Study of Charged Particle Storage Ring as Detector of Gravitational Waves

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Since Einstein predicted the existence of gravitational waves in nature, more than eighty years passed. Several methods such as Weber antenna, laser interference to directly detect the gravitational waves have been proposed and developed. Unfortunately, untill now, no confirmative gravitational wave signal has been observed directly. Here, we propose a new method to directly detect the gravitational waves by monitoring the motion of charged particles in the storage ring.

Suppose that a plane gravitational wave is incident on a simple quadruple oscillator consisting of charged particles moving in a storage ring. The metric of spacetime with a plane gravitational wave propagating along z-axis is given in c=1 unit by

$$ds^2 = -dt^2 + (1+h_+)dx^2 + (1-h_+)dy^2 + 2h_\times dxdy + dz^2 \tag{1}$$

where $h_+(<<1)$ and $h_\times(<<1)$ are two independent polarizations. The equantion of the deviation n^μ for the moving particles with the rest mass m_0 and the 4-velocity U^μ is

$$\frac{D^2 n^\mu}{d\tau^2} + R^\mu_{\alpha\beta\nu} U^\alpha \left(x^\beta - x_0^\beta\right) U^\nu = \frac{1}{m_0}\frac{DF^\mu}{dx^\kappa} n^\kappa \tag{2}$$

where $R^\mu_{\alpha\beta\nu}$ is the curvature tensor of the spacetime, x_0^β the coordinates of the center of the storage ring, and F^μ is the four-vector Lorentz force. When the storage ring is placed on the x-y plane with the center at the origin of the coordinates, it may reduce to

$$\frac{d^2 n^1}{dt^2} + \omega_a^2 n^1 = \frac{1}{2}\left(\frac{\partial^2 h_+}{\partial t^2}\cdot x + \frac{\partial^2 h_\times}{\partial t^2}\cdot y\right) \tag{3}$$

$$\frac{d^2 n^2}{dt^2} + \omega_a^2 n^2 = \frac{1}{2}\left(\frac{\partial^2 h_\times}{\partial t^2}\cdot x - \frac{\partial^2 h_+}{\partial t^2}\cdot y\right) \tag{4}$$

under the simplified assumptions that (i) the magnetic field B is in the z-axis, (ii) n^0 and n^3 are negligible, and (iii) the variations of U^0 and U^3 are negligible. In Eqs. (3) and (4), $\omega_a^2 = \omega_0^2 + \omega_0\omega_B$, where $\omega_0 = qB/m$ is the angular velocity of the charged particles in the storage ring, $\omega_B = qRB'(r)/m$ is the angular velocity induced by the gradient of B in radial direction on the x-y plane, q and $m = m_0\Big/\sqrt{1-\omega_0^2 R^2}$ are the charge and the moving mass of each charged particle, respectively, and R is the radius of the orbit of the charged particles.

When the incident gravitational wave has only one polarization h_+ and takes the cosine form with the angular frequency ω_g and the wavelength λ_g, i.e., $h_+ = h\cdot\cos(\omega_g t - 2\pi z/\lambda_g)$. The solution of the Eqs.(3) and (4) is

$$n^1 = A_1 \cos(\omega_a t + \varphi_1) + \frac{h\omega_g^2 x}{2(\omega_g^2 - \omega_a^2)} \cos(\omega_g t) \quad (5)$$

$$n^2 = A_2 \cos(\omega_a t + \varphi_2) - \frac{h\omega_g^2 y}{2(\omega_g^2 - \omega_a^2)} \cos(\omega_g t). \quad (6)$$

If we only consider the deformation generated by gravitational waves. When there is no gravitational wave, the changes of beam position n^1 and n^2 should be zero, $A_1 = A_2 = 0$ can be obtained.

Obviously, both the deviations n^1 and n^2 caused by the gravitational wave have sharp maximums at the angular frequency $\omega_g \approx \omega_a$. Namely, the gravitational wave with such an angular frequency is in resonance with the oscillations of the charged partcle in the storage ring. If the damping rate of the system, for example caused by beam position measurement, is ζ, the changes of the beam position is larger than the amplitude of motion for a free particle by a factor ω_a / ζ, assuming $\zeta << \omega_a$. Therefore, one might detect the gravitational wave with the dimensionless amplitude h of 10^{-18} if the $\omega_a / \zeta \sim 10^9$ and the beam position can be determined within the precision of 1ppm.

The change of circumference per turn corresponding to the close orbit distortion (COD) in accelerator, due to the perturbation of the gravitational wave is

$$\Delta l = \frac{hR\omega_0\omega_g^3}{2(\omega_g^2 - \omega_a^2)(4\omega_0^2 - \omega_g^2)} \cdot \sin\frac{2\pi\omega_g}{\omega_0}. \quad (7)$$

Eq. (7), shows that the COD signal Δl is independent from time. It seems that the storage ring cannot be used as the detector of gravitational waves by monitoring the COD signal. However, since the Earth spins with angular velocity ω_e, Let $(\alpha_r, \beta_r, \gamma_r)$ and $(\alpha_g, \beta_g, \gamma_g)$ stand for the Euler angles of the normal of the storage ring and the propagating direction of the gravitational waves, respectively. Then Δl should be reduced by a factor of $[\cos\gamma_g \cdot \cos\gamma_r + \sin\gamma_g \cdot \cos(\omega_e t)]^2$. Namely, Δl will change in the period of 12 hours and 24 hours. Therefore, one might still detect the gravitational waves by monitoring the COD signal of the charged particles moving in a storage ring.

CONCLUSION

We are considering and hope developing a new method of gravitational waves detection by using the charged particles storage ring. One might have chance to pickup gravitaional wave signals by monitoring the beam position. In this case, the position of all the magnets and the BPMs should be given continuely online, and the BPM resolution should be better than 1PPM.