

Measurement of the Muon $(g - 2)$ -Value

B. Lee Roberts on behalf of the Muon $(g - 2)$ Collaboration ^{a * †}

^aDepartment of Physics, Boston University,
590 Commonwealth Avenue, Boston, MA 02215 USA

The muon $(g - 2)$ experiment is described, and the recent results are presented. These results represent the final measurement for the positive muon.

1. Introduction

The measurement of magnetic moments has been important in advancing our knowledge of sub-atomic physics since the famous 1921 paper of Stern,[1] which laid out the principles of what we now call the ‘‘Stern-Gerlach experiment’’. The experimental and theoretical developments in the study of the electron’s anomalous magnetic moment represent one of the great success stories of modern physics. The experimental accuracy has reached a relative accuracy of ~ 4 parts in 10^9 (parts per billion),[2] and the theory is constrained by our knowledge of the fine-structure constant α , rather than by the eight-order and tenth-order QED calculations.[3]

*R.M. Carey, E. Efsthadiadis, M.F. Hare, X. Huang, F. Krienen, A. Lam, I. Logashenko, J.P. Miller, J. Paley, Q. Peng, O. Rind, B.L. Roberts, L.R. Sulak, A. Trofimov - Boston University; G. Bennett, H.N. Brown, G. Bunce, G.T. Danby, R. Larsen, Y.Y. Lee, W. Meng, J. Mi, W.M. Morse, D. Nikas, C. Özben, R. Prigl, Y.K. Semertzidis, D. Warburton - Brookhaven National Laboratory; Y. Orlov - Cornell University; K. Jungmann -KVI Groningen; A. Grossmann, P. von Walter, G. zu Putlitz - University Heidelberg; P.T. Debevec, W. Denninger, F. Gray D.W. Hertzog, C. Onderwater, C. Polly, M. Sosong, D. Urner, - University of Illinois, Urbana Campaign; A. Yamamoto - KEK B. Bousquet, P. Cushman, L. Duong, S. Giron, J. Kindem, I. Kronkvist, R. McNabb, T. Qian, P. Shagin - University of Minnesota; V.P. Druzhinin, G.V. Fedotovitch, D. Grigoriev, B.I. Khazin, N.M. Ryskulov, Yu.M. Shatunov, E. Solodov - Budker Institute; M. Iwasaki - Tokyo Institute of Technology; H. Deng, M. Deile, S.K. Dhawan, F.J.M. Farley, M. Grosse-Perdekamp, V.W. Hughes, D. Kallow, J. Pretz, S.I. Redin, E. Sichtermann, A. Steinmetz - Yale University

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The gyromagnetic ratio g is defined by

$$\vec{\mu}_s = g\left(\frac{e}{2m}\right)\vec{s}, \quad (1)$$

where \vec{s} is the spin angular momentum, and $\vec{\mu}$ is the magnetic moment resulting from this angular momentum. The Dirac equation for point particles predicts that $g \equiv 2$, but radiative corrections increase the value at the part per mil level. The magnetic moment μ is defined as $\mu = (1 + a)e\hbar/2m$, where $a = (g - 2)/2$ is the anomalous magnetic moment (or simply the anomaly).

When E821 began in the early 1980s, a_μ was known to 7.3 parts per million (ppm).[4] The E821 Collaboration has reported four new measurements with relative accuracies of 13, 5, 1.3 and 0.7 ppm respectively.[5,6,7,8]

To the level of the experimental accuracy, the electron anomaly can be described by the QED of e^\pm and photons, with the contribution of heavier virtual particles entering at a level of around 3 ppb. The larger mass of the muon permits heavier virtual particles to contribute, and the enhancement factor is $\sim (m_\mu/m_e)^2 \sim 40,000$. The CERN measurement observed the effect on a_μ of virtual hadrons at the 8 standard deviation level.[4] The standard model value of a_μ consists of QED, strong interaction and weak radiative corrections, and a significant deviation from the calculated standard model value would represent a signal for non-standard model physics. At this conference we have heard much discussion of the theory of $(g - 2)$,[9,10,11,12] so it will not be discussed further in this talk.[13]

2. The Experimental Technique

The method used in the third CERN experiment and the BNL experiment are very similar, save the use of direct muon injection into the storage ring which was developed by the E821 collaboration. These experiments are based on the fact that for $g \neq 2$ (or more precisely $a_\mu > 0$) the spin precesses faster than the momentum vector when a muon travels transversely to a magnetic field. The Larmor and Thomas spin-precession and the momentum precession frequencies are

$$\omega_S = \frac{geB}{2mc} + (1 - \gamma) \frac{eB}{\gamma mc}; \quad \omega_C = \frac{eB}{mc\gamma} \quad (2)$$

and the difference frequency gives the frequency with which the spin precesses relative to the momentum,

$$\omega_a = \omega_S - \omega_C = \left(\frac{g-2}{2}\right) \frac{eB}{mc} \quad (3)$$

which is proportional to the anomaly, rather than to the full magnetic moment. A precision measurement of a_μ requires precision measurements of the precession frequency ω_a and the magnetic field, which is expressed as the free-proton precession frequency ω_p in the storage ring magnetic field.

The muon frequency can be measured as accurately as the counting statistics and detector apparatus permit. The design goal for the NMR magnetometer and calibration system was a field accuracy of about 0.1 ppm. The B which enters in Eq. 3 is the average field seen by the ensemble of muons in the storage ring, $\langle B \rangle_\phi = \frac{1}{\pi R^2} \int_0^R \int_0^{2\pi} M(r, \theta) B(r, \theta) r dr d\theta >_\phi$ where ϕ is the azimuthal angle around the ring, r, θ are the coordinates at a single slice of azimuth centered at the middle of the 90 mm diameter muon storage region. $M(r, \theta)$ is the moment (multipole) distribution of the muon beam, and couples multipole by multipole with the magnetic field multipoles. In the analysis, the field is averaged over the data collection time as well.

The need for vertical focusing implies that a gradient field is needed, but the usual magnetic gradient used in storage rings is ruled out in our case. A sufficient magnetic gradient for vertical

focusing would spoil the ability to use NMR to measure the magnetic field to the necessary accuracy. Furthermore, it is very difficult to obtain adequate information on the higher moments of the muon distribution in the storage ring, so the presence of higher multipoles in $\langle B \rangle$ is also undesirable for this reason. A round beam-profile was chosen, since sharp corners would imply large higher moments for $M(r, \theta)$.

An electric quadrupole is used for vertical focusing, taking advantage of the “magic” $\gamma = 29.3$ at which an electric field does not contribute to the spin motion relative to the momentum. In the presence of an electric and a magnetic field, the spin difference frequency is given by

$$\vec{\omega}_a = \frac{e}{mc} \left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \vec{\beta} \times \vec{E} \right], \quad (4)$$

which reduces to Eq. 3 in the absence of an electric field. Note that for muons with $\gamma = 29.3$ in an electric field alone, the spin would follow the momentum vector.

The arrangement of a magnetic dipole field combined with an electric quadrupole field is called a Penning trap in atomic physics. However with a 14 m diameter and ~ 700 T weight, the scale of our trap is quite different from the usual one.[2]

In order to meet the conditions discussed above, a goal of ± 1 ppm uniformity of the $\langle B \rangle$ -field over the storage region was set and met. Given the projected knowledge of the muon distribution, the allowable strength of the quadrupole and higher magnetic multipoles was also determined.

A kick of about 0.1 Tm is needed to bring the beam onto a stable orbit. This is achieved with three 1.7 m long ferrite-free kickers,[15] which can be thought of as single-loop pulsed magnets carrying a current of 4,200 A. The minimum inductance achievable of 1.6 μ H limited the peak current to 4200A, and resulted in a current-pulse width ~ 2.5 times greater than the cyclotron period of 150 ns. The phase space mis-match between the size of the inflector exit and the storage region, and multiple scattering in the inflector end, reduces the calculated injection efficiency to $\sim 8.7\%$. The less than optimal kicker pulse

further reduces the injection efficiency to about 7.3%. Nevertheless the effective data rate per fill is almost a factor of 100 over that available in the final CERN experiment,[4] a factor of 10 coming from muon injection, and a factor of 10 coming from the AGS intensity. With direct muon injection, the injection-related background seen by the detectors is down by a factor of 50.[5,6]

The $(g - 2)$ ring functions as a weak focusing storage ring with the field index

$$n = \frac{\kappa R_0}{\beta B_0}, \quad (5)$$

where κ is the electric quadrupole gradient. Several n - values were used for data acquisition: $n = 0.137, 0.142$ and 0.122 , the latter two having been used for μ^- . The horizontal (radial) and vertical betatron frequencies are (approximately) given by

$$f_x = f_C \sqrt{1 - n} \simeq 0.93 f_C; \quad f_y = f_C \sqrt{n} \simeq 0.37 f_C \quad (6)$$

where f_C is the cyclotron frequency and the numerical values assume $n = 0.137$.

The experimental signal is the e^\pm from μ^\pm decay, which were detected by lead-scintillating fiber calorimeters.[17] The time and energy of each event was stored for analysis offline. Muon decay is a three-body decay, so the 3.1 GeV muons produce a continuum of positrons (electrons) from the end-point energy down. Since the highest energy e^\pm are correlated with the muon spin, if one counts high energy e^\pm as a function of time, one gets an exponential from muon decay modulated by the $(g - 2)$ precession. The expected form for the positron time spectrum is

$$f(t) = N_0 e^{-\lambda t} [1 + A \cos(\omega_a t + \phi)] \quad (7)$$

However, a Fourier analysis of the residuals from this five parameter fit (see Fig. 1) shows a number of frequency components which can be understood from the beam dynamics in the ring.[8,18] The most prominent frequencies in the residuals come from the coherent oscillation of the beam.

While the frequencies given in Eq. 6 describe the motion of a single beam particle, the aperture mis-match between the inflector exit and the storage aperture, along with an imperfect kick and

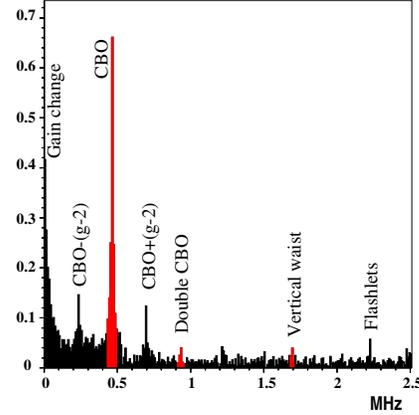


Figure 1. A Fourier Transform of the fit residuals from a 5-parameter fit to the 1999 data set. To adequately describe the data, additional parameters were necessary (see Eq. 9).

momentum dispersion in the ring, will cause the beam to undergo coherent radial oscillations. The detector acceptance depends on the radial position of the muon when it decays, so any coherent radial beam motion will amplitude modulate the decay e^\pm distribution. The principal frequency will be the “Coherent Betatron Frequency”

$$f_{CBO} = f_C - f_x = (1 - \sqrt{1 - n}) f_C \quad (8)$$

which is the frequency a single fixed detector sees the beam moving coherently back and forth. This motion can be understood by the cartoon in Fig. 2. It is this CBO frequency and its sidebands from beating with the $(g - 2)$ frequency which are clearly visible in the Fourier spectrum of Fig. 1. $n = 0.137$ was chosen to avoid storage ring resonances, however the resulting CBO frequency was close to the second harmonic of $f_a (= \omega_a / 2\pi)$ putting the difference sideband close to f_a . The n -value was changed for the 2001 data collection period to reduce our sensitivity to this difficulty.

The CBO modified the positron time spectrum by

$$N_p = N_0 e^{-\frac{t}{\tau}} (1 + A' \sin(\omega_a t + \phi')) \times (1 + A_{CBO}(t) \cos(\omega_{CBO} t + \phi_{CBO})), \quad (9)$$

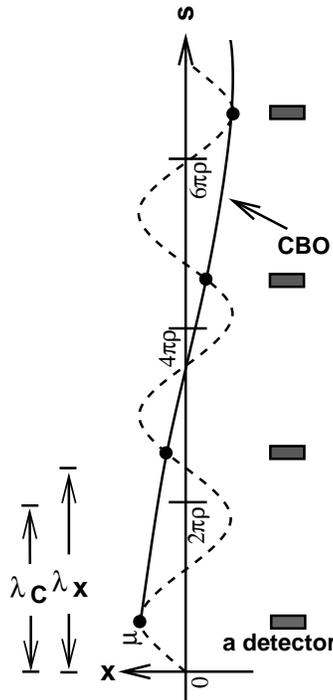


Figure 2. A cartoon showing the beam motion (dashed line), and the apparent motion (solid line) seen by a single detector. The vertical axis is distance along the storage ring (shown for 4 turns) and the horizontal axis shows the radial betatron amplitude. $\lambda_C = 2\pi\rho$ is the cyclotron wavelength, λ_x is the radial betatron wavelength, and ρ is the central orbit radius.

where

$$A' = A(1 + A_1(t) \cos(\omega_{CBO}t + \phi_1)) \quad (10)$$

$$\phi' = \phi(1 + A_2(t) \cos(\omega_{CBO}t + \phi_2)) \quad (11)$$

The terms A_1 and A_2 can cause artificial shifts in a_μ up to 4 ppm in individual detectors when not accounted for in the fitting procedure. The final fitting procedure included these, and other effects such as pulse pile-up, detector gain changes and muon losses from the ring other than by decay.

3. Results and Conclusions

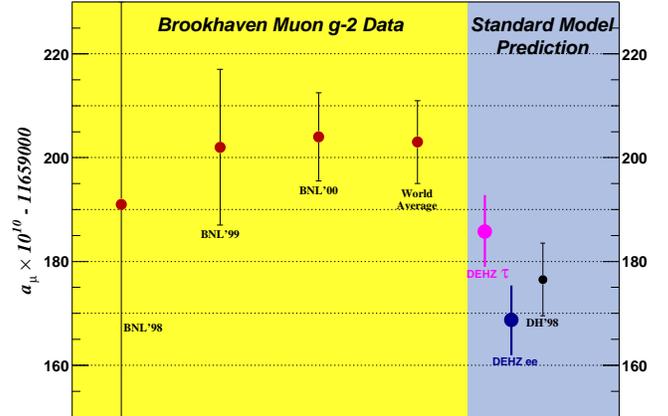


Figure 3. E821 measurements of a_μ carried out with direct muon injection into the storage ring. The relative uncertainties are ± 5 ppm (98), ± 1.3 ppm (99), ± 0.7 ppm (00), where the number in parentheses is the year when the data were collected. All measurements are for μ^+ . The three theory points use the lowest order hadronic contribution from DH98[19] and the separate DEHZ evaluations[13] from τ and e^+e^- hadronic data. The DH98 evaluation includes both τ and e^+e^- data.

After consistent results were obtained in four independent (and three different) analysis procedures for ω_a , and two independent studies of ω_p , the offsets were removed and the value

$$a_{\mu^+} = 11\,659\,204(7)(5) \times 10^{-10} \quad (0.7 \text{ ppm}) \quad (12)$$

was obtained.[8] Excellent agreement was found with the previous measurements from Brookhaven and from CERN.

The E821 results are displayed graphically in Fig. 3 along with the Davier-Höcker(98)[19] first order hadronic contribution, and the most recent results from τ -decay and e^+e^- annihilation reported at this conference.[10,11,12,13]

Now that the τ -decay value of $a_\mu(\text{Had}; 1)$ disagrees with the value obtained from electron positron annihilation, the situation is somewhat confused. While the path from hadronic electron-positron annihilation to a value of $a_\mu(\text{Had}; 1)$ is somewhat more theoretically direct than from hadronic tau decay, the extraction of $a_\mu(\text{Had}; 1)$ from tau-decay data has been carefully studied, with all the expected effects included.

However, as we have heard at this meeting, there are systematic differences between other quantities when e^+e^- annihilation and τ -decay are compared. Since the low-energy e^+e^- data are dominated by the recent precise data from Novosibirsk, it is fortunate that further work on e^+e^- -annihilation is being carried out at Frascati, Belle and BaBar, and they will either confirm or disagree with the recent Novosibirsk data. Comparison with the new DEHZ evaluations[13] shows either a 1.6 or 3 standard deviation discrepancy. An independent analysis[20] of the e^+e^- data agrees well with the equivalent DEHZ analysis. Thus the significance for an indication of new physics will have to wait for clarification of the correctness of the hadronic contribution.

Nevertheless, a recent very conservative evaluation of the impact of $(g-2)$ on the constraining of supersymmetry parameters shows that even with the current uncertainties, $(g-2)$ already rules out a “substantial region of (susy) parameter space... that has not been probed by any previous experiment”.[21]

The E821 collaboration has one additional data set, which was taken with μ^- in the ring. The experimental uncertainty will be on the order of 0.8 ppm. Obviously we will have to stay tuned for further clarification on the hadronic contribution, and the analysis of the μ^- data.

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