THE METHOD OF THE DESIGN FOR SURVEY NETWORK BY Q MATRICES

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1. INTRODUCTION

When composing a complicated survey network, these surveying data must be computed by least square method. As measurements always have errors, to minimize these errors, it must be measured with some precise survey instruments. The observation data must be processed by the least mean square method as well as it.

When the surveying data are computed by least square method, these errors describe standard deviation of coordinates. And these are computed by the equation $m_s = m_o\sqrt{Q}$. Where m_s is standard deviation of coordinates, m_o is accuracy of measurements and $\sqrt{Q}$ means sub matrices with diagonal components in the coefficient matrix on the normal equation.

This $\sqrt{Q}$ matrix consist of observed sides. Therefore, $\sqrt{Q}$ matrix can be calculated before observation, and optimize survey network.

This paper discuss the method of the design for survey network by $\sqrt{Q}$

2. PRINCIPLES OF LEAST SQUARES

In surveying, the measurements must often satisfy established numerical relationships known as geometric constraints. And that the number of normal equations in a parametric least square adjustment is always equal to the number of unknown variables. Often, the system of normal equations becomes quite large.

Matrix algebra provides at least two important advantages

(1) It enables reducing complicated systems of equations to simple expressions that can be visualized and manipulated more easily.

(2) It provides a systematic, mathematical method for solving problems that is well adapted to computers.

Problems are frequently encountered in surveying and geodesy that require the solution of large systems of equations.

2.1 Observing Equation

To make the matrix expression for performing least square adjustment, analogy will be made with the systematic procedures. The system of observation equations is presented by matrix notation.

$$V = AX - L \qquad (1)$$

where

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix},$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad L = \begin{bmatrix} l_1 \\ l_2 \\ \vdots \\ l_m \end{bmatrix}, \quad V = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \end{bmatrix}$$

2.2 Normal Equation

Subjecting the matrices above to the manipulations given in the following expression, the normal equations are (2).

$$A^T A X = A^T L \qquad (2)$$

Equation (2) can also expressed as

$$NX = A^T L \qquad (3)$$

It has been demonstrated that equations (2) and (3) produce the normal equations of a least squares adjustment. By using matrix algebra, the solution of normal equations like equation (2) is

$$X = (A^T A)^{-1} A^T L \qquad (4)$$

2.3 Observation Equation for Distance

In adjusting trilateration surveys by parametric least squares, observation equations are written that relate the observed quantities and their inherent random errors to the most probable values for the x and y coordinates (the parameters) of the stations involved. Referring to fig.1, the following distance equation can be written for any line P_iP_j.

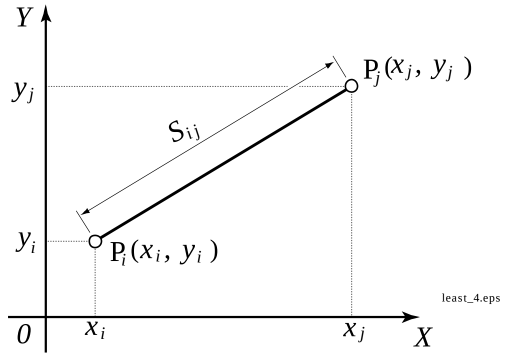


Fig.1 Measurement of a distance

$$S_{ij} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} \qquad (5)$$

where S_{ij} is the ideal length of a line between monuments P_i and P_j by survey network design.

The adjusted length $S_{(adj)ij}$ is

$$S_{(adj)ij} = S_{(obs)ij} + v_{ij} \qquad (6)$$

where v_{ij} is the residual in the measurement, and $S_{(obs)ij}$ is the measured length of a line between monuments P_i and P_j.

On the other hand, dx_i, dy_i, dx_j and dy_j are the corrections to the initial approximations such that

$$\begin{aligned} x_{(adj)i} &= x_i + dx_i \quad , \quad x_{(adj)j} = x_j + dx_j \\ y_{(adj)i} &= y_i + dy_i \quad , \quad y_{(adj)j} = y_j + dy_j \end{aligned} \qquad (7)$$

Equation (5),(6),(7) yields

$$\begin{aligned} S_{(adj)ij} &= \sqrt{\left\{(x_j + dx_j) - (x_i + dx_i)\right\}^2 + \left\{(y_j + dy_j) - (y_i + dy_i)\right\}^2} \\ &= \sqrt{\left\{(x_j - x_i)^2 + (y_j - y_i)^2\right\} + \left\{(dx_j - dx_i)^2 + (dy_j - dy_i)^2\right\}} \qquad (8) \\ &\simeq S_{ij} + dS_{ij} \end{aligned}$$

where dS_{ij} is

$$\begin{aligned} dS_{ij} &= \left(\frac{\partial S_{ij}}{\partial x_j}\right) dx_i + \left(\frac{\partial S_{ij}}{\partial x_i}\right) dx_j + \left(\frac{\partial S_{ij}}{\partial y_j}\right) dy_i + \left(\frac{\partial S_{ij}}{\partial y_i}\right) dy_j \\ &= \frac{x_j - x_i}{S_{ij}} dx_j - \frac{x_j - x_i}{S_{ij}} dx_i + \frac{y_j - y_i}{S_{ij}} dy_j - \frac{y_j - y_i}{S_{ij}} dy_i \end{aligned} \qquad (9)$$

Thus, equation (6) and (8) yields

$$S_{ij} + dS_{ij} = S_{(obs)ij} + v_{ij} \qquad (10)$$

Replacing equation (10) by $l_{ij} = Sij - S_{(obs)ij}$.

$$\begin{aligned} v_{ij} &= dS_{ij} - (S_{(obs)ij} - S_{ij}) \\ &= dS_{ij} - l_{ij} \qquad (11) \\ &= \frac{x_j - x_i}{S_{ij}} dx_j - \frac{x_j - x_i}{S_{ij}} dx_i + \frac{y_j - y_i}{S_{ij}} dy_j - \frac{y_j - y_i}{S_{ij}} dy_i - l_{ij} \end{aligned}$$

Therefore, equation (11) is the observation equation for distance.

2.4 Description by Matrices of Observation Equation for Distance

The equation (11), which is the observation equation for the distance, can be described under below by the matrix.

$$\begin{bmatrix}\vdots\\ v_{ij}\\ \vdots\end{bmatrix} = \begin{bmatrix} \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ \vdots & & & \ddots & & & \vdots \\ \cdots & -\dfrac{x_j-x_i}{S_{ij}} & -\dfrac{y_j-y_i}{S_{ij}} & \cdots & \dfrac{x_j-x_i}{S_{ij}} & \dfrac{y_j-y_i}{S_{ij}} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}\begin{bmatrix}\vdots\\ dx_i\\ dy_i\\ \vdots\\ dx_j\\ dy_j\\ \vdots\end{bmatrix} - \begin{bmatrix}\vdots\\ l_{ij}\\ \vdots\end{bmatrix} \qquad (12)$$

The equation (12) is equal to the equation (1), which is observation equation by the matrix. Therefore, The normal equation of equation (12) is the equation (4).

3. SURVEY NETWORK DESIGN BY $\sqrt{Q}$

3.1 The Standard Deviation for the Coordinate of Monuments, The standard Deviation for the Observation and $\sqrt{Q}$

Rewriting the normal equation (4)

$$X = (A^T A)^{-1} A^T L = N^{-1} A^T L \qquad (13)$$

In equation (13), X is the unknown coordinates vector for the monuments, L is the vector for observed lengths.

The standard deviation for the X , which is the coordinates of monuments, is described such that

$$m_s = m_o \sqrt{Q} \qquad (14)$$

where m_o is the standard deviation for the observed length, and $\sqrt{Q}$ is the elements of the matrix $N^{-1}A^T$ in the equation (13). These elements of the matrix correspond to the monuments ID. As mention above, the matrix $\boldsymbol{A}$ is composed of designed coordinates of monuments. Therefore, the elements of the matrix $\sqrt{Q}$ can know composing of observing sides in advance.

3.2 Design for the Survey Network by $\sqrt{Q}$

In the equation (14), m_o is the standard deviation of the observing sides. In the case of the design and preanalysis for the survey network, m_o is supposed to the precision of the distance measurement instrument. For example, MEKOMETER ME5000 is $\pm(0.2mm + 0.2ppm \times Dist.)$, Laser Tracker is $\pm 20mm$.

When required the precision of monuments is m_s, the survey network is determined as following below

$$m_s \geq m_o \sqrt{Q} \qquad (15)$$

In the equation (15), m_o is the accurate of the instruments. Therefore, the equation (15) is rewriting as following below

$$\frac{m_s}{m_o} \geq \sqrt{Q} \qquad (16)$$

Thus, the optimal survey network is determined by $\sqrt{Q}$ which is satisfying by the equation (16).

4. THE SUBSTANTIATION ON THE SYNCHROTRON SURVEY NETWORK

the Siam Photon Project is the complex of the storage ring at Thai Land. There is no obstacle for the survey sights. It can observe from one monument to all monuments. Hence, it confirm the accuracy of the monuments, which is the error ellipses, by the $\sqrt{Q}$.

4.1 The Substantiation on the Synchrotron Survey Network at Siam Photon Project

(1) CASE 1 (Fig.2,3)

This survey network is the most simple. It is surveyed between neibour vertex of octagon, and from the center to the vertex (Fig.2).

(2) CASE 2 (Fig.4,5)

The difference of $\sqrt{Q_x}, \sqrt{Q_y}$ become smaller, and $\sqrt{Q}$ is improved (Fig.4). The results of the error ellipses are smaller than CASE 1 (Fig.3).

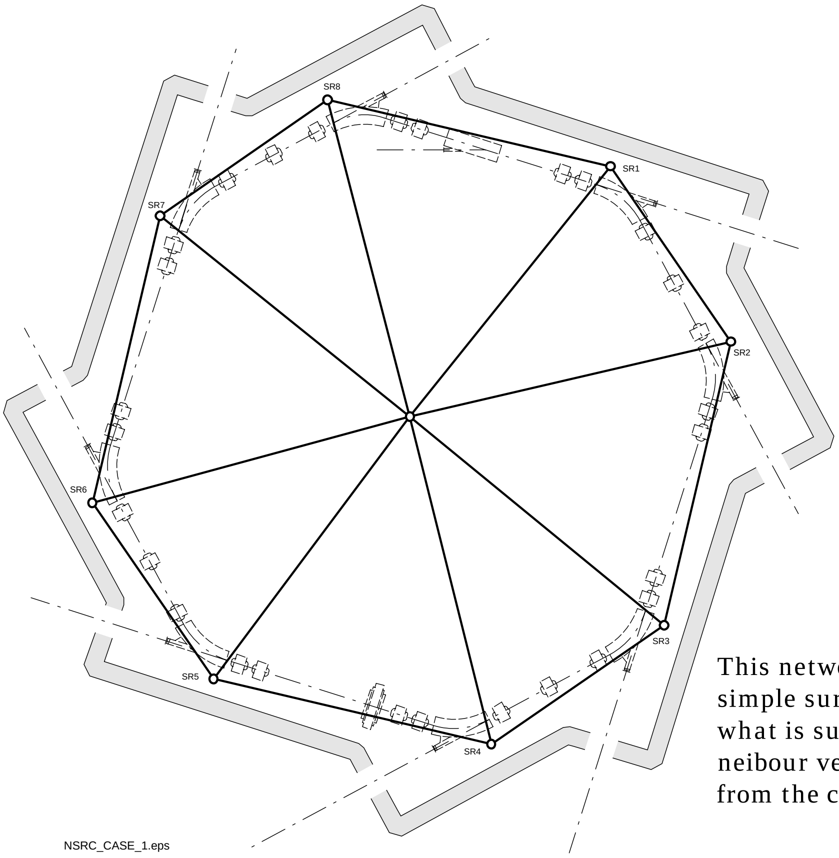
(3) CASE 3 (Fig.6,7)

This survey network is the most complicate (Fig.6). It is surveyed all sights. $\sqrt{Q}$ is the smallest of all cases. The difference of $\sqrt{Q_x}, \sqrt{Q_y}$ is the smallest. The results of the error ellipses are smallest (Fig.7). And the error ellipses is not ellipses but circle. That is to say the most precise monuments.

(4) CASE 4 (Fig.8,9)

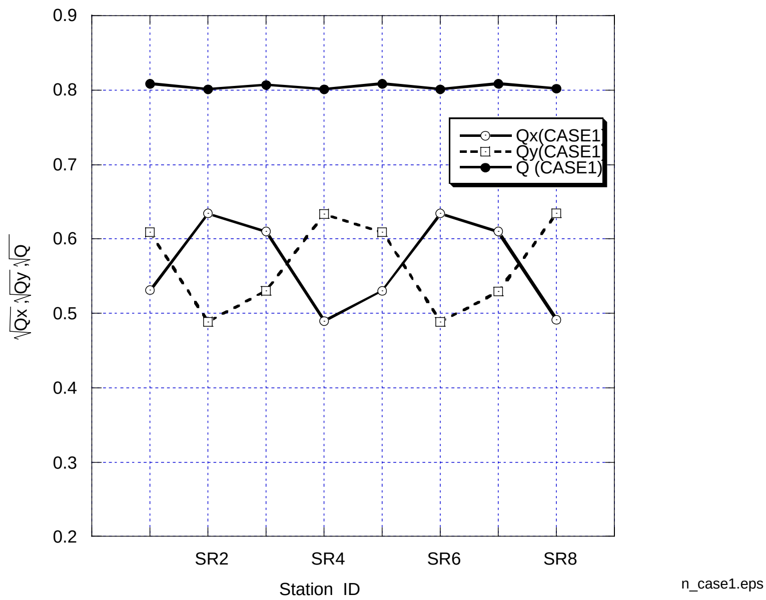
This survey network is eliminated sights from center to vertices of octagon and from vertices to vertices. This case assume large ring (Fig.8). The difference of $\sqrt{Q_x}, \sqrt{Q_y}$ is the largest. The results of error ellipses are is too difficult to secure the precision of the direction for the radius of the ring (Fig.9).

Thus, the relationship between the error ellipses and the $\sqrt{Q}$ have been confirmed.



This network is most simple survey network what is surveyed btween neibour vertices and from the center to vertices.

(a) Survey network of CASE 1



(b) $\sqrt{Q}$ of monuments on the CASE 1

Fig. 2 The survey network and $\sqrt{Q}$ on the CASE 1

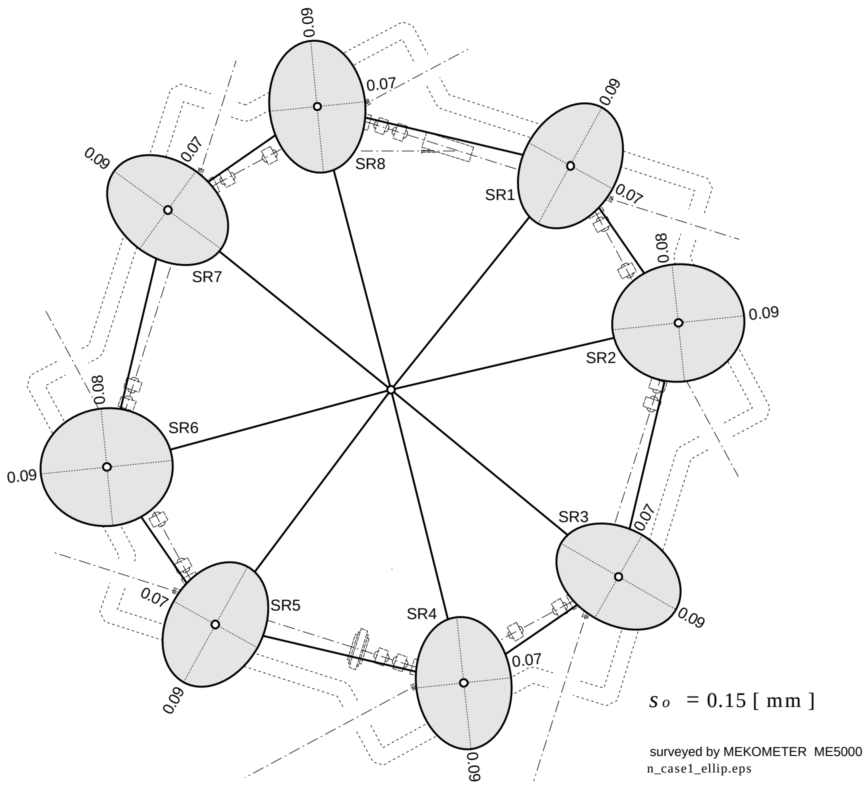
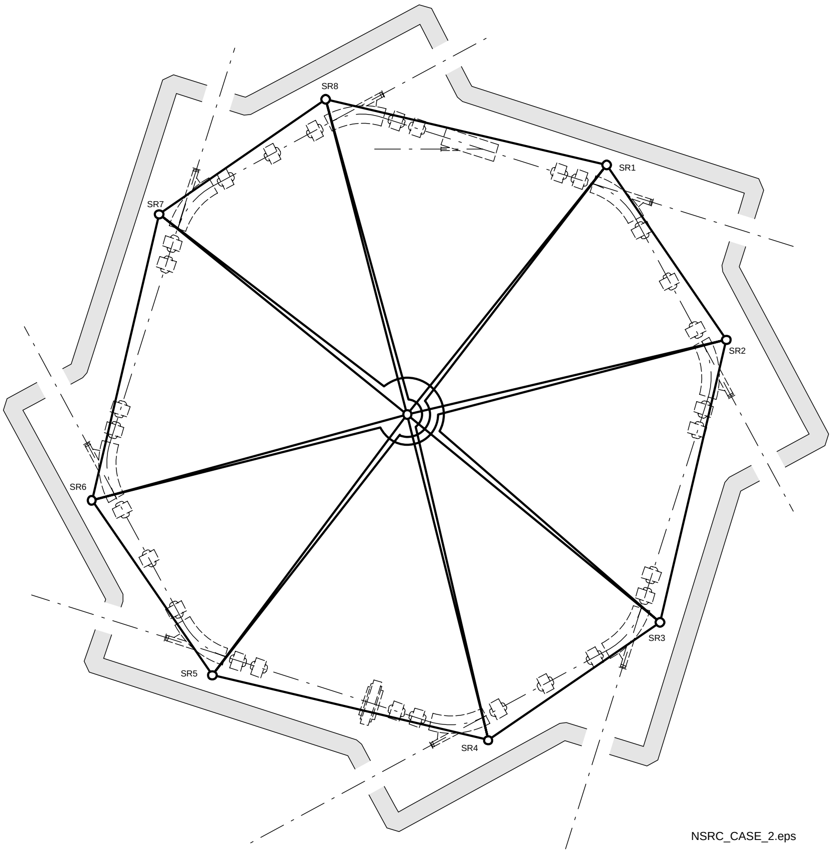
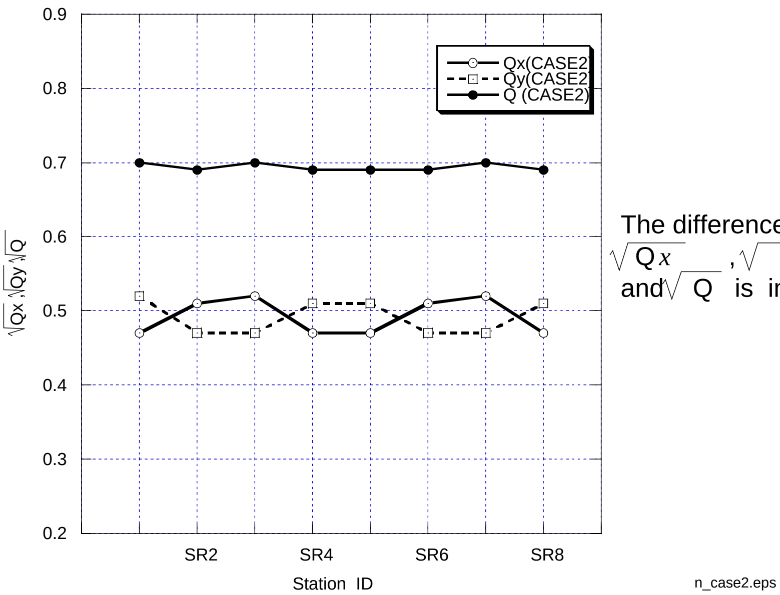


Fig.3 The results of the errors ellipses



(a) Suevey network of CASE 2



The differences of $\sqrt{Qx}$, $\sqrt{Qy}$ become smaller and $\sqrt{Q}$ is imprved

(b) $\sqrt{Q}$ of monuments on the CASE 2

Fig.4 The survey network and $\sqrt{Q}$ on the CASE 2

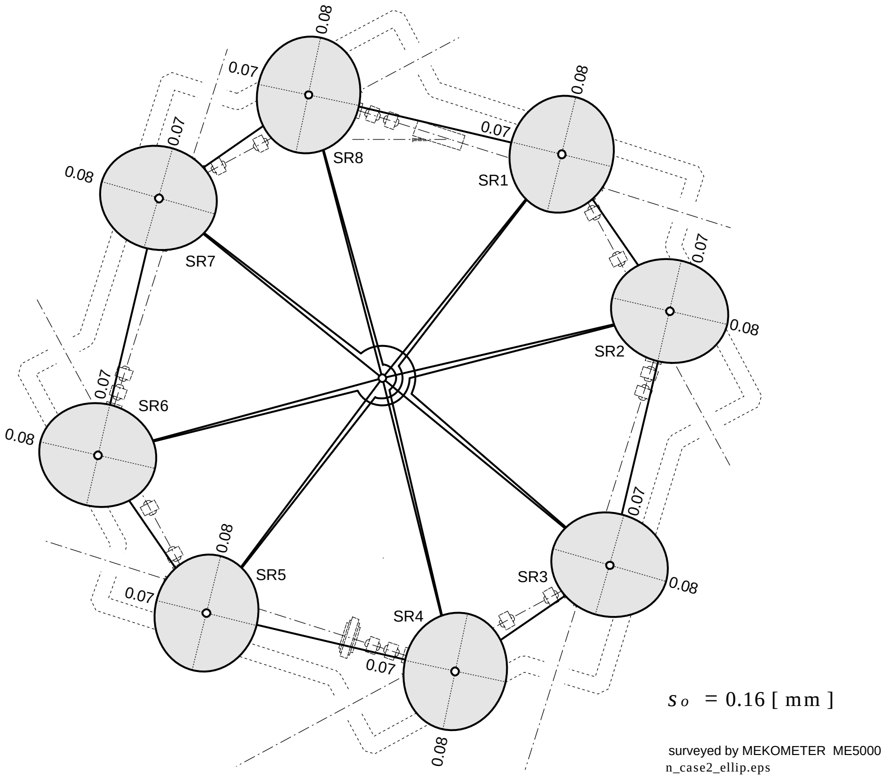
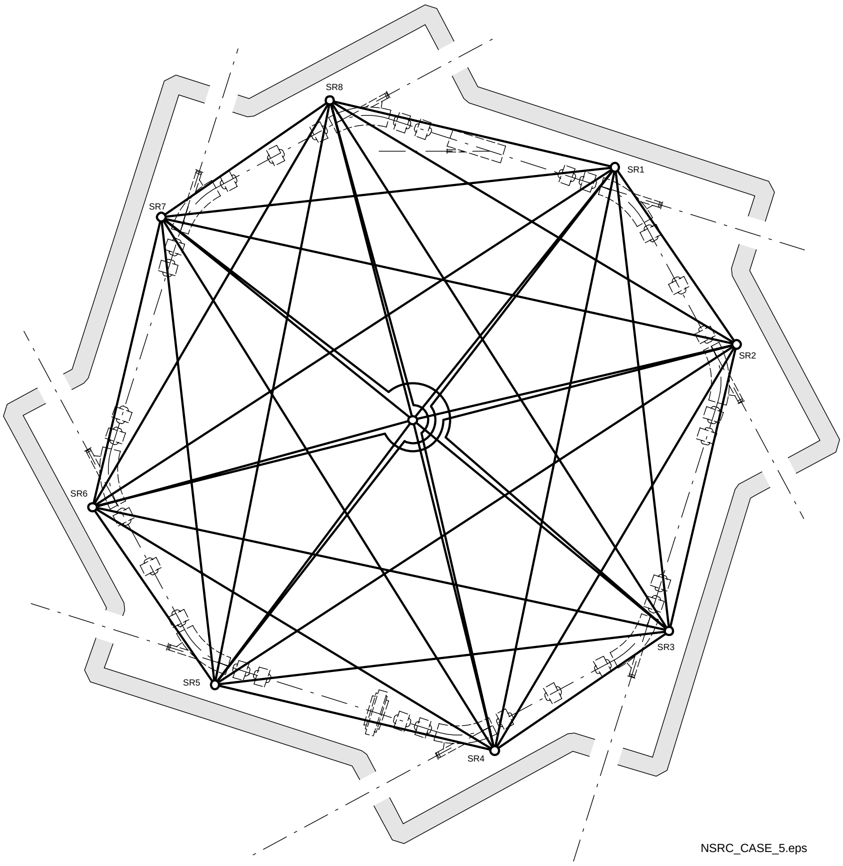
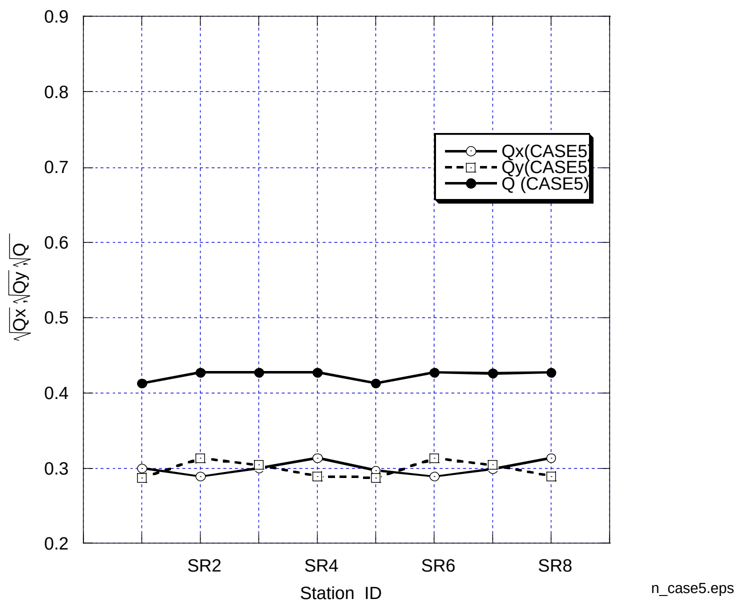


Fig.5 The survey network and $\sqrt{Q}$



(a) Survey network of CASE 3



(b) $\sqrt{Q}$ of monuments on the CASE 3

Fig.6 The survey network and $\sqrt{Q}$ on the CASE 3

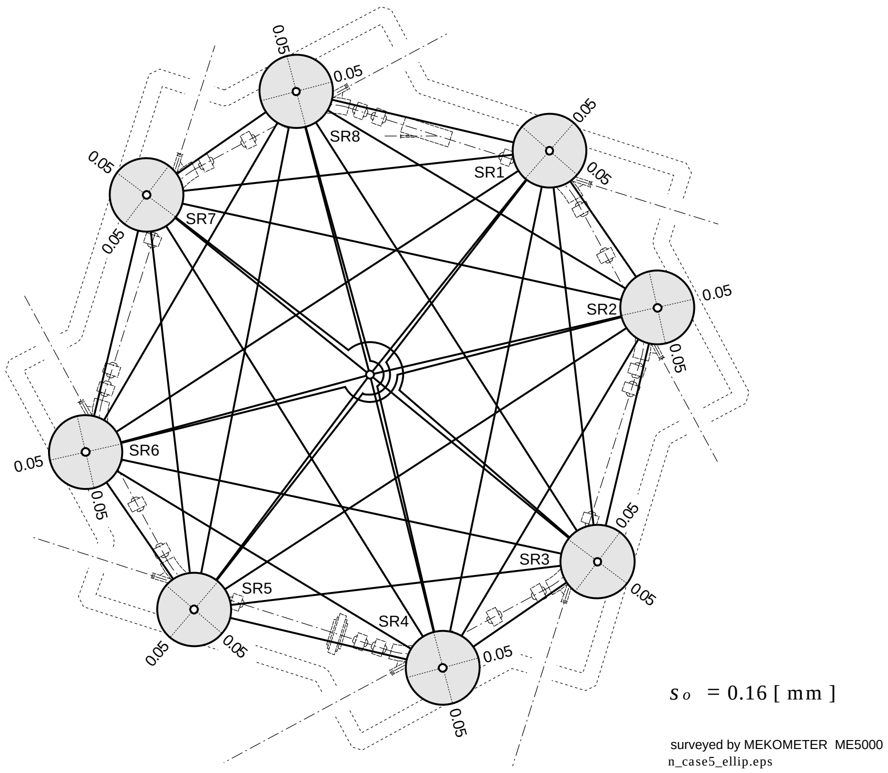
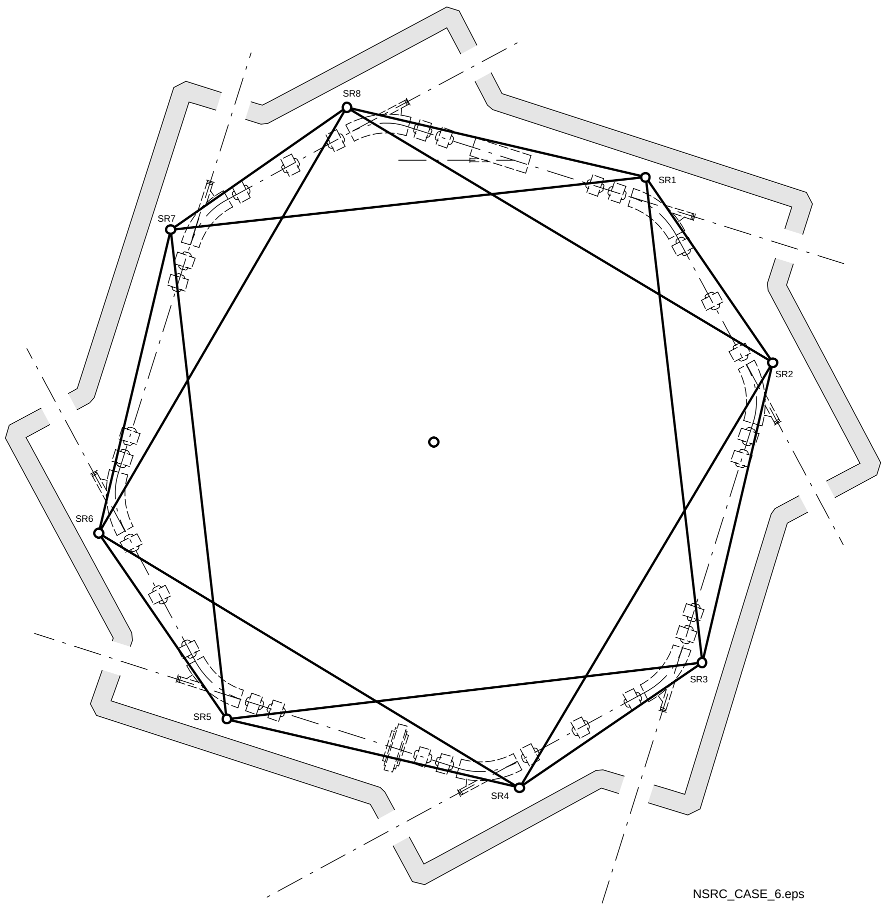
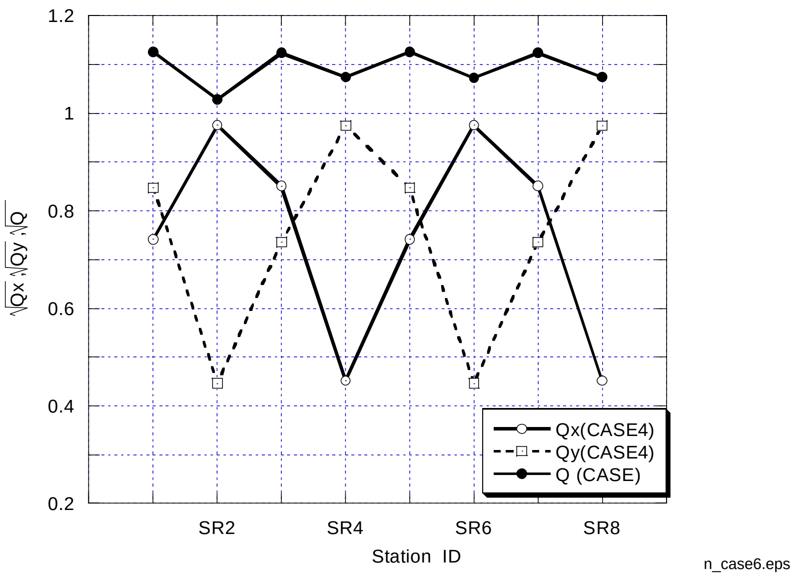


Fig.7 The results of the errors ellipses



(a) Survey network of CASE 4



(b) $\sqrt{Q}$ of monuments on the CASE 4

Fig.8 The survey network and $\sqrt{Q}$ on the CASE 4

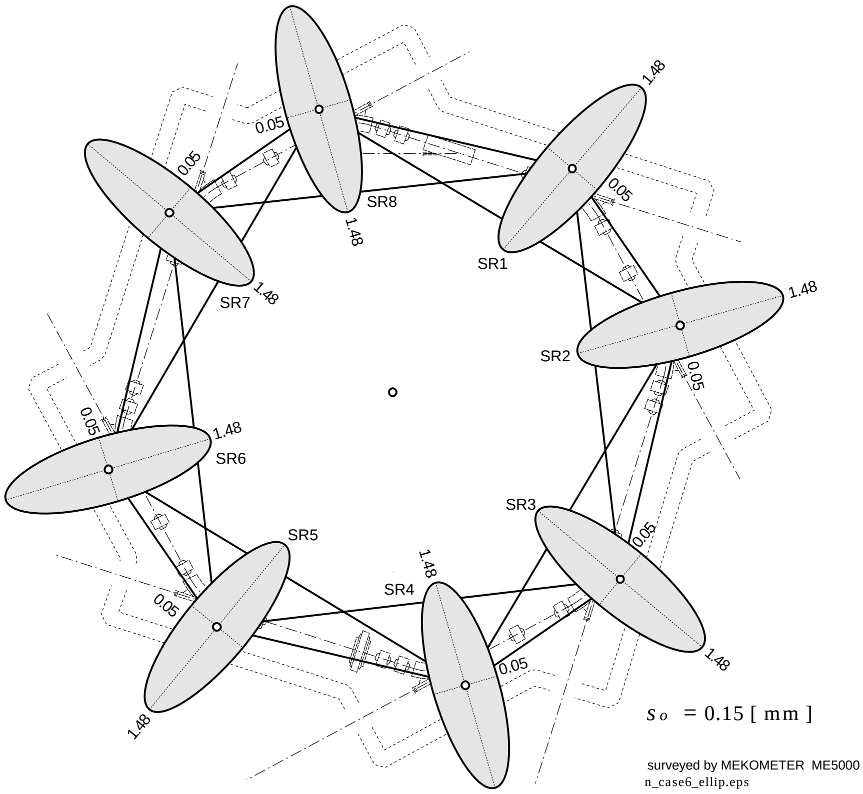


Fig.9 The results of the errors ellipses

5. CONCLUSION

The sizes, shapes and orientations of error ellipses are dependent on (1) the control used to constrain the adjustment, (2) the observational precisions, and (3) the geometry of the survey network. The last two of these elements are variables that can be readily altered in the design of a survey in order to produce optimal results. Especially, in the equation (14), the method by $\sqrt{Q}$ which describe the characteristic of the survey network for the geometry is very effective to the preanalysis, the alignment plan and the survey network design.

6. REFERENCES

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[2] K. Mishima, *Study on the Precise Alignment of Accelerator (in Japanese)*, thesis for dissertation, 2002