

# The Strange Quark Mass, $\alpha_s$ , and the Chiral Limit Electroweak Penguin $K \rightarrow \pi\pi$ Matrix Elements From Hadronic $\tau$ Decay Data

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Hadronic  $\tau$  decay data provides access to the light quark vector (V) and axial vector (A) spectral functions. This makes possible investigations of the dynamics of QCD at intermediate scales and improved determinations of certain QCD/Standard Model parameters. We discuss three such applications: (1) the investigation of the nature of duality violation in QCD at intermediate scales (and its relation to the determination of  $\alpha_s$ ); (2) the extraction of  $m_s$  from flavor-breaking differences of  $ud$  and  $us$  data; and (3) the determination of the dimension  $D = 6$  term in the OPE for the flavor  $ud$  V-A correlator difference. The latter is relevant to the evaluation of the chiral limit values of the  $K \rightarrow \pi\pi$  electroweak penguin matrix elements, and hence to our understanding of expectations for  $\epsilon'/\epsilon$  in the Standard Model.

## 1. INTRODUCTION/BACKGROUND

As is well known, the ratio,  $R_{V/A;ij}$ , of the flavor  $ij = ud, us$  vector (V) or axial vector (A) current-mediated hadronic  $\tau$  decay rate to the corresponding electron decay rate,

$$R_{V/A;ij} = \frac{\Gamma[\tau^- \rightarrow \nu_\tau \text{ hadrons}_{V/A;ij}(\gamma)]}{\Gamma[\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e(\gamma)]}, \quad (1)$$

(with  $(\gamma)$  denoting extra photons and/or lepton pairs) can be written as a sum of weighted integrals over the spin  $J = 0$  and 1 hadronic spectral functions[1,2]. Explicitly,

$$\begin{aligned} \frac{R_{V/A;ij}}{[12\pi^2|V_{ij}|^2 S_{EW}]} &= \int_0^1 dy_\tau (1 - y_\tau)^2 \\ &\left[ (1 + 2y_\tau) \rho_{V/A;ij}^{(0+1)}(s) - 2y_\tau \rho_{V/A;ij}^{(0)}(s) \right] \\ &\equiv \int_0^{m_\tau^2} ds \frac{dR_{V/A;ij}(s)}{ds} \end{aligned} \quad (2)$$

where  $y_\tau = s/m_\tau^2$ ,  $V_{ij}$  is the flavor  $ij$  CKM matrix element,  $S_{EW}$  is an electroweak correction, and  $\rho_{V/A;ij}^{(J)}(s)$ , with  $(J)$  the spin of the hadronic system, is the spectral function of the corresponding spin  $(J)$  part of the flavor  $ij$  V/A correlator,

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$\Pi_{V/A;ij}^{(J)}(s)$ .<sup>2</sup> The  $\Pi_{V/A;ij}^{(0,1)}(s)$  are defined by

$$\begin{aligned} i \int d^4x e^{iq \cdot x} \langle 0 | T \left( J_{V/A}^\mu(x) J_{V/A}^\nu(0) \right) | 0 \rangle = \\ (q^\mu q^\nu - q^2 g^{\mu\nu}) \Pi_{V/A}^{(1)}(q^2) + q^\mu q^\nu \Pi_{V/A}^{(0)}, \end{aligned} \quad (3)$$

where  $J_{V/A}^\mu$  are the standard V and A currents. For a given channel, the kinematically-weighted linear combination of  $(J) = (0 + 1)$  and  $(0)$  spectral functions appearing in Eq. (2) can thus be extracted from the corresponding bin-by-bin experimental decay distribution.

For the V channel,  $\rho_{V;ij}^{(0)}$  is proportional to  $(m_i - m_j)^2$ , and hence numerically negligible for  $ij = ud$ . For the A channel,  $\rho_{A;ij}^{(0)}$  is saturated by the flavor  $ij$  Goldstone boson pole in the chiral limit and has non-Goldstone-boson contributions proportional to  $(m_i + m_j)^2$ . Apart from the  $\pi$  pole contribution,  $\rho_{A;ud}^{(0)}$  is, thus, also numerically negligible.  $m_s$  is not sufficiently small that  $\rho_{V;us}^{(0)}$  and the non- $K$ -pole contributions to  $\rho_{A;us}^{(0)}$  can be safely neglected. The spin separation of the  $\rho_{V/A;ij}^{(0)}(s) + \rho_{V/A;ij}^{(1)}(s) \equiv \rho_{V/A;ij}^{(0+1)}(s)$ . This combination, and also  $s\rho_{V/A;ij}^{(0)}(s)$ , correspond to scalar correlators,  $\Pi_{V/A;ij}^{(0+1)} \equiv \Pi_{V/A;ij}^{(0)} + \Pi_{V/A;ij}^{(1)}$  and  $s\Pi_{V/A;ij}^{(0)}$ , having no kinematic singularities.

$us$  experimental data is straightforward for the  $K$  and  $K^*$  contributions, but not yet available for the data above the  $K^*$ . Flavor  $ud$  V/A separation for states consisting only of pions can be accomplished using G-parity.

Cauchy's theorem, together with analyticity, implies that correlators  $\Pi$  having no kinematic singularities satisfy finite energy sum rules (FESR's). For weight functions,  $w(s)$ , analytic in the region  $|s| < S$  of the complex- $s$  plane, and any  $s_0 < S$ , these have the form

$$\int_0^{s_0} ds \rho(s) w(s) = \frac{-1}{2\pi i} \oint_{|s|=s_0} ds \Pi(s) w(s). \quad (4)$$

For  $s_0$  large enough that  $\Pi$  on the RHS of Eq. (4) may be approximated by its OPE, one obtains a relation between spectral data and OPE parameters. The well-known predictions for the inclusive flavor  $ij = ud, us$  hadronic  $\tau$  decay widths in terms of the parameters occurring in the OPE of the corresponding V and A correlators (dominantly  $\alpha_s$ ) [1],

$$R_{V/A;ij} = 6\pi i S_{EW} |V_{ij}|^2 \oint_{|y|=1} dy (1-y)^2 \times \left[ (1+2y) \Pi_{V/A;ij}^{(0+1)}(s) - 2y \Pi_{V/A;ij}^{(0)}(s) \right] \quad (5)$$

are examples of such a relation. The modified version of this relation obtained by multiplying the integrands in both the spectral and OPE representations of  $R_{V/A;ij}$  by the factor  $(1-y_\tau)^k y_\tau^m$  is called "the  $(k, m)$  spectral weight sum rule" [3]. Such sum rules have the advantage that a spin separation of the  $ij = us$  spectral data is not required in order to evaluate their spectral sides.

The spectral integrals required for various V and A FESR's can be evaluated, using hadronic  $\tau$  decay data, for any  $s_0 \leq m_\tau^2$ . So long as there exists a window of  $s_0$  values below  $m_\tau^2$  for which the breakdown of the OPE representation of the relevant correlator ("duality violation") is negligible, the values of basic QCD parameters, such as  $\alpha_s$  and  $m_s$ , appearing on the OPE side of the FESR relation can be determined using spectral integral data. The existence of such a window of  $s_0$  values is, of course, crucial to the reliability of such a determination. We will concentrate on three analyses of this type below.

The rest of the paper is organized as follows. In Section 2 we discuss the issue of duality violation, and the closely related issue of the determination of  $\alpha_s$ , using the very precise  $ud$  V+A spectral data. In Section 3, we discuss some complications, and the current status, of attempts to extract  $m_s$  from the flavor-breaking difference of  $ud$  and  $us$  V+A correlators. In Section 4, we discuss some recent work on the determination of the dimension 6 term in the OPE of the  $ud$  V-A correlator. This quantity bears a special relation to the chiral limit values of the  $K \rightarrow \pi\pi$  electroweak penguin (EWP) operator matrix elements, and hence to expectations for the value of  $\epsilon'/\epsilon$  in the Standard Model. Finally, in Section 5, we consider what improvements in these determinations may be possible in the near future.

## 2. DUALITY VIOLATION AND THE EXTRACTION OF $\alpha_s$

For very large  $s_0$ , the OPE is expected to provide a reliable representation of hadronic correlators over the entire circle  $|s| = s_0$  in the complex  $s$ -plane. For such  $s_0$ , the corresponding spectral function,  $\rho(s_0)$ , will also be well-represented by its OPE form. This is conventionally referred to as the regime of the validity of "local duality" (LD). As one moves to lower  $s_0$ , the arguments of Poggio, Quinn and Weinberg [4] suggest that the OPE will break down first for those  $s$  on  $|s| = s_0$  near the timelike real axis. We thus expect there to exist a regime of "intermediate" scales,  $s_0$ , for which the OPE, while not reliable near the timelike real axis, continues to be reliable over most of the rest of the circle  $|s| = s_0$ . We will refer to the scales for which this is true as the regime of the validity of "semi-local duality" (SLD). Eventually, for sufficiently small  $s_0$ , we expect the OPE to become unreliable over the whole of the circle  $|s| = s_0$ . Were all  $s_0 < m_\tau^2$  to lie in this final regime, it would be impossible to use hadronic  $\tau$  decay data to determine parameters appearing in the OPE; it is therefore important to verify that some of the scales kinematically accessible in  $\tau$  decay lie, at the very least, in the region of validity of SLD.

The flavor  $ud$  V, A and V+A correlators pro-

vide an excellent laboratory for studying the nature of duality violation. The reason is that, for scales  $\sim 2 - 3 \text{ GeV}^2$ , the OPE for these correlators is completely dominated by the  $D = 0$  term, which is known to  $O(\alpha_s^3)$  [5]. Since  $\alpha_s$  is measured with good accuracy at the  $Z$  scale and the QCD  $\beta$ -function is known to 4-loop order [6], so that  $\alpha_s$  can be run down reliably to scales below  $2 \text{ GeV}^2$ , the OPE in these cases is known with good accuracy down to scales significantly below  $m_\tau^2$ . One can, therefore, compare the spectral and OPE integrals for various FESR weight choices and determine the extent to which the OPE provides a good representation of the spectral integral data.

The non-perturbative terms of the OPE relevant to this comparison are [7]

$$\left[ \Pi_{V/A;ud}^{(0+1)} \right]_{D=4} = \frac{(1 - \frac{11}{18}a)}{12Q^4} \langle aGG \rangle + \dots \quad (6)$$

$$\left[ \Pi_{V/A;ud}^{(0+1)} \right]_{D=6} = \frac{\pi\rho\alpha_s\langle\bar{q}q\rangle^2}{Q^6} \left( \frac{(\mp 288 + 64)}{81} \right) \quad (7)$$

where  $a \equiv a(Q^2) = \alpha_s(Q^2)/\pi$ ,  $\rho \equiv \rho_{VSA}$  expresses the deviation of the four-quark condensates from their vacuum saturation approximation (VSA) values,  $+\dots$  in Eq. (6) stands for numerically small terms involving the quark condensates and fourth powers of the light quark masses, and, in Eq. (7), the upper (lower) sign refers to the V (A) case. The dominant  $D = 0$  contribution may be expressed in terms of the Adler function

$$\begin{aligned} D_{V/A}(Q^2) &\equiv -Q^2 \left[ \frac{d\Pi_{V/A}^{(0+1)}}{Q^2} \right]_{D=0} \\ &= \frac{1}{4\pi^2} \sum K_n a(Q^2)^n, \end{aligned} \quad (8)$$

where, in the  $\overline{MS}$  scheme,  $K_0 = K_1 = 1$ ,  $K_2 = 1.63982$ ,  $K_3 = 6.37101$  [5], and the higher  $K_n$  are unknown.

To be specific, we take as OPE input (1)  $\alpha_s(m_\tau) = 0.334 \pm 0.022$  [8,9], (2)  $\langle \frac{\alpha_s}{\pi} G^2 \rangle = (0.009 \pm .015) \text{ GeV}^4$ ,<sup>3</sup> (3)  $\rho_{VSA} = 1 \pm 5^4$  and use

<sup>3</sup>This value encompasses the ranges obtained in three different analyses,  $0.021 \pm 0.03 \text{ GeV}^4$  [10],  $0.006 \pm 0.012 \text{ GeV}^4$  [11] and  $0.009 \pm 0.007 \text{ GeV}^4$  [12].

<sup>4</sup>The VSA-violating parameter,  $\rho_{VSA}$ , for the V-A corre-

$K_4 = 25 \pm 50$  as a means of estimating the  $D = 0$  truncation error [2]. In the V+A case, the size of the VSA  $D = 6$  contribution is quite small, so the conservative error on  $\rho_{VSA}$  has little impact on the full theory errors, which are dominated by the uncertainty in  $\alpha_s(m_\tau)$  and the estimated truncation error. Possible  $D = 8$  and higher terms are neglected. Errors associated with the uncertainty in each of the inputs are combined in quadrature to obtain the total theory error. Spectral integrals are evaluated using the ALEPH data [8,9], which produces the smallest errors<sup>5</sup>. The corresponding errors are computed using the ALEPH covariance matrix. The final errors on the ratios of OPE to data integrals, which provide a measure of duality violation, are obtained by combining theory and experimental errors in quadrature.

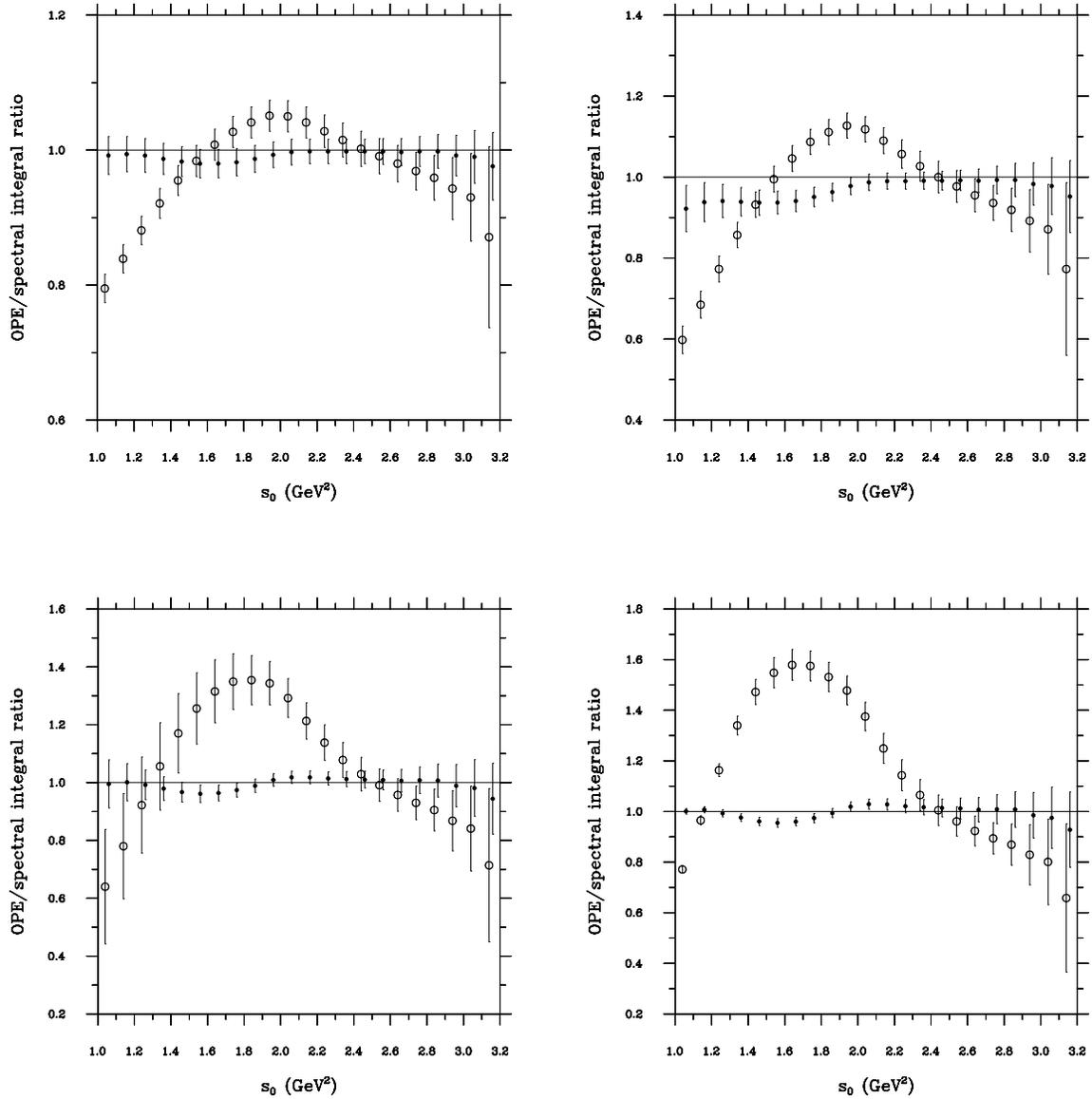
Making the OPE/data comparison first for the V channel, we discover that  $s_0 < m_\tau^2$  [15] is not a region of the validity of LD for the  $ud$  V correlator. Indeed, the OPE and spectral integrals for FESR's based on the weights  $w(s) = s^k$  (which do not suppress contributions from the vicinity of the timelike real axis) are in rather poor agreement. This is illustrated in Figure 1, which displays the OPE-to-spectral-integral-ratio (which should be 1 if LD is valid) as a function of  $s_0$ , for  $k = 0, \dots, 3$ . Also shown are the corresponding results for the  $ud$  V+A combination. One can see that the level of duality violation in the V+A correlator is much smaller than that in the V and A correlators separately.

The strong duality violations in the  $s^k$ -weighted FESR's for the  $ud$  V correlator are not a general feature of FESR's involving this correlator. Indeed, the FESR for  $R_{V;ud}$ , Eq. (5), (whose "kinematic" weight,  $w_\tau(y_\tau) = (1 - y_\tau)^2(1 + 2y_\tau)$ , is a linear combination of the four  $s^k$ -weighted FESR's shown in Figure 1) is known to be in good agreement with experiment [8,9]. The difference has to do with the form

lator is known to be  $\sim 1.6$  [13]. Significant cancellation, however, occurs in the VSA expression for the V+A sum; the expanded error,  $\pm 5$ , on the VSA value  $\rho_{VSA} = 1$  is meant to deal with this in a conservative manner.

<sup>5</sup>See also Refs. [14] for the OPAL data on the  $ud$  V and A spectral functions. The spectral integrals using the OPAL data are in good agreement with those using ALEPH data.

Figure 1. Ratios of OPE to data integrals as a function of  $s_0$  for the flavor  $ud$  V (open circles) and V+A correlators (solid circles). The top left, top right, bottom left and bottom right figures correspond to  $w(s) = 1$ ,  $w(s) = s$ ,  $w(s) = s^2$  and  $w(s) = s^3$ , respectively.



of the weight: because the point  $s = m_\tau^2$ , where the circle  $|s| = m_\tau^2$  crosses the timelike real axis, lies at the edge of hadronic phase space,  $w_\tau$  has a (double) zero at  $s = m_\tau^2$ . This zero suppresses OPE contributions from the part of the integration region near the timelike real axis where use of the OPE is expected to be potentially most problematic. It is presumably this feature of  $w_\tau$  which is responsible for the success of the  $R_{V;ud}$  FESR.

If, as this argument suggests, the success of the  $R_{V;ud}$  FESR is a reflection of the localization of the breakdown of the OPE to the vicinity of the timelike real axis for scales  $s_0 \simeq m_\tau^2$ , then other FESR's with suppressions of OPE contributions from this region should also be well satisfied. This hypothesis was tested in Ref. [15]. FESR's for the V channel were considered for a range of  $s_0$  different from  $m_\tau^2$ , and a range of alternate weights,  $w(s)$ , still satisfying  $w(s = s_0) = 0^6$ . The resulting pFESR's were all found to be well satisfied for all  $s_0 > 2 \text{ GeV}^2$ . This suppression of duality violations, which is a generic feature of weights of the forms

$$w(y) = (1 - y)^2(1 + Ay) \quad (9)$$

$$w(y) = (1 - y)(1 + Ay) , \quad (10)$$

having either a single or double zero at  $s = s_0$  (where  $A$  is a free parameter) can, in favorable cases (including that of the kinematic weight), extend to much smaller  $s_0$ . This is illustrated for the pFESR's based on the weights  $w(y) = (1 - y)^2(1 + 2y)$  and  $w(y) = 1 - y$  in the left and right panels of Fig. 2, respectively. Also shown for comparison in each panel are the results for the corresponding  $s^k$ -weighted (unpinched) FESR's, where  $k$  is the degree of the pinched weight in question. The  $w(y) = 1 - y$  pFESR receives no contributions from the  $D = 6$  OPE term and hence has reduced OPE errors at low  $s_0$ . The absence of duality violation in this case extends to very low  $s_0$ , despite the fact that the weight has only a single zero at  $s = s_0$ .

The results above provide a specific illustration of the dangers of attempting to extract OPE pa-

<sup>6</sup>We refer to such weights as “pinched” weights, and the FESR's based on them as “pinched FESR's”, or pFESR's. The natural variable for use in such pFESR's is  $y \equiv s/s_0$ .

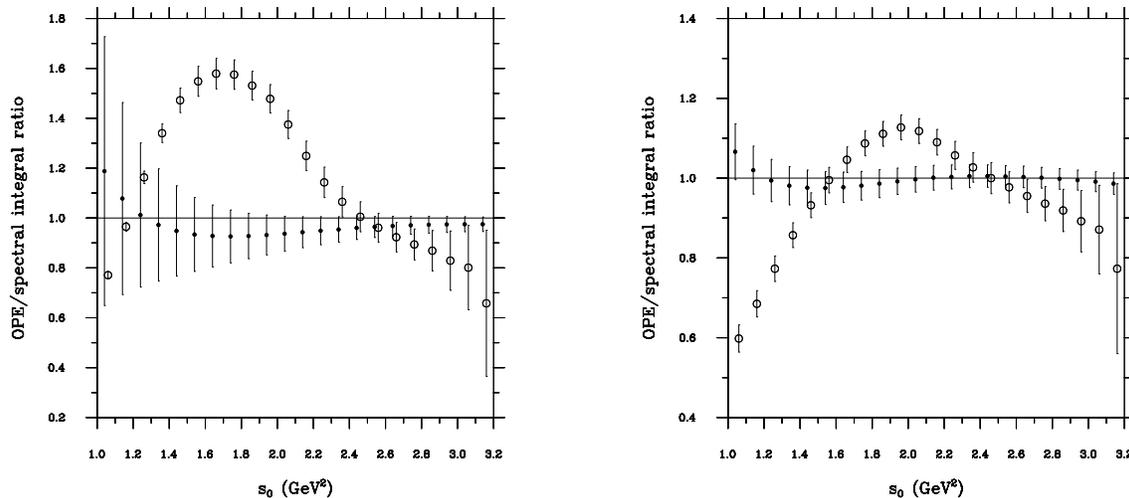
rameters from a sum rule analysis at intermediate scales when duality violations are present. Were one to take a typical scale  $s_0 < m_\tau^2$  and “extract”  $\alpha_s(s_0)$  by matching the OPE and spectral sides of one of the  $s^k$ -weighted FESR's, one would find a value which, when run up to higher scales, failed to agree with that obtained in direct measurements, for example, at the Z scale. In this case we do not actually need to know the high-scale value beforehand in order to expose the presence of duality violation: the  $s_0$ -dependence of the  $\alpha(s_0)$  extracted at different  $s_0$ , but using the same  $s^k$  weight, can be seen to be in very poor agreement with that predicted by 4-loop running in QCD. Similar “internal” tests can be performed for any FESR. Passing such a test is of course a necessary, but not sufficient, condition for demonstrating the absence of duality violations in the analysis in question.

Let us now turn to  $\alpha_s$ . In order to reduce the difficult-to-quantify theoretical systematic error associated with possible residual duality violations, it is useful to work with correlators which display a reduced level of duality violation to begin with. The results of Figure 1 show that the  $ud$  V+A combination is favored in this regard<sup>7</sup>. Similarly, pFESR's are favored over their unpinched analogues at intermediate scales such as those forced on us by  $\tau$  decay kinematics. Since there is still significant uncertainty in the value of the gluon condensate, which dominates the non-perturbative corrections on the OPE side, it is useful to work with pFESR weights which strongly suppress the  $D = 4$  contribution (Eq. (9) with  $A = 2$  and Eq. (10) with  $A = 1$ ).

Bearing in mind the necessity of using the data not only to extract  $\alpha_s$ , but also to verify that duality violations are not obviously present, the following procedure seems optimally conservative. We first extract  $\alpha_s$  using the  $w(y) = (1 - y)^2(1 + 2y)$  pFESR at  $s_0 = m_\tau^2$ . A weight with a double zero and the highest possible value for  $s_0$  are chosen to increase the likelihood that duality violation will be negligible. Neglect of the  $D = 8$  OPE contribution (which is not suppressed

<sup>7</sup>This combination also has reduced  $D = 6$  contributions on the OPE side, and reduced errors for  $s > 2 \text{ GeV}^2$  on the experimental side.

Figure 2. Ratios of OPE to data integrals as a function of  $s_0$  for the pinched and unpinched FESR's involving the  $ud$  V correlator. The left figure shows the results for  $w(y) = (1-y)^2(1+2y)$  (solid circles) and  $w(s) = s^3$  (open circles), the right figure the results for  $w(y) = 1-y$  and  $w(s) = s$ .



by any factor of  $\alpha_s$  for this weight choice, but which scales as  $1/s_0^3$ ) is also most reliable for the highest possible  $s_0$ . Using the contour improved perturbation theory (CIPT) scheme [16,3] for the  $D = 0$  contribution, the result is

$$\alpha_s(m_\tau) = 0.345 \pm 0.026 \quad (11)$$

where experimental and theoretical errors have been combined in quadrature. If instead of CIPT one employs fixed order perturbation theory (FOPT), the central value is reduced to  $0.326^8$ . The method, and results, are similar to those of earlier analyses [17,18,8,9,14].

Having extracted  $\alpha_s$  using only one pFESR, and one value of  $s_0$ , we can then use other pFESR's and other  $s_0$  values to check for possible duality violation and/or the presence of neglected higher dimension contributions<sup>9</sup>. Here we compare the OPE predictions (obtained using the

<sup>8</sup>FOPT involves an expansion in  $\alpha_s(s_0)$ , CIPT a summing of logs point-by-point around the integration contour by the scale choice  $\mu^2 = Q^2$ .

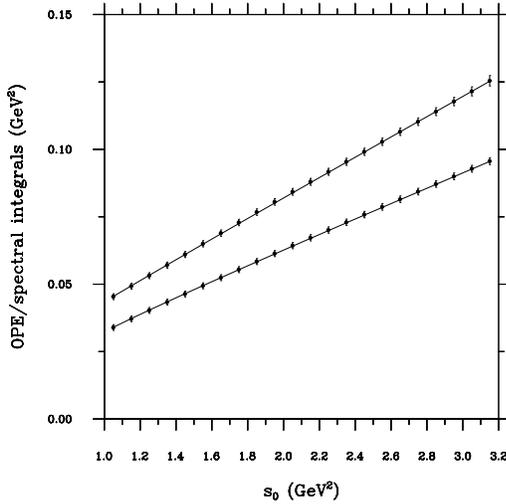
<sup>9</sup>Contributions to the OPE side from terms of dimension  $D = 2k + 2$  scale as  $1/s_0^k$ , allowing one to distinguish

value of  $\alpha_s$  just extracted) to the corresponding data integrals for (1)  $w(y) = (1-y)^2(1+2y)$  (at  $s_0 < m_\tau^2$ ) and (2)  $w(y) = (1-y)(1+y)$  (also as a function of  $s_0$ ). The  $s_0$  dependence of the OPE prediction depends crucially on the form of the 4-loop running of  $\alpha_s$ . The results of the tests for possible duality violation are shown in Figure 3. There is no evidence of such violations, within experimental errors, even down to rather low scales. While this does not prove that they are absent, it does give us additional confidence in the fit value, Eq. (11).

We conclude this section with a comment on the use of higher degree spectral weights for analyses of the type described above. Weights with degrees  $k > 2$  in principle involve OPE contributions with  $D = 8, \dots, 2k+2$ . Usually these contributions are assumed to be negligible, at least for  $s_0 = m_\tau^2$ . Although the values of the relevant condensate combinations are not known for the  $ud$

contamination by an operator neglected when it should not have been, at least if one considers the data integrals over a sufficiently large window of  $s_0$  values.

Figure 3. The OPE and spectral integral sides of two pFESR's for the  $ud$  V+A correlator. The value  $\alpha_s(m_\tau) = 0.345$  obtained by the fitting procedure described in the text is used as input on the OPE sides. The upper curve corresponds to  $w(y) = (1-y)(1+y)$ , the lower curve to  $w(y) = (1-y)^2(1+2y)$ .



V+A correlator, the analogous V-A combinations have been extracted in Ref. [13]. If we take these as representative of the scale of higher  $D$  contributions for the V+A correlator, we can make a rough estimate of the expected size of higher  $D$  contributions, and hence determine the likelihood that neglect of such contributions is safe. For the  $(0,0)$  spectral weight,  $w(y) = (1-y)^2(1+2y)$ , this yields an estimate of  $\sim 0.3\%$  of the OPE total for  $s_0 = m_\tau^2$ , making neglect of such contributions quite safe. For the  $(1,0)$  spectral weight,  $w(y) = (1-y)^3(1+2y)$ , the estimated  $D = 8, 10$  contributions, at  $s_0 = m_\tau^2$ , are both  $\sim 1.5\%$  of the OPE total, while for the  $(2,0)$  spectral weight,  $w(y) = (1-y)^4(1+2y)$ , the estimated  $s_0 = m_\tau^2$   $D = 8, 10, 12$  contributions are  $\sim 3\%, 6\%$  and  $2\%$  of the OPE total, respectively. Because of

the growth in the size of the relevant polynomial coefficients, the neglect of unknown higher  $D$  contributions thus becomes progressively less safe as one goes to higher spectral weights.

### 3. THE STRANGE QUARK MASS FROM FLAVOR BREAKING IN HADRONIC $\tau$ DECAY

The  $D = 0$  (mass-independent) part of the perturbative contribution to the flavor-breaking difference  $\Delta\Pi_{V/A}^{(J)}(Q^2) \equiv [\Pi_{V/A;ud}^{(J)} - \Pi_{V/A;us}^{(J)}](Q^2)$  vanishes in the  $SU(3)$  flavor limit. The formally leading (in dimension) term in the OPE of  $\Delta\Pi_{V/A}^{(J)}$  is therefore the  $D = 2$  term resulting from flavor breaking in the  $m_q^2$ -dependent perturbative contributions. To the extent that  $\hat{m} \equiv (m_u + m_d)/2 \ll m_s$  [19], this contribution is proportional to  $m_s^2$ . The basic idea of using hadronic  $\tau$  decay data to determine  $m_s$  is then to construct flavor-breaking differences of  $ud$  and  $us$  spectral integrals and equate these, using the basic FESR relation, to equivalent OPE contour integrals involving the unknown parameter  $m_s$ .

A simple way to construct such flavor-breaking differences is to note that, from Eq. (2), the  $ud$  decay distribution, rescaled by  $1/|V_{ud}|^2$ , and the  $us$  decay distribution, rescaled by  $1/|V_{us}|^2$ , become equal in the  $SU(3)_F$  limit. Weighted integrals of the difference of the rescaled spectral functions then produce flavor-breaking observables of the type amenable to a pFESR extraction of  $m_s$ .

Such differences can, in principle, be constructed separately for V, A and V+A combinations, and also separately for  $(J) = (0+1), (0)$ . Since the V/A separation has not been performed for the  $us$  spectrum, and even for the  $ud$  spectrum the V+A spectral distribution is better determined than are the separate V and A distributions, it is preferable to work with the V+A combinations for both  $ud$  and  $us$ . Bearing in mind that a  $J = 0/J = 1$  spin separation is not currently available above  $\sim 1$  GeV in the  $us$  channel (where the  $J = 0$  spectral contributions to the experimental decay distributions may not be totally negligible) the natural choices for flavor-breaking observables are those constructable

from the kinematically-weighted combinations,  $(1 - y_\tau)^2 \left[ (1 + 2y_\tau) \rho_{V+A;ij}^{(0+1)}(s) - 2y_\tau \rho_{V+A;ij}^{(0)}(s) \right]$ , which are determined directly from the experimental  $ij = ud$  and  $us$  decay distributions. Most analyses in the literature have employed the  $(k, n)$  spectral weight versions of this construction,

$$\delta R^{(k,n)} \equiv \frac{R_{V+A;ud}^{(k,n)}}{|V_{ud}|^2} - \frac{R_{V+A;us}^{(k,n)}}{|V_{us}|^2}, \quad (12)$$

where

$$R_{V+A;ij}^{(k,n)} \equiv \int_0^{m_\tau^2} ds (1 - y_\tau)^k y_\tau^n \frac{dR_{V+A;ij}}{ds}. \quad (13)$$

The approach to determining  $m_s$  just outlined, though straightforward in principle, turns out to have non-trivial complications.

On the experimental side, the first complication is the limited accuracy of the current determination of the  $us$  spectral distribution (known to  $\sim 6 - 8\%$  in the  $K^*$  region, and to  $\sim 20 - 30\%$  above  $1 \text{ GeV}^2$  [9]). This situation will improve dramatically as analyses of the  $\tau$  decay data from the B factory experiments begin to come online. The second complication arises from the rather close cancellation between  $ud$  and  $us$  contributions occurring on the spectral sides of the flavor-breaking pFESR's employed in the literature (typically to the  $\sim 10\%$  or less level). Such close cancellation makes the  $ud$ - $us$  spectral difference sensitive to both uncertainties in  $|V_{us}|^2$  and (apparently) small changes in the measured  $us$  branching fraction. For example, with  $ud$ - $us$  cancellation to the 10% (5%) level, the  $\sim 2.5\%$  uncertainty in the value of  $|V_{us}|^2$  [20] translates into a 25% (50%) uncertainty in the value of the integrated  $ud$ - $us$  spectral difference. The small differences between the preliminary ALEPH determination  $R_{us} \equiv R_{V+A;us} = 0.155 \pm 0.006$  [21], the final published version,  $R_{us} = 0.161 \pm 0.007$  [9], and Davier's update at Tau'2000,  $R_{us} = 0.163 \pm 0.006$  [22–24], also produce significant shifts in the central value of  $m_s$  extracted in a given pFESR analysis for the same reason.

Table 1 displays the central values of  $m_s$  obtained in a number of independent analyses reported in the literature, all nominally based on the “same” (ALEPH)  $us$  data. Also shown

are the central values employed for the input quantities  $R_{us}$  and  $|V_{us}|$ . The entries are labelled by their analysis type. Table 2 shows the same results converted to common input, and also to a common  $D = 2$  OPE truncation scheme, for two choices of the CKM input: (1) the central values of the three-family unitarity-constrained PDG2002 fit,  $|V_{ud}| = 0.9734$  and  $|V_{us}| = 0.2225$  (CKMU) and (2) the best fit independent PDG2002 central fit values,  $|V_{ud}| = 0.9749$  and  $|V_{us}| = 0.2196$  (CKMN). The sensitivity to  $R_{us}$  and  $|V_{us}|$ , as well as the good consistency between the different analyses *when the same input is employed*, is evident from the tables<sup>10</sup>.

On the OPE side, the major complication has to do with the bad behavior of the various weighted integrals of the  $D = 2$  part of the longitudinal ( $(J) = (0)$ ) contribution,

$$\oint_{|y|=1} ds w(y) \left[ \Delta \Pi_{V+A}^{(0)}(s) \right]_{D=2}. \quad (14)$$

The expression for  $\left[ \Delta \Pi_{V+A}^{(0)}(s) \right]_{D=2}$  in terms of the running quark masses  $m_q(Q^2)$  and running coupling  $\alpha_s(Q^2)$ , is known to  $O(\alpha_s^3)$  [31]. It turns out that, for all of the weights studied in the literature, the integrated version of this series is non-converging, even at the highest scale ( $s_0 = m_\tau^2$ ) allowed by kinematics [32,27,33]. This is true whether the integrated series is organized using FOPT or CIPT<sup>11</sup>. Most of the recent analyses in the literature deal with this problem by nonetheless retaining the longitudinal contributions out to  $O(\alpha_s^3)$ , combining them with the  $(0 + 1)$  contributions, and considering the sum, truncated at  $O(\alpha_s^3)$ . Attempts are made to assign errors to this truncated sum which are sufficiently conservative to take into account the bad behavior of the  $(0)$  part of the series [27,9,28,22,29]. Exceptions are (1) Ref. [26], which makes a subtraction of the

<sup>10</sup>For further details on the variations in input among the different analyses, the impact of these variations on the extracted values of  $m_s$ , and the conversion to common input and truncation scheme, see Ref. [25].

<sup>11</sup>As an example of the bad convergence, for  $s_0 = m_\tau^2$  and the kinematically-weighted  $(0, 0)$  spectral weight pFESR, the integrated series behaves as  $\sim 1 + 0.99 + 1.24 + 1.59$  in FOPT and  $\sim 1 + 0.78 + 0.78 + 0.90$  in CIPT [9].

Table 1

Central values for  $m_s(2 \text{ GeV})$  (in MeV) in the  $\overline{MS}$  scheme from various analyses.  $(k, 0)$  labels analyses which combine the  $(0, 0)$ ,  $(1, 0)$  and  $(2, 0)$  spectral weights. The 1999 ALEPH analysis, labelled by  $(k, n)$ , is a combined analysis using the  $(0, 0)$ ,  $(1, 0)$ ,  $(2, 0)$ ,  $(1, 1)$ ,  $(1, 2)$  spectral weights. The  $w_{20}$  label for KM00 denotes a non-spectral weight, the details of which may be found in Ref. [26]; the analysis in this case is of the non-inclusive  $(0 + 1)$  type. The various analyses also display some differences in their truncation procedures, especially in the case of ALEPH99, and some differences in their treatments of  $D = 4$  contributions. Small differences in the central values employed for  $|V_{ud}|$  also exist. Details may be found in Ref. [25].

Reference	Type	$R_{us}$	$ V_{us} $	$m_s(2 \text{ GeV})$ [MeV]
CKP98 [27]	$(0, 0)$	0.155	0.2213	145
ALEPH99 [9]	$(k, n)$	0.161	0.2218	144
PP99 [28]	$(k, 0)$	0.161	0.2218	114
KKP00 [29]	$(0, 0)$	0.161	0.2218	125
KM00 [26]	$w_{20}$	0.161	0.2196	115
DHPPC00 [22]	$(k, 0)$	0.163	0.2225	108

Table 2

Central values for  $m_s(2 \text{ GeV})$  (in MeV) obtained by updating analyses in the literature to common CKM input, the most recent value of  $R_{us}$  and, where possible, a common truncation scheme. Possible  $D = 8$  and higher terms have been neglected throughout. Since DHPPC00 represents a combined update of both ALEPH99 and PP99, we display only the update of DHPPC00 in this table. KKP00 employs an expansion in an effective coupling which is different from that used in the other analyses. KM00 is a  $(0 + 1)$  analysis, whereas the others are inclusive. In Ref. [30] a  $k$ -dependent truncation scheme has been used for the  $(k, 0)$  spectral weights; this scheme has been retained in converting to CKMU and CKMN input. A discussion of the issues underlying this choice of truncation scheme is given below. For further details see Ref. [25].

Reference	Type	CKMU input	CKMN input
CKP98 [27]	$(0, 0)$	$116 \pm 31$	$99 \pm 34$
KKP00 [29]	$(0, 0)$	$120 \pm 28$	$106 \pm 32$
KM00 [26]	$w_{20}$	$110 \pm 16$	$100 \pm 18$
DHPPC00 [22]	$(0, 0)$	$124 \pm 32$	$106 \pm 37$
	$(1, 0)$	$113 \pm 32$	$102 \pm 21$
	$(2, 0)$	$99 \pm 21$	$91 \pm 21$
CDGHPP01 [30]	$(0, 0)$	$126 \pm 31$	$107 \pm 35$
	$(1, 0)$	$116 \pm 19$	$106 \pm 19$
	$(2, 0)$	$113 \pm 22$	$103 \pm 22$

longitudinal contributions to the spectrum based on sum rule analyses of the strange scalar and pseudoscalar channels [34], and then works with pFESR's for the better behaved (0+1) correlator and (2) Ref. [30], which truncates the (0,0) and (1,0) spectral weight pFESR's at  $O(\alpha_s^3)$ , but the (2,0) spectral weight pFESR at  $O(\alpha_s^2)$ <sup>12</sup>.

The badly-converged  $D = 2$  longitudinal OPE series for the  $(k, 0)$  spectral weight pFESR's turns out to have another problem, namely that, when employed with either of the truncation schemes noted above, it produces an unphysical decrease with  $k$  in the extracted value of  $m_s$  [34]. For such analyses there is thus an additional theoretical systematic error not accounted for in the errors quoted in the literature.

It is easy to see how the problem arises. Since, apart from the pion pole, the  $ud$  ( $J = 0$ ) spectral function is negligible, the non-pole ("continuum") part of the spectral function of  $\Delta\Pi_{V+A}^{(0)}$ ,  $\Delta\rho_{V+A}^{(0)}$ , is negative definite. Thus, in the  $(k, 0)$  spectral weight pFESR, where it occurs weighted by  $-2y_\tau(1-y_\tau)^{2+k}$ , it yields a strictly positive contribution to the integrand over the whole of the continuum region  $s > s_{th} = (m_K + m_\pi)^2$ . Since  $0 < 1 - y_\tau < 1 - s_{th}/m_\tau^2 = 0.87$  in this region, the integrated continuum contribution,  $[\Delta^{(k,0)}]_L^c$ , is *necessarily* a decreasing function of  $k$ , and must satisfy the rigorous inequality

$$[\Delta^{(k+1,0)}]_L^c < 0.87 [\Delta^{(k,0)}]_L^c . \quad (15)$$

The sum of the longitudinal pion and kaon pole contributions is also positive, and a (slowly) decreasing function of  $k$ .

The bound in Eq. (15), though rigorous, is overly conservative. One would, in fact, expect the continuum contribution to be dominated by the  $K_0^*(1430)$  and  $K(1460)$  resonances. Since the masses and widths of the two resonances happen

<sup>12</sup>For the (0,0) and (1,0) spectral weights, the series for the sum of the (0+1) and (0)  $D = 2$  contributions is decreasing with increasing order out to  $O(\alpha_s^3)$ ; for the (2,0) spectral weight, however, the bad behavior of the longitudinal part wins out earlier, and the  $O(\alpha_s^3)$  term is larger than the  $O(\alpha_s^2)$  term. The  $k$ -dependence of the truncation scheme of Ref. [30] for the  $(k, 0)$  spectral weight pFESR's results from the ansatz of truncating the series at the point beyond which the terms begin to increase in size.

to be comparable, one can make an improved estimate for the relation of the  $[\Delta^{(k,0)}]_L^c$  for different  $k$  by integrating over a Breit-Wigner profile with the average mass and width. The result is that, in the limit of resonance dominance of the continuum longitudinal contributions, one would expect [34]

$$\begin{aligned} [\Delta^{(1,0)}]_L^c &\simeq 0.44 [\Delta^{(0,0)}]_L^c \\ [\Delta^{(2,0)}]_L^c &\simeq 0.22 [\Delta^{(0,0)}]_L^c . \end{aligned} \quad (16)$$

We can contrast these physical constraints with what is implied by the truncated OPE representation. To do so we consider the integrated  $(k, 0)$  OPE representations (using the combined fit  $m_s$  central value of Ref. [28] as input, to be specific) and subtract from them the very accurately known pion and kaon pole spectral terms. This leaves the implicit OPE representation for the longitudinal continuum contributions. The resulting longitudinal contributions turn out to be comparable to, or larger than, the corresponding longitudinal pole contributions for all the cases under consideration ( $k = 0, 1, 2$ ). The longitudinal contributions are also larger than the (0+1) contributions for all cases. With the uniform-in- $k$  truncation scheme, one finds that the continuum longitudinal contributions predicted by the OPE are in the ratios 1 : 1.16 : 1.42 for the (0,0), (1,0) and (2,0) cases, respectively. This fails to satisfy even the weak rigorous inequalities implied by Eq. (15), let alone the expectations based on resonance dominance given in Eqs. (16). If one instead uses the  $k$ -dependent truncation scheme of Ref. [30], the truncated OPE implies continuum longitudinal contributions in the ratios 1 : 1.16 : 0.83, still rather far from those implied by the physical constraints.

The failure of the OPE to satisfy the physical constraints (which must necessarily be reflected in the spectral integrals) means that the  $m_s$  values extracted in the separate  $(k, 0)$  spectral weight analyses will themselves have an unphysical  $k$ -dependence. Since, for fixed  $m_s$ , the higher  $k$  OPE contributions are too large relative to the (0,0) contributions, successively smaller values of  $m_s$  will be required to produce a match

between the OPE and spectral integrals as  $k$  is increased. This trend is seen clearly in the results of the uniform-in- $k$  truncation scheme for the  $(k, 0)$  analyses reported in the literature, and in Table 2 above. The size of the resulting systematic uncertainty on  $m_s(2 \text{ GeV})$  is likely to be at least 20–25 MeV for the  $(1, 0)$  and  $(2, 0)$  analyses [34].

In order to avoid the problems with the OPE representation of the longitudinal contributions, one can work with pFESR's for the  $(0+1)$  correlator difference alone. Since the  $us$  spin separation above 1 GeV is not available, however, some external input is required in order to allow a determination of the longitudinal continuum subtraction to be performed in converting from the measured to the purely  $(0+1)$  spectral distribution.

One possibility, tried in Ref. [9], is to identify experimentally the  $K_0^*(1430)$  and  $K(1460)$  decay modes (expected to dominate the non-pole part of the  $(J) = (0)$  spectral integral), and subtract these contributions, plus the longitudinal pole contributions, from the measured spectrum. Since neither the  $K_0^*(1430)$  nor  $K(1460)$  decays of the  $\tau$  have been detected to date, this is not yet practical, but may well become so with the new B factory data.

It is also possible to compute the continuum  $(J) = (0)$  subtraction if one knows the  $K_0^*(1430)$  and  $K(1460)$  decay constants [34]. These can be estimated with  $\sim 10 - 20\%$  accuracy using sum rules for the correlators involving the divergences of the  $us$  vector and axial vector currents [34,35]. An alternate possibility for the strange scalar continuum contributions is to use analyticity, unitarity and the existence of an Omnes relation for the timelike scalar  $K\pi$  form factor to compute the strange scalar spectral function. This was done, ignoring the effects of channel coupling above the  $K_0^*(1430)$ , in Ref. [36]. An improved version of this analysis, which includes the effects of channel coupling using a model constrained by ChPT and known short-distance physics, has also recently been performed [37]. The effect of channel coupling on the spectrum in the  $K_0^*(1430)$  region is found to be small. The  $K_0^*(1430)$  decay constants found in the sum rule analysis of

Refs. [34,35] and the coupled channel analysis of Ref. [37] are in good agreement within errors, suggesting that the uncertainties in the theoretical determination of the longitudinal subtraction are under control, certainly at the  $\sim 20 - 40\%$  level. Since, for practical reasons, the weights employed in current pFESR analyses necessarily strongly suppress contributions from the region above 1 GeV<sup>2</sup>, where  $us$  spectral errors are large, such a level of uncertainty on the longitudinal subtraction produces an uncertainty in the extracted value of  $m_s$  which is negligible compared to the other sources of error [26,25,34].

An analysis of the  $(0+1)$  type was performed in Ref. [26]. The analysis employs three weights, constructed so as to (1) strongly suppress  $us$  contributions from the region above 1 GeV<sup>2</sup>, (2) improve the convergence of the integrated  $(0+1)$  OPE  $D = 2$  series, (3) reduce to some extent the strong  $ud-us$  cancellation present for the kinematically-weighted  $(0+1)$  spectral integral, and (4) suppress, as much as possible, potential contributions from unknown higher dimension ( $D > 6$ ) condensates. The second point is of relevance because the known terms of the  $D = 2$  series for  $\Delta\Pi_{V+A}^{(0+1)}$  suggest potentially slow convergence<sup>13</sup>. Explicitly, in the  $\overline{MS}$  scheme [31],

$$\left[ \Delta\Pi_{V+A}^{(0+1)}(Q^2) \right]_{D=2} = \frac{3m_s^2(Q^2)}{2\pi^2 Q^2} \left[ 1 + \frac{7}{3}a + 19.9332a^2 + \dots \right]. \quad (17)$$

Both the running coupling and running mass are known to 4-loop order [6,38]. Details of the treatment of higher  $D$  contributions may be found in Ref. [26]. Regarding the fourth point, note that, for the higher degree spectral weights employed in the literature, the modified transverse kinematic weights,

$$w_{(0+1)}^{(k,n)}(y) = \underline{y^n (1-y)^{2+k} (1+2y)}, \quad (18)$$

<sup>13</sup>In estimating the  $D = 2$  truncation errors it is important to bear in mind that, for the  $(k, 0)$  spectral weight cases, there is an accidental suppression of the second order CIPT contour integral caused by cancellations between contributions from different parts of the contour. This cancellation does not persist to higher orders, so taking the second order term as an estimate of the truncation error is likely to produce a significant underestimate of this uncertainty.

involve uncomfortably large coefficients of the  $y^m$  terms with  $m > 2$ , which correspond to integrated OPE contributions proportional to condensate combinations with  $D = 2m + 2 > 6$ . This includes the  $(1, 0)$  and  $(2, 0)$  spectral weights, whose pFESR's are favorable from the point of view of reduced  $ud-us$  cancellation. These pFESR's are thus unfavorable from the point of view of possible unknown higher dimension OPE contributions. The weights of Ref. [26] have been designed to avoid large coefficients for this reason.

Especially after the most recent update of  $R_{us}$ , the  $ud-us$  cancellation creates sizeable errors for the weights  $w_{10}$  and  $\hat{w}_{10}$  of Ref. [26]. The remaining weight,  $w_{20}(y)$ , whose form is given explicitly in Ref. [26], has a profile in the spectral integral region intermediate between that of  $w_{(0+1)}^{(1,0)}(y)$  and  $w_{(0+1)}^{(2,0)}(y)$ . As such, it has a lesser degree of  $ud-us$  cancellation than the former, but somewhat greater degree of cancellation than the latter. Whereas  $w_{(0+1)}^{(2,0)}(y) = 1 - 2y - 2y^2 + 8y^3 - 7y^4 + 2y^5$  has two uncomfortably large coefficients, 8 and  $-7$ , however, the largest coefficient in  $w_{20}$ , associated with  $D = 8$  contributions, is 2.1. Updating this analysis for the new  $R_{us}$  value (following the procedure described in Ref. [22] for the modification of the  $us$  spectral distribution), truncating the  $D = 2$  OPE series at  $O(a^2)$ , and representing the  $D > 6$  contributions by an effective  $D = 8$  term, fit to data, we find

$$m_s(2 \text{ GeV}) = 123 \pm 18 \pm 15 \text{ MeV} , \quad (19)$$

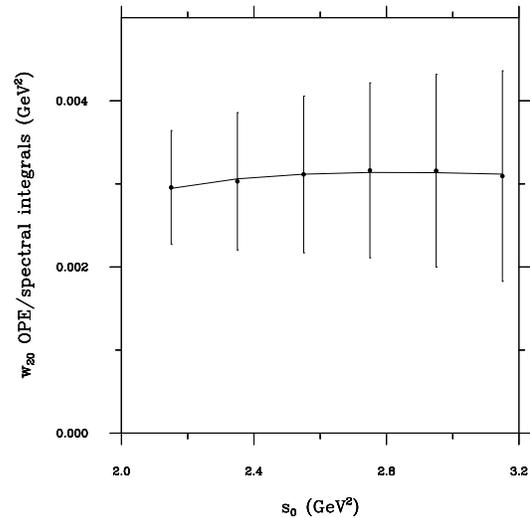
for CKMU input and

$$m_s(2 \text{ GeV}) = 104 \pm 18 \pm 17 \text{ MeV} , \quad (20)$$

for CKMN input. The first error in each case is experimental, the second theoretical. The theoretical error is dominated by the estimate of the error associated with truncating the  $D = 2$  series. The size of the data errors, and the quality of the match between the OPE and hadronic sides of the  $w_{20}$  FESR which results, are shown in Figure 4.

Possibilities for improving the  $(0 + 1)$  determination of  $m_s$  will be described in the final section.

Figure 4. Optimized OPE fit to the spectral integrals for the  $w_{20}$  pFESR. The  $(D = 4)$  and  $D = 6$  OPE contributions have been subtracted from the data integrals. The solid line corresponds to the best fit for  $m_s$  and the coefficient for the  $D_{eff} = 8$  effective operator used to represent  $D > 6$  effects. The data integrals and errors correspond to the ALEPH data and covariance matrix.



#### 4. CHIRAL LIMIT VALUES OF THE $K \rightarrow \pi\pi$ EWP MATRIX ELEMENTS

In the Standard Model, the effective strangeness-changing non-leptonic decay Hamiltonian takes the form

$$H_{w,eff}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{k=1}^{10} [z_k + \tau y_k] Q_k , \quad (21)$$

where  $\tau \equiv -V_{td} V_{ts}^* / V_{ud} V_{us}^*$  and the explicit forms of the effective operators  $Q_1, \dots, Q_{10}$  may be found in Ref. [39]. The Wilson coefficients,  $z_k$  and  $y_k$ , are known to NLO in  $\alpha_s$  [39–41]. The task of determining physical weak decay amplitudes, such as those relevant to  $\epsilon'/\epsilon$ , is thus reduced to the problem of evaluating the relevant

low-energy matrix elements of the operators  $Q_k$ .

Since CP violation in the Standard Model requires the participation of all three generations,  $\epsilon'/\epsilon$  receives contributions only from the gluonic penguin operators  $Q_{3-6}$  and EWP operators  $Q_{7-10}$ . For the large physical top quark mass, it turns out that the dominant contributions are those associated with  $Q_6$  and  $Q_8$  [39]. In the  $\overline{MS}$  NDR scheme, for example [39],

$$\begin{aligned} \frac{\epsilon'}{\epsilon} = & 20 \times 10^{-4} \left( \frac{\text{Im}\lambda_t}{1.3 \cdot 10^{-3}} \right) [-2.0 \text{ GeV}^{-3} \\ & \times \langle (\pi\pi)_{I=0} | Q_6 | K^0 \rangle_{2 \text{ GeV}} (1 - \Omega_{IB}) \\ & - 0.50 \text{ GeV}^{-3} \cdot \langle (\pi\pi)_{I=2} | Q_8 | K^0 \rangle_{2 \text{ GeV}} \\ & - 0.06] , \end{aligned} \quad (22)$$

where  $\Omega_{IB}$  is an isospin-breaking correction [42] and  $\lambda_t = V_{td}V_{ts}^*$ .

By good fortune it turns out to be possible to use hadronic  $\tau$  decay data to determine the values of the  $K \rightarrow \pi\pi$  EWP matrix elements  $\langle \pi\pi | Q_{7,8} | K \rangle$  in the chiral limit. The  $Q_8$  matrix element is a crucial ingredient in our understanding of the Standard Model prediction for  $\epsilon'/\epsilon$ . Although the  $Q_7$  matrix is not of equivalent phenomenological interest, it is likely to be of relevance as a test of lattice techniques for the evaluation of weak matrix elements, particularly the reliability of the extrapolations to physical light quark masses. We briefly review the fortuitous circumstances which make the evaluation of these matrix elements possible.

From the explicit form of  $Q_7$ ,  $Q_8$ ,

$$Q_7 = \frac{3}{2} \bar{s}_a \gamma_\mu (1 - \gamma_5) d_a \sum_q e_q \bar{q}_b \gamma^\mu (1 - \gamma_5) q_b \quad (23)$$

and

$$Q_8 = \frac{3}{2} \bar{s}_a \gamma_\mu (1 - \gamma_5) d_b \sum_q e_q \bar{q}_b \gamma^\mu (1 - \gamma_5) q_a , \quad (24)$$

where  $a, b$  are color labels, one sees that both transform as  $8_L \times 8_R$  under  $SU(3)_L \times SU(3)_R$ . As a result, the  $K \rightarrow \pi\pi$  matrix elements survive in the chiral limit (in contrast to the gluonic penguin matrix elements) and hence can be evaluated using soft pion/kaon techniques. The results, for

the  $I = 2$  final  $\pi\pi$  state, are [43]

$$\langle (\pi\pi)_2 | Q_8 | K^0 \rangle_\mu = -\frac{2}{3F_0^3} \left[ \langle O_1 \rangle + \frac{3}{2} \langle O_8 \rangle \right]_\mu \quad (25)$$

and

$$\langle (\pi\pi)_2 | Q_7 | K^0 \rangle_\mu = -\frac{2}{F_0^3} \langle O_1 \rangle_\mu , \quad (26)$$

where  $F_0$  is the pion decay constant in the chiral limit and the operators are given by  $O_{1,8}$  are

$$\begin{aligned} O_1 & \equiv \bar{q} \gamma_\mu \frac{\tau_3}{2} q \bar{q} \gamma^\mu \frac{\tau_3}{2} q \\ & \quad - \bar{q} \gamma_\mu \gamma_5 \frac{\tau_3}{2} q \bar{q} \gamma^\mu \gamma_5 \frac{\tau_3}{2} q , \\ O_8 & \equiv \bar{q} \gamma_\mu \lambda^a \frac{\tau_3}{2} q \bar{q} \gamma^\mu \lambda^a \frac{\tau_3}{2} q \\ & \quad - \bar{q} \gamma_\mu \gamma_5 \lambda^a \frac{\tau_3}{2} q \bar{q} \gamma^\mu \gamma_5 \lambda^a \frac{\tau_3}{2} q . \end{aligned} \quad (27)$$

The connection to  $\tau$  decay data arises from the fact that precisely the same vacuum matrix elements  $\langle O_{1,8} \rangle$  determine the  $D = 6$  part of the flavor  $ud$  V-A correlator,  $\Delta\Pi_{ud} \equiv \Pi_{V;ud}^{(0+1)} - \Pi_{A;ud}^{(0+1)}$ . Explicitly [44]

$$[\Delta\Pi_{ud}]_{D=6} = a_6(\mu) + b_6(\mu) \log(Q^2/\mu^2) , \quad (28)$$

with

$$\begin{aligned} a_6(\mu) & = 2 \left[ \langle (2\pi\alpha_s + A_8\alpha_s^2) O_8 \rangle \right. \\ & \quad \left. + A_1 \langle \alpha_s^2 O_1 \rangle \right]_\mu , \\ b_6(\mu) & = 2 \left[ \alpha_s^2 (B_8 \langle O_8 \rangle + B_1 \langle O_1 \rangle) \right]_\mu , \end{aligned} \quad (29)$$

where  $A_{1,8}$  and  $B_{1,8}$  are coefficients depending on the renormalization scheme, the prescription for the treatment of  $\gamma_5$ , and the evanescent operator basis [39]. Values employing the same evanescent operator basis as used for computing the Wilson coefficients of  $H_{w,eff}$  [40,41] and for both the HV and NDR prescriptions for  $\gamma_5$ , may be found in Ref. [44].  $\langle O_1 \rangle$  is suppressed relative to  $\langle O_8 \rangle$  in the large  $N_C$  expansion. Dispersive determinations (to be discussed below) also indicate that it is much smaller numerically. Thus, both the dominant CP violating  $K \rightarrow \pi\pi$   $Q_8$  matrix element and the  $D = 6$  part of the  $ud$  V-A correlator are essentially determined by the same quantity,  $\langle O_8 \rangle$ . One can therefore construct either dispersive sum rules or pFESR's for the V-A correlator

in an attempt to extract this matrix element using the  $ud$  V-A hadronic  $\tau$  decay spectral data.

Dispersive sum rules for  $\langle O_{1,8} \rangle$  were first considered in Ref. [43] and re-analyzed in Ref. [44], taking into account the  $D > 6$  contributions discussed in Ref. [45] and NLO radiative corrections. With  $\Delta\rho(s)$  the spectral function of  $\Delta\Pi_{ud}$ , one finds, for  $\langle O_1 \rangle$ ,

$$\langle O_1 \rangle_\mu - \frac{3C_8}{8\pi} \langle \alpha_s O_8 \rangle_\mu = \bar{I}_1(\mu) \quad (30)$$

where the scheme-dependent coefficient  $C_8$  is given in Ref. [44], and

$$\bar{I}_1(\mu) = \frac{3}{(4\pi)^2} [I_1(\mu) + H_1(\mu)] , \quad (31)$$

with

$$\begin{aligned} I_1(\mu) &= \int_0^\infty ds s^2 \ln\left(\frac{s+\mu^2}{s}\right) \Delta\rho(s) \\ H_1(\mu) &= \int_{\mu^2}^\infty dQ^2 Q^4 [\Delta\Pi_{ud}(Q^2)]_{D>6} . \end{aligned} \quad (32)$$

Similarly, for  $\langle O_8 \rangle$ , one has

$$\begin{aligned} \langle (2\pi\alpha_s + \alpha_s^2) O_8 \rangle_\mu + A_1 \langle \alpha_s^2 O_1 \rangle_\mu \\ = 2\pi\alpha_s(\mu) \bar{I}_8(\mu) \end{aligned} \quad (33)$$

where

$$\bar{I}_8(\mu) = \frac{1}{2\pi\alpha_s(\mu)} \left[ I_8(\mu) - H_8(\mu) \right] , \quad (34)$$

with

$$\begin{aligned} I_8(\mu) &= \int_0^\infty ds \frac{s^2\mu^2}{s+\mu^2} \Delta\rho(s) \\ H_8(\mu) &= \mu^6 [\Delta\Pi_{ud}(\mu)]_{D>6} . \end{aligned} \quad (35)$$

Although the spectral integrals  $I_{1,8}(\mu)$  extend over the whole range  $0 < s < \infty$  and not just  $0 < s < m_\tau^2$ , it turns out to be possible to use the relations above, together with hadronic  $\tau$  decay data, to evaluate these integrals with good accuracy. This is made possible by the fact that the  $ud$  V-A correlator satisfies three classical chiral sum rules in the chiral limit, the two Weinberg sum rules [46] and the sum rule for the chiral limit pion electromagnetic splitting [47]. These sum rules involve integrals over the  $ud$  V-A spectral function,  $\Delta\rho(s)$ , but with weights  $w(s) = 1, s$

and  $s \log(s/\Lambda^2)$  (for any  $\Lambda$ ), respectively, rather than those relevant to  $\langle O_{1,8} \rangle$ .

Any weighted integral of the form  $\int_0^\infty ds w(s) \Delta\rho(s)$  can be re-written as

$$\begin{aligned} \int_0^\infty ds \left[ c_1 + c_2 s + c_3 s \log\left(\frac{s}{\Lambda^2}\right) \right] \\ + \int_0^\infty ds \Delta w(s) \Delta\rho(s) \end{aligned} \quad (36)$$

where  $\Delta w(s) = w(s) - c_1 - c_2 s - c_3 s \log(s/\Lambda^2)$ . The first integral in Eq. (36) is determined, for any  $c_{1-3}$ , by the chiral limit values of  $f_\pi$  and the pion squared electromagnetic mass splitting, as a consequence of the three classical chiral sum rules. So long as it is possible to make  $\Delta w(s)$  small in the region above  $s = m_\tau^2$  by an appropriate choice of  $c_{1-3}$ , the original integral can be evaluated in terms of the  $\tau$  spectral data and the chiral limit input. The details of how this ‘‘residual weight method’’ (RWM), is implemented, and the procedure for minimizing the resulting errors, may be found in Ref. [44]. See also Refs. [48–50] for other approaches to employing dispersive sum rules, or FESR’s, to evaluate the chiral limit  $Q_{7,8} K \rightarrow \pi\pi$  matrix elements.

The RWM method turns out to produce errors on the spectral integrals  $I_{1,8}(\mu)$  which increase as  $\mu$  is increased. To be able to work at lower scales, where the errors are small, however, one has to worry about the presence of the higher dimension  $D > 6$  contributions  $H_{1,8}(\mu)$ . An alternate approach to determining the  $D = 6$  term in the  $ud$  V-A OPE is to employ appropriately designed pFESR’s. These have the advantage of requiring data only over a limited range  $s < s_0$ , which may be chosen to lie entirely in the range allowed by  $\tau$  decay kinematics. In addition, they can, in principle, be employed to extract the higher dimension coefficients in the V-A OPE which are needed for an empirical determination of  $H_{1,8}$ . Such a determination would then allow for a low-scale version of the RWM analysis. Since, at scales  $\mu \sim 2$  GeV the contributions to  $I_{1,8}(\mu)$  associated with the chiral constraints are more than 50% again larger than the integrals of the residual weights over the range of  $s$  covered by the  $\tau$  decay data, the pure pFESR and low-scale RWM ‘‘hybrid’’ analyses are largely independent, and provide a useful

self-consistency check on one another. Details of these two analyses may be found in Refs. [13].

The preliminary results from this analysis show excellent consistency between the two methods for the  $Q_8$  matrix elements, and are listed in Table 3. Note that the  $\langle O_1 \rangle$  contribution to  $a_6$  is only  $\sim -3\%$  of that associated with  $\langle O_8 \rangle$ . The direct pFESR extraction thus determines only  $\langle O_8 \rangle$ <sup>14</sup>. Thus, although one may perform a hybrid analysis for  $\langle O_1 \rangle$ , no pure pFESR analysis is possible. An independent test of the hybrid extraction of  $\langle O_1 \rangle$  can, however, be obtained by performing a RWM analysis for  $\langle O_1 \rangle$  at a scale sufficiently high that  $H_1(\mu)$  may be taken to be zero (say  $\mu \sim 4$  GeV) and then evolving the results down to  $\mu = 2$  GeV using the known anomalous dimension matrix [51]. The results of this test are shown in Table 4.

The results of the pFESR analysis correspond to an EWP contribution to  $\epsilon'/\epsilon$ , in the chiral limit, of

$$[\epsilon'/\epsilon]_{EWP}^{\chi L} = (-16.2 \pm 3.4) \times 10^{-4}, \quad (37)$$

and to deviations from the vacuum saturation value for the  $Q_8$  matrix element  $B_8 = 1.7$  and  $1.9$  for the NDR and HV schemes, respectively. These are a factor of  $\sim 2$  larger than the values produced by most models employed previously in the literature. The result of Eq. (37) is similar in magnitude, but opposite in sign, to the current experimental determination

$$[\epsilon'/\epsilon]_{exp} = (18 \pm 4) \times 10^{-4}. \quad (38)$$

A rapid change in  $\epsilon'/\epsilon$  as  $m_s$  is varied from 0 to its physical value is thus required if the Standard Model is to explain the experimental result.

## 5. COMMENTS/PROSPECTS

The hadronic  $\tau$  decay data base will increase significantly with the data from CLEO-C, the new B factory experiments and (eventually) the Beijing  $\tau$ -charm factory. In this section the question of what improvements in the determinations

<sup>14</sup>The smallness of the  $\langle O_1 \rangle$  contribution is a result of two factors: the color suppression of  $\langle O_1 \rangle$  relative to  $\langle O_8 \rangle$  and the additional factor of  $\alpha_s$  in the coefficient of  $\langle O_1 \rangle$ . The results of the dispersive analysis are, in fact, used to evaluate the tiny  $\langle O_1 \rangle$  contribution to  $a_6$ .

discussed above are likely to be made possible by this new data is briefly discussed.

Regarding  $\alpha_s$ , it appears unlikely that significant further improvement can be made since the dominant error, even with the existing data, is theoretical, associated with the truncation of the  $D = 0$  series [9,14]. This shows up in the direct estimates of the truncation error based on variations in the unknown  $O(a^4)$  term in the  $D = 0$  part of the Adler function and/or the renormalization scale, and also in the deviations between the solutions for  $\alpha_s$  obtained using different methods of handling the truncated  $D = 0$  series<sup>15</sup>.

The situation is quite different for  $m_s$ , where significant progress can be expected in the near future. The B factory data should drastically reduce the errors on the  $us$  spectral distribution. In addition to the obvious advantage of producing a reduction in the size of the errors on  $m_s$  for existing analyses, such improvements should also make possible the use of alternate sum rules not practically useful at present because of the size of the spectral integral errors when evaluated with current data.

This latter point may be relevant to improving less-than-optimal features of existing analyses. An example is the current sensitivity to  $|V_{us}|$ . At present, the large  $us$  experimental errors above the  $K^*$  force one to work with weights which strongly suppress the high- $s$  part of the spectrum. These weights turn out to produce a rather high level of  $ud$ - $us$  cancellation, and hence a significant sensitivity to  $|V_{us}|$ . It is likely that, with improved data, the reduction in the level of high- $s$  suppressions required will allow the construction of alternate weights with a reduced level of  $ud$ - $us$  cancellation.

An important point to be stressed regarding

<sup>15</sup>The CIPT scheme is based on the truncation of the Adler function at fixed order, followed by integration around the contour  $|s| = s_0$ , the FOPT scheme on the expansion of the integrated Adler function to fixed order in  $\alpha_s(s_0)$ . Since the integral of  $[\alpha_s(Q^2)]^k$  can be written as a series in  $\alpha_s(s_0)$  beginning at order  $k$ , the two schemes differ by terms higher order in  $\alpha_s(s_0)$ . The difference between the results  $\alpha_s(m_\tau) = 0.345$  and  $\alpha_s(m_\tau) = 0.326$ , obtained from the CIPT and FOPT treatments of the same pFESR, at the same value of  $s_0$ , thus provides one estimate for the size of the truncation error.

Table 3

pFESR and “hybrid” results for the matrix elements  $M_{7,8} \equiv \langle (\pi\pi)_{I=2} | Q_{7,8} | K^0 \rangle_{2 \text{ GeV}}$ . The hybrid analysis is an RWM analysis at  $\mu = 2 \text{ GeV}$ , using pFESR input to evaluate  $H_{1,8}(2 \text{ GeV})$ . Results are in units of  $\text{GeV}^6$ .

Scheme	Method	$M_8(2 \text{ GeV})$	$M_7(2 \text{ GeV})$
NDR	hybrid	$1.65 \pm 0.45$	$0.21 \pm 0.03$
	pFESR	$1.62 \pm 0.34$	–
HV	hybrid	$1.84 \pm 0.46$	$0.46 \pm 0.08$
	pFESR	$1.80 \pm 0.36$	–

Table 4

Comparison of the “hybrid” determination of  $\langle O_1(2 \text{ GeV}) \rangle$  and the results obtained by evolving the  $\mu = 4 \text{ GeV}$  scale RWM results to  $\mu = 2 \text{ GeV}$  using the known anomalous dimension matrix. Results are in units of  $10^{-4} \text{ GeV}^3$ .

Scheme	Hybrid Analysis	Evolution from $\mu = 4 \text{ GeV}$
NDR	$-(0.70 \pm 0.11)$	$-(0.53 \pm 0.34)$
HV	$-(1.52 \pm 0.27)$	$-(1.64 \pm 0.18)$

the extraction of  $m_s$  is that, given the sizeable theoretical systematic problems associated with the behavior of the OPE representation of the longitudinal  $us$  contributions, the inclusive spectral weight analysis method should almost certainly be abandoned. There are two ways to deal with the longitudinal subtraction that must be made if one wishes pursue sum rules based on the  $(0+1)$  correlator.

One is the theoretical approach outlined above. Note that the continuum longitudinal subtraction does not need to be known to very high accuracy since it actually has little impact on the  $(0+1)$  sum rules, at least for the weights studied so far in the literature. For example, using the  $K(1460)$  and  $K_0^*(1430)$  decay constants extracted from the sum rule analyses of the scalar and pseudoscalar channels [35] to make the subtraction, one finds that the shift in the values of the  $(0+1)$  spectral integrals induced by the subtraction is less than 4% for  $s_0$  in the range  $s_0 > 2 \text{ GeV}^2$ , even for the least rapidly falling of the  $(0+1)$  weights employed in the literature (the  $(0,0)$  spectral weight). Thus even a 100% uncer-

tainty in the longitudinal subtraction corresponds to an error vastly smaller than that associated with the use of the badly-behaved longitudinal OPE representation.

The second approach is experimental, and likely to be feasible, at least to some extent, with the new B factory data. If current theoretical estimates for the  $K_0^*(1430)$  decay constant [35,37] are correct, the  $\tau \rightarrow \nu_\tau K_0^*(1430)$  branching fraction should be  $\sim 6 \times 10^{-5}$  (a factor of  $\sim 8$  smaller than the current experimental upper bound), a level which may be reachable in the new B factory experiments. Since the  $K_0^*(1430)$  decays essentially entirely to  $K\pi$ , the only experimental complication (apart from rate) is the  $\sim 7\%$   $K\pi$  branch of the  $K^*(1410)$ . Based on the central value of the ALEPH determination,  $B(\tau \rightarrow K^*(1410)\nu_\tau) = (1.5_{-1.0}^{+1.4}) \times 10^{-3}$ , one would expect a  $K\pi$  background in the  $K_0^*(1430)$  region at the  $\sim 1 \times 10^{-4}$  level, and with a width somewhat smaller than that of the  $K_0^*(1430)$ . The  $\tau \rightarrow \nu_\tau K(1460)$  mode appears less amenable to experimental identification; confirmation of the theoretical estimates for the  $K_0^*(1430)$  decay constant would, however,

serve as strong evidence in favor of the corresponding  $K(1460)$  decay constant estimate. Even an upper bound on  $B(\tau \rightarrow K_0^*(1430)\nu_\tau)$  at the  $\sim 1 \times 10^{-4}$  level would be useful since the impact of the longitudinal subtraction on the  $(0+1) m_s$  extraction, at the predicted level for the two decay constants, is already known to be small. It is thus likely that the uncertainty in the impact of the longitudinal subtraction can be made rather small *experimentally* in the near future.

Regarding the  $K \rightarrow \pi\pi$  matrix elements of the EWP operators, the main improvement to be hoped for is reduced errors in the  $ud$  V-A spectral function in the region above  $2 \text{ GeV}^2$ . This would require an improved V/A separation of the contributions to states containing a  $K\bar{K}$  pair. Such an improvement might be possible with the enhanced statistics expected from the B factory experiments, at least for quasi-two-body modes. A reduction of the errors on the V-A spectral function would allow one not only to reduce the errors on the extracted EWP matrix elements, but also to sharpen tests for the absence of duality violation. Recall that both the  $ud$  V and A correlators display, in general, significant duality violation for  $s_0 < m_\tau^2$ . Since the V and A duality violations largely cancel in the V+A sum, the intrinsic level of duality violation in the V-A correlator must be expected to be large. The analysis of Ref. [13] tests for the presence of such violations in the pFESR's employed, and finds none, within experimental errors, for  $s_0$  above  $\sim 1.8 \text{ GeV}^2$ . The larger errors in the upper part of the  $s_0$  window, however, leave room for improvement in these tests, particularly in the case of the pFESR's employed to extract the  $D > 6$  contributions needed as input to the hybrid RWM/pFESR version of the analysis.

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