ON THE INFLUENCE OF THE MEASUREMENT BIASES AT THE ACCELERATOR ALIGNMENT

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At the construction and alignment of the circular and linear accelerators of charged particles a primary task of geodesy is to ensure an accurate mutual location of magnetic element chain. As a rule, when designing the geodetic work, an evaluation of accuracy is based on the analysis of action of random errors of measurement. Herewith a priori one expects an absence or negligible values of biases (values of systematic errors). But the biases are, certainly, present in measurements, and omitting by their values can lead to irreparable consequences, particularly, large amount of measurements at the construction and alignment of modern large accelerators.

Let us estimate an influence of the measurement bias in comparison with the influence of random errors on the standard geodetic measurement scheme.

Ensuring an precision mutual location for the accelerator magnetic elements suggests a strategy of measurements, in which a basic principle is a precise position measurement of each element with respect to two nearby. Then by results of measurements, the element chain is aligned up on design curve or straight line, corresponding to design line of particle trajectory.

Such measurements are described by the following scheme (see figure 1). The straight line or curve is leant on two reference points. The points, corresponding to quadrupole element centres, for instance, are based between the reference points. The purpose of measurements is to measure the relative position of internal points and to calculate their position from the design line.

According to results of calculations, the quadrupole centres are moved to the design line.

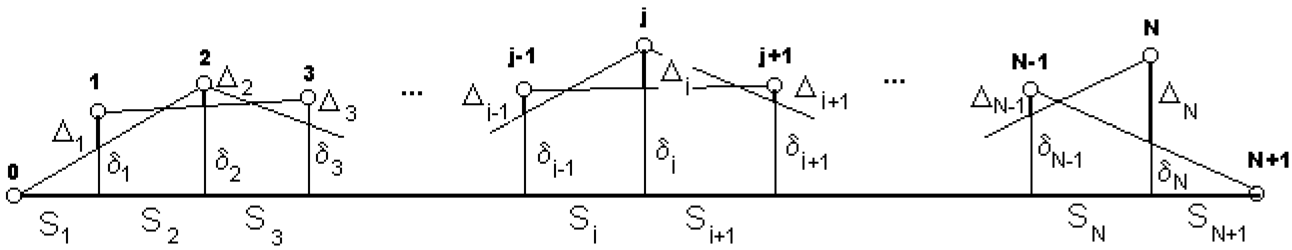

Figure 1. The measurement scheme of the mutual position of quadrupoles.

If amount of internal points is N, the reference points are marked as 0 and N+1. The order of measurements is the following: measure the position Δ_1 of point 1 with respect to the straight line, passing through points 0-2, then get the position Δ_2 of point 2 with respect to the straight line 1-3, position Δ_3 of point 3 with respect to straight line 2-4 and so on until the measurement of position Δ_N of point N with respect to straight line (N-1) - (N+1).

The vector of distances δ of each point to the design line 0 – (N+1) is calculated with help of vector of measured relative positions of internal points Δ by solving a system of equations:

$$A\delta = \Delta, \tag{1}$$

where

$$\Delta = \begin{pmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \cdots \\ \Delta_N \end{pmatrix} \qquad \delta = \begin{pmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \cdots \\ \delta_N \end{pmatrix}$$

$$A =$$

$$\begin{pmatrix}
1 & -a_1 & 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 \\
-(1-a_2) & 1 & -a_2 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 \\
0 & -(1-a_3) & 1 & -a_3 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
0 & 0 & 0 & 0 & \ldots & -(1-a_j) & 1 & -a_j & \ldots & 0 & 0 \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & -(1-a_N) & 1
\end{pmatrix}$$

$$a_1 = \frac{S_1}{S_1 + S_2}, \ldots, a_j = \frac{S_j}{S_j + S_{j+1}}, \ldots, a_N = \frac{S_N}{S_N + S_{N+1}}.$$

If distances between the internal points along the design line are equal to each other $S_1 = S_2 = S_3 = \ldots = S_N$, the matrix of coefficients will take the following form:

$$A = \begin{pmatrix}
1 & -0{,}5 & 0 & 0 & 0 & \ldots & 0 & 0 \\
0 & -0{,}5 & 1 & -0{,}5 & 0 & \ldots & 0 & 0 \\
0 & 0 & -0{,}5 & 1 & -0{,}5 & \ldots & 0 & 0 \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
0 & 0 & 0 & 0 & -0{,}5 & 1 & -0{,}5 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -0{,}5 & 1
\end{pmatrix} \tag{2}$$

Values δ are found by solving the equation system (1):

$$\delta = A^{-1}\Delta.$$

Solution of the equation system for point j will be written in the following form [1]:

$$\delta_j = \frac{2}{N+1}\left[\sum_{K=1}^{K=j-1}(N-j+1)K\Delta_K + \sum_{K=j}^{K=N}(N-K+1)j\Delta_K\right] \quad (3)$$

Let us use the formula (3) for the evaluation of the influence of random errors and biases on the position of the central internal point: $j = \frac{N+1}{2}$.

For the central point the formula (3) after easy transformations takes the following form:

$$\delta_j = \left[\sum_{K=1}^{K=j-1} K\Delta_K + \sum_{K=j}^{K=N}(N-K+1)\Delta_K\right]. \quad (4)$$

<u>The accumulation of biases.</u>

Let us find the position error of central point because of the influence of measurement biases by substituting for values Δ_K the values of their measurement biases d_K in the expression (4). For the simplicity we put all biases equal to each other: $d_1 = d_2 = d_3 = \ldots = d_N = d$. In this case after some transformations we will find the position error of central point:

$$D_M = d\left[\sum_{K=1}^{K=j-1} K + \sum_{K=j}^{K=N}(N-K+1)\right]$$

The above expression can be easily rewritten as:

$$D_M = d(1+2+3+\ldots+N),$$

or finally:

$$D_M = \frac{N(N+1)}{2}d. \quad (5)$$

<u>The accumulation of random errors.</u>

The formula (4) for random measurement errors with r.m.s m_K has the following form:

$$M_K^2 = \left[\sum_{K=1}^{K=j-1} K^2 m_K^2 + \sum_{K=j}^{K=N}(N-K+1)^2 m_K^2\right].$$

At equal conditions of measurements one can consider that r.m.s. values of random measurement errors are equal to each other: $m_1 = m_2 = m_3 = \ldots = m_N = m$. In this case after some transformations we get:

$$M_M = m\sqrt{(1^2+2^2+3^2+\ldots+N^2)},$$

or finally:

$$M_M = m\sqrt{\frac{N(N+1)(2N+1)}{6}} \qquad (6)$$

The position error of central point (magnetic element) of accelerator section was calculated by final formulas (5) and (6) for the different amount of internal points (magnetic elements). The r.m.s. value of random errors and bias value have put equal 0.1 mm: $d = m = 0{,}1$ мм. The calculation results are presented in table and in figure 2.

The influence of random error and biases on the position error of central point of accelerator section

Amount of internal points, N	Random errors M_M, mm	Biases D_M, mm
1	0	0
5	1	2
10	2	6
15	4	12
20	5	21
25	7	33
30	10	47
35	12	63
40	15	82
45	18	104

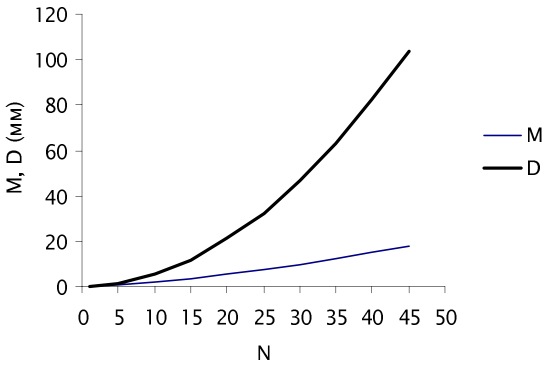

Figure 2. The compare of action of biases and random errors.

The results of calculations show that accumulation of biases in the centre of section essentially exceeds an accumulation of random errors. We should expect it. Never the less, the influence of small value biases leads to a smooth geometry distortion of particles orbit that is not so critical for the accelerator functioning. In contrast to biases, random errors, though with a small probability, can cause rather sharp mutual displacements of magnets from the design position. So the influence of random errors is more critical and methods of measurements are exactly directed on their value restriction.

However, we see, that small value of bias can lead to a significant position error in the central section point. In this case, the trajectory, assigned by magnetic element centres, can simply be beyond the operation limits of adjustment system. This limits, as a rule, are short, particularly at magnet movement in the plan.

So, designing a strategy measurement at the accelerator construction and alignment one should undertake all possible cares to reduce values of biases: tend to reducing a number of measuring absolute values, produce measurements under favourable temperature modes, create such conditions of measurements, which excluding the lateral refraction of sight ray and etc.

BIBLIOGRAPHY

1. Gorelov V.A. Analysis and processing the results of high precision range measurements. Analytical review. CNIIatominform, MOSCOW, 1976.