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The Maximum-Likelihood Fit Program ckmLfit

Motivation and Goal
Our Likelihood Approach
Inputs
Program Status
Results and Future

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Lfit

Motivation:

1. Starting from the idea of the BABAR Physics Book to write the measured values m for the CKM triangle fit as

$$m = e \cdot t\ , \quad e = \mu \pm \sigma\ , \quad t = t_{\min} \div t_{\max}$$

with Gaussian errors σ on the experimental factors e and ranges t_{max}-t_{min} on the theoretical factors t, and

2. having done a first CKM triangle fit in1992,
[K.R.Schubert and M.Schmidtler, Z.f.Physik C 53 (1992) 347]

we wrote a new fit procedure,
based on a standard maximum-likelihood language,
optimized in computation time,
and with refined output presentation.

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Goal:

Based on the hypothesis that the Standard Model is true,
i. e. that all observations of the charged weak interaction
are calculable with unique values of the 18 SM parameters,
we perform a parameter estimation („fit“)
of 4 of these parameters (A, ρ, η, m_t) and
of 3 extra parameters $(B_K, f_B\sqrt{B_B}, \xi)$
with a standard maximum likelihood technique.

The (Classical) Likelihood Approach:

A measurement m yields information on a parameter p.

$$L(p|m) = const \cdot pdf(m|p).$$

The likelihood for p is proportional to
the probabitity density for finding m if p is true.

$$L(p|m) = const \cdot pdf(m|p).$$

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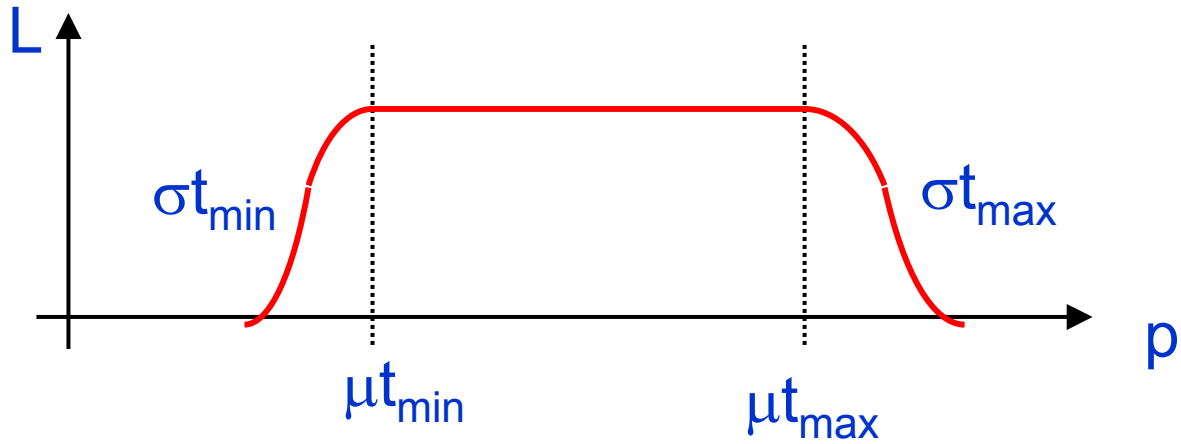
L is not a pdf . L has a finite value for every p.
There are no intervals dp. There is no pdf for p.
A given p is either true or false.

$$L(p|m = e \cdot t) = ?$$

For fixed t, L is Gaussian:

$$L(p|m = e \cdot t_0) = \exp\left[-(p - \mu \cdot t_0)^2 / 2\sigma^2\right]$$

If t has a range, the likelihood for p gets wider. Combining with every accepted t between t_{max} and t_{min} gives:



An estimation of M parameters p_a by N independent measurements $m_b = e_b\, t_b$ is obtained by multiplying the N likelihood functions $L_b(p_1 ... p_M \mid m_b)$. The resulting product has either a single maximum or a region with constant maximum L. Parameter estimation means point and interval estimation, in the latter case only interval estimation. We aim for three intervals: $L = L_{max}$, $L > L_{max}\, e^{-1/2}$, $L > L_{max}\, e^{-2}$, corresponding to best values and >68% and >95% confidence (for one parameter).

Inputs:

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Parameter	Value	Error (flat)	Error (gauss)	Unit
λ	0.2231	-	-	
A	0.818	0.010	$^{+\ 0.014}_{-\ 0.020}$	
$\lvert V_{ub}\rvert_{incl}$	4.11	0.55	0.25	$\cdot 10^{-3}$
$\lvert V_{ub}\rvert_{excl}$	0.46	0.46	0.20	$\cdot 10^{-3}$
$F_B = f_{B_d}\sqrt{B_d}$	0.223	0.012	0.033	GeV
ξ	1.24	0.06	0.04	
B_K	0.87	0.13	0.06	
Δm_s	Amplitude	Spectrum	(Fit Function)	
Δm_d	0.489	-	0.008	ps^{-1}
sin2β	0.735	-	0.055	
m_t	166.0	-	5.0	GeV/c^2
m_c	1.3	-	0.1	GeV/c^2
$\lvert\epsilon_K\rvert$	2.282	-	-	$\cdot 10^{-3}$
m_{B_d}	5.2793	-	-	GeV/c^2
m_{B_s}	5.3696	-	-	GeV/c^2
m_K	0.497672	-	-	GeV/c^2
Δm_K	3.4938	-	-	$\cdot 10^{15}$ GeV/c^2
f_K	0.1598	-	-	GeV
m_W	80.419	-	-	GeV/c^2
η_t	0.574	-	-	
η_c	1.38	-	-	
η_{ct}	0.47	-	-	
η_B	0.55	-	-	

ICHEP2002

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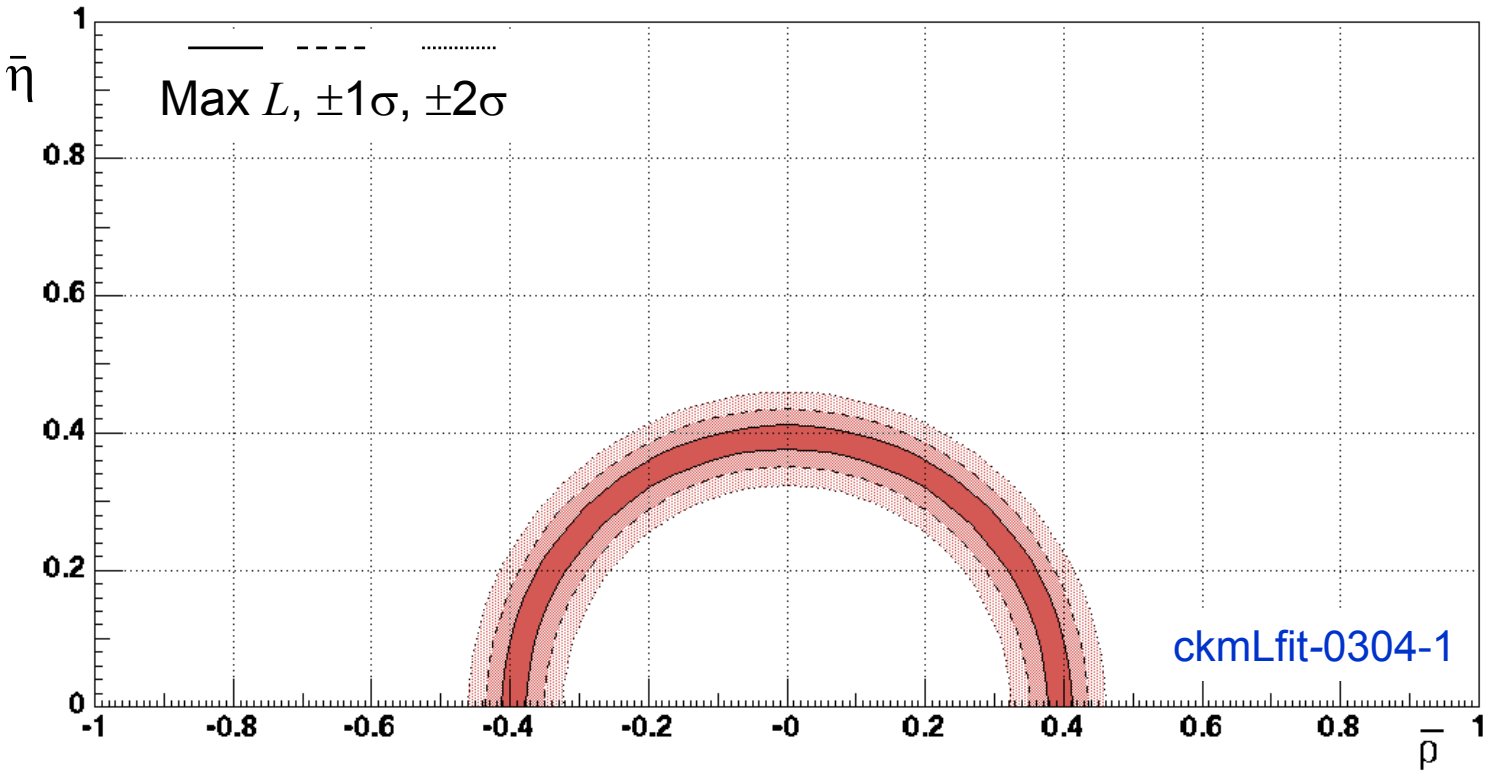
Status of the Program ckmLfit:

Written in C++ by René Nogowski using ROOT frame.
Runs under RedHat Linux 7.3 (compiler gcc 2.96);
planned to also run under Windows.
Plot parameters are not ρ and η, but

$$\bar{\rho} \quad and \quad \bar{\eta} \,.$$

Maximum and 1-σ and 2-σ contours are obtainable for any input combinations. Contours are found by an interval approximation method on 90 ϕ-rays starting from the L_{max} point in the $\bar{\rho},\bar{\eta}$ plane. The 90 points are connected with a smooth line by ROOT.
The ROOT frame (now v.3.03/09) also offers easy choice of any combination of plots.
Time for compile + execute = *o* (1 min) with 1.2 GHz

Results:


Max L, ±1σ, ±2σ
$\bar{\eta}$
$\bar{\rho}$
1
0.8
0.6
0.4
0.2
0
-1
-0.8
-0.6
-0.4
-0.2
-0
0.2
0.4
0.6
0.8
1
ckmLfit-0304-1

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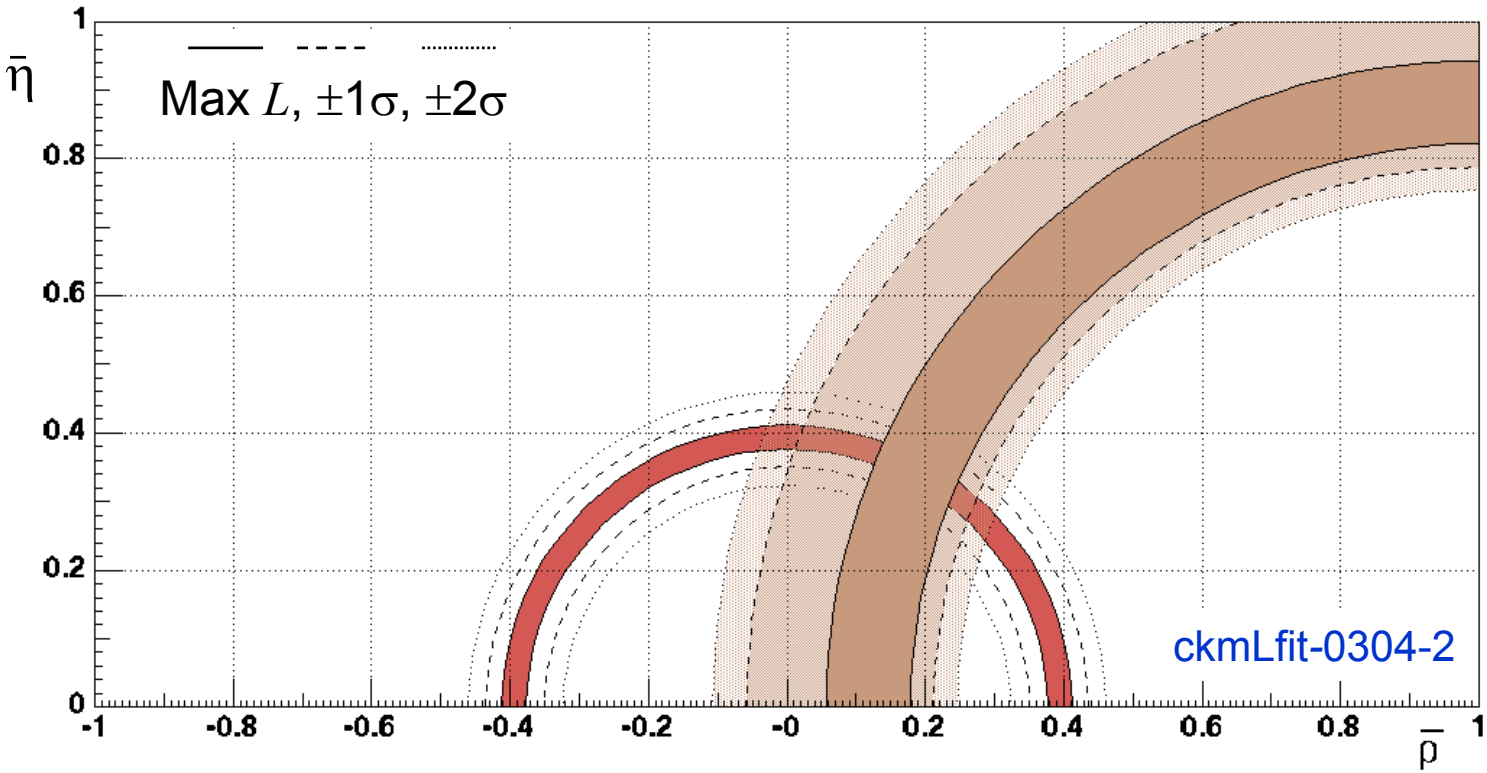
Max L, ±1σ, ±2σ
$\bar{\eta}$
$\bar{\rho}$
1
0.8
0.6
0.4
0.2
0
-1
-0.8
-0.6
-0.4
-0.2
-0
0.2
0.4
0.6
0.8
1
ckmLfit-0304-2

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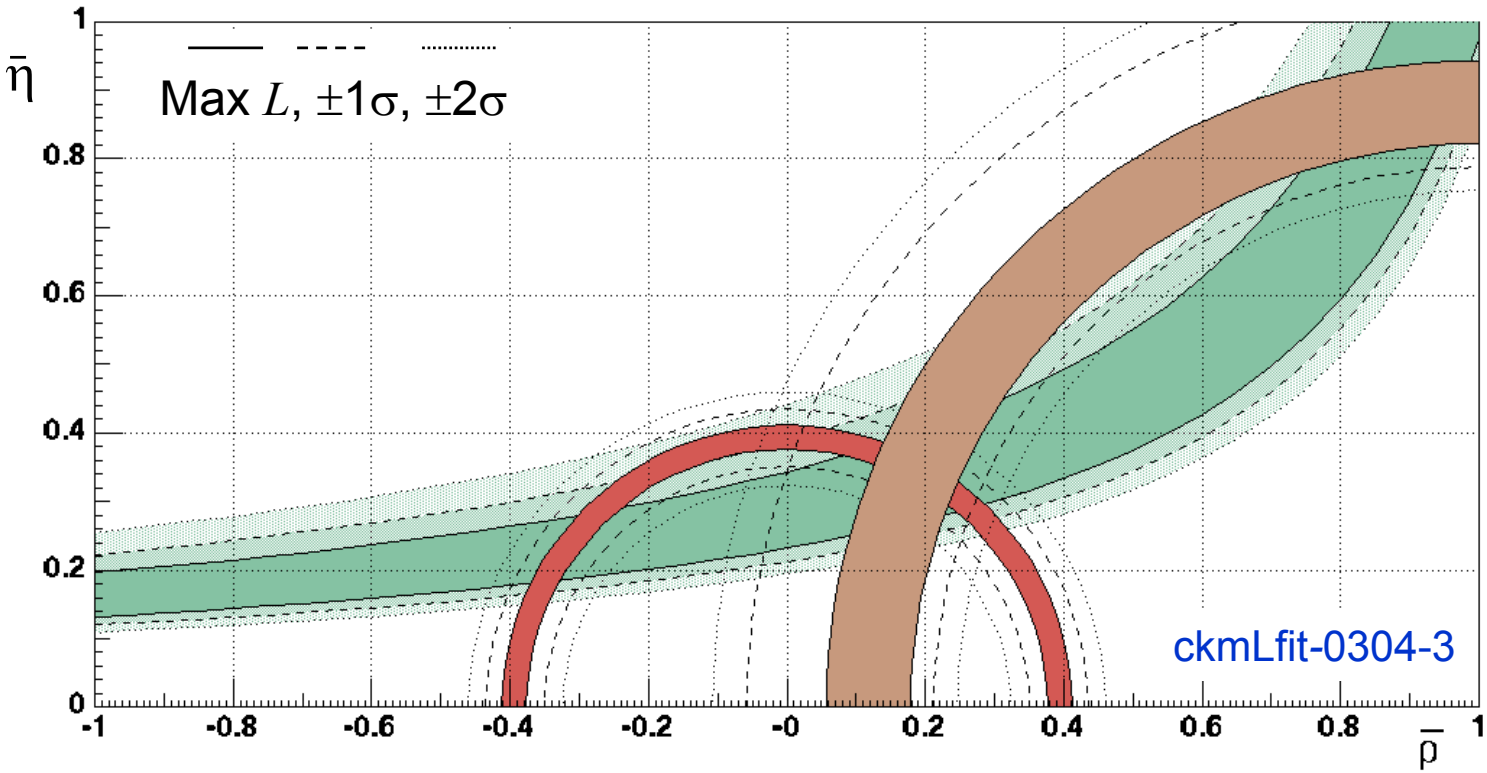
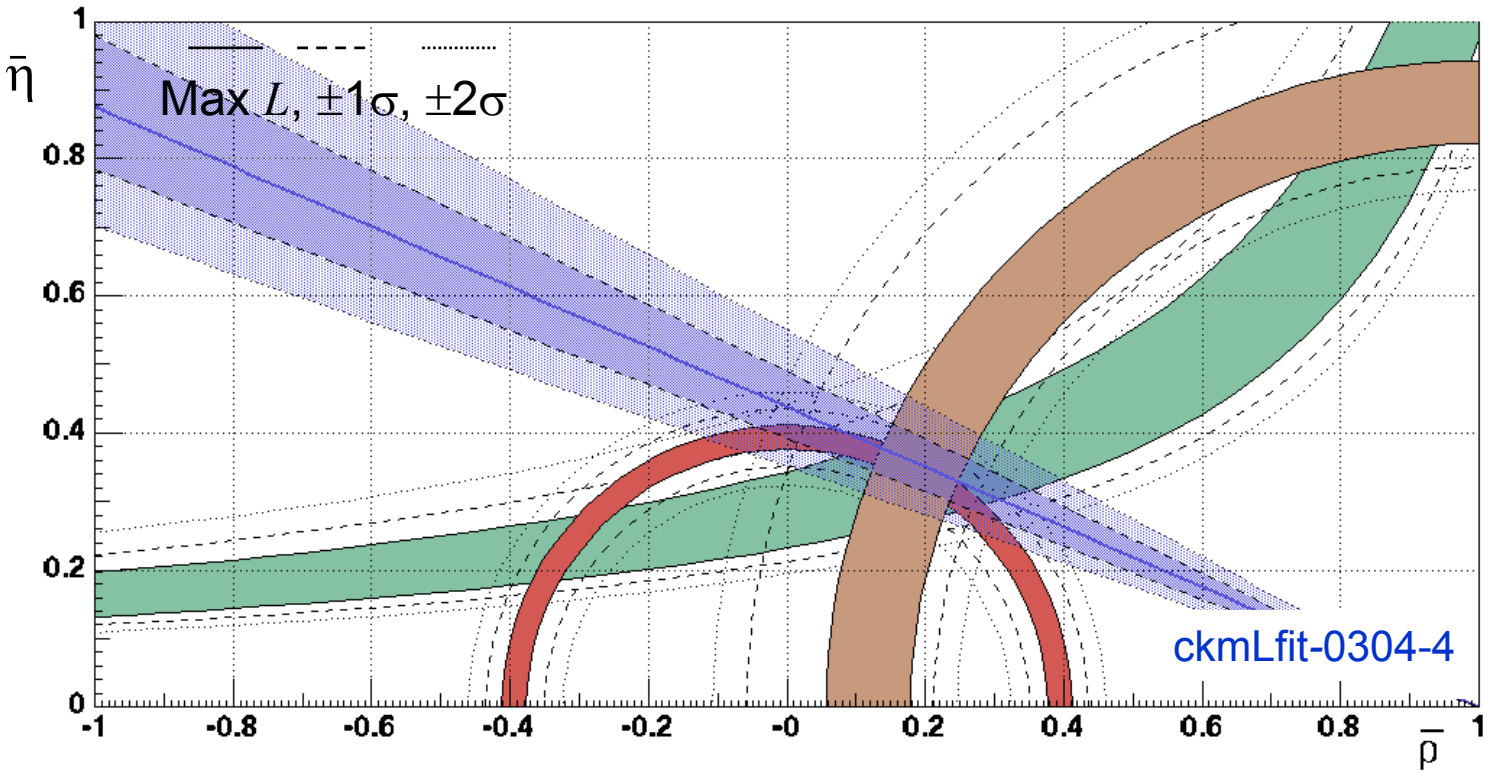
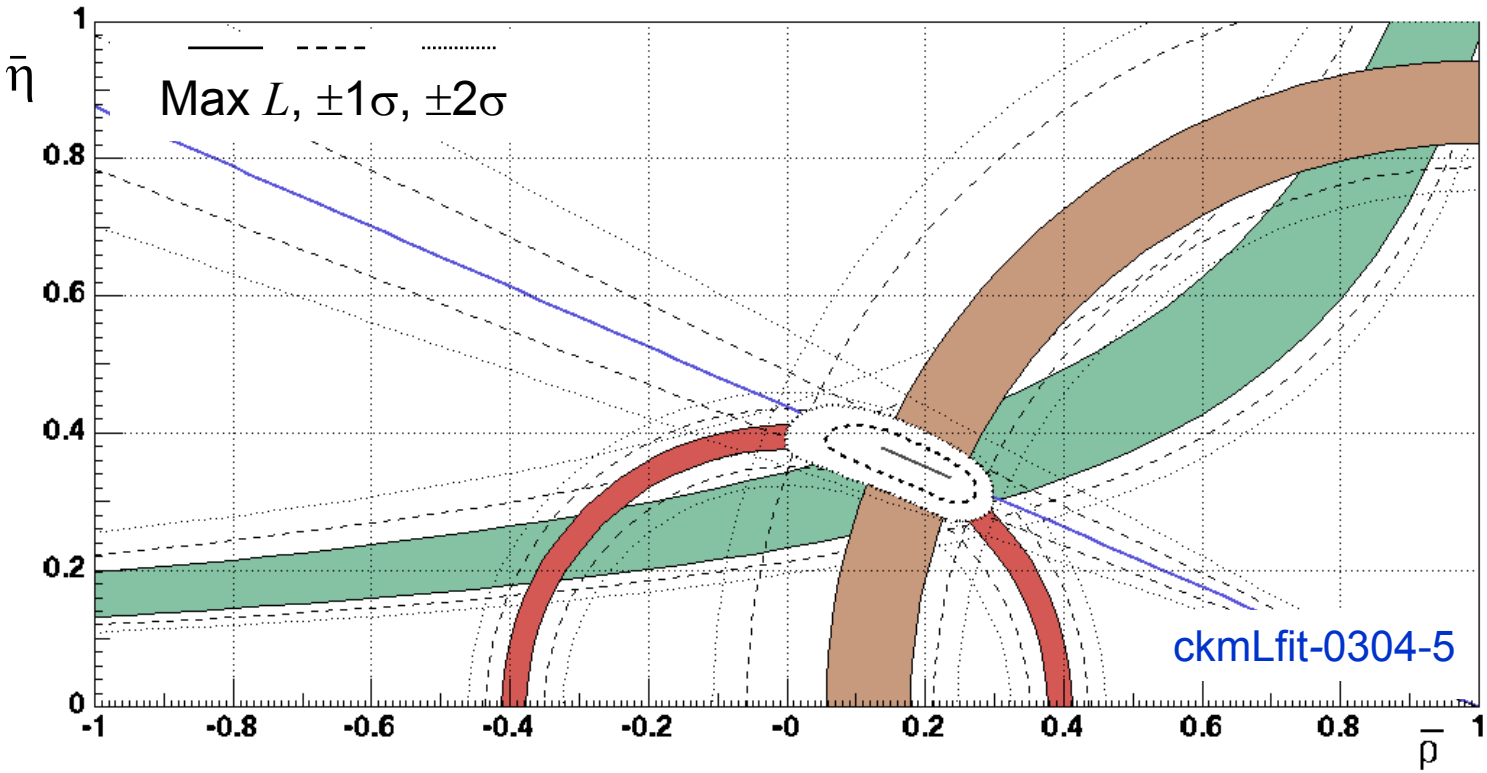

Max L, ±1σ, ±2σ
$\bar{\eta}$
$\bar{\rho}$
1
0.8
0.6
0.4
0.2
0
-1
-0.8
-0.6
-0.4
-0.2
-0
0.2
0.4
0.6
0.8
1
ckmLfit-0304-3


Max L, ±1σ, ±2σ
$\bar{\eta}$
$\bar{\rho}$
ckmLfit-0304-4
1
0.8
0.6
0.4
0.2
0
-1
-0.8
-0.6
-0.4
-0.2
-0
0.2
0.4
0.6
0.8
1

Final Fit:



$$\overline{\rho} = 0{,}187^{+0{,}036}_{-0{,}082} \pm 0{,}051$$

$$\overline{\eta} = 0{,}356^{+0{,}034}_{-0{,}036} \pm 0{,}022$$

Output and Input Informations on all used Parameters:

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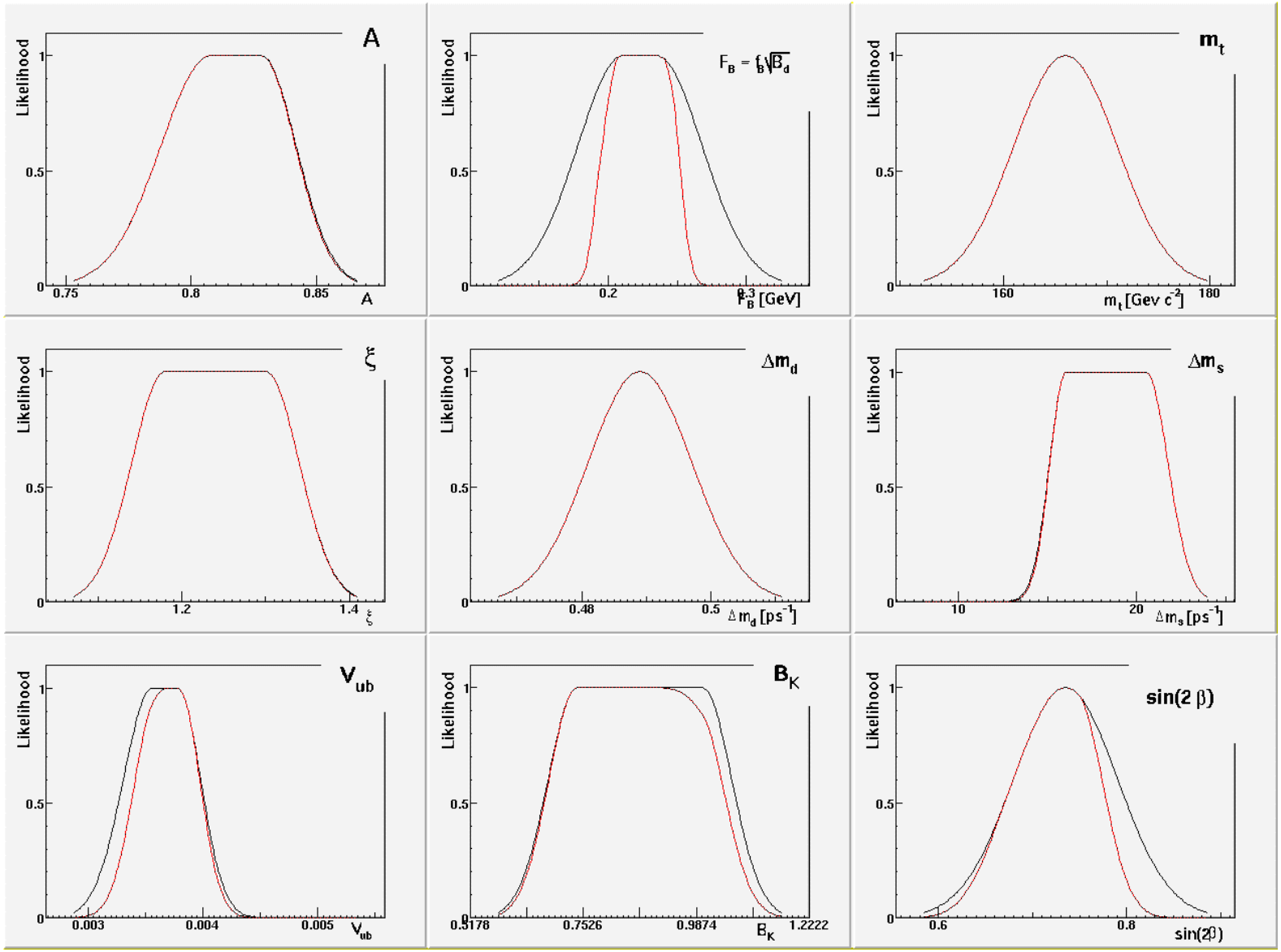

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Further Fit Results:

No changes in A, B_K, Δm_d, Δm_s, ξ, m_t

$$
\begin{aligned}
|V_{ub}| &= \left(3{,}74^{+0{,}19}_{-0{,}29} \pm 0{,}04\right)\times 10^{-3} \\
f_{B_d}\sqrt{B_d} &= \left(0{,}223^{+0{,}016}_{-0{,}015} \pm 0{,}012\right)\mathrm{GeV} \\
B_K &= 0{,}81^{+0{,}16}_{-0{,}06} \pm 0{,}07 \\
\sin(2\beta) &= 0{,}735^{+0{,}039}_{-0{,}055} \\
\beta &= \left(23{,}7^{+1{,}6}_{-2{,}3}\right)^\circ \\
\sin(2\alpha) &= -0{,}06 \pm 0{,}55 \\
\alpha &= (92 \pm 17)^\circ
\end{aligned}
$$

To be done:

More inputs. More than 7 parameters.
And already mentioned: Running under Windows.