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Exclusive Radiative B Decays: $B \to V\gamma$ at NLO

Stefan W. Bosch[a] and Gerhard Buchalla[b]

[a]Newman Laboratory for Elementary-Particle Physics, Cornell University, Ithaca, NY 14853, U.S.A.

[b]Ludwig-Maximilians-Universität München, Sektion Physik, Theresienstraße 37, D-80333 München, Germany

We discuss the exclusive radiative decays $B \to K^*\gamma$ and $B \to \rho\gamma$ in QCD factorization within the Standard Model. The analysis is based on the heavy-quark limit of QCD. Our results for these decays are complete to next-to-leading order in QCD and to leading order in the heavy-quark limit. Phenomenological implications for branching ratios and isospin breaking effects are discussed. Special emphasis is placed on constraining the CKM unitarity triangle from these observables.

1 Introduction

The rare radiative B decays belong to the most valuable probes of the quark flavour sector (see [1] for a recent review). The inclusive $b \to s\gamma$ mode, showing good agreement of theoretical next-to-leading-logarithmic (NLL) QCD prediction and experimental measurements, puts stringent bounds on physics beyond the standard model. CP-averaged branching ratios of exclusive radiative channels are measured to be $B(B^0 \to K^{*0}\gamma) = (4.18 \pm 0.23) \cdot 10^{-5}$ and $B(B^+ \to K^{*+}\gamma) = (4.14 \pm 0.33) \cdot 10^{-5}$ [2], and bounded with 90% confidence level as $B(B^0 \to \omega^0\gamma) < 1.0 \cdot 10^{-6}$, $B(B^0 \to \rho^0\gamma) < 1.2 \cdot 10^{-6}$ and $B(B^+ \to \rho^+\gamma) < 2.1 \cdot 10^{-6}$ [3].

Whereas the inclusive decay can be treated perturbatively, bound state effects are essential for the exclusive modes and have to be described by some nonperturbative quantities like hadronic form factors and light-cone distribution amplitudes (LCDAs). However, in the heavy-quark limit $m_b \gg \Lambda_{\rm QCD}$ a systematic treatment of exclusive B decays is possible within QCD [4]: Perturbatively calculable contributions to the matrix elements can be factorized from nonperturbative form factors and universal light-cone distribution amplitudes. We use the QCD factorization technique for the exclusive radiative decays $B \to K^*\gamma$ and $B \to \rho\gamma$ as in [5–7]. The ratio of the $B \to \rho\gamma$ and $B \to K^*\gamma$ branching fractions is, at leading order in α_s, directly proportional to the side R_t in the standard unitarity triangle (UT), where

$$R_t \equiv \sqrt{(1-\bar\rho)^2 + \bar\eta^2} = \frac{1}{\lambda}\left|\frac{V_{td}}{V_{ts}}\right| \qquad (1)$$

Having the complete NLL result for the decay amplitudes in $B \to V\gamma$ at hand, we can calculate α_s corrections to their relation with R_t and evaluate the implications in the $(\bar\rho, \bar\eta)$ plane [8,9].

2 $B \to V\gamma$ at NLO in QCD

The effective weak Hamiltonian for $b \to s/d\gamma$ transitions is

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(s/d)} \left(\sum_{i=1}^{2} C_i Q_i^p + \sum_{j=3}^{8} C_j Q_j \right) \qquad (2)$$

where $\lambda_p^{(s/d)} = V^*_{ps/d} V_{pb}$. The relevant operators are the current-current operators $Q^p_{1,2}$, the QCD-penguin operators $Q_{3...6}$, and the electro- and chromomagnetic penguin operators $Q_{7,8}$. To evaluate the hadronic matrix elements of these operators we employ the heavy-quark limit $m_b \gg \Lambda_{\rm QCD}$ to get the factorization formula [5,6]

$$\begin{aligned}&\langle V\gamma(\epsilon)|Q_i|\bar B\rangle = \qquad (3)\\ &= \left[F^{B\to V} T_i^I + \int_0^1 d\xi\, dv\, T_i^{II}(\xi, v)\Phi_B(\xi)\Phi_V(v)\right]\cdot \epsilon\end{aligned}$$

where ϵ is the photon polarization 4-vector. Here $F^{B\to V}$ is a $B \to V$ transition form factor, and Φ_B, Φ_V are leading twist light-cone distribution amplitudes of the B meson and the vector meson V, respectively. These quantities are universal, nonperturbative objects. They describe the long-distance dynamics of the matrix elements, which is factorized from the perturbative, short-distance interactions expressed in the hard-scattering kernels T_i^I and T_i^{II}. To leading order in QCD and leading power in the heavy-quark limit, Q_7 gives the only contribution to the $B \to V\gamma$ amplitude. At $\mathcal{O}(\alpha_s)$ the operators $Q_{1...6}$ and Q_8 start contributing and the factorization formula becomes nontrivial.

The relevant diagrams for the NLO hard-vertex corrections T_i^I were computed in [10] to get the virtual corrections to the matrix elements for the inclusive

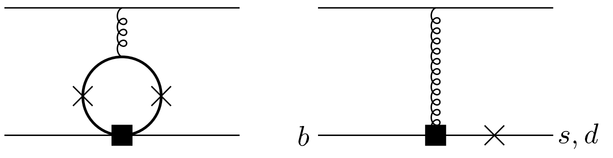

Figure 1. *$\mathcal{O}(\alpha_s)$ contribution at leading power to the hard-scattering kernels T_i^{II} from four-quark operators Q_i (left) and from Q_8. The crosses indicate the places where the emitted photon can be attached.*

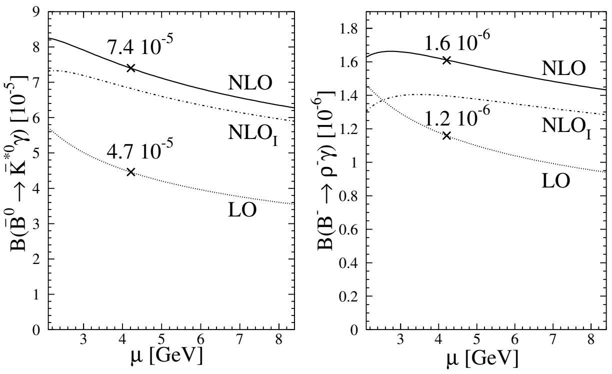

Figure 2. *Dependence of the branching fractions $B(\bar{B}^0 \to \bar{K}^{*0}\gamma)$ and $B(B^- \to \rho^-\gamma)$ on the renormalization scale μ. The dotted line shows the LO, the dash-dotted line the NLO result including type-I corrections only and the solid line shows the complete NLO result.*

$b \to s\gamma$ mode at next-to-leading order. For the exclusive modes the same corrections play the role of the perturbative type I hard-scattering kernels. The non-vanishing contributions to T_i^{II} where the spectator participates in the hard scattering are shown in Fig. 1. We can express both the type I and type II contributions to the matrix elements $\langle Q_i \rangle$ in terms of the matrix element $\langle Q_7 \rangle$, an explicit factor α_s, and hard-scattering functions G_i and H_i which are given explicitely in [6,11].

Weak annihilation contributions are suppressed by one power of $\Lambda_{\rm QCD}/m_b$ but they are nevertheless calculable in QCD factorization because in the heavy-quark limit the colour-transparency argument applies to the emitted, highly energetic vector meson. Including them we become sensitive to the charge of the decaying B meson and thus to isospin breaking effects.

3 Results

The total $\bar{B} \to V\gamma$ amplitude then can be written as

$$A(\bar{B} \to V\gamma) = \frac{G_F}{\sqrt{2}} \left[\lambda_u a_7^u + \lambda_c a_7^c\right] \langle V\gamma|Q_7|\bar{B}\rangle \tag{4}$$

where the factorization coefficients $a_7^p(V\gamma)$ consist of the Wilson coefficient C_7, the contributions from the type I and type II hard-scattering and annihilation corrections. One finds a sizeable enhancement of the leading order value, dominated by the T^I-type correction. The net enhancement of a_7 at NLO leads to a corresponding enhancement of the branching ratios, for fixed value of the form factor. This is illustrated in Fig. 2, where we show the residual scale dependence for $B(\bar{B} \to \bar{K}^{*0}\gamma)$ and $B(B^- \to \rho^-\gamma)$ at leading and next-to-leading order. Our central values for the $B \to K^*\gamma$ branching ratios are higher than the quoted experimental measurements. The dominant uncertainty in the theoretical values comes from the $B \to V\gamma$ form factors. We used the light-cone sum rule (LCSR) results $F_{K^*} = 0.38 \pm 0.06$ and $F_\rho = 0.29 \pm 0.04$ from [12]. A recent preliminary lattice QCD determination, $F_{K^*} = 0.25 \pm 0.05 \pm 0.02$ [13], would give a better agreement with the experimental central values.

The charge averaged isospin breaking ratio can be defined as

$$\Delta(V\gamma) = \frac{\Gamma(B^0 \to V^0\gamma) - v\Gamma(B^\pm \to V^\pm\gamma)}{\Gamma(B^0 \to V^0\gamma) + v\Gamma(B^\pm \to V^\pm\gamma)} \tag{5}$$

with $v = 1$ for $V = K^*$ and $v = 1/2$ for $V = \rho$. This ratio has a reduced sensitivity to the nonperturbative form factors. As already discussed, in our approximations, isospin breaking is generated by weak annihilation contributions. Kagan and Neubert found a large effect from the penguin operator Q_6 on the isospin asymmetry $\Delta(K^*\gamma)$ [14]. The prediction $\Delta(K^*\gamma) = (3.9^{+3.1}_{-1.9})\%$ is in agreement with the experimental value $\Delta(K^*\gamma)_{exp} = (4.5 \pm 4.9)\%$. For $B \to \rho\gamma$ we find a strong dependence of the isospin asymmetry on the angle γ of the unitarity triangle. The γ dependence is in particular pronounced for the zero crossing of $\Delta(\rho\gamma)$ around $\gamma = 60°$, the value favoured by the standard UT fits.

4 Implications for the UT Analysis

We can use observables in the $B \to V\gamma$ decay modes to get information on parameters in the $(\bar{\rho}, \bar{\eta})$ unitarity triangle plane. The cleanest observable is the ratio of the neutral $B \to \rho\gamma$ and $B \to K^*\gamma$ branching ratios. (We choose here the neutral modes because annihilation effects could be sizeable in $B^\pm \to \rho^\pm\gamma$. However, those effect can be estimated and the charged mode can also be used for a similar analysis.)

We define

$$R_{00} \equiv \frac{B(B^0 \to \rho^0\gamma)}{B(B^0 \to K^{*0}\gamma)} \tag{6}$$

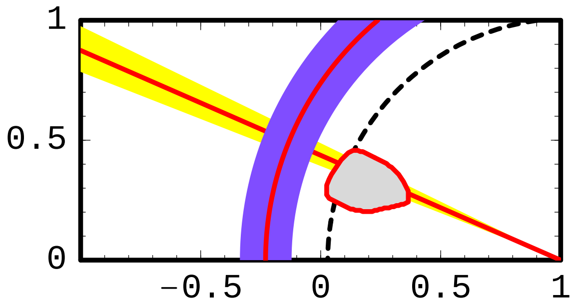

Figure 3. *Impact of the experimental upper bound on R_{00} in the $(\bar{\rho}, \bar{\eta})$ plane. The width of the dark band reflects the variation of ξ. The intersection with the light-shaded $\sin(2\beta)$ band defines the apex of the unitarity triangle and the length R_t.*

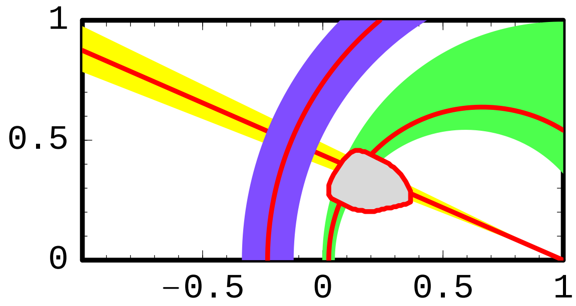

Figure 4. *Same as Fig. 3 including the implication of a measurement of $\Delta(\rho\gamma)^{exp} = 0$ (curved band on the right). The width of the band reflects the theoretical uncertainties from varying the hadronic parameter λ_B and the renormalization scale μ. (The effect of isospin breaking in the form factors is neglected here.)*

$$\approx \quad \frac{1}{2}\left|\frac{V_{td}}{V_{ts}}\right|^2 \xi^{-2}\left(1+\Delta(\bar{\rho},\bar{\eta})\right) \tag{7}$$

where CP averaged branching fractions are understood. Here $\Delta(\bar{\rho},\bar{\eta})$ is a small perturbative correction [9] and $\xi = F_{K^*}/F_\rho$, the ratio of the form factors, is essentially the only source of theoretical uncertainty. We use the LCSR estimate $\xi = 1.33 \pm 0.13$ [12]. A preliminary lattice value is $\xi = 1.1 \pm 0.1$ [13]. Experimentally, so far only an upper limit on R_{00} exists. Because the $B \to \omega\gamma$ branching ratio is up to tiny corrections the same as the one for $B^0 \to \rho^0\gamma$ [9] and its experimental limit is tighter, we use it to get $R_{00}^{\rm exp} < 0.024$. If we use in addition the experimental measurement $\sin(2\beta) = 0.734 \pm 0.054$ [15] we can construct the overlap of the R_{00} and $\sin(2\beta)$ bands and extract the length R_t as shown in Fig. 3. The dashed curve in Fig. 3 was obtained setting $\xi = 1$ in (7). This can be viewed as the lowest possible value. If R_{00} were measured at its current experimental bound $R_{00} = 0.024$, the dashed line would correspond to a conservative lower bound on R_t. The value $R_t < 1.24$ from R_{00} is already becoming comparable with the constraints from ΔM_{B_d} and ΔM_{B_s}. It is possible that the experimental measurement of R_{00} may actually be achieved before the measurement of ΔM_{B_s}. Once a measurement of both the charged and neutral $B \to \rho\gamma$ modes is available, one can also use $\Delta(\rho\gamma)$ to constrain the unitarity triangle. For illustration purposes we plot in Fig. 4 in addition to the R_{00} and $\sin(2\beta)$ bands the implication of an assumed measurement of $\Delta(\rho\gamma)^{\rm exp} = 0$, which would correspond to the Standard Model prediction for a CKM angle $\gamma = 60°$.

5 Conclusions and Outlook

We have discussed a systematic and model-independent NLL framework for the rare radiative decays $B \to V\gamma$ based on the heavy-quark limit $m_b \gg \Lambda_{\rm QCD}$. As observables of primary interest we considered the branching fractions, the ratio R_{00} of the neutral $B \to \rho\gamma$ and $B \to K^*\gamma$ branching fractions, and the isospin breaking ratios $\Delta(K^*\gamma)$ and $\Delta(\rho\gamma)$. Our main focus was on the implications of measurements of these quantities on the $(\bar{\rho},\bar{\eta})$ plane of the CKM unitarity triangle. The theoretically cleanest quantities are R_{00} and $\Delta(\rho\gamma)$ which, however, are not yet measured experimentally. We hope that they will become accessible in the near future with new data from the B factories.

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