EIGENVALUE PROBLEM IN PRECISE MAGNET ALIGNMENT

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Abstract

For the alignment of the accelerator components at the designed coordinates, a matrix connecting observables and coordinates can be treated as an eigenvalue problem and has its own characteristics depending on the structure of the survey network. Once its eigenvectors are determined, the misalignment of the magnets is expressed by their linear combination. By decomposing the magnet misalignments into the eigenmodes and evaluating each contribution of the eigenmode to the closed orbit distortion (COD), it is possible to find which mode affects significantly to the beam in the synchrotron. This approach is applied to the synchrotron rings with the typical symmetric and asymmetric magnet arrangements to which several survey networks are adopted and gives the useful information for the magnet precise alignment, to say, the misalignment modes which are less significant to the beam can be safely omitted. In this work, three kinds of survey networks, which were applied for the real synchrotron, are treated in details.

1. INTRODUCTION

An alignment matrix equation is obtained from the triangulation of the survey networks for the precise alignment of the synchrotron magnets. The horizontal misalignments, the spatial deviations of magnet from the planned positions in the horizontal plane, are expressed by (x, y) or (r, θ) coordinates with an origin at the center of the synchrotron ring. From the survey of the physical distances such as the distance of the adjacent quadrupoles and the perpendicular from the quadrupole to the straight line stretched between the quadrupoles at both sides, the deviations of these distances from the design values are expressed approximately as a linear function of the misalignments of the related quadrupoles. These relations for all quadrupoles are merged into a global matrix equation termed here as an alignment equation. The matrix depends on both the magnet lattice design and the survey network and connects the magnet misalignment with the deviations of the surveyed distances, or the survey variables.

As the magnet lattice is fixed, the various kinds of the survey network formed on the targets of quadrupoles or the representative points close to each quadrupole introduce the variance in the matrix of the alignment equation. The survey network or mesh should be determined according to the magnet lattice structure and the easiness of the survey measurement in the narrow synchrotron tunnel.

If the target is placed on each quadrupole magnet, an accurate drilled hole where the target is inserted with a minimal clearance is required. The misalignment is corrected by moving the quadrupole so that the target sits at the designed coordinate. If the representative points are used, each quadrupole is aligned by referring these points.

Large colliding beam synchrotron with the experimental insertions is composed of the arced and straight sections. The arced section has a regular magnet lattice and quadrupoles are placed equidistant, but the experimental straight section has different quadrupole arrangements because it accommodates the RF cavities and mini-beta quadrupole magnet system to squeeze the beam at the collision point. The synchrotron ring in which the quadrupoles are not placed on the common circle is classified here as an asymmetric ring (inhomogeneous ring). To the contrary the hypothetical ring where all quadrupoles sit on the common circle is called here as a symmetric ring (homogeneous ring).

2. DERIVATION OF THE ALIGNMENT EQUATION

Assuming that either symmetric or asymmetric synchrotron ring has N quadrupoles and their N survey targets ($i = 1, 2, \cdots, N$) are on the designed orbit, at least two survey variables S_i and P_i are necessary to each quadrupole. These variables are expressed as a function of R_i and Θ_i as

$$S_i = F(R_i, \Theta_i) \text{ and } P_i = G(R_i, \Theta_i), \qquad (1)$$

where the subscript i of R and Θ is considered for all possible range. For an example, if S_i is a distance between the adjacent quadrupole magnets, $S_i = F(R_i, R_{i+1}, \Theta_i)$. Differentiating the above relations to the first order,

$$s_i \equiv \Delta S_i = \sum_i \frac{\partial F}{\partial R_i}\Delta R_i + \sum_i \frac{\partial F}{\partial \Theta_i}\Delta\Theta_i = \sum_i \frac{\partial F}{\partial R_i} r_i + \sum_i \frac{\partial F}{\partial \Theta_i}(\theta_{i+1} - \theta_i) \qquad (2)$$

and

$$p_i \equiv \Delta P_i = \sum_i \frac{\partial G}{\partial R_i}\Delta R_i + \sum_i \frac{\partial G}{\partial \Theta_i}\Delta\Theta_i = \sum_i \frac{\partial G}{\partial R_i} r_i + \sum_i \frac{\partial G}{\partial \Theta_i}(\theta_{i+1} - \theta_i), \qquad (3)$$

where s_i and p_i are the deviations from the design value which are determined from the precise survey, and r_i and θ_i the quadrupole displacements to the radial and azimuthal direction, respectively. Deriving relations (2) and (3) for all quadrupoles,

$$\begin{pmatrix} p_1 \\ : \\ p_N \\ s_1 \\ : \\ s_N \end{pmatrix} = \begin{pmatrix} a_{11} & \cdot\cdot & a_{1N} & c_{11} & \cdot\cdot & c_{1N} \\ : & & : & : & & : \\ a_{N1} & \cdot\cdot & a_{NN} & c_{N1} & \cdot\cdot & c_{NN} \\ b_{11} & \cdot\cdot & b_{1N} & d_{11} & \cdot\cdot & d_{1N} \\ : & & : & : & & : \\ b_{N1} & \cdot\cdot & b_{NN} & d_{N1} & \cdot\cdot & d_{NN} \end{pmatrix} \begin{pmatrix} r_1 \\ : \\ r_N \\ \theta_1 \\ : \\ \theta_N \end{pmatrix} \qquad (4)$$

where the 2N x 2N matrix is obtained for the above relations [1][2]. If 3 variables S_i, P_i and Q_i are considered, the matrix becomes 3N x 2N and the left column vector has 3N components. In any case the matrix equation (4) is expressed simply as

$$\mathbf{p} = (A)\mathbf{r} \qquad (5)$$

and solved by the least squares method as

$$\mathbf{h} \equiv (A^T)\mathbf{p} = (A^T A)\mathbf{r} \tag{6}$$

where (A^T) is the transposed matrix of (A) and $(A^T A)$ is the 2N x 2N matrix. The solution is given by

$$\mathbf{r} = (A^T A)^{-1}\mathbf{h}, \tag{7}$$

if the matrix is not singular. In general, however, the matrix (A) is singular. Therefore one must consider to reduce the rank of the matrix by fixing several coordinates, for an example, $r_1 = 0$, $r_2 = 0$ and $\theta_1 = 0$ without loosing generality. The number of coordinates to be fixed depends on the adopted survey network. Another method to solve (6) is to use the generalized inverse matrix. Even if the matrix is singular, its inverse matrix is determined uniquely following the method of the singular value decomposition. Replacing $(A^T A)$ with (H) and assuming $(H)^+$ the generalized inverse matrix, the solution of (6) is simply given by

$$\mathbf{r} = (H)^+\mathbf{h}. \tag{8}$$

In this manuscript the singular matrix of (4) is reduced to a lower rank to obtain the non-singular matrix so as to analyze its properties. From the relation (6) the precise magnet alignment problem can be treated as an eigenvalue problem,

$$(H)\mathbf{v}_i = \lambda_i \mathbf{v}_i, \tag{9}$$

where λ_i is an eigenvalue belonging to the eigenvector $\mathbf{v}_i$. If all components of eigenvector are multiplied by the coefficients and are combined linearly, the displacements of the quadrupole magnets (r_i and θ_i) are obtained as a linear combination of the eigenvectors,

$$\mathbf{r} = \sum_k c_k \mathbf{v}_k, \tag{10}$$

where c_k is the coefficient. From (6) and (10),

$$\sum_k (H) c_k \mathbf{v}_k = \mathbf{h}. \tag{11}$$

The linear coefficient of (10) is

$$c_k = (\mathbf{v}_k^T \mathbf{h}) / \lambda_k, \tag{12}$$

where $\mathbf{v}_k^T$ is the transpose of the k-th eigenvector $\mathbf{v}_k$. The j-th component of the displacement vector is

$$r_j = \sum_k \frac{(\mathbf{v}_k^T \mathbf{h})}{\lambda_k} v_{kj} \text{ or } r_j = \sum_k \frac{(\mathbf{v}_k^T A^T \mathbf{p})}{\lambda_k} v_{kj}, \tag{13}$$

where v_{kj} is the j-th component of the eigenvector $\mathbf{v}_k$ [3].

3 SURVEY NETWORKS

In many cases of the synchrotron ring, all magnets are aligned in the narrow tunnel and the survey network for the measurements of distances between magnets is restricted. Nowadays very precise instruments such as a laser tracker and a mekometer are available, but the maximum length to be measured is limited to maintain accuracy. Thus the survey network must be established considering the distribution of the survey points which can attain the required accuracy.

From (13), the displacement error Δr due to the survey error Δp is estimated by

$$\Delta r_j = \sum_k \frac{\sum_{i,m} a_{im} \Delta p_i v_{km}}{\lambda_k} v_{kj} = \sum_i \left(\sum_m a_{im} \sum_k \frac{v_{kj} v_{km}}{\lambda_k} \right) \Delta p_i \,. \qquad (14)$$

Then the standard deviation of Δr_j is expressed as follows,

$$\sigma_{r_j}^2 = \sum_i \left(\sum_{k,m} \frac{a_{im} v_{kj} v_{km}}{\lambda_k} \right)^2 \sigma_{p_i}^2 \,, \qquad (15)$$

where σ_{p_i} is the expected error for each p_i, the statistical error for repeated distance measurements between the same points. Between the standard deviations of the coordinates and distance measurement errors for the survey network there is a relation mediated by the network structure [4]. The smaller coordinate error is expected if omitting the contributions by eigenmodes with small eigenvalues.

For large synchrotrons there are not so many variations for the survey networks, because the number of the collimation axes extended from each point to another is limited. The typical survey networks considered here for the synchrotron are

(Case #1) Short chord and perpendicular,
(Case #2) Long chord and perpendicular, and
(Case #3) Short chord and 2 perpendiculars,

as shown in Fig.1, where the survey variables are given by the solid lines. Differences of the numbers of eigenmodes and vector elements are compared in Table 1.

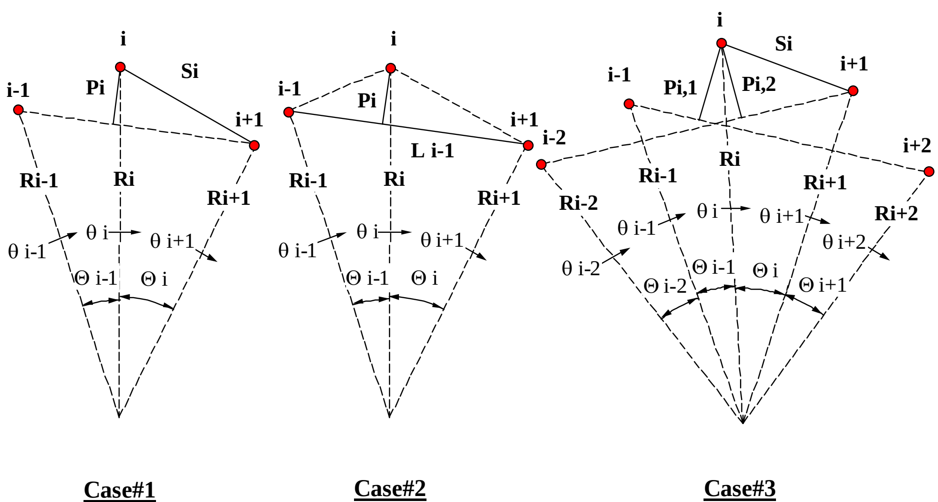


Fig.1 Survey variables (solid lines) for the different survey networks.

The matrix equations for 3 cases are derived for both symmetric and asymmetric ring, assuming both rings have the same diameter (480 m) and the same number of the quadrupole magnets (392 quads). The asymmetric ring is an electron-positron colliding synchrotron called TRISTAN of KEK which has the long experimental insertions at both sides of 4 interaction points. To the contrary, the symmetric ring is the hypothetical synchrotron of which all quadrupole magnets locate on the same circle at equidistant all around the ring.

The eigenvectors for 2 largest and 2 smallest eigenvalues are shown for the symmetric 3 cases in Fig.2 and for the asymmetric 3 cases in Fig.3. In these figures the radial and azimuthal displacements are given in seperate both in the order of quadrupole magnet configuration.

Table 1 Differences of the numbers of eigenmodes and vector elements for 3 survey networks. N is the number of the quadrupole magnets.

	Case #1	Case #2	Case #3
Number of eigenmodes	2N-3	2N-4	2N-3
Number of the eigenvector elements of each eigenmode	2N-3	2N-4	2N-3

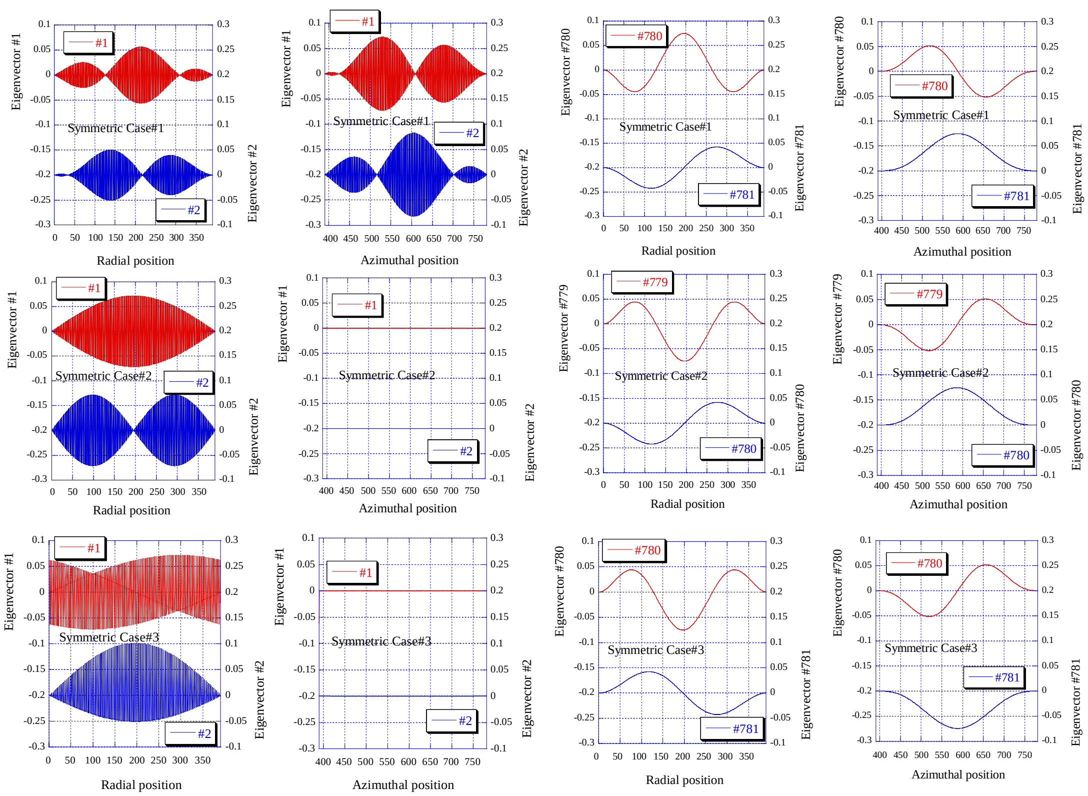


Fig.2 Eigenvectors with 2 largest and 2 smallest eigenvalues for the symmetric cases. Eigenvector #1 has the largest eigenvalue and eigenvector #781 (#780 for case#2) the smallest one. These eigenvectors give the magnet misalignment patterns to the radial and azimuthal directions in the order of magnet configuration. Eigenvectors for (a) Upper: Symmetric case#1, (b) Middle: Symmetric case#2, and (c) Lower: Symmetric case#3.

In every case the same eigenvector is obtained for the smallest eigenvalue. It gives the quadrupole displacements forming a sinusoidal undulation along the synchrotron ring in both radial and azimuthal directions. Generally for larger eigenvalue the number of the sinusoidal displacement undulations along the ring increases according to the magnitude of the eigenvalue. They give the characteristic envelope patterns of the sinusoidal displacements for large eigenvalues. However, these misalignment undulations are obtained without considering the betatron motion of the particle. As each eigenvector has its own pattern of the magnet misalignment, if the betatron oscillation number becomes similar to the quadrupole magnet alignment pattern gives the relative large COD to the beam.

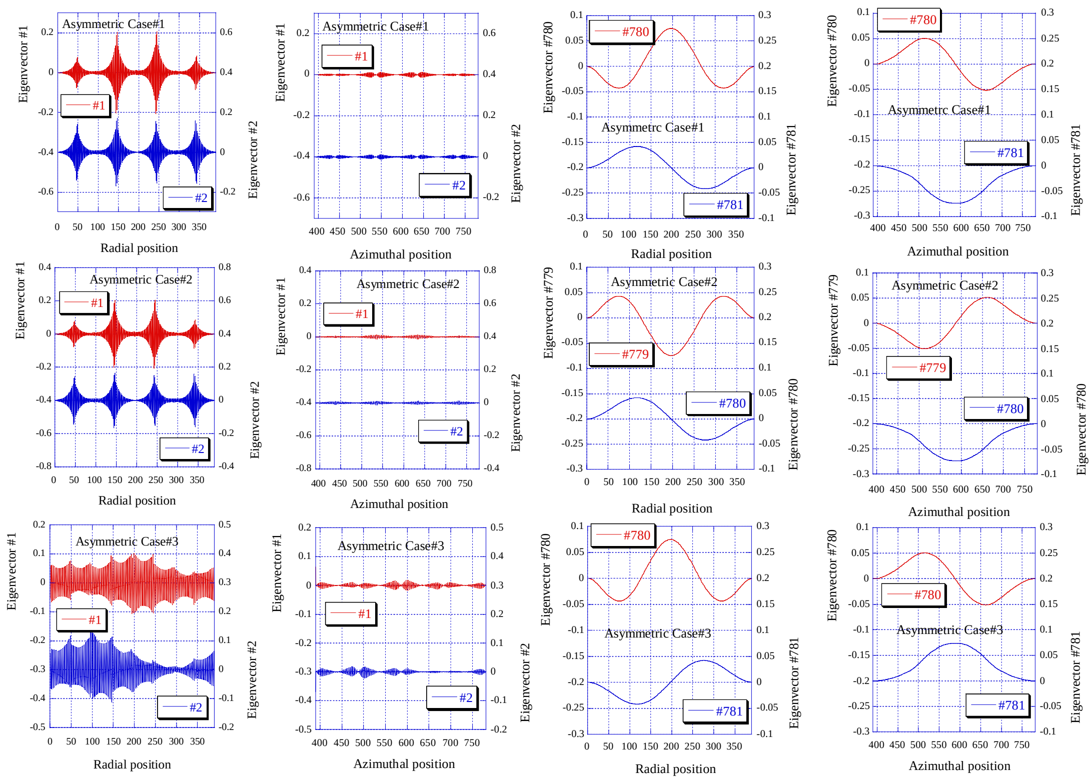


Fig.3 Eigenvectors with 2 largest and 2 smallest eigenvalues for the asymmetric cases. Eigenvector #1 has the largest eigenvalue and eigenvector #781 (#780 for the case#2) the smallest one. These eigenvectors give the magnet misalignment patterns to the radial and azimuthal directions in the order of magnet configuration. Eigenvectors for (a) Upper: Asymmetric case#1, (b) Middle: Asymmetric case#2, and (c) Lower: Asymmetric case#3.

4. RMS CLOSED ORBIT DISTORSION (COD)

To estimate the rms COD the magnitude of the misalignment is normalized to

$$\sqrt{\sum_{i=1}^{2N} \frac{(\Delta x_i)^2}{2N}} = 0.1mm, \qquad (16)$$

after giving the radial and azimuthal misalignments to every quadrupole magnet. If $\sqrt{\sum_{i=1}^{N}(\Delta x_i)^2 / N} = 0.1mm$ is assumed only for the radial displacements, an extraordinary azimuthal displacement is introduced even for the small radial misalignment. To avoid this problem, the normalization over both radial and azimuthal displacements is adopted.

For the calculation of COD the beam optics code is used [5]. An essential feature of the COD calculation depends on the transfer matrices of all machine elements including quadrupoles, dipoles and drift spaces that the circulating beam encounters when traveling the synchrotron orbit. The transfer matrix through one period can be rewritten by using the twiss parameters and the betatron phase advances. Once the twiss parameters are obtained at a point, those at other points are easily calculated. A matrix to calculate COD at the beam position monitors due to the misalignments of the quadrupole magnets can be derived by using the twiss parameters. If each eigenvector is applied to this matrix, its contribution to the beam is estimated by using the rms value of COD.

Every quadrupole is shifted to the radial and longitudinal directions according to the assumed misalignment in the horizontal plane and the drift space lengths are adjusted to make up for the longitudinal shift of quadrupoles before the COD calculation.

By investigating the relations between the misalignment modes and the beam orbit deformations, the underlying rules for the accelerator alignment issue will be found. In the synchrotron the charged particles perform the betatron oscillation which is determined by the magnetic field gradient and the configuration of the quadrupole magnets. The COD is defined as the orbit of the gravitational center of the circulating particles. If the undulation number of the misalignment pattern coincides with that of the betatron oscillation, particles suffer the Lorenz force due to the systematically varying magnetic field of the displaced quadrupole magnet and will result in the large orbit distortion. .

4.1 The rms COD's for each eigenmode

The rms COD is defined here as

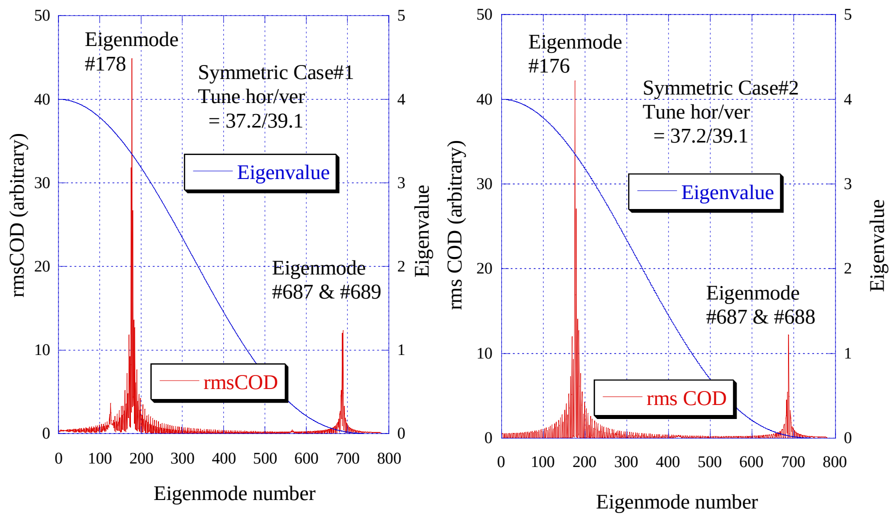
$$< r^k >= \sqrt{\frac{1}{N}\sum_{i=1}^{N}(r_i^k)^2}\ , \qquad (17)$$

where r_i^k is the radial COD at the i-th beam position monitor and $< r^k >$ the corresponding rms COD originated from the k-th eigenvector. The rms COD may be used to estimate the effect to the beam orbit by each eigenmode.

4.1.1 Symmetric cases

In general, the rms COD responses are different depending on the survey networks. However, there is no structural difference between cases #1 and #2 for eigenvalues as shown in Fig.4. The undulation number of the eigenvector (eigenmode #687) shown in Fig.5 is very close to the betatron oscillation frequency. The eigenmode #687 of the symmetric case#2 has the same undulation pattern as Fig.5. The largest peak corresponding to the eigenmode #178 has the eigenvector shown in Fig.6. This eigenmode has about 4 times of the undulation number of the eigenmode #687 which is close to the horizontal betatron frequency. As almost all dots shown in Fig.7 fit on the phase of the misalignment pattern of the eigenmode #178 which has the largest rms COD response, the eigenmode #178 is expected to have a large contribution to COD. The same relation concerning to the largest rms COD peak holds on other cases.

The eigenmode #176 of the symmetric case#2 have the same undulation pattern except for the azimuthal part which has 71 undulations.



(a) Symmetric case#1 (b) Symmetric case#2

Fig.4 The rms COD responses of the symmetric cases #1 and #2.

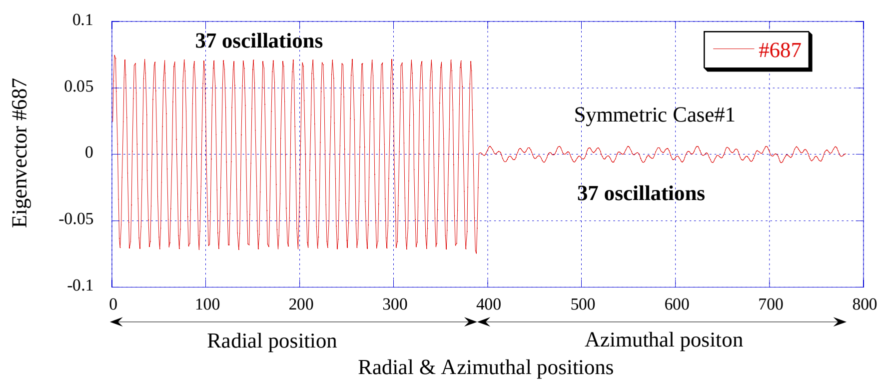


Fig.5 Undulation pattern of the eigenvector #687 (2^{nd} peak) of the symmetric case#1.

The survey network of the symmetric case #3 has the 3^{rd} peak as shown in Fig.8 which does not exist in cases #1 and #2. The 2^{nd} peak corresponds to the eigenmode #652 having the undulation number close to the betatron frequency (Fig.9). But the 1^{st} and 3^{rd} peaks have larger undulation numbers as shown in Fig.10 and Fig.11, respectively.

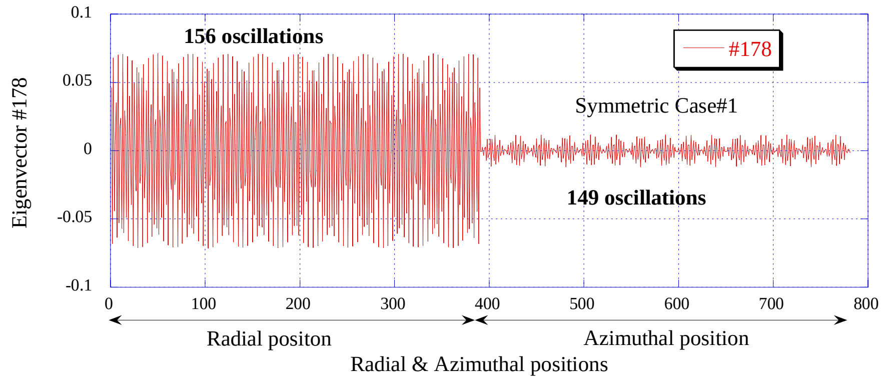


Fig.6 Undulation pattern of the eigenvector #178 (1st peak) of the symmetric case#1

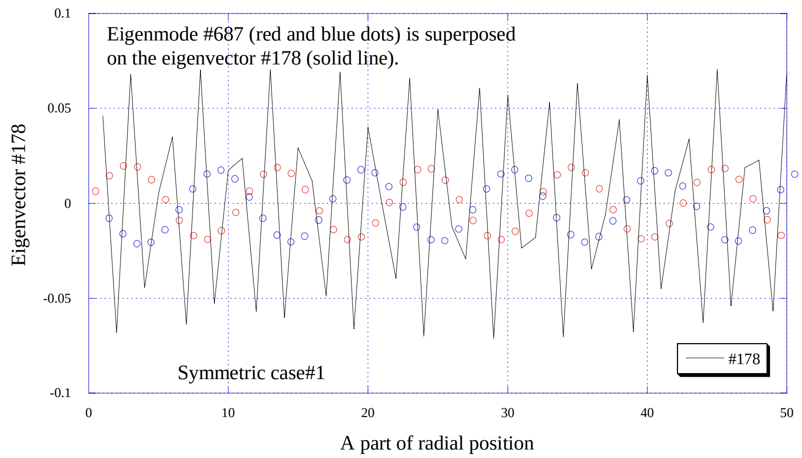


Fig.7 Eigenmode #178 (solid line) has 4 times of the undulation number of the eigenmode #687 (red and blue dots) which is close to the horizontal betatron frequency in the symmetric case#1. Almost all dots fit on the phase of the misalignment pattern of the eigenmode #178 which has the largest rms COD response. Only one fourth of the circumference is shown for the explanation.

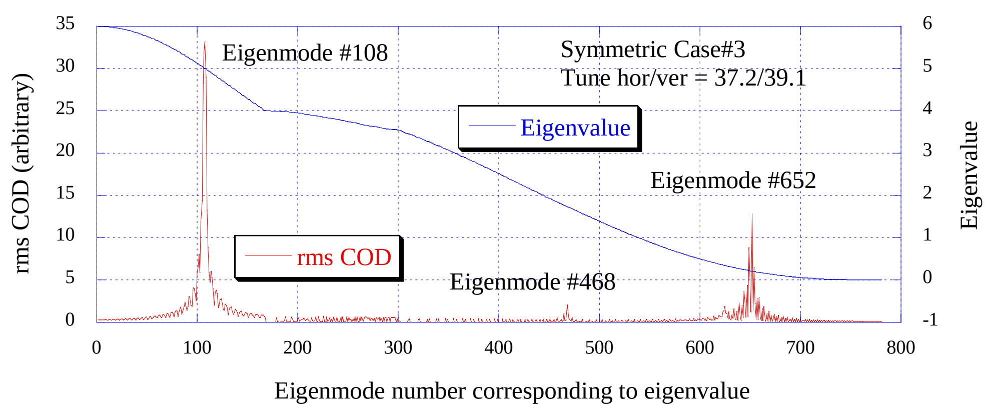


Fig.8 The rms COD response of the symmetric case #3.

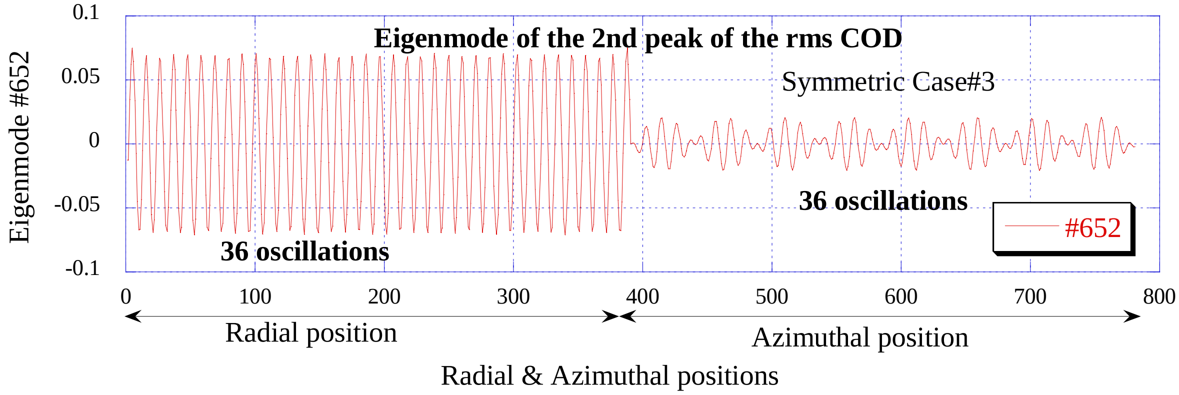


Fig.9 Undulation pattern of the eigenvector #652 (2^{nd} peak) of the symmetric case#3.

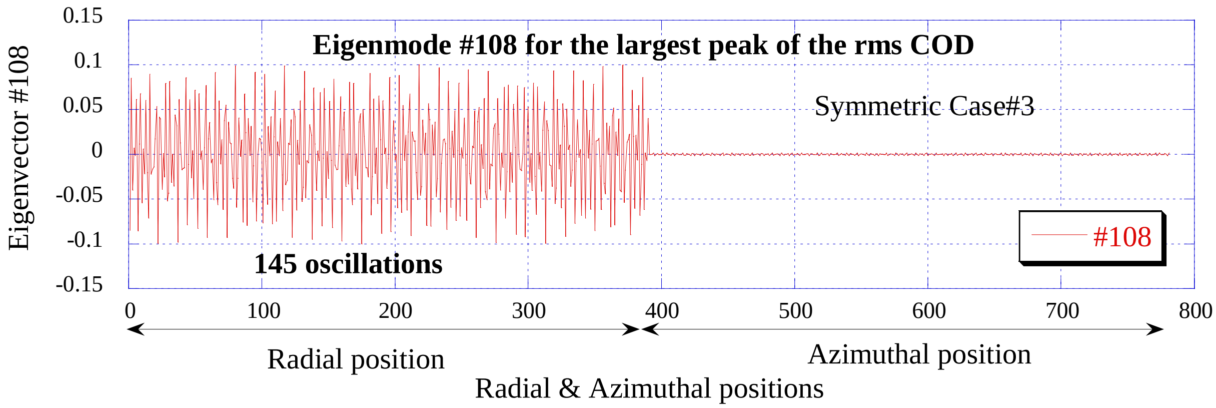


Fig.10 Undulation pattern of the eigenvector #108 (1^{st} peak) of the symmetric case#3.

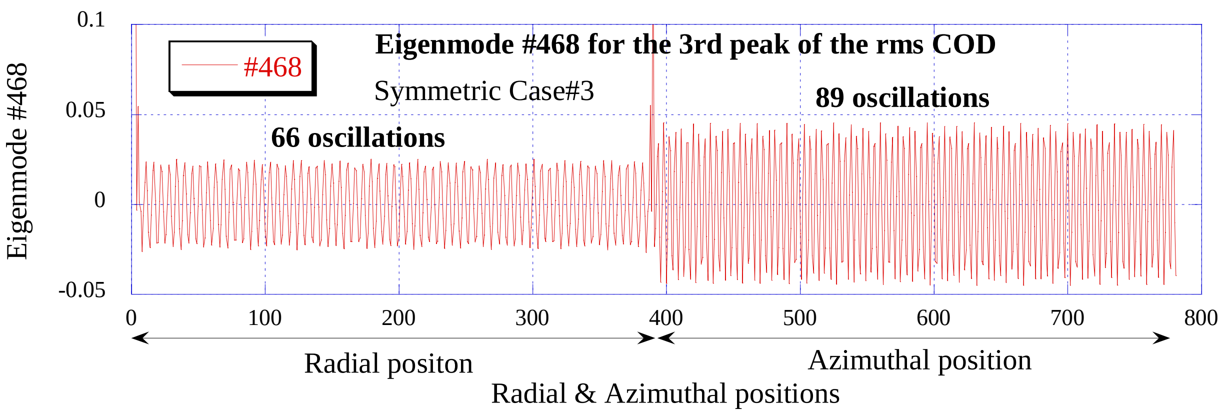
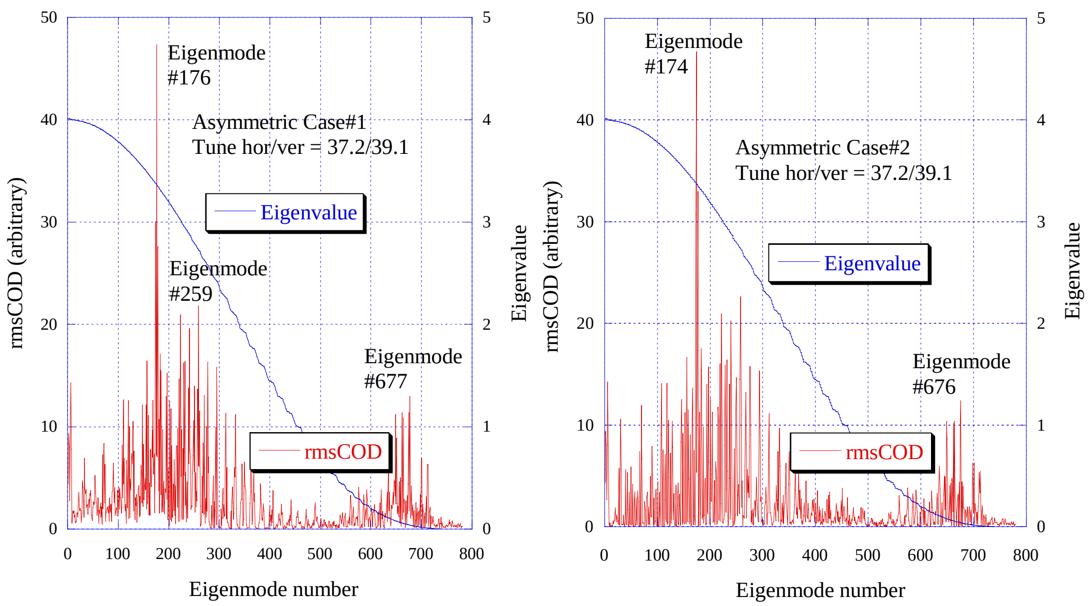


Fig.11 Undulation pattern of the eigenvector #468 (3^{rd} peak) of the symmetric case#3.

4.1.2 Asymmetric cases

In the asymmetric cases there appear many peaks in the rms COD response spectra. It seems that the irregularity in distance between quadrupole magnets plays a role. As shown in Fig.12 the cases #1 and #2 have almost the same rms COD responses and the peaks around the eigenmode #677 correspond to the eigenvector close to the betatron frequency as shown in Fig.13. The eigenmode for the largest peak (Fig.14) has a similar eigenvector as Fig.6.



(a) Asymmetric case#1 (b) Asymmetric case#2

Fig.12 The rms COD responses of the asymmetric cases #1 and #2.

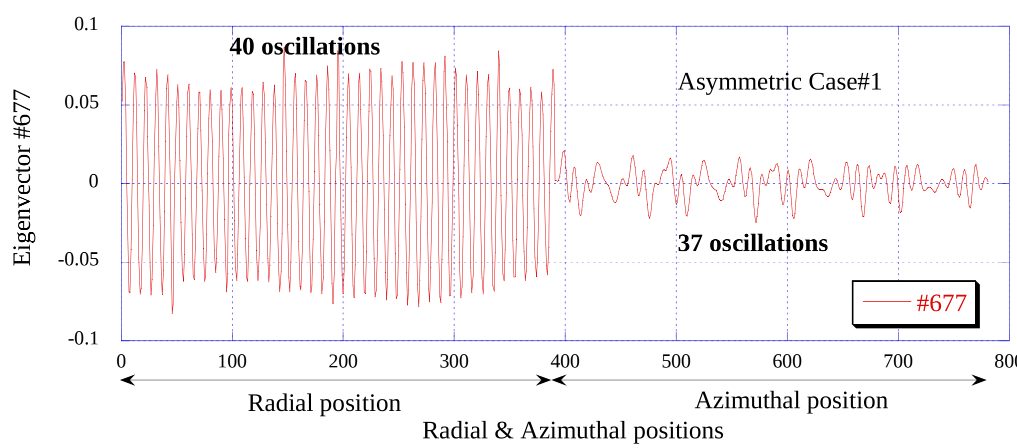


Fig.13 Undulation pattern of the eigenvector #677 of the asymmetric case#1.

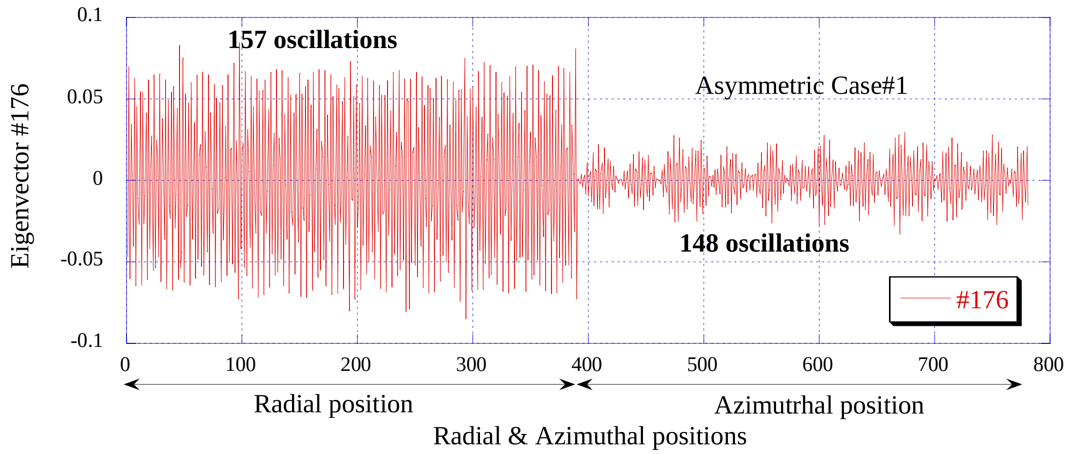


Fig.14 Undulation pattern of the eigenvector #176 of the asymmetric case#1.

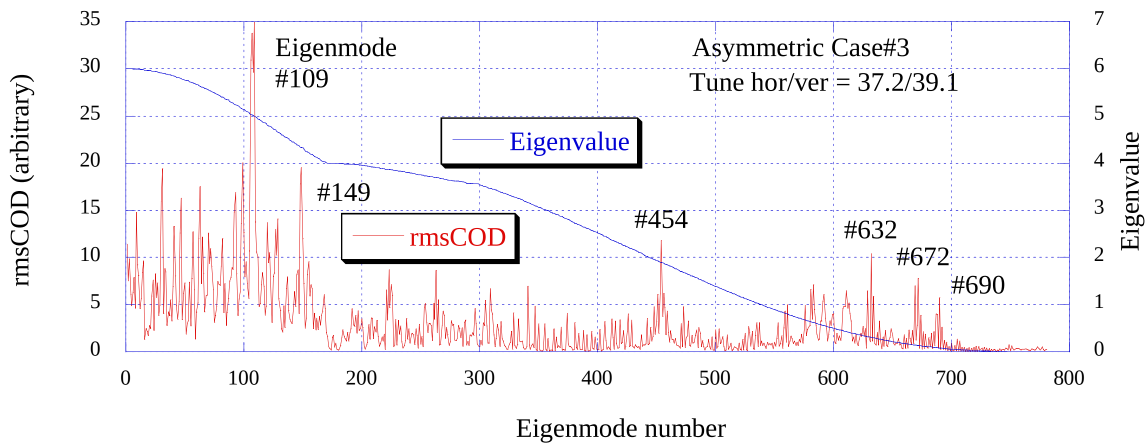


Fig.15 The rms COD response of the asymmetric case #3.

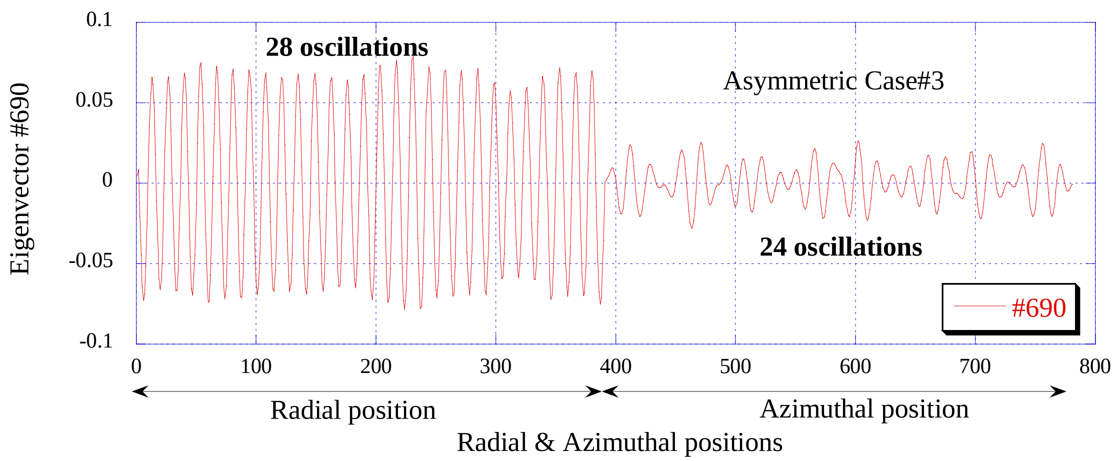


Fig.16 Undulation pattern of the eigenvector #690 of the asymmetric case#3.

For the asymmetric case#3 has the rms COD response spectrum different from Fig.12 as shown in Fig.15. The peaks of the rms COD responses for the eigenmodes corresponding to the betatron frequency shift to larger eigenvalues. The undulation number of the eigenmode #690 (Fig.16) is less than the betatron frequency which corresponds to the eigenmode #632 (Fig.17).

Comparing the cases #1, #2 and #3, there are big differences as shown in Fig.12 and Fig.15. However, it seems little effect to COD by omitting the corrections of eigenmodes (more than #720) with small eigenvalues. In this way the beam sensitive eigenmodes, which must be corrected, are selected through the investigation of the eigenmode problem.

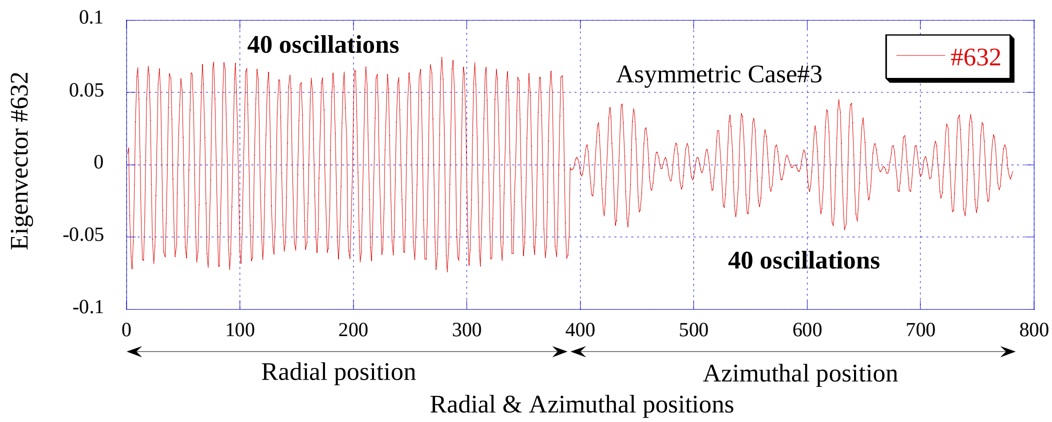


Fig.17 Undulation pattern of the eigenvector #632 of the asymmetric case#3.

4.2 Solving the eigenmode based alignment problem

If several eigenmodes are selected as the quadrupole alignment errors, the corresponding eigenvectors can be combined linearly assuming arbitrary coefficients and transformed to the survey variables such as the perpendicular and short chord lengths in the case#1. According to (13) the difference of the survey data from the design values can be processed to obtain the quadrupole misalignments. An example for the asymmetric case#1 is given in Fig.18, where the 677-th eigenmode is assumed as the misalignment mode and the difference between the misalignment and the corresponding eigenvector is shown by the blue line. Most significant contribution to the misalignment is the 677-th eigenmode as shown in Fig.18 by the peak of the green line.

Other eigenmodes at the right side of the absolute coefficient (green line) also contribute the misalignment but these eigenmodes arise by solving the singular matrix problem by the rank reduction method. The survey variables, perpendicular and short chord length errors, are obtained from the 677-th eigenvector as given in Fig.19.

Even if two eigenmodes are linearly combined, the same procedure can be applied to obtain the misalignment. Combining the 176-th and the 677-th eigenmodes with the equal weight but the different signs for the asymmetric case#1 (Fig.20), both eigenmodes have significant contributions to the misalignment and eigenmodes at the right side of the absolute coefficient (green line) also have relatively large contributions which are also introduced through the rank reduction of the singular matrix. The difference between the calculated misalignment and the combined eigenvector arises from the excitation of the lower undulation frequency modes with small eigenvalues. The eigenmodes with the undulation frequency much lower than the betatron frequency and with the small rms COD response do not contribute to the beam motion and can be safely disregarded from the misalignment correction.

If the random misalignment is assumed to obtain the perpendicular and short chord errors, most eigenmodes contribute equally to the misalignment as shown in Fig.21 for the asymmetric case#1.

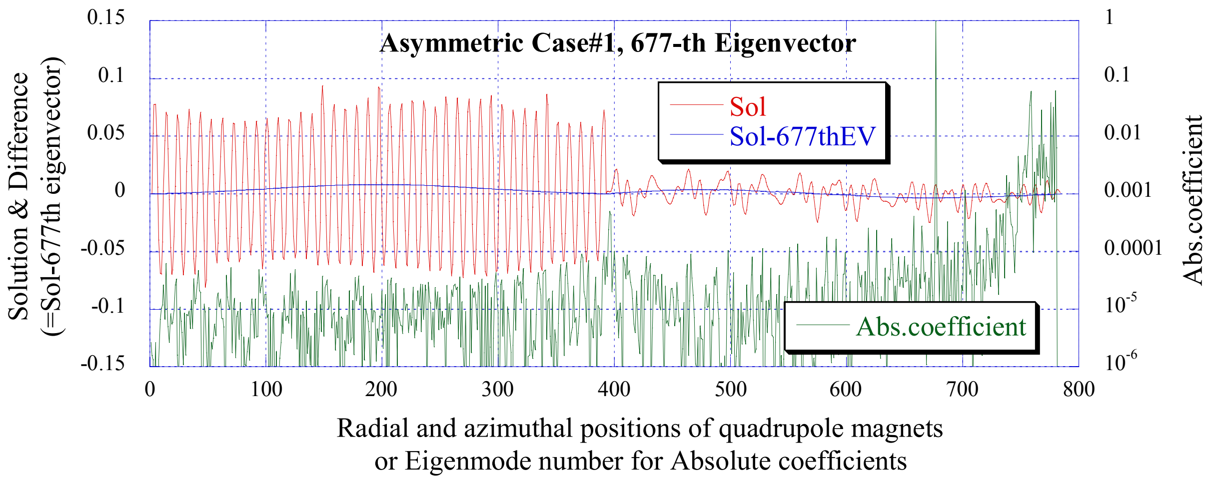


Fig.18 Misalignment due to the 677-th eigenmode of the asymmetric case#1. The red line is the quadrupole misalignment and the blue line is the difference between the calculated misalignment and the 677-th eigenvector. The green line is the absolute coefficients combining all eigenvectors to make up the misalignment of the red line.

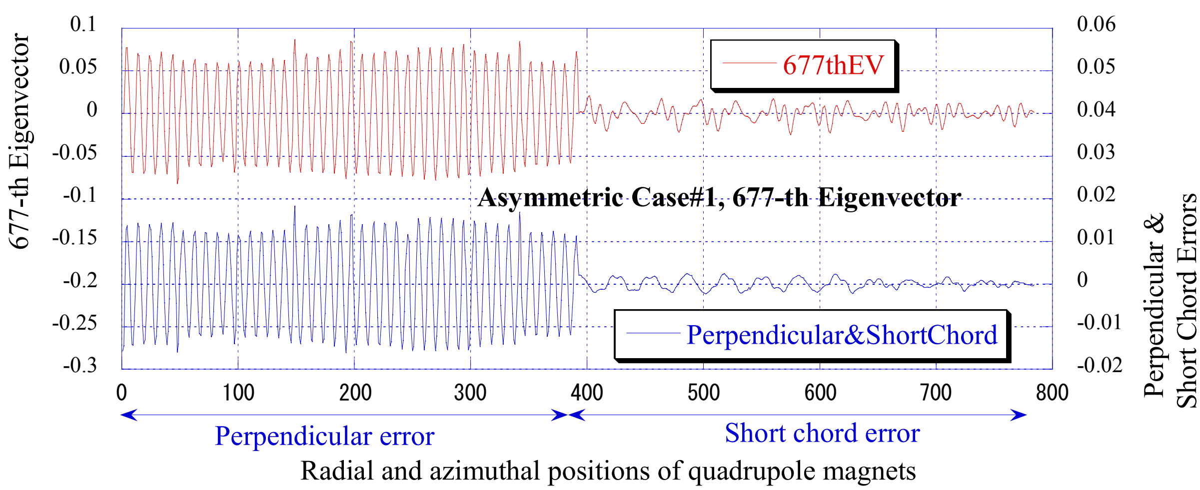


Fig.19 Comparison of the 677-th eigenvector and the corresponding survey variables obtained from this eigenvector for the asymmetric case#1. The perpendicular and short chord errors (blue line) give the solution (red line) in Fig.18.

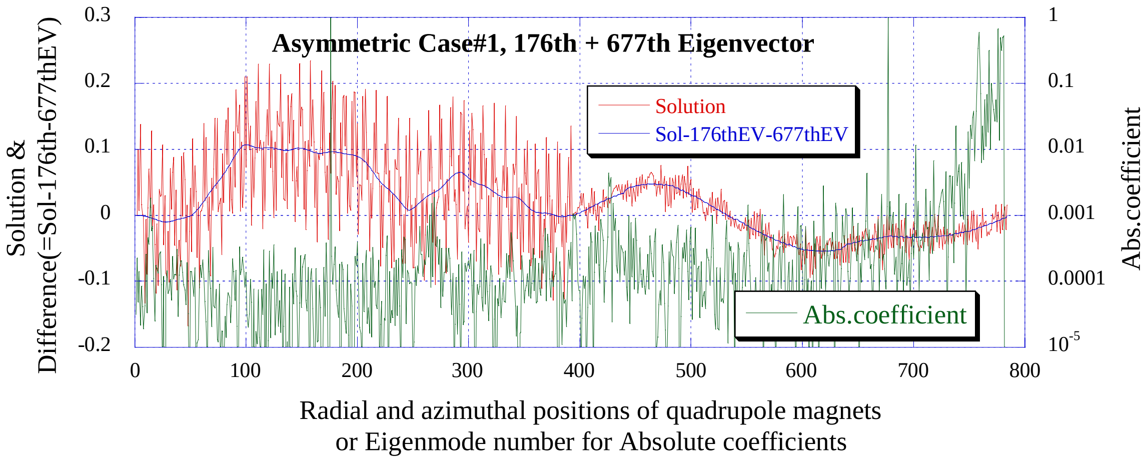


Fig.20 Misalignment due to the combination of the 176-th and the 677-th eigenmodes.

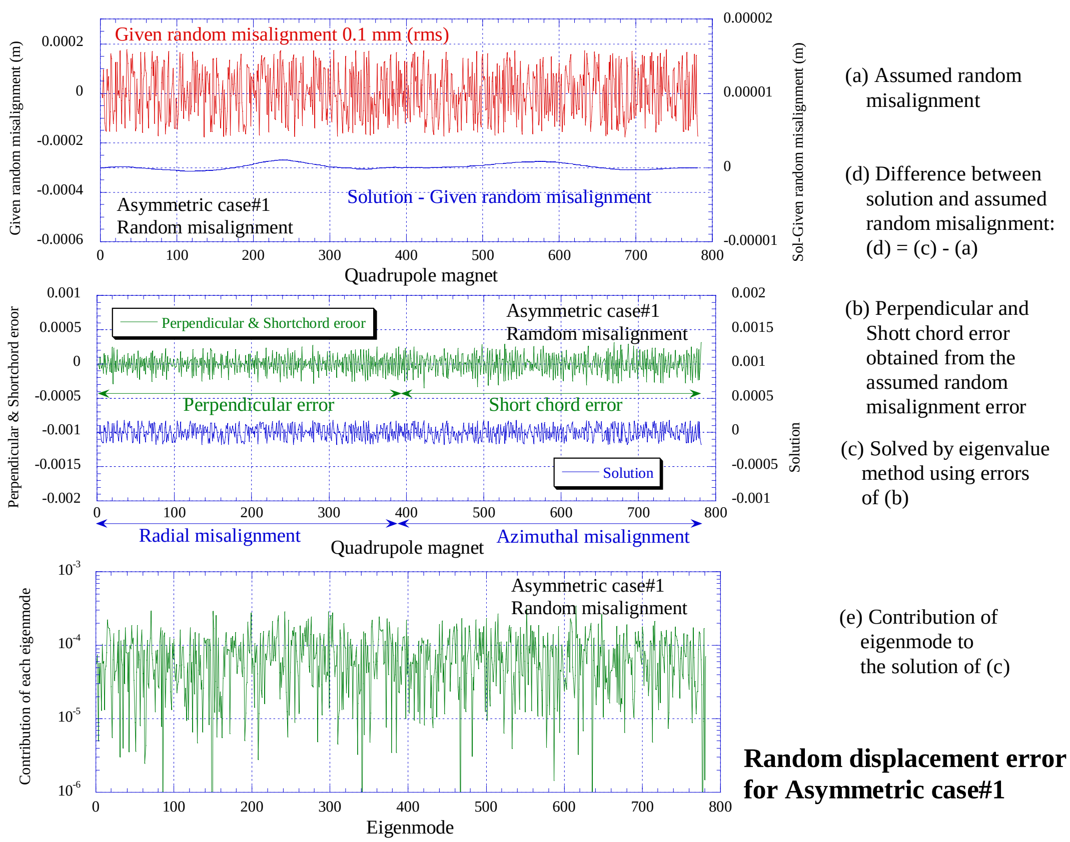
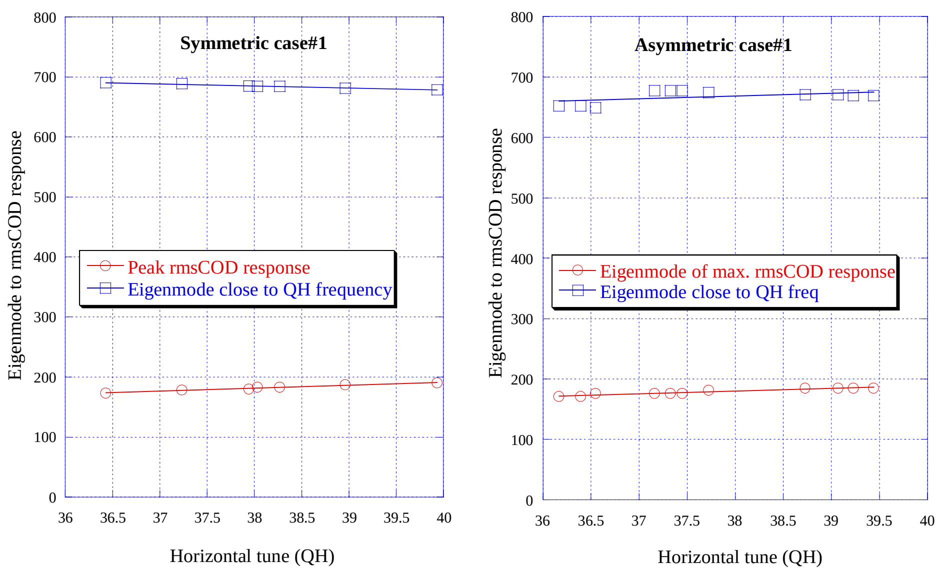


Fig.21 Random misalignment of 0.1 mm (rms) is assumed for the asymmetric case#1. Most eigenmodes contribute equally to the misalignment and it also gives the similar difference between the assumed and calculated misalignment.

4.3 Effect of the betatron tunes to rms COD

The peaks of the rms COD response shown in Fig.4, Fig.8, Fig.12 and Fig.15 shift, if the horizontal betatron frequency changes. Examples for the asymmetric and symmetric case#1 are given in Fig.22. The sharp rms COD response peaks are obtained for the symmetric cases which are better to get the relation between the betatron tune and eigenmode. If the horizontal tune increases, the maximum rms COD response peaks shift towards larger eigenmode number which has smaller eigenvalue and the rms COD response peaks in the symmetric case#1 close to the horizontal betatron tune shift to the smaller eigenmode number which has larger undulation frequency.



(a) Symmetric case#1

(b) Asymmetric case#1

Fig.22 Dependence of the rms COD response peaks on the horizontal betatron tune for the symmetric and asymmetric case#1.

5. CONCLUSION

As the eigenvectors of the matrix which is derived for the survey network correspond to the components of the misalignment, it is possible to estimate their effects on COD by treating each eigenvector as a misalignment mode in the beam optics code. In this study the following results were obtained.

1) There are some peaks in the rms COD response spectrum corresponding to the eigenmodes of which sinusoidal undulation numbers are close to or multiple of the horizontal betatron oscillation frequency. These peaks shift depending on the horizontal tune.

2) For the inhomogeneous quadrupole configuration, there are many rms COD response peaks, whereas for the homogeneous one only few peaks appear.

3) In practice, the misalignment modes with relatively large rms COD response should be corrected. Considering the possible extent of the betatron tune variation, the misalignment correction should be made for all eigenmodes that the large rms COD response is expected.

4) Lower modes, having small eigenvalue which have small number of the sinusoidal undulations, are inevitably introduced through the reduction of the rank when obtaining non-singular matrix, but these modes can be left uncorrected because their effects on the rms COD response are very small.

5) The largest peak of the rms COD response corresponds to the undulation number close to 4 times of the horizontal betatron frequency. This misalignment pattern will fit well to the misalignment mode of which undulation number is close to the horizontal betatron frequency.

6. REFERENCES

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