CONSEQUENCES OF PERTURBATIONS OF THE GRAVITY FIELD ON HLS MEASUREMENTS

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1. INTRODUCTION

For this study, the specifications and characteristics of the CLIC are used.

The Compact Linear Collider (CLIC) Team at CERN is studying the technical feasibility of building a 3 TeV centre-of-mass $e^{\pm}$ linear collider as a possible new experimental particle physics facility for the post-LHC era. The CLIC consists of two linear accelerators (linacs), one to accelerate electrons (e-) and the other to accelerate positrons (e+). Each linac is 14 km long. The final focusing of the beams onto the interaction point is carried out over 7 km, which brings the overall length up to 35 km [1].

The initial pre-alignment of components guarantees that when the first beams are injected into the linacs, they will not be too far off the design trajectory, and will produce signals in the Beam Position Monitors (BPMs) that can be used in a second phase to drive the more accurate beam-based alignment system [2].

The pre-alignment tolerances on transverse positions for the girders, quadrupoles and BPMs is presently 10 μm over 200 m.

It is desired to use the hydrostatic alignments with measurements from Hydrostatic Levelling Systems (HLSs). These HLSs define the vertical reference frame. The different paths followed by a water surface at rest and by a straight line are shown in Fig. 1.

The difference in height at the end of the straight line is far too great to be able to consider using only a single hydrostatic line over the whole length of a linac. In order to restrict the vertical offsets separating the HLSs from the linac to reasonable values, several hydrostatic lines will be required.

A set-up like that shown in Fig. 2 allows the relative positions of successive lines to be determined and thus ensures the continuity of the geometric reference system.

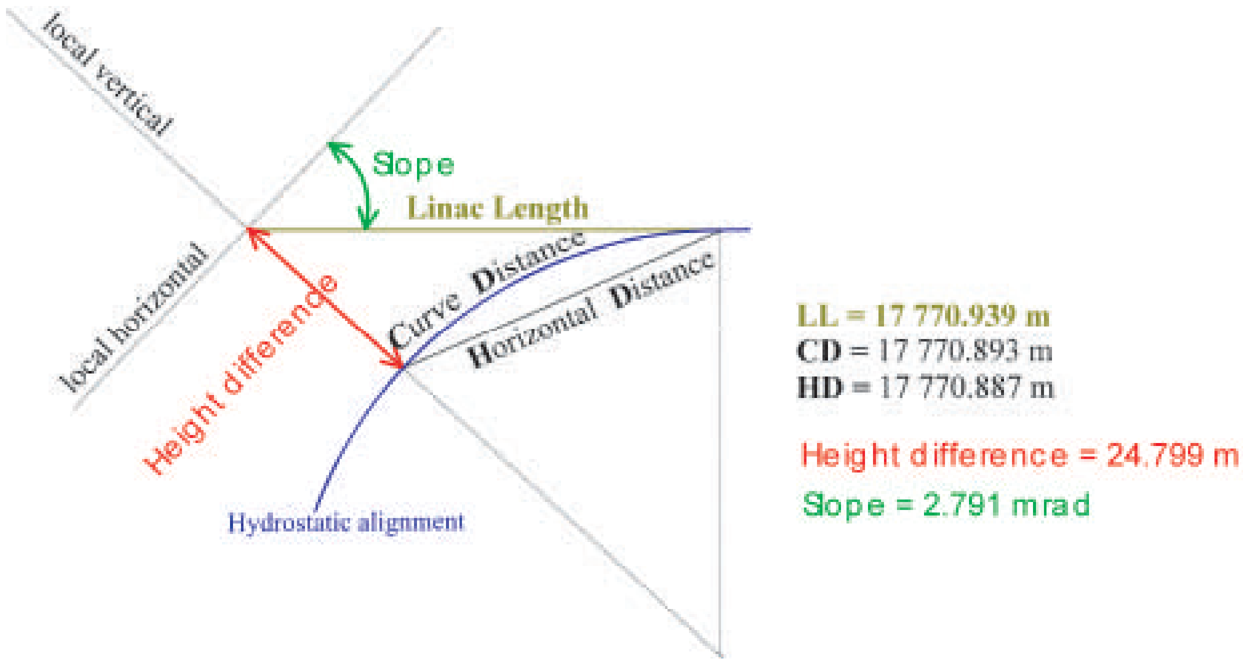

Fig. 1 Differences between a straight line and a hydrostatic alignment

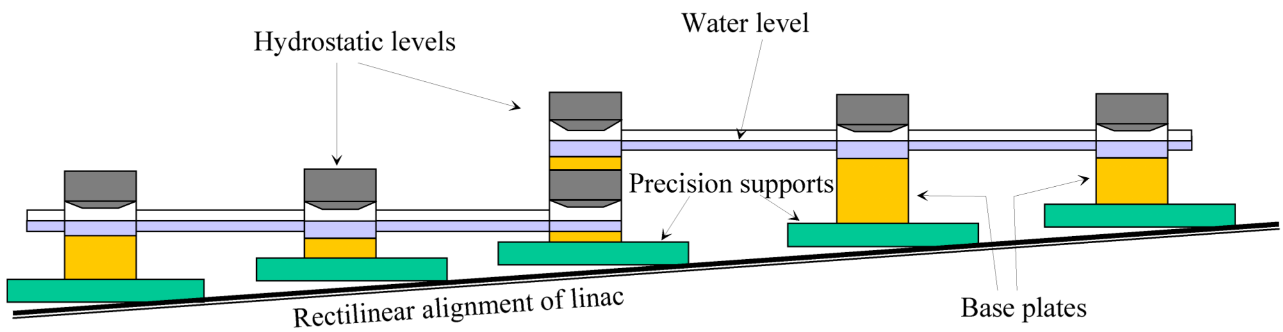

Fig. 2 Start and end points of hydrostatic alignments

Advantages of hydrostatic alignments

In the present case the use of hydrostatic alignments in the geometric reference frame has two advantages:

- they can provide height measurements to micron precision;
- the long and continuous reference surface provide by the HLS allows height differences to be determined very accurately over much greater ranges than optical levelling.

Disadvantages of hydrostatic alignments

The major disadvantage of using hydrostatic alignments for vertical referencing is that water levels follow equipotential surfaces of the earth's gravitational field. It is difficult to determine the geometry of such surfaces in relation to a reference frame so as to allow a straight line to be established.

2. STUDY OF THE PERTURBATIONS OF GRAVITY AND THEIR CONSEQUENCES

In order to derive any benefit from the high accuracy achieved by our sensor measurements relative to the geometric reference systems, it is essential that these should themselves be defined with micrometre accuracy in a reference frame that allows us to establish a straight line such as that of the linac.

As these reference frames are for the most part sensitive to gravity, it is not possible in this situation to consider the earth's gravity field as uniform and spherical, nor even everywhere perpendicular to an ellipsoidal model of the earth.

2.1 Perturbing effects

Two phenomena disturb the gravity field:

- the distribution of mass in the neighbourhood;
- the attraction of the moon and the sun.

2.1.1 The distribution of mass in the neighbourhood

Topographic relief and differences in density of the body of the earth can be considered as anomalies that result in a gravity field that is different from that of a homogenous ellipsoid. Thus, for example, the mountains in the neighbourhood of CERN exercise on any mass m, an attraction dg that must be combined with the attraction γ, normal to the ellipsoid, to obtain the true attraction g (Fig. 3).

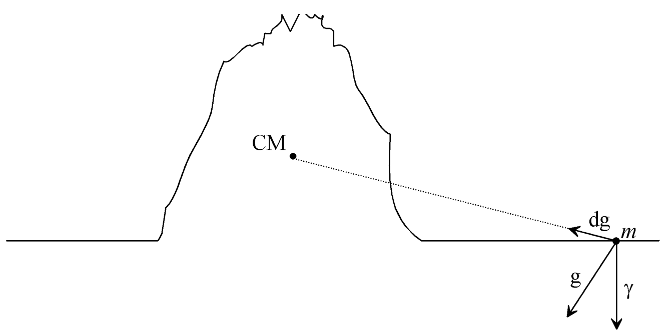

Fig. 3 Effect of a nearby mass anomaly

A study of the gravity field in the neighbourhood of CERN is given in Bell, 1985 [3]. The results of a simulation using a model of the masses covering an area of 70×50 km indicate a maximum deviation of the vertical of 15″ (sexagesimal seconds) relative to the ellipsoid of the CERN system, at the level of the topographic surface, oriented perpendicular to the line of the Jura mountains (the upward vertical is deflected by 7″ to the south and 13″ to the east). We will use this maximum value in the following section in order to analyse the effects of masses in the neighbourhood.

Undulations of an equipotential surface with respect to a reference ellipsoid

We have calculated a 40 km geoid profile of a possible CLIC installation site. This profile (fig. 4) is based on the geoid model CHGEO98 established by the Federal Topography Office in Bern.

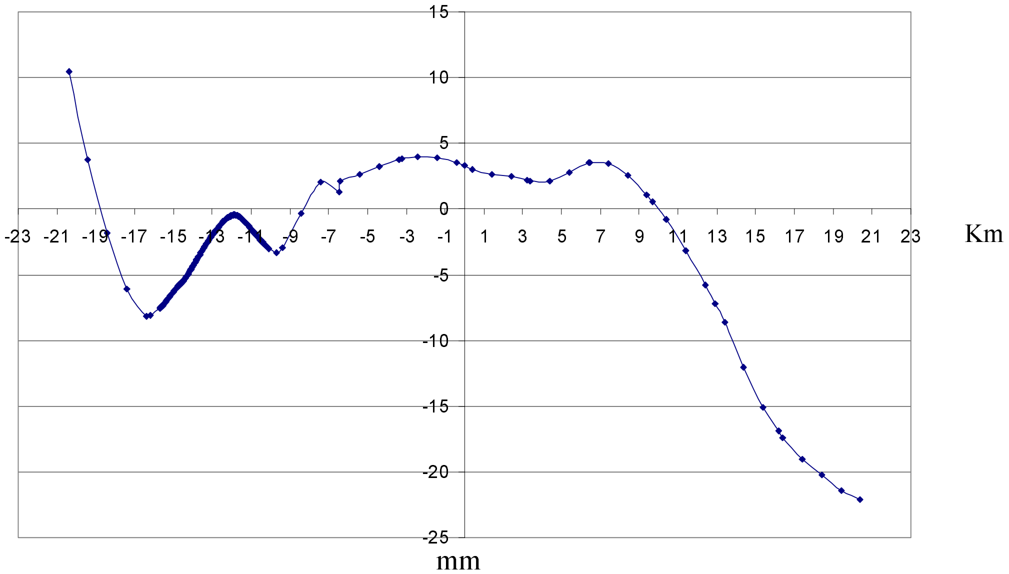

Fig. 4 Geoid Profile along the CLIC

The geoid is a particular equipotential surface, and we can therefore reasonably suppose that the profile in figure 4 may be considered to be that of the water in the HLS tubes.

It is important to realise that one can interpret this profile as that of an accelerator in a Cartesian reference frame if we started from the principle that hydrostatic lines follow a trajectory parallel to an ellipsoid surface, and that this hypothesis forms the basis for the determination of the corrections to be made in order to return to a straight line.

As one can see, the profile includes a section where the density of points is higher, where there are points every 20 or 40 m, and two other sections where there is only a point every kilometre.

We have used this denser sprinkling of points to evaluate the geoidal undulations, which give a good indication of the straightness that an accelerator would have if one supposed that the hydrostatic lines had a perfectly elliptical curve.

For each 200 m segment included in the denser section (Fig.5), the maximum offset of the geoid profile has been calculated with respect to a straight line passing through the point at each end of the segment. These defects are relatively large, even though they are less than the ±10 μ*m* threshold over 200 m. In fact, these would be the defects in the CLIC if it could be perfectly aligned with respect to an ellipsoidal surface. Don't forget however that the measurements and the mechanical links of the metrological network will also introduce their own errors.

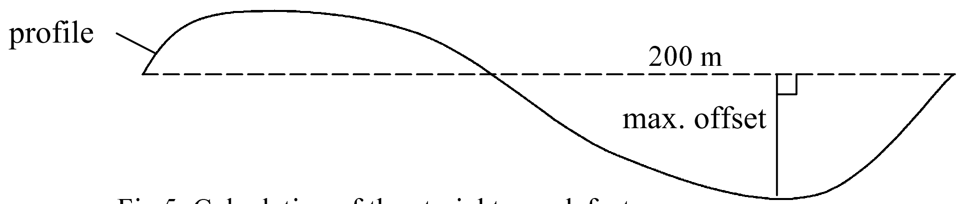

Fig.5 Calculation of the straightness defects

The results of these calculations are presented in figure 6. The rms of the straightness defects over 200 m is ± 8.4 μm.

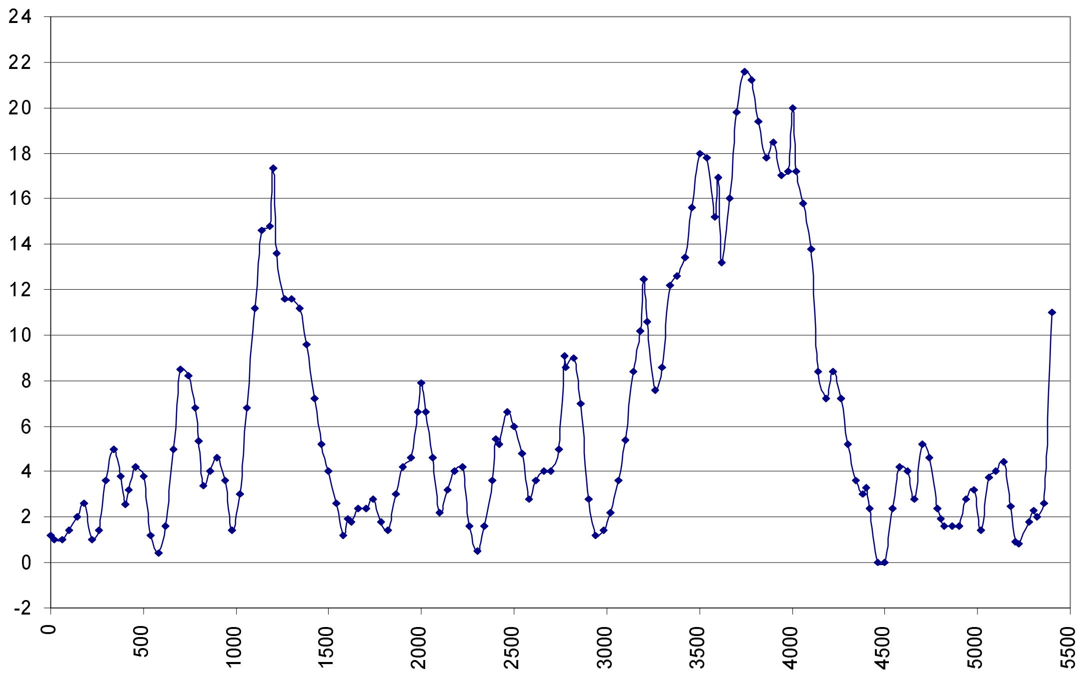

Fig. 6 Defects in the straightness of the profile in micrometres

The approximations made in neglecting that the hydrostatic lines do not have an elliptic curvature are therefore too large, even more so since the calculations that have been made are based upon a geoid model likely to show differences with respect to the real situation. *Knowledge of the equipotential surfaces of the gravity field is consequently indispensable to the use of HLSs for this alignment*. The required precision (several microns in the height differences between two points a hundred metres or so apart) is very unusual in gravimetry, and the CERN survey group has undertaken steps to understand the best precision achievable with current techniques for this type of determination.

2.1.2 Attraction of the moon and the sun

Acting as perturbing masses, the moon and the sun modify the terrestrial gravitational field. Their peculiarity is that their effect at a given point varies continuously as their position in relation to the earth is varying.

The acceleration due to gravity $\vec{g}$ thus changes continuously both in direction and in intensity. The intensity varies within a range of $\pm 2.4 \cdot 10^{-6} \, m \cdot s^{-2}$, while the direction can vary by $\pm 0''05$ (Melchior, 1973, chap. 1.5 [4]).

Elsewhere, marine tides are another more conspicuous consequence of lunar and solar attraction. It is in part because of its response to the continuous variations in the gravitational potential that the free surface of the oceans is in constant motion. If the terrestrial globe is considered as an elastic fluid, this also deforms continuously under the influence of the moon and the sun. This is the "earth tide" phenomenon, studied by Melchior, 1966 5 and Jober and Coulomb, 1973 [6].

Repercussions of these perturbations for the alignment of a linac

The perturbing effects described above are liable to have consequences for the geometry of the ground surface and hence also for the geometry of the accelerator itself, and for the use of the hydrostatic levels.

Deformation of the ground

The ground surface deforms continuously under the influence of the moon and the sun. Since earth tides are a periodic phenomenon, the accelerator, rigidly fixed within the ground, could be subjected to deflections that are incompatible with its proper operation. These deformations would then have to be corrected by the alignment system.

A preliminary approximate computation allows us to analyse the possible deformations due to earth tides over the range of 35 km. According to Melchior, 1973, the maximum amplitude of this wave is about ± 40 cm and it can be broken down into elementary sinusoidal components. The elementary components with the largest amplitudes have periods of about 12 hours. Let us consider the most unfavourable situation, with a sinusoidal wave of amplitude 40 cm and a period of 12 h, propagated along the alignment of CLIC. This situation is illustrated in Fig.7, which shows the plane section through the centre of the earth that contains the accelerator.

The 35 km line has a maximum curvature when its mid point is at the peak of this oscillation, as shown in Fig. 8. The deflexion due to the earth tide can thus be estimated by simple differences of ordinates on this sine wave.

From an approximate computation, the value of the deflection over 35 km is 6.0 µm and thus remains far smaller than the accuracy of ±10 µm over 200m which has to be maintained. The deformations of CLIC due to ground movements engendered by the attraction of the moon and the sun are therefore negligible.

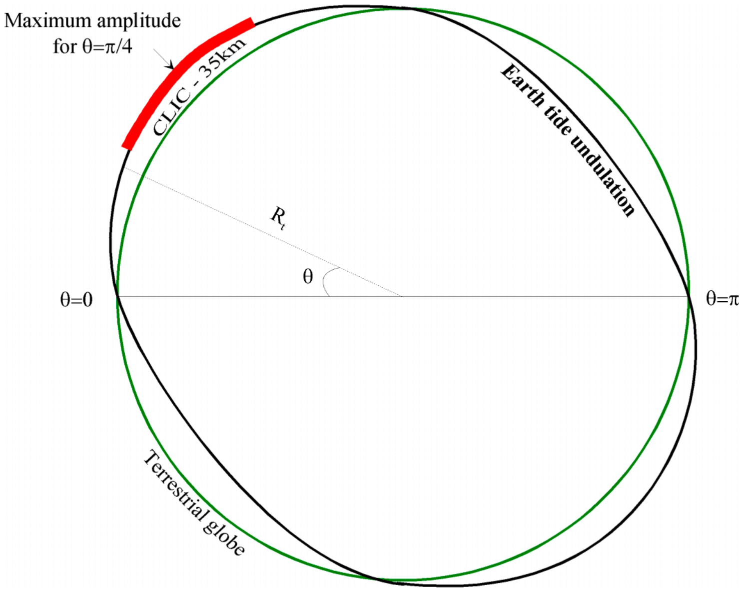

Fig. 7 Simplified tidal waveform propagated along the alignment of the CLIC

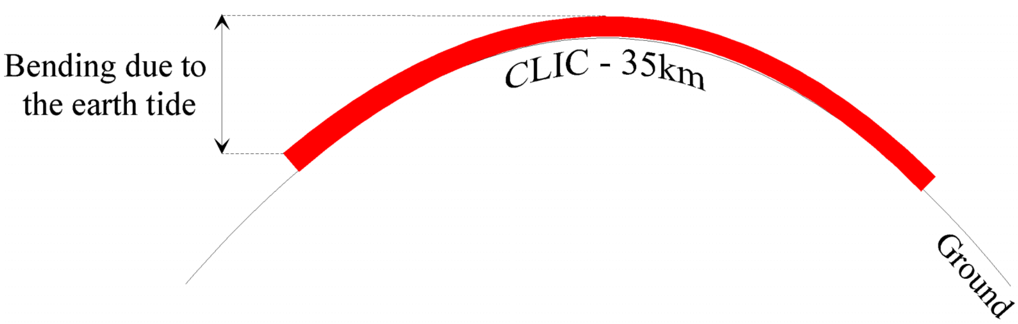

Fig. 8 Bending of the accelerator due to the periodic undulations of the earth tide

3. USE OF THE HYDROSTATIC LEVELLING SYSTEMS

The water at rest in the network of pipes connecting the HLS vessels is intended to provide a reference surface for the vertical alignment of CLIC. This obviously only makes sense if the geometry of the reference surface itself, which is an equipotential of the gravitational field, is known to the required degree of accuracy. The deformation of such equipotential surfaces by nearby masses is constant, whilst the deformations induced by the attraction of the moon and the sun are variable with time. The repercussions of these two phenomena for the use of the HLS will now be examined.

3.1 Effect of nearby masses

In the presence of a topographic anomaly the equipotential surfaces of the gravity field are deformed, as shown in Fig.9.

Among results given by Bell 1985 are the differences between the equipotential surfaces and the ellipsoid model of the earth used for geodetic computations at CERN. Although these differences are almost zero at the Meyrin site, they can exceed 20 cm nearer the Jura.

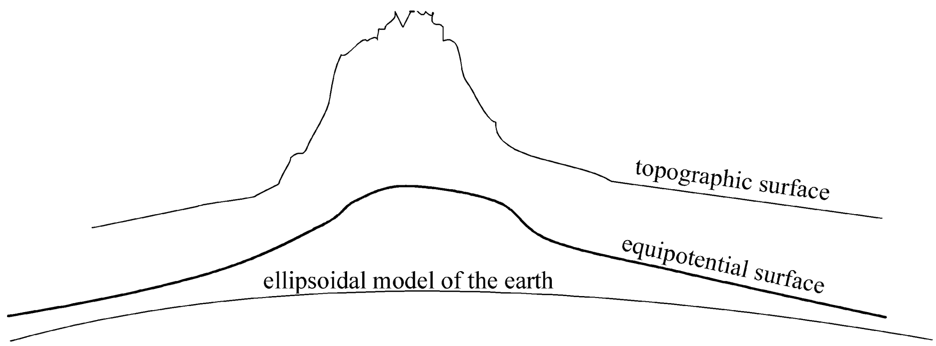

Fig. 9 Equipotential surface deformed by a topographic anomaly

The heights of a point above the ellipsoid and the geoid can thus be significantly different. However the primary interest is the height differences between pairs of points. If the height differences in respect to each of the two surfaces were sufficiently similar for each point, the undulations of the equipotential surface could be ignored and the raw height differences obtained with the HLS could be considered to refer to the ellipsoid.

To answer this question the following procedure has been adopted:

- Selection of two sets of points (100 m grid) defined in geocentric coordinates and contained in the plane of the LEP accelerator; the first set relates to an area where there is only slight ground relief, while the second set is at the foot of the Jura;

- With coordinate transformation programs and making use of the correct geoid and ellipsoid for CERN, computation of the heights of the points in relation to each of the reference surfaces (Fig. 10);
- Calculation and comparison of the two types of height difference between pairs of points 100 m apart.

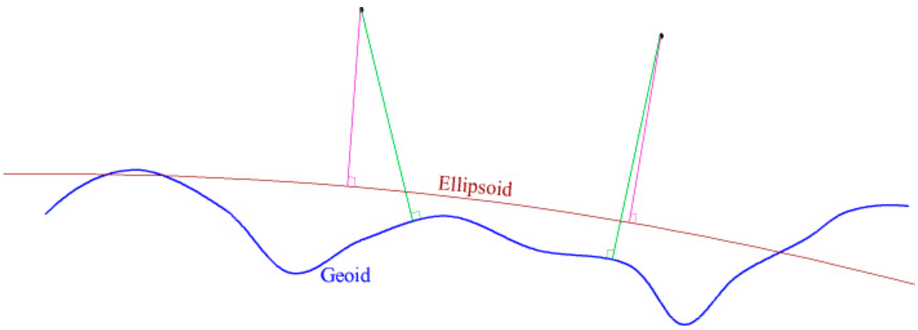

Fig. 10 Heights above the ellipsoid and the geoid

The results of this computation are given in Table 1.

These results show us that it will not be possible to disregard the undulations of the hydrostatic surfaces with respect to the ellipsoid. *Knowledge of the geometry of the equipotential surfaces due to gravity is thus essential for the use of the HLSs for the alignment.* The required accuracy (a few micrometres in height differences between points about a hundred meters apart) is very unusual in gravimetry. Steps have been taken to find out the best accuracy available from current methods for this type of determination.

Table 1: Comparison of height differences referred to the ellipsoid and the geoid

	Flat ground on the surface	***Hilly ground on the surface***
Number of pairs of points	110	110
Mean difference (µm)	153	366
Maximum difference (µm)	246	458
Minimum difference (µm)	60	273

3.2 Effects of the attraction of the moon and the sun

Oceanic and earth tides both affect the HLSs because the water in the pipes and the ground to which the whole system is fixed are being continuously deformed under the influence of the moon and the sun and are thus modifying the values recorded by the sensors.

We have seen that earth tides do not affect the straightness of the alignment of the accelerator, but merely vary the inclination of the whole system. Tidal effects on the HLS must thus be corrected and not interpreted as alignment errors.

The theoretical values of the tides that result from the attraction of the moon and the sun will now be examined and compared with the readings that have been taken on a hydrostatic line at CERN.

Theory of tides

Tides result from the fact that water surfaces at rest, together with the earth considered as an elastic fluid, deform in response to the continuously changing gravitational field. This change results primarily from the continuously varying positions of the moon and the sun relative to the earth.

According to Melchior, 1966, p.15, the disturbing attraction of the moon or the sun is derived from the potential W_2 as follows:

$$W_2 = \frac{GM_e\left(M_c/M_e\right)}{2} \cdot \frac{a^2}{r^3} \cdot \left(3\cos^2 z - 1\right) \tag{1}$$

G : Newton'gravitational constant,
M_e, M_c : masses of the earth and of a given celestial body,
a : distance from a given point to the centre of the earth,
r : distance from the celestial body to the earth,
z : zenith angle of the celestial body at the point.

G, M_e, M_c and Mc are known, a can be calculated from the coordinates of the point in question using geodetic formulae, and z and r may be calculated using astronomical ephemerides.

From (1) it can be deduced that the resultant deformation ξ of the gravitational equipotential surfaces, given by Melchior, 1966, p.16 is as follows:

$$\xi_o = W_2/g \tag{2}$$

Knowing that :

$$g = \frac{GM_e}{a^2} \tag{3}$$

It can be shown that :

$$\xi = \frac{m_l \cdot a^4}{2r_l^3}\left(3\cos^2 z_l - 1\right) + \frac{m_s \cdot a^4}{2r_s^3}\left(3\cos^2 z_s - 1\right) \tag{4}$$

m_l, m_s: ratio of the masses of the moon and the sun to that of the earth,

z_l, z_s: zenith angles of the moon and the sun at the point,

r_l, r_s: distances from the moon and the sun to the earth.

The computation leads to values of ξ which can reach several tens of centimetres in absolute terms. The variations in the height of a liquid at rest (which is known to conform to an equipotential surface) do not reach these values because the liquid changes its potential rather than following the same equipotential with its movements. On the other hand one can observe variations in the differences of level of a liquid that illustrate the changing deformations of equipotential surfaces and of the ground.

The difference between the readings of two HLS vessels thus combines the tidal effects on both the water in the pipes and on the ground that supports the whole system. These tilt effects must be corrected. In fact, if this is not done, the height differences obtained from the HLS could be interpreted as departures of the accelerator components from a straight line, whereas it has been shown that the tidal flexing of the accelerator is negligible.

Love's numbers

Love's numbers [Melchior, 1966, partie 2, chap.1] are used to formalise the expression of different aspects of earth tides. These will be useful for the interpretation of HLS data.

The application of Love's numbers is very straightforward and each type of elastic deformation due to tidal effects can be represented by a combination of these numbers. They are linked by rather complex differential equations to the distribution of densities and of modules of rigidity within the earth, but can be considered as functions solely of the distance from the point in question to the centre of the earth.

On the earth's surface and for the phenomena of interest the two Love numbers which will be used are known as *h* and *k*. *h* represents the ratio of the height of the earth tide to the height of the corresponding tide of an ocean at rest.

Moreover the effects of the density variation that accompanies the deformation and the displacement of masses due to the attraction of the sun and the moon can be modelled by means of an additional potential. *k* represents the ratio of this additional potential caused by the deformation itself to the disturbing potential W_2.

Thus the total disturbing potential is $(1+k)W_2$, and the tide observed on an ocean at rest would have a height of $(1+k)\cdot W_2/g$ However, if this tide is observed by means of a reference mark attached to the earth's surface, the mark is itself displaced because of the earth tide by a height of $h\cdot W_2/g$ in relation to the surface of the earth (considered as spherical). The observed deformation ξ_0 will thus be:

$$\xi = (1+k-h)\cdot\frac{W_2}{g} \qquad (5)$$

The values usually accepted for h and k are as follows [Jobert and Coulomb, 1973]:

$(1+k)=1.3$ and $h=0.6$ hence $\boxed{(1+k-h)=0.7}$

This implies that the tide observed on an ocean at rest by means of a reference mark attached to the earth's surface would be 70% of that which would be observed if the earth were infinitely rigid (*h*=*k*=0).

Knowing these values it is then possible to calculate the corrections to be applied to readings taken on the HLS.

Correction of HLS readings for tidal effects

A hydrostatic line about 70 m long has been installed in a part of CERN which is stable and free of vibration. HLS vessels were placed at each end in order to read the distance separating their reference surfaces from the plane of a free water surface (Fig.11)). Recordings of the sensor values were taken at the desired frequency. Knowing the geodetic coordinates of the two HLS, the values of the tides have been calculated in order to correct the differences between the readings $l_{HLS2}-l_{HLS1}$.

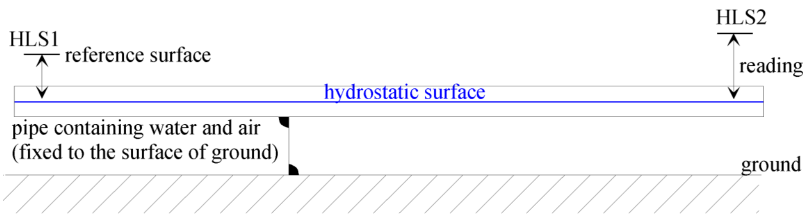

Fig. 11: Device used for HLS trials

The HLS are used to determine height differences by means of differences between the readings taken on each. With neither an earth nor a water tide, this difference would give us the height difference between the points directly, relative to a gravitational equipotential whose geometry would be invariant with time. But in reality the difference between the readings also includes the differences between the amplitudes of the water and earth tides as shown in Fig. 12.

In Fig. 12(a), the water tide alone is considered, and it is assumed that the attraction of the moon and the sun is greater at HLS2 than at HLS1. At 2 the water will thus be attracted more strongly towards the reference surface of the HLS vessel than at 1, and, if one ignored the tide, 2 would appear to be too low compared with 1. The error in the height difference $l_{HLS2} - l_{HLS1}$ would be $(1+k)(W_{22}/g - W_{21}/g)$ (W_{22} and W_{21} denote the tidal potentials at point 2 and 1 respectively, see (1)).

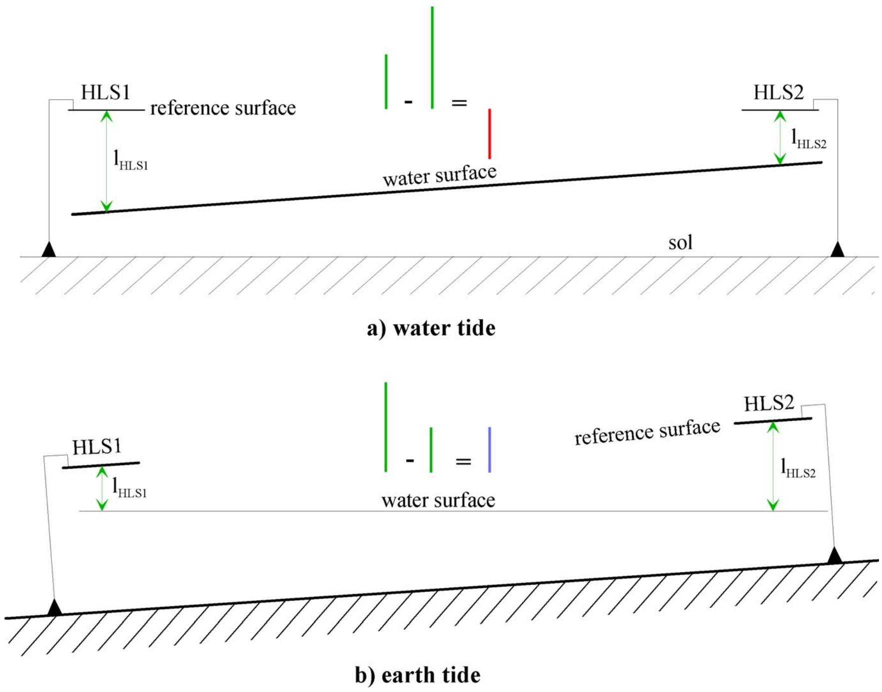

Fig. 12 Effect of tides on HLS readings HLS

In Fig. 12(b), the earth tide alone is considered, and it is again assumed that the attraction of the moon and the sun is greater at point 2 than at 1. As the HLS vessels are rigidly attached to the ground, which is raised more at 2 than at 1, the reference surface will move further away from the water surface at 2 than at 1, and if one ignored the tide, 2 would appear to be too high compared with 1. The error in the height difference $l_{HLS2} - l_{HLS1}$ in this case would be $-h(W_{22}/g - W_{21}/g)$) (opposite sign from case (a)).

It can be seen that the earth and water tides, although always of the same sign, have opposite effects on HLS readings. It is this that explains why one subtracts h from $1+k$ in the determination of the amplitude of tides observed by means of a reference mark attached to the ground as in the present case. If we denote the HLS readings at points 1 and 2 by l_{HLS1} and l_{HLS2}, the corrected height difference Δ^1_2 is obtained from:

$$\Delta^1_2 = (l_{HLS1} - l_{HLS2}) - (1 + k - h)(W_{22}/g - W_{21}/g) \quad (6)$$

Figure 13 illustrates the results that have been obtained. The test, which will be now described, covered a period of four days (a total of 393 readings taken at quarter hour intervals).

It can be seen that the difference HLS1 -HLS2, corrected for the tidal effects, shows both a more general diminishing movement together with residual semi-diurnal undulations with amplitudes of over 2 µm. The reduction appears to express a more general movement of the ground. The residual movements can be attributed in part to the accuracy of the data used in (4), but also relate to other phenomena which have yet to be addressed and which could involve alignment errors. These might include periodic ground movements other than tidal effects (such as thermal or diurnal effects) or local water heating.

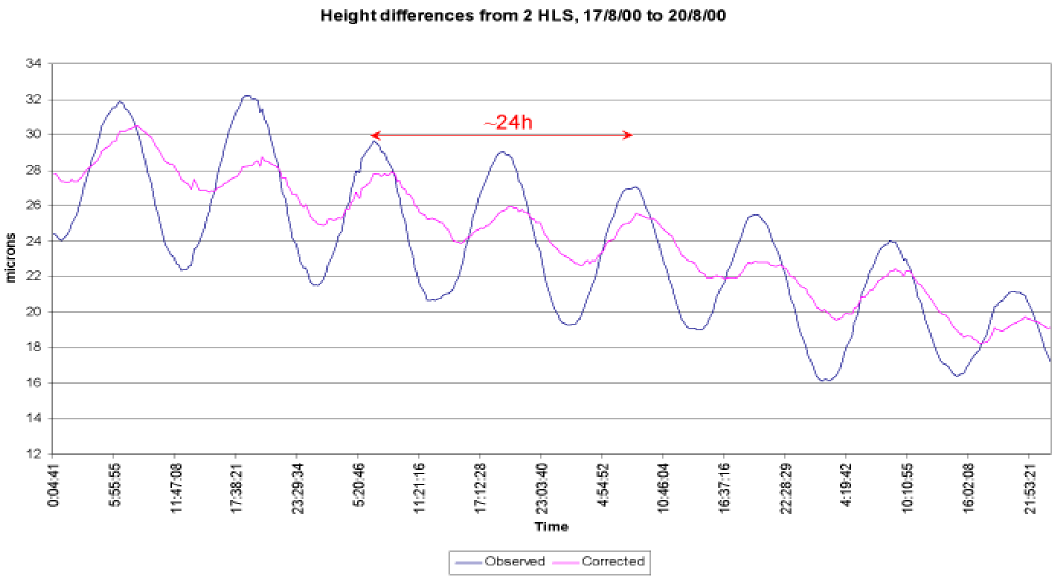

Fig. 13 Differences HLS 1 - HLS 2: observed, and corrected for earth and water tide effects

An important question is whether this corrected signal is a good representation of the variations of height in relation to a straight line that rocks under the effect of tides, or whether the correction applied needs to be improved. Formula (6) is in fact very general, and by analysing in situ readings covering a period of one to three months it is possible to define tidal parameters for a particular site [Ducarme, 2001, 7]. This is a solution that will certainly be used for CLIC. In fact the HLS will be among the first equipment to be installed, and enough time will therefore be available to collect the data needed to establish a local tidal model.

The next section examines whether the parameters used to compute the corrections are known with sufficient accuracy.

Accuracy of corrections

It is necessary to evaluate the accuracy of the corrections calculated using (4) and (5). Table 2 shows the accuracies of the parameters used in the test illustrated in Fig. 9. The accuracies shown for z_l and z_s are in fact averages calculated over a period of a year. For a given celestial body, z is obtained as follows:

$$z = \sin\varphi \cdot \sin\delta + \cos\varphi \cdot \cos\delta \cdot \cos(\theta - \alpha) \qquad (7)$$

Where: φ is the latitude of the point in question $\sigma\varphi \cong \pm 5$ m (for our test),
δ is the angle between the body in question and the plane of the equator,
$\sigma_{\delta_l} = \pm 0.03"$ for the moon et $\sigma_{\delta s} = \pm 0.3"$ for the moon,
θ is the sidereal time at the point in question $\sigma_\theta = \pm 1.2 \cdot 10^{-7}\,h$
α is an angle between the body in question and the prime meridian,
$\sigma_{\alpha l} = \pm 0.03"$ for the moon and $\sigma_{\alpha s} = \pm 0.3"$ for the sun.

Love's numbers are known to sufficient accuracy to consider them to be exact (they are now known to six decimal places but one significant figure is sufficient for the present). The a-priori accuracies of the corrections applied, for a period of a year, have been calculated. Fig.14 illustrates the results obtained.

Table 2: Accuracies of parameters used to compute tidal corrections

Parameter	***Type***	***Accuracy***
m_l	Physical constant	$\pm 1 \cdot 10^{-9}$
m_s	Physical constant	$\pm 1 \cdot 10^{-2}$
z_l	Ephemerides and geodesy	$\pm 0{,}14"$
z_s	Ephemerides and geodesy	$\pm 0{,}32"$
r_l	Ephemerides	± 4 m
r_s	Ephemerides	± 15km
α	Geodesy	± 5 m

It can be seen that the a priori accuracies of the corrections that have been applied are always better than ± 1 μm. This result is entirely satisfactory at present but it can be further improved if necessary. In fact the points which were used for the trial were only fixed to ± 5m in relation to the centre of the earth, while current geodetic methods easily allow an accuracy of better than ± 1m to be achieved. As a guide, with the latter value, the accuracy of corrections to be applied to HLS readings would be of the order of ± 0.3 μm.

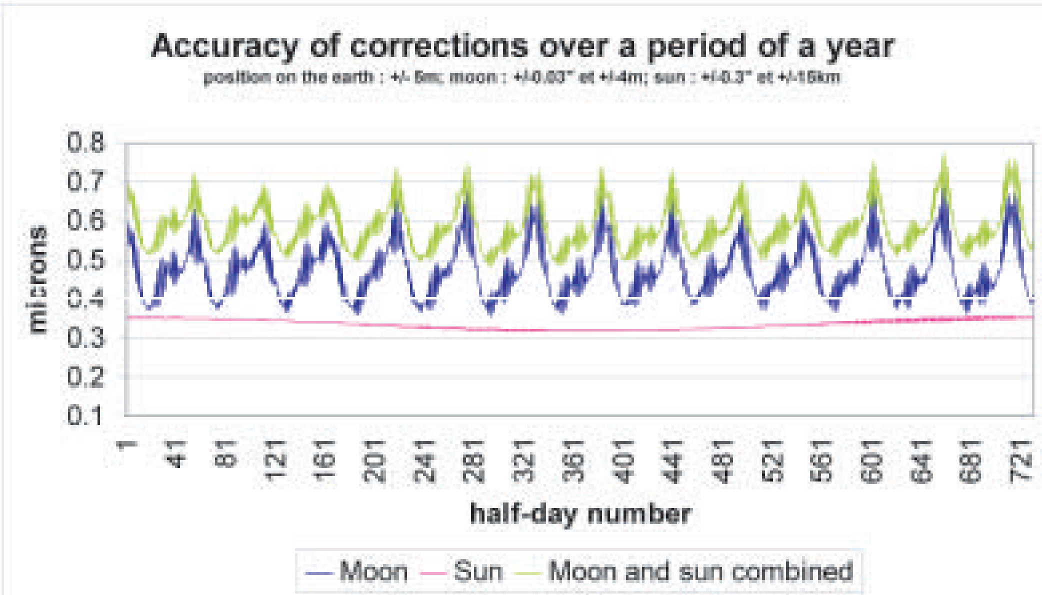

Fig. 14 Accuracy of the corrections applied to HLS readings

4. CONCLUSIONS

The hydrostatic system that has been devised meets the specifications for the vertical alignment of the CLIC machine, but it is subject to requiring an exceptionally precise knowledge of the geometry of the gravitational field. Research has been identified to determine whether such a degree of understanding can be achieved. If this proves to be possible, the solution that is proposed should provide a satisfactory response to the vertical alignment requirements of CLIC.

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1 J .P. Delahaye, I. Wilson, et al., *Performances obtained with the CERN linear Collider Test Facility,* European Particle Accelerator Conference, 1994.

2 W. Coosemans and H. Mainaud, *Pre-alignment of CLIC using the Double-Wire Method,* CERN CLIC Note 343, 1997.

3 B.A. Bell, *A simulation of gravity field around LEP,* 1985.

4 P. Melchior, *The Tides of the Planet Earth,* Pergamon Press, 1973.

5 P. Melchior, *The Earth Tides,* Pergamon Press, 1966.

6 G. Jobert and J. Coulomb, *Traité de Géophysique Interne, Tome I: Sismologie et Pesanteur,* chapter 18, Editions Masson, 1973.

7 B. Ducarme, (Observatoire Royal de Belgique) *Communications personnelles*, 2001.