PRIMARY HYDROKINETICS STUDY AND EXPERIMENT ON THE HYDROSTATIC LEVELING SYSTEM

C. Zhang, K. Fukami, S. Matsui
JASRI/SPring-8, Mikazuki Kouto1-1-1, Sayo-gun, Hyogo 679-5198, Japan

1. INTRODUCTION

The SPring-8 is a third generation synchrotron radiation source. Its storage ring, with a circumference of 1.5 kilometers, is constructed on hard rock or partly man-made rock to ensure the stable operation. The extreme small beam emittance requires that the sources that cause the growth of emittance or the drift of beam obit being eliminated to the utmost. For this purpose a project team was formed to deal with various problems of beam obit variations, including very broad spectrum from low frequent drift to high frequent vibrations.

In the storage ring there are no absolute level references for the magnets, the relative levels of the magnets are measured every summer with the Wild N3 or Zeiss DiNi11. In the first year after the tunnel completion we did measurement in winter also. From accumulated observation data some features of the level variations become clear to us.

- The consistence of the magnet level was surveyed within ±0.4 mm for the first year after the tunnel completion [1]. It is still kept in ±1 mm according to recent observation, even we did not do any realignment.
- The areas that had large movements in the initial stage keep large movement after all.
- Some areas that have special underground structures, such as rf wave-guide pits, underpass, drain pipes, or the junctions of constructing zones show apparent atmospheric temperature dependence. These local level variations are about several hundred microns from summer to winter or in reverse. Daily change of ~2 μm is predicted in these areas.

On the other hand, the improvements we should make in level measurement are considered in two aspects. Firstly, the circumference of the ring is so long that the measurement, with an error about ±0.2 mm, is difficult to give an absolute figure of level variations because of small amounts of the changes. Secondly, off-line measurement do not reflects local or short-term level variation. It is desirable that we could detect the variations on real-time.

As a consequence a hydrostatic leveling system (HLS) is being examined at the SPring-8. It should be a solution to above problems. Moreover, our interest is to explore the correlation between the beam obit variation and the tunnel ground movement, the system that set on to the ground is being considered.

Because the target is to find the places and amounts of the ground variations the measurement resolution of this system to the level variation should be less than 1 μm.

The problem coming consequently is what will be the response capability for a 1.5-kilometers fluid system. Once the equilibrium of hydrostatics is broken by external disturbances, how long we have to wait before the system can be used for another measurement.

An example is that the system is usually connected with 12 mm in diameter pipes between sensors. In this case you need at least one hour before the disturbed water surface getting quiet for a one-kilometer system.

Basically, there are two different types of hydrostatic leveling systems: full-fill and half-fill. The former fills up the communicating pipe with fluid while the latter fills only half of the pipe. The half-filled system is less sensitive to environmental temperature change, therefore possesses higher accuracy. But its response is slow. The Half and the Full have quite different motions of fluid (water) which determine the response capability of the system to level variations. Thereinafter, primary hydrokinetics studies as well as the experiments on the two types of hydrostatic leveling system are carried out.

2. FULL-FILLED HYDROSTATIC LEVELING SYSTEM

2.1 Oscillation of the fluid in full-fill

Consider two vessels separated by distance L are connected with a pipe of d in diameter as shown in Fig.1. The fluid was initially at rest. After an external disturbance, for instance because of ground movement, the surface of the fluid in one sensor will be higher than that of another. The fluid begins flow along the pipe axis. According to Euler's equation of motion, one dimension flow between two arbitrary points 1 and 2 is given as [2]

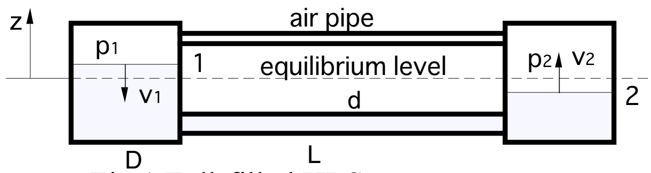

Fig.1 Full-filled HLS

$$\frac{v_1^2}{2g}+\frac{p_1}{\gamma}+z_1=\frac{v_2^2}{2g}+\frac{p_2}{\gamma}+z_2+\frac{1}{g}\int_{s1}^{s2}\frac{\partial v}{\partial t}ds+\sum h_f \tag{1}$$

where, v is average velocity of flow positive to the direction of flow; p is pressure on the surface; z is height from the equilibrium level here; $\gamma=\rho g$ (density of fluid and acceleration of gravity); $\sum h_f$ is total friction loss; s is distance along the streamline.

The sensors are usually connected with an air pipe to get equivalent pressure ($p_1= p_2$). And, for incompressible fluid we have $v_1= v_2$ as long as the two sensors are the same diameter. Eq. (1) than becomes

$$z_2-z_1+\frac{1}{g}\int_{s1}^{s2}\frac{\partial v}{\partial t}ds+\sum h_f=0 \tag{2}$$

Since the rate of flow is constant, the velocity in communicating pipe is $v=v_1A/a$. (A and a are areas of the sensor and the pipe respectively.) Integrating (2) along streamline one gets

$$\frac{dv_1}{dt}+\frac{g}{\sigma}\sum h_f+\frac{2g}{\sigma}z=0 \tag{3}$$

where, $\sigma=2h+lA/a\fallingdotseq lA/a$ (l: length of pipe; h: height of the fluid in sensor).

With Poisenille's Law we have the total friction loss for a horizontal pipe in form

$$\sum h_f=\frac{32\mu l}{\gamma d^2}v \tag{4}$$

where, μ is coefficient of viscosity of fluid. The friction gradient is therefore

$$J = \sum h_f \big/ l = \frac{32\mu}{\gamma d^2} v \tag{5}$$

After substituting Eq. (4) into (3) one gets

$$\frac{d^2 z}{dt^2} + \frac{32\mu}{\rho d^2}\frac{dz}{dt} + \frac{2g}{\sigma} z = 0 \tag{6}$$

That is, the motion of fluid in full-filled system is a damped oscillation, with damping coefficient β of

$$2\beta = \frac{32\mu}{\rho d^2} = \frac{gJ}{v} \tag{7}$$

and eigenfreqency ω_0 of $\omega_0^2 = 2g/\sigma$.

The solutions to the oscillation of Eq. (6) are

$$\begin{cases} z = \dfrac{\omega_0 H_0}{\sqrt{\omega_0^2 - \beta^2}} e^{-\beta t} \cos(\sqrt{\omega_0^2 - \beta^2}\; t - tg^{-1} \dfrac{\beta}{\sqrt{\omega_0^2 - \beta^2}}) & (\beta < \omega_0) \\ z = H_0 (1 + \beta t) e^{-\beta t} & (\beta = \omega_0) \\ z = \dfrac{\omega_0 H_0}{\sqrt{\beta^2 - \omega_0^2}} e^{-\beta t} \sinh(\sqrt{\beta^2 - \omega_0^2}\; t + \tanh^{-1} \dfrac{\sqrt{\beta^2 - \omega_0^2}}{\beta}) & (\beta > \omega_0) \end{cases} \tag{8}$$

where, H_0 is the initial height of the fluid above equilibrium level.

2.2 Optimum diameter of the pipe and the system response capability

When $\beta = \omega_0$ in Eq.(6):

$$\frac{16\mu}{\rho d^2} = \sqrt{\frac{2g}{\sigma}} \tag{9}$$

the system will behave a critical oscillation as the formula in Eq. (8). The critical (optimum) diameter of pipe is than obtained as

$$d_c = (\frac{16\mu D}{\rho})^{\frac{1}{3}} (\frac{l}{2g})^{\frac{1}{6}} \tag{10}$$

With the critical diameter the oscillation will be damped out fastest to come to rest.

Correspondingly the eigenfrequency and the oscillation period are as

$$\omega_c = \frac{d_c}{D}\sqrt{\frac{2g}{l}} \quad ; \qquad T_c = \frac{2\pi}{\omega_c} = \frac{\pi D}{d_c}\sqrt{\frac{2l}{g}} \tag{11}$$

respectively.

For critical oscillation the period is the time elapsed when oscillation amplitude damped to 1.3% of its initial value, for example from 100 μm to 1 μm. For a full-filled HLS the oscillation period is proportional to the root of pipe length.

Figure 2 shows the critical (optimum) diameter of pipe and the period of oscillation for different pipe length of a full-filled water system. For example, for 100 meters system one should use ϕ14-mm pipe of which oscillation period is about 80 seconds. For a 1km system a ϕ21-mm pipe should be used and the oscillation period is about 172 seconds.

Distance (m)	Diameter (m)	Period (sec)
1	0.0066	17.1
5	0.0086	29.3
10	0.0097	36.9
50	0.0126	63.2
100	0.0142	79.6
500	0.0186	136.2
1000	0.0209	171.7

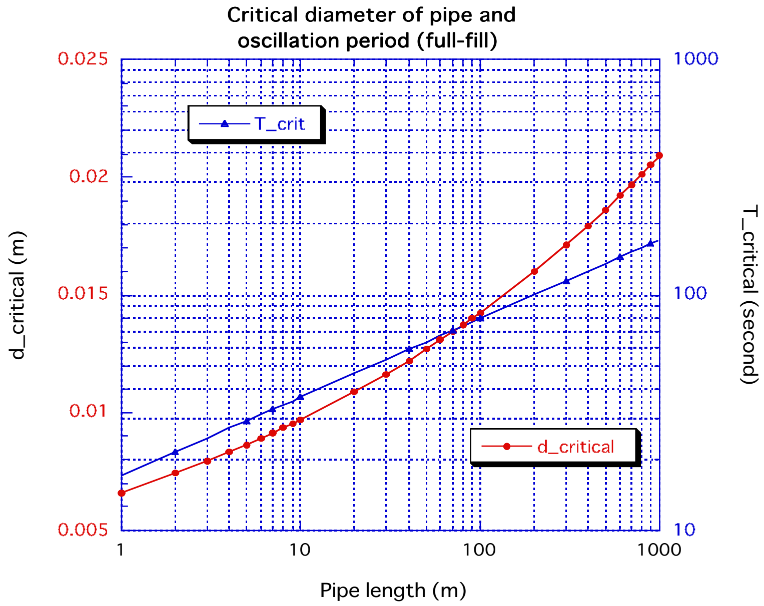

Fig.2 The critical (optimum) diameter of pipe and the period of oscillation for different length of pipe in a full-filled system.

3. HALF-FILLED HYDROSTATIC LEVELING SYSTEM

3.1 Oscillation of the fluid in half-fill

Consider two vessels separated by distance L are connected with a half-filled pipe as shown in Fig.3. In this model the sensors are connected to main communicating pipe with short pipes.

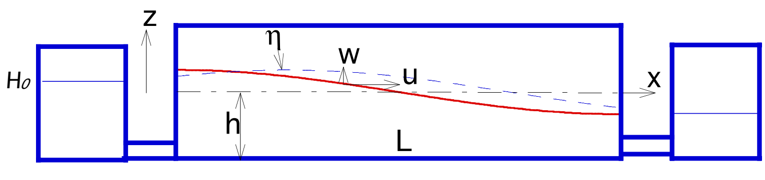

Fig.3 Half-filled HLS.

The fluid was initially at rest. External disturbance, for instance because of ground declination, usually makes the system inclined and the fluid as a whole will move inside pipe. For a two-dimension motion of irrotational ideal fluid, the velocities of flow can be written as the functions of velocity potentials:

$$u=\frac{\partial\phi}{\partial x}\ ,\qquad w=\frac{\partial\phi}{\partial z} \tag{12}$$

where, u and w are horizontal and vertical velocities of flow. Euler's continuous equation becomes [3]

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2}=0 \tag{13}$$

Consider the solution for Eq.(13) has the form:

$$\phi(x,z,t)=A(z)\sin(kx-\omega t) \tag{14}$$

where, k is wave number ($=2\pi/\lambda$).

At the bottom of pipe the velocity in vertical is zero, the same in horizontal for the two ends.

$$w=\frac{\partial\phi}{\partial z}\ \Big|_{z=-h}=0\ ,\qquad u=\frac{\partial\phi}{\partial x}\ \Big|_{x=0,l}=0 \tag{15}$$

And, for the free surface of the fluid we have

$$\frac{\partial^2\phi}{\partial t^2}+g\frac{\partial\phi}{\partial z}=0 \tag{16}$$

With the boundary conditions Eq.(15) and (16), and substituting Eq.(14) into (13) one can derive that the oscillations in half-filled pipe are standing waves with dispersed eigenfrequencies:

$$\omega_m=\sqrt{m\frac{\pi g}{l}\tanh(m\frac{\pi h}{l})}\qquad\qquad (\mathrm{m}=1,2,\ ...) \tag{17}$$

where, l and h are length of pipe and depth of fluid inside the pipe as shown in Fig.3. The fundamental frequency is

$$\omega_1=\frac{\pi}{l}\sqrt{gh} \tag{18}$$

This fundamental mode has longest oscillation period.

Further detail investigation leads to two conclusions. Firstly, if the oscillation starts from the initial condition of

$$\eta=H_0\cos(\frac{\pi}{l}x) \tag{19}$$

which is an approximation to declined ground that results one end of the pipe higher than another by $2H_0$ (red surface in Fig.3; η denotes z at the surface), the shape of free surface will be a simple harmonic oscillation of

$$\eta_1=H_0\cos(\frac{\pi}{l}x-\omega t) \tag{20}$$

at any arbitrary point along the pipe.

Secondly, the vertical velocity is much smaller than that of the horizontal

$$\frac{w}{u}\approx\frac{\pi h}{l}<<1 \tag{21}$$

Therefore, main friction loss will be in horizontal direction along pipe axis.

TOC

With above conclusions, we can treat the two-dimension motion of fluid in half-filled pipe using one-dimension model as long as we know the damping coefficient in the system.

It can be derived that for a half-filled circular pipe the friction loss is in the form of

$$\sum h_f = \frac{8\mu l}{\gamma r^2} v \tag{22}$$

where, r is radius of pipe and also represents the depth of fluid. Similar to full-filled system, the damping coefficient β can be deduced from the coefficient of the friction gradient J (refer to Eq.7). That is,

$$2\beta = \frac{gJ}{v} = g\frac{8\mu}{\gamma r^2} = \frac{8\mu}{\rho r^2} \tag{23}$$

The coefficient of damping therefore has the form:

$$\beta = \frac{4\mu}{\rho r^2} = \frac{4\mu}{\rho h^2} \tag{24}$$

With the β and ω_1 , the motion of fluid of a half-filled system can be calculated using Eq.(8) of damped oscillations.

3.2 Optimum depth of the fluid in pipe and the system response capability

When $\beta = \omega_1$, that is, $4\mu/\rho h^2 = \pi/l\sqrt{gh}$, one obtains critical (optimum) depth of fluid as

$$h_c = \left(\frac{4\mu l}{\pi\rho\sqrt{g}}\right)^{\frac{2}{5}} \tag{25}$$

With the critical depth the oscillation will be damped out fastest to come to rest. Than, the fundamental frequency becomes

$$\omega_c = \frac{\pi}{l}\sqrt{gh_c} \tag{26}$$

Correspondingly, the critical period of oscillation will be

$$T_c = \frac{2\pi}{\omega_c} = \frac{2l}{\sqrt{gh_c}} = 0.76 l^{\frac{4}{5}} \left(\frac{\rho}{\mu}\right)^{\frac{1}{5}} \tag{27}$$

Different from that of the full-filled system, the oscillation period of the half-fill is directly proportional to pipe length.

Figure 4 shows the critical depth of water and the period of oscillation for different pipe length of a half-filled water system. For example, for a 100-meter system the optimum depth is 17.5 mm of which oscillation period is 484 seconds. For a 1km system the water depth should be 44 mm (88 mm pipe in diameter) to obtain a critical oscillation in the period of about 3050 seconds. Comparing to full-filled system the Half need much longer time before the fluid comes to rest. If one do care about the response capability for a very large accelerator it is better to use full-fill, or use overlapped multi-section half-filled system.

Distance (m)	Depth of water (m)	Period (sec)
1	0.0028	12
5	0.0053	44
10	0.0070	76
50	0.0133	277
100	0.0175	484
500	0.0334	1752
1000	0.0440	3050

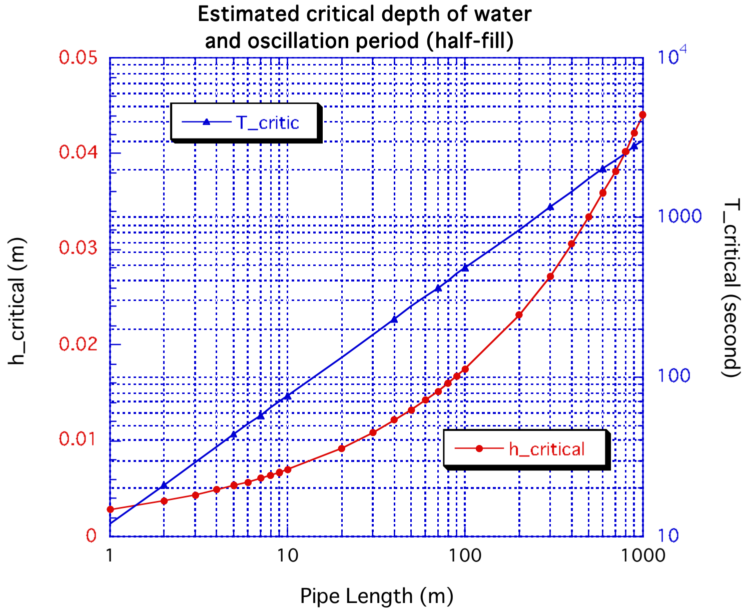

Fig.4 The critical (optimum) depth of water and the period of oscillation for different length of pipe in a half-filled system.

4. EXPERIMENTS ON THE RESPONSES OF THE TWO SYSTEMS

The response capabilities of the HLS of the two types were tested to verify above analysis. For this purpose a 5-m test bench was made to hold four sensors communicated with water of full and half fill respectively (Fig.5). The bench as a whole can be tilted with a z-stage on the

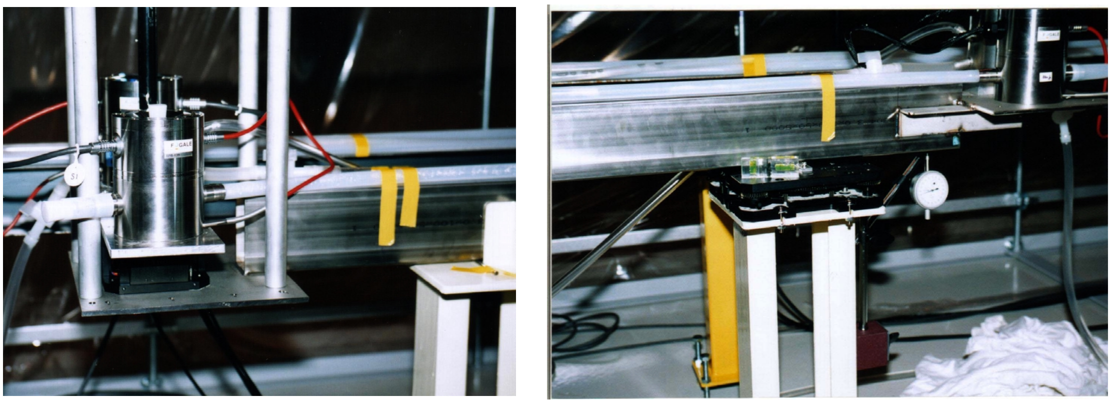

Fig.5 HLS of two types are set up on a common bench. The one in this side is half; outer side is the full filled system.

right support pillar. The sensors are FOGALE capacitance sensors whose measurement ranges are 2.5 mm.

The first experiment was using ϕ9-mm tube for the Full and ϕ19-mm tube for the Half. Test bench was tilted by ~0.1 mm and returned to initial level after 4 minutes. Oscillations of the water level inside the sensors were measured as shown in Figure 6.

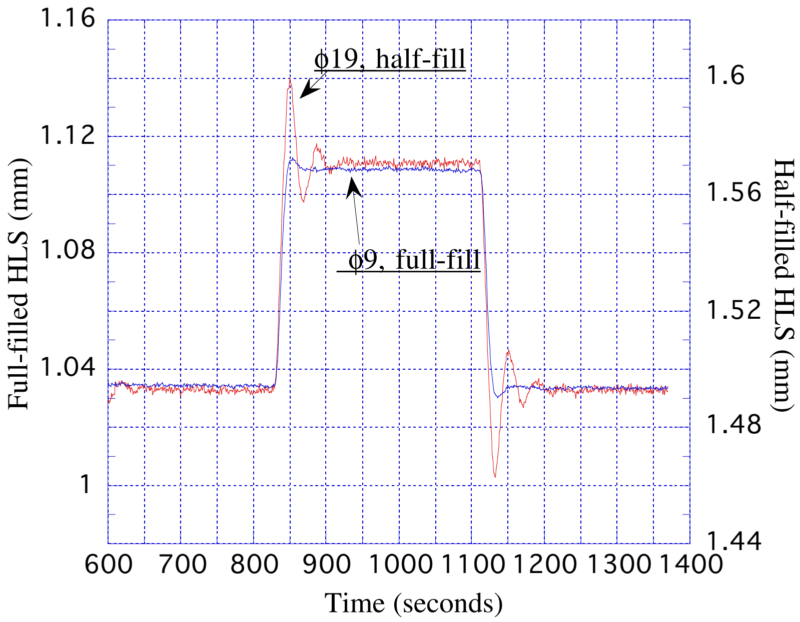

Fig.6 Responses of the HLS of the two types to inclination movement. The red is the oscillation of the water surface in half-fill; the blue is that in full-fill.

By referring to figure 2 and 4, the critical diameter for the Full is about 9 mm and about 11 mm (2 times of the depth of water) for the Half. As expected, the Full shows almost a critical

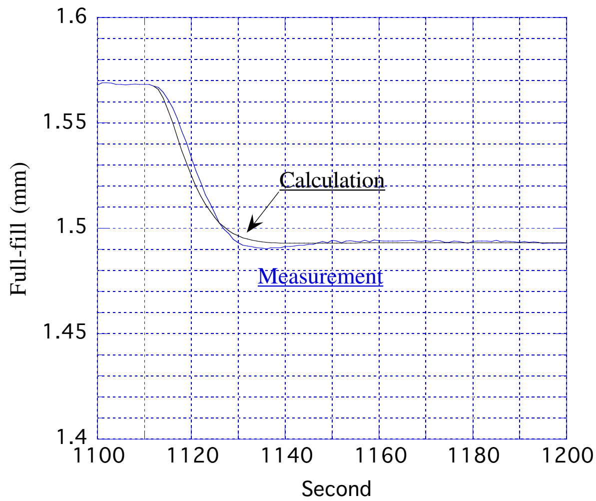

Fig.7 Comparison between measurement and calculation for the damping of the Full.

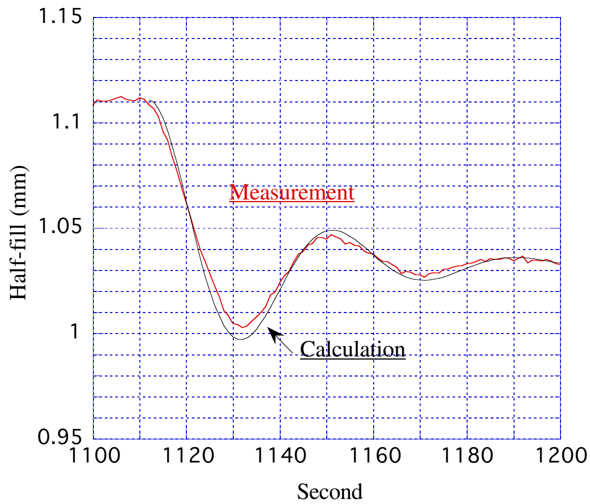

Fig.8 Comparison between measurement and calculation for the damping of the Half.

oscillation because of its optimum diameter. The Half gives a less damped oscillation. In figure 7 and figure 8 the measurements are compared with the calculations for the data between 1100 and 1200 seconds. If gives attention to the slope of falls and the period of oscillations, one can see that the two are almost coincident with each other. It is need to say that the inner surface roughness of the pipe has more or less influence on the experiment result.

The second experiment was changing the tubes to ϕ12-mm both for the Full and the Half. In this case, the diameter of the Half is near to its critical. Figure 9 and 10 show the measurements as well as the calculations for the oscillations of the water inside the sensors.

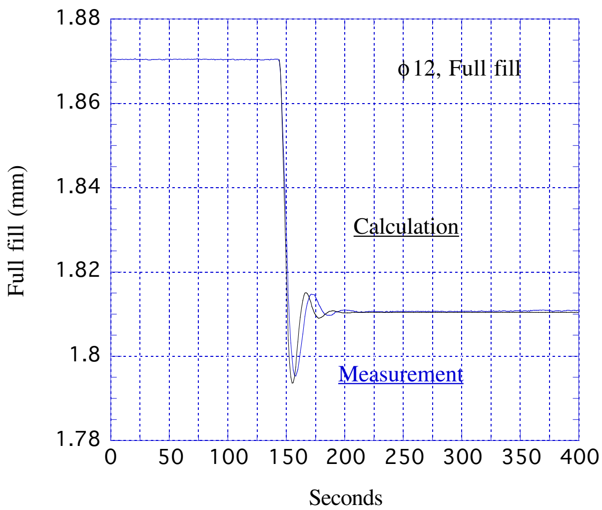

Fig.9 Comparison between measurement and calculation for the Full (continue).

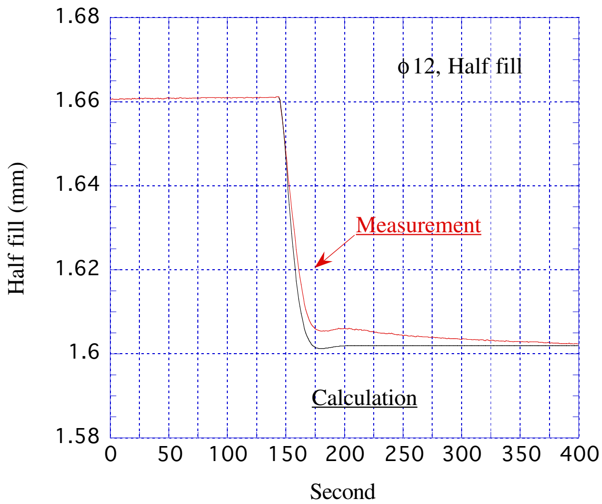

Fig.10 Comparison between measurement and calculation for the Half (continue).

It is the same that the measurements are similar to the calculations for the slope of falls and the period of oscillations. And, the Half, as expected, is a nearly critical oscillation.

In figure 10 the level of the water inside the sensor did not reach its equilibrium level for a while because the outlet of the sensor was so small that the water can not flow out immediately. This is beyond the discussion here.

Above experiments illustrate that the hydrokinetic analysis in Sections 2 and 3 are correct and there exist critical parameters both for the full and the half-filled HLS.

5. 10-METER MOVABLE HLS

Hereinafter, we would like to give a brief show of a Movable HLS. We intend to move it around the tunnel of the storage ring to find the places and the amounts of the ground variations. We had planned to make a system of 60 meters and to set sensors in an interval of 10 meters.

A preceding ten meters of the movable HLS is shown in figure 11. It is a half-filled system with dual pipes. The inner pipe is for the fluid and the outer is the supporter. Pipe segments of five meters are connected with flexible tubes and can be moved by hands here and there.

For testing, this 10-meter HLS was first set up in the experiment hall of the storage ring. The data for several months were taken for the ground movement. Figure 12 shows the measurement of the inclination of the ground. The inclination movement of the ground is usually in a circle of 24 hours because of the temperature circle in the experiment hall. From figure 12 one can see besides the period of 24 hours there is clearly a variation in 12 hours period. This is the movement of the earth tide. For comparison, the calculated earth tide of the ground inclination is also plotted in the figure, which is about in 1x10^{-7} radian peak to peak.

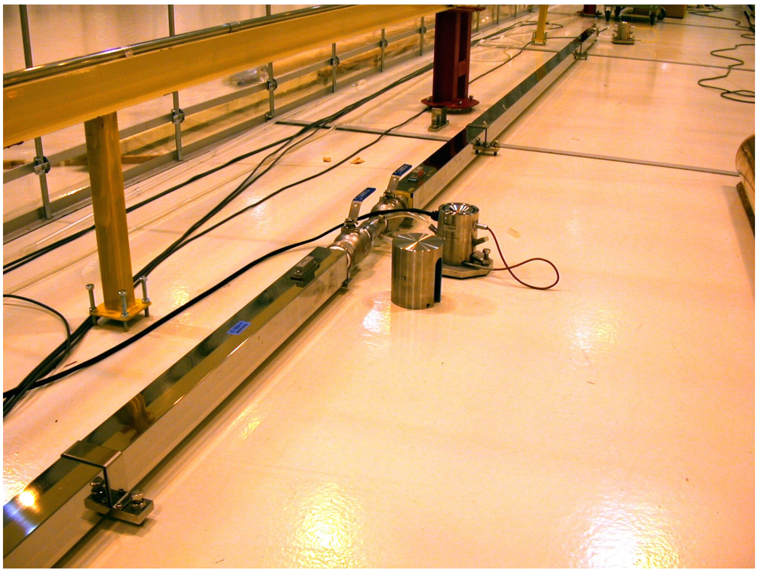

Fig.11 Ten meters of movable HLS were set in the experiment hall for testing.

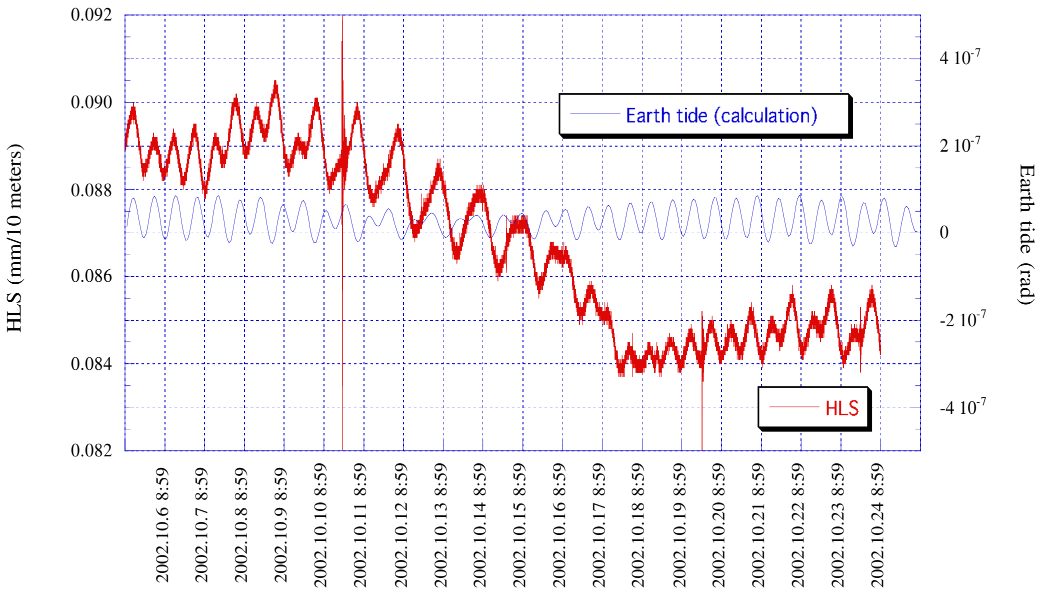

Fig.12 the measurement of the inclination movement of the ground and the calculations result of the earth tide.

The recording of the earth tide is a good illustration of the system resolution. That is, the resolution of the movable HLS is better than 1 micron.

The problem we have for the time being is the long-term drift of the measurement. It can be seen in the same figure and is about several microns one month. The reason for the drift is not yet clear.

6. CONCLUSION

To optimize the parameters of the communicating pipe of the hydrostatic level systems, we studied the motions of fluid both for full-filled and half-filled system. The diameter of pipe or depth of fluid is the determinative parameter for the response capability of the system to level variation.

For the full-filled HLS, external disturbance, either for the height change of a sensor, or for the inclination of whole system (sensors and pipe), will cause the fluid moved as a damped oscillation. The oscillation period is proportional to the root of pipe length. Corresponding to different length there is a critical or optimum diameter that makes the oscillation damped out fastest to come to rest.

For the half-filled HLS, external disturbance usually makes the system inclined and causes the fluid oscillated inside the pipe with dispersed eigenfrequencies or oscillation periods. With the fundamental mode of the motions, a critical or optimum depth of fluid can be derived for corresponded length of pipe. The oscillation period of the half-fill is directly proportional to pipe length. Comparing to full-filled system it needs much more time before the fluid comes to rest.

Experiments were made both for the full and the half fill HLS. In the experiments the system response capabilities to inclination movements were measured. The results are coincident with the calculations. As expected, the critical oscillations both for the full and the half fill HLS are verified.

7. REFERENCES

[1] S. Matsui, C. Zhang et al, *Elevation changes of the storage ring magnets*, Annual Report 1996, SPring-8.
[2] Walther Kaufmann, *Fluid Mechanics*.
[3] Imai Kou, *Fluid Mechanics* (in Japanese).