values of the parameters relevant for neutrino masses, e.g. $f_{\alpha\beta} \sim 10^{-5}$ and $M \sim 1 \text{ TeV}$, one typically finds $\text{BR} \left( \mu \to e\gamma \right) \sim (10^{-25} - 10^{-30})$. This is well above the contribution of the PMNS matrix itself, although still orders of magnitude below the experimental limit. The reason is that small neutrino masses require small values of $f_{\alpha\beta}/M$. A similar conclusion holds for models in which neutrino Majorana masses are generated from Yukawa couplings involving an $SU(2)_L$ Higgs triplet [19]; however, one can play with the value of the triplet vev so as to increase the value of the flavour violating Yukawa couplings.

There are many other models which realize the idea of generating neutrino masses through loop diagrams, and may lead to larger branching ratios for LFV processes. Let us just mention an interesting possibility that arises within supersymmetric extensions of the Standard Model without $R$-parity [20], a discrete symmetry usually imposed in order to forbid dangerous baryon number and lepton number violating couplings. In these theories, lepton number violating couplings such as $\lambda_{ijk} \tilde{e}_j L \tilde{e}_k R \nu_i L$ and $\lambda_{ijk} \tilde{d}_j L \tilde{d}_k R \nu_i L$, where $\tilde{e}_L$ and $\tilde{d}_L$ are the supersymmetric scalar partners of the left-handed charged leptons and down quarks, respectively, induce (Majorana) neutrino masses at the one-loop level [21] and can lead to a viable mass and mixing pattern. They also contribute to a number of LFV processes [21] which could well be accessible experimentally, such as $\mu \to e\gamma$, $\mu \to eee$ and $\mu - e$ conversion in nuclei (see e.g. Ref. [22]). The phenomenology of supersymmetry without $R$-parity is actually very rich and signatures at high-energy colliders are also expected if $R$-parity violation is responsible for neutrino masses.

---

4Since the simultaneous presence of baryon number and lepton number violating couplings could lead to an unacceptably short proton lifetime, we assume baryon number conservation.
### 3.2 Supersymmetric extensions of the Standard Model

In supersymmetric extensions of the Standard Model like the MSSM (Minimal Supersymmetric Standard Model), new sources of lepton flavour violation, and of CP violation, can be present in the slepton (the supersymmetric scalar partners of the leptons) sector. Indeed, while at the supersymmetric level sleptons and leptons are degenerate in mass, supersymmetry breaking generates new contributions to the slepton mass matrices of the form:

\[
(m^2_L)_{\alpha\beta} \tilde{L}^\dagger_{\alpha} \tilde{L}_{\beta} + \left( m^2_\nu \right)_{\alpha\beta} \tilde{\nu}^*_{\alpha} \tilde{\nu}_{\beta} + \left( A^e_{\alpha\beta} \tilde{e}_{\alpha} \tilde{e}^*_{\beta} + \text{h.c.} \right),
\]

where \( \tilde{L}^T = (\tilde{\nu}_L e_L) \) (resp. \( \tilde{e}_R) \) is the supersymmetric partner of the lepton doublet \( L^T = (\nu_L e_L) \) (resp. of the lepton singlet \( e_R) \), \( m^2_L \) and \( m^2_\nu \) are 3 \times 3 hermitean mass matrices, \( A^e_{\alpha\beta} \) is a 3 \times 3 complex matrix, and \( \nu_L \) is the vev of \( H_u \), the Higgs doublet which couples to down quarks and charged leptons in the MSSM. The last term in Eq. (14) mixes "left" and "right" states and is known as A-term. Since the mechanism responsible for supersymmetry breaking is not known, the soft supersymmetry breaking parameters \( m^2_L \), \( m^2_\nu \) and \( A^e_{\alpha\beta} \) are arbitrary matrices in flavour space. In particular, they need not be diagonal in the flavour basis defined by the charged lepton mass eigenstates, and this results in flavour violating couplings of sleptons to leptons and charginos/neutralinos (mass eigenstate combinations of the supersymmetric partners of the gauge and Higgs bosons), such as \( e^+ \mu^- \tilde{\chi}^0 \) or \( \tilde{\nu}_e \mu^- \tilde{\chi}^+ \). These couplings induce large contributions to LFV processes [23], potentially above the present experimental limit.

In practice, one often works in the mass insertion approximation [24], which allows to relate the rates for LFV processes to the off-diagonal entries of the slepton mass matrices:

\[
\delta_{LL}^{\alpha\beta} \equiv \frac{(m^2_L)_{\alpha\beta}}{m^2_L}, \quad \delta_{RR}^{\alpha\beta} \equiv \frac{(m^2_\nu)_{\alpha\beta}}{m^2_\nu}, \quad \delta_{RL}^{\alpha\beta} \equiv \frac{A^e_{\alpha\beta} \nu_L}{m_R m_L}, \quad \delta_{LR}^{\alpha\beta} \equiv \delta_{RL}^{\beta\alpha}, \quad (\alpha \neq \beta)
\]

where \( m^2_L \) (resp. \( m^2_\nu \)) is the average left (resp. right) slepton mass. For instance, the branching ratio for \( \mu \rightarrow e\gamma \) is given by, at leading order [25, 26]:

\[
\text{BR} (\mu \rightarrow e\gamma) = \frac{3\pi\alpha^3}{4 G_F \cos^4 \theta_W} \left\{ \left| f_{LL} \delta_{12}^{LL} + f_{LR} \delta_{12}^{RR} \right|^2 + \left| f_{RR} \delta_{12}^{LR} + f_{LR} \delta_{21}^{LR} \right|^2 \right\} \tan^2 \beta,
\]

where \( f_{LL}, f_{RR} \) and \( f_{LR} \) are functions of the superpartner masses and of the ratio of the vevs of the two MSSM Higgs doublets, \( \tan \beta = <H^0_u> / <H^0_d> \). For moderate and large values of \( \tan \beta \), say \( \tan \beta > 10 \), \( \text{BR} (\mu \rightarrow e\gamma) \) approximately scales as \( \tan^2 \beta \), unless \( |\delta_{12}^{LR}| \gg |\delta_{12}^{LL}|, |\delta_{12}^{RR}| \). From the present experimental limit on \( \text{BR} (\mu \rightarrow e\gamma) \), one can extract upper bounds on the \( \delta_{12} \)'s as functions of the supersymmetric parameters. This is illustrated in Fig. 1, in which mSUGRA relations between the soft terms, i.e. universality relations at the Planck scale, have been assumed. We can see that the \( \delta_{12} \)'s are constrained to be rather small, unless the supersymmetric partners are very heavy [27]. The upper bounds on the \( \delta_{23} \)'s and \( \delta_{13} \)'s, which come from the present experimental limit on \( \text{BR} (\tau \rightarrow \mu\gamma) \) and \( \text{BR} (\tau \rightarrow e\gamma) \), respectively,
are significantly weaker, though quite stringent. This shows that supersymmetry can lead to observable LFV processes, but their rates are controlled by the mechanism responsible for supersymmetry breaking rather than by the parameters associated with neutrino masses. The requirement that the corresponding rates are below the experimental limits actually constitutes a strong constraint on supersymmetry breaking.

Interestingly, the above conclusion can be evaded if the mechanism responsible for supersymmetry breaking is flavour-blind, as might be necessary in order to satisfy all constraints from quark and lepton flavour violating processes. If neutrino masses are generated from the seesaw mechanism [28], the LFV rates are then controlled by the seesaw parameters [29], as we discuss now. In the seesaw mechanism, the smallness of neutrino masses naturally arises from
the couplings of the ordinary LH neutrinos to heavy Majorana RH neutrinos:

\[ Y_{\kappa\alpha} \bar{N}_{Rk} L_{\alpha} H_u + \frac{1}{2} (M_R)_{ki} N_{Rk}^T C N_{Rl} + \text{h.c.}, \]

(17)

where \( Y \) is the Dirac mass matrix and \( M_R \) the RH neutrino Majorana mass matrix. At low energy, the effective light neutrino mass matrix is given by (from now on, we work in the bases in which both \( M_R \) and the charged lepton mass matrix \( M_e \) are diagonal):

\[ M_\nu = -Y^T M_R^{-1} Y = U^* \text{Diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) U^\dagger. \]

(18)

For Dirac couplings of order one, \( m_{\nu_3} \approx \sqrt{\Delta m^2_{\text{atm}}} \) is obtained for right-handed neutrino masses of the order of \( 5 \times 10^{14} \) GeV, remarkably close to the scale at which gauge couplings unify in the MSSM (\( 2 \times 10^{16} \) GeV). Now if at some high scale \( M_U > M_R \) the soft supersymmetry breaking masses are universal in the slepton sector:

\[ (m^2_{L\alpha\beta})_{\alpha\beta} = (m^2_{\bar{e}})_{\alpha\beta} = m^2_0 \delta_{\alpha\beta}, \quad A^e_{\alpha\beta} = A_0 Y^e_{\alpha\beta}, \]

(19)

the Dirac couplings \( Y_{\kappa\alpha} \), which violate lepton flavour, will induce flavour off-diagonal entries through loops of heavy RH neutrinos. One thus obtains, at low energy:

\[ (m^2_{L\alpha\beta})_{\alpha\beta} \approx -\frac{3m^2_0 + A^2_0}{8\pi^2} C_{\alpha\beta}, \quad (m^2_{\bar{e}})_{\alpha\beta} \approx 0, \quad A^e_{\alpha\beta} \approx -\frac{3}{8\pi^2} A_0 y_{e\alpha} C_{\alpha\beta}, \]

(20)

where the coefficients \( C_{\alpha\beta} = \sum_k Y^*_{\kappa\alpha} Y_{k\beta} \ln(M_U/M_k) \) encapsulate the dependence on the seesaw parameters. Putting back Eq. (20) into Eqs. (15) and (16), one sees that \( \text{BR} (\mu \rightarrow e\gamma) \propto |C_{12}|^2 \), \( \text{BR} (\tau \rightarrow \mu \gamma) \propto |C_{23}|^2 \) and so on. Fig. 2 shows the upper bound on \( |C_{12}| \) associated with the present experimental limit on \( \text{BR} (\mu \rightarrow e\gamma) \), and the upper bound on \( |C_{23}| \) that would be obtained if the experimental limit on \( \text{BR} (\tau \rightarrow \mu \gamma) \) were improved by 3 orders of magnitude.

Since the neutrino mass matrix \( M_\nu \) and the coefficients \( C_{\alpha\beta} \) depend on different combinations of the seesaw parameters, it is not possible to relate the rates of LFV processes to the observed values of neutrino masses and mixing angles. Rather experimental limits on LFV processes can be used to discriminate between different classes of seesaw models. In particular, in models in which the heaviest right-handed neutrino contributes significantly to the atmospheric neutrino mass scale, the relation \( |C_{23}| \sim |Y_{33}|^2 \ln(M_U/M_3) \) holds. Assuming that \( Y_{33} \) is of order one, as happens e.g. in \( SO(10) \) Grand Unified Theories, this implies that \( \text{BR} (\tau \rightarrow \mu \gamma) > 10^{-9} \) over a large portion of the MSSM parameter space [30]. Therefore, if the experimental sensitivity reaches \( 10^{-9} \) and still \( \tau \rightarrow \mu \gamma \) is not observed, this class of models will be disfavoured over most of the MSSM parameter space. As for \( \mu \rightarrow e\gamma \), its branching ratio is generally predicted to be large in supersymmetric seesaw models (see e.g. Refs. [25, 31]), with good chances of being detected in forthcoming experiments, but it is much more model-dependent [30].

### 4 Dipole electric moments (EDMs) of charged leptons

While charged lepton EDMs arise only at the multiloop level in the Standard Model, and are out of reach of foreseen experiments, they are generated at the one-loop level in its supersymmetric extensions [32] and can have much larger values. One can distinguish between two types of contributions:
Figure 2: Upper bounds on $|C_{32}|$ and $|C_{12}|$ associated with, respectively, $\text{BR}(\tau \rightarrow \mu \gamma) < 10^{-9}$ and $\text{BR}(\mu \rightarrow e\gamma) < 10^{-11}$, as functions of the average right slepton mass $m_{\tilde{e}_R}$ and of the bino mass $M_1$, for $\tan \beta = 10$ and $A_0 = m_0$. From Ref. [30].

- flavour conserving contributions from phases in flavour diagonal parameters, i.e. the $A$-terms and the supersymmetric Higgs mass parameter $\mu$;
- flavour violating contributions from phases in the $\delta$'s (off-diagonal entries of the slepton mass matrices).

At present, only the experimental limit on the electron EDM yields significant constraints on these phases, but future experiments may be sensitive to values of the muon EDM that are typically expected in supersymmetric models.

The flavour conserving contributions are proportional either to $\text{Im}(A_i - \mu \tan \beta) m_{e_i}$ or to $\text{Im}(\mu) \tan \beta m_{e_i}$. Unless there is a strong hierarchy among the $A_i$, it follows that the charged lepton EDMs approximately satisfy the scaling relation $d_i \propto m_{e_i}$, where $d_i$ is the EDM of the $i$th charged lepton, and $m_{e_i}$ is its mass. The experimental limit on the electron EDM, $|d_e| < 1.6 \times 10^{-27}$ (90 % C.L.) [33], strongly constrains the phase of the $\mu$ parameter (see Fig. 3), while the constraint on $\text{Im}A_e$ is much weaker. Note that the upper bound on $\sin \Phi_{\mu}$ cannot be relaxed by lowering the value of $|\mu|$, since the latter is constrained by the condition of electroweak symmetry breaking. The present experimental limits on $d_\mu$ and $d_\tau$ do not yield any significant constraint on $\sin \Phi_{\mu}$ and $\text{Im}A_{\mu,\tau}$. Moreover, the scaling relation $d_i \propto m_{e_i}$, together with the experimental limit on the electron EDM, implies a strong upper bound on the muon EDM, which is smaller by 7 orders of magnitude than the present experimental limit and lies below the sensitivity of the planned BNL experiment [34]:

$$d_\mu|_{\text{th(FC)}} < \frac{m_\mu}{m_e} \left| d_e \right|_{\text{exp}} = 3 \times 10^{-25} \text{ e.cm} .$$

This fact is illustrated in Fig. 3, where the upper bound on $\sin \Phi_{\mu}$ that would correspond to a limit of $10^{-25}$ e.cm on the muon EDM (right) is compared with the upper bound obtained from the present experimental limit on the electron EDM (left).
Since EDMs are flavour diagonal quantities, the flavour violating contributions necessarily involve two $\delta$'s. For instance, the following combinations contribute to $d_i$: $\sum_k \text{Im}(\delta^{LL}_{ik} \delta^{LR}_{ki})$, $\sum_k m_k \text{Im}(\delta^{LL}_{ik}(A_k - \mu \tan \beta)\delta^{RR}_{ki})$, $\sum_k m_k \text{Im}(\delta^{LR}_{ik}(A_k - \mu^* \tan \beta)\delta^{LR}_{ki})$. These contributions do not satisfy the scaling relation $d_i/d_j \approx m_{e_i}/m_{e_j}$; in particular, it is possible to have $d_\mu \gg m_{\mu}/d_{e|\text{exp}}$, in the sensitivity range of the future BNL experiment [35]. Note that the experimental upper limit on the electron EDM provides better constraints on the (imaginary part of the) products $\delta_{13}\delta_{31}$ than the LFV decay $\tau \to e\gamma$.

Let us finally add that, in supersymmetric seesaw models with flavour-blind supersymmetry breaking, the phases present in the Dirac couplings can induce complex off-diagonal slepton masses, connecting the values of the charged lepton EDMs to the seesaw parameters.

5 Conclusions

We have seen in this short review that flavour and CP violation have a very different status in the lepton and in the quark sectors. Indeed, if the PMNS matrix is the only source of flavour and CP violation in the lepton sector, neutrino oscillations are likely to be the only manifestation of lepton flavour violation that can be accessed experimentally, as well as the only place (together with neutrinoless double beta decay if neutrinos are Majorana particles) where one can possibly test leptonic CP violation.

LFV processes involving charged leptons and charged lepton EDMs are therefore a unique probe of new physics. The observation of e.g. $\mu \to e\gamma$, or the measurement of a nonzero muon EDM would definitely testify for physics beyond the Standard Model. A significant improvement of the experimental upper limits on BR ($\mu \to e\gamma$) and BR ($\tau \to \mu\gamma$) would already provide strong constraints on supersymmetry breaking and on supersymmetric seesaw models.
References


