

Theoretical Expectations for Rare and Forbidden Tau Decays

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Given the experimental evidence for $\nu_\mu - \nu_\tau$ oscillations, the existence of lepton flavor violation in τ decays is a theoretical certainty. In this brief review, I consider the connection between models of neutrino mass and the expected observability of some τ decays.

1. INTRODUCTION

In the minimal standard model (SM) with $m_\nu = 0$, the 3 lepton numbers L_e , L_μ , and L_τ are separately conserved. However, given the present neutrino-oscillation data, which imply that neutrinos have mass and mix with one another, the only conserved lepton number is $L = L_e + L_\mu + L_\tau$ if m_ν is Dirac, and $(-1)^L$ if m_ν is Majorana. As a result, lepton flavor violation must occur in τ decays, which may be classified into 3 groups:

(I) $\Delta L = 0$, and to be specific $\Delta L_\tau = -1$.

$$\Delta L_\mu = 1, \Delta L_e = 0 : \tau^- \rightarrow \mu^- X,$$

$$\Delta L_\mu = 0, \Delta L_e = 1 : \tau^- \rightarrow e^- X,$$

$$\Delta L_\mu = 2, \Delta L_e = -1 : \tau^- \rightarrow \mu^- \mu^- e^+,$$

$$\Delta L_\mu = -1, \Delta L_e = 2 : \tau^- \rightarrow e^- e^- \mu^+,$$

where $X = \gamma, \mu^- \mu^+, e^- e^+, q\bar{q}$.

(II) $\Delta L = \pm 1$.

$$\tau^- \rightarrow n\pi^- \quad [\Delta L = -1, \Delta B = 1],$$

$$\tau^+ \rightarrow p\pi^0 \quad [\Delta L = +1, \Delta B = 1].$$

(III) $\Delta L = \pm 2$ $[\Delta(-1)^L = +]$.

$$\tau^- \rightarrow \mu^+ (e^+) d\bar{u}d\bar{u}.$$

Group (I) is rare, but hopefully observable. Group (II) is forbidden, to the extent that baryon number is conserved. Group (III) is somewhere in between.

2. MINIMAL EXPECTATIONS

The present data on atmospheric and solar neutrino oscillations imply that, to a very good approximation, the neutrino mixing matrix is given by

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \simeq \begin{pmatrix} c & -s & 0 \\ s/\sqrt{2} & c/\sqrt{2} & -1/\sqrt{2} \\ s/\sqrt{2} & c/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \quad (1)$$

What does that tell us about τ decay? Since lepton flavor is violated, $\tau \rightarrow \mu X$ and $\tau \rightarrow e X$ must exist, but they are not necessarily observable.

In analogy, consider the well-known $b \rightarrow s\gamma$ decay in the quark sector. This proceeds in one-loop order through the exchange of a W boson and t, c, u quarks. The reason that it is observable is twofold: (1) $V_{tb} \simeq 1$, $V_{ts} \simeq 0.04$, and (2) $m_t^2/M_W^2 \simeq 4.7$. On the other hand, the $\tau \rightarrow \mu\gamma$ decay involves neutrino intermediate states. Although the mixing angle is maximum, i.e. $V_{\tau 3} = 1/\sqrt{2}$, $V_{\mu 3} = -1/\sqrt{2}$, the ratio m_3^2/M_W^2 is at best 10^{-21} which renders it totally unobservable.

The above argument applies if m_ν is Dirac, but if it is Majorana, then there is another contribution. To see this, consider the famous seesaw

neutrino mass matrix:

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix}. \quad (2)$$

Let it be diagonalized by the unitary matrix:

$$U = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \quad (3)$$

where $AA^\dagger + BB^\dagger = 1$. Now B is not zero but of order m_D/M , so the matrix A which was used to connect τ and μ to the light neutrinos is not quite unitary. The heavy neutrinos must also be considered as intermediate states. Theoretically, $M > 10^{13}$ GeV is usually assumed, in which case $\tau \rightarrow \mu\gamma$ is again negligible. However, if M is allowed to be much smaller, then there is some hope. Recently, it has been shown [1] that with certain assumptions of fine tuning and for $M < 10$ TeV, the largest $B(\tau \rightarrow \mu\gamma)$ is 10^{-9} and the largest $B(\tau \rightarrow 3\mu)$ is 10^{-10} , whereas the present experimental upper bounds are 1.1×10^{-6} and 1.9×10^{-6} respectively. [New (preliminary) limits from BELLE reported at this Workshop are 6.0×10^{-7} and 3.8×10^{-7} .]

Another recent analysis [2] considers the effective operator $(\bar{\mu}\Gamma\tau)(\bar{q}^\alpha\Gamma q^\beta)$. Using the various experimental upper bounds on $B(\tau \rightarrow \mu + \text{hadrons})$, the scale of new physics is constrained up to about 10 TeV.

3. INDUCED FLAVOR VIOLATION

To have observable lepton flavor violation, there must be new physics at or below the TeV scale. How is that possible if the scale associated with neutrino mass is very high? The answer is supersymmetry where slepton flavor violation at the TeV scale can induce $\tau \rightarrow \mu(e)$ transitions. I will discuss two examples.

3.1. Universal Soft Terms at the Planck Scale

Consider the minimal supersymmetric standard model (MSSM) with the addition of 3 heavy singlet neutrino superfields (N^c). The seesaw mechanism works as in the SM and the particle content of this model at low energies is the same

as that of the MSSM. There is however an important difference, namely the slepton mass matrix. The reason is as follows. Because of the existence of N^c , there is a Yukawa coupling matrix Y_ν which links $L = (\nu, e)$ to N^c . Since neutrinos mix with one another, this matrix is not diagonal. This means that from the Planck scale (M_P) to M , the slepton mass matrix will pick up off-diagonal entries even though it starts out as universal, i.e. the supergravity scenario. Specifically,

$$(\Delta m_L^2)_{ij} \simeq -\frac{\ln(M_P/M)}{16\pi^2} [6m_0^2(Y_\nu^\dagger Y_\nu)_{ij} + 2(A_\nu^\dagger A_\nu)_{ij}], \quad (4)$$

where $i \neq j$, and A_ν is the soft supersymmetric breaking trilinear scalar coupling matrix. The assumed boundary condition at M_P is of course $m_L^2 = m_0^2$.

To make contact with the actual neutrino mass matrix, the M matrix itself has to be known, the simplest assumption being $M_1 = M_2 = M_3$. If it is also assumed that A_ν is proportional to Y_ν , then the above equation may be evaluated to find m_L^2 at the TeV scale. The off-diagonal entries enable τ to decay into μ or e in one-loop order through the exchange of sleptons and gauginos, which have masses at or below the TeV scale. Many studies have been made regarding this possibility. Typically [3], for a large class of neutrino textures,

$$B(\tau \rightarrow \mu\gamma) < 10^{-8}, \quad (5)$$

although 10^{-7} is possible at the edge of the allowed parameter space.

If the boundary conditions at M_P are not enforced, there will not be any prediction. In that case, it is possible [4] to have $B(\tau \rightarrow \mu\gamma)$ up to the present experimental bound, without any problem phenomenologically.

Going back to the case with universal boundary conditions, it has also been shown recently [5] that the neutral Higgs particles of the MSSM

obtain off-diagonal couplings, thus allowing

$$B(\tau \rightarrow 3\mu) \simeq 10^{-7} \left(\frac{\tan \beta}{60} \right)^6 \left(\frac{100 \text{ GeV}}{m_A} \right)^4. \quad (6)$$

Similarly, $B(\tau \rightarrow \mu\eta) \simeq 8.4B(\tau \rightarrow 3\mu)$ is also possible [6].

Note added: After this talk was given, a preprint appeared [7] which pointed out that in the above scenario, the corresponding $\tau \rightarrow \mu\gamma$ branching fraction would be much above the present experimental bound, thus $B(\tau \rightarrow 3\mu) < 4 \times 10^{-10}$ is expected, rather than Eq. (6).

3.2. Radiatively Corrected Neutrino Mass Matrix

Suppose that at some high scale, when the charged lepton matrix is diagonalized, the Majorana neutrino mass matrix is of the form

$$\mathcal{M}_\nu = \begin{pmatrix} m_0 & 0 & 0 \\ 0 & 0 & m_0 \\ 0 & m_0 & 0 \end{pmatrix}. \quad (7)$$

Consider then the most general one-loop radiative corrections to the above. It is easily shown [8] that \mathcal{M}_ν becomes

$$m_0 \begin{pmatrix} 1 + 2\delta + 2\delta' & \delta'' & \delta''^* \\ \delta'' & \delta & 1 + \delta \\ \delta''^* & 1 + \delta & \delta \end{pmatrix}, \quad (8)$$

where only δ'' is complex. This matrix is very special, because in the limit δ'' is real, it is exactly diagonalized by Eq. (1). In other words, the right \mathcal{M}_ν has been obtained almost without trying.

In the context of supersymmetry, δ comes from $\mu - \tau$ mixing in the slepton mass matrix, but unlike the previous example, it is not induced by the corresponding Yukawa matrix between M_P and M . In other words, the origin of soft supersymmetry breaking is not specified in this case; it is simply taken to be whatever is allowed by phenomenology. Let the two mass eigenvalues of the sleptons be $\tilde{m}_{1,2}$ and their mixing angle be θ , then in the approximation $\tilde{m}_1^2 \gg \mu^2 \gg M_{1,2}^2 = \tilde{m}_2^2$, where μ is the Higgsino mass and $M_{1,2}$ are the gaugino masses, the parameter δ is given by

$$\delta = \frac{\sin \theta \cos \theta}{16\pi^2} \left[(3g_2^2 - g_1^2) \ln \frac{\tilde{m}_1^2}{\mu^2} \right.$$

$$\left. - \frac{1}{4} (3g_2^2 + g_1^2) \left(\ln \frac{\tilde{m}_1^2}{\tilde{m}_2^2} - \frac{1}{2} \right) \right]. \quad (9)$$

Using $\Delta m_{32}^2 \simeq 2.5 \times 10^{-3} \text{ eV}^2$ from atmospheric neutrino data, the condition

$$\begin{aligned} & \ln(\tilde{m}_1^2/\mu^2) - 0.3[\ln(\tilde{m}_1^2/\tilde{m}_2^2) - 1/2] \\ & \simeq (0.535/\sin \theta \cos \theta)(0.4 \text{ eV}/m_0)^2 \end{aligned} \quad (10)$$

is obtained. This is easily satisfied if m_0 (the common mass measured in neutrinoless double beta decay) is not much less than the present experimental bound of about 0.4 eV.

Using the same approximation which leads to Eq. (9), the $\tau \rightarrow \mu\gamma$ amplitude is then given by

$$\frac{e(3g_2^2 + g_1^2)}{3072\pi^2} \sin 2\theta \frac{m_\tau}{\tilde{m}_2^2} \epsilon^\lambda q^\nu \bar{\mu} \sigma_{\lambda\nu} (1 + \gamma_5) \tau. \quad (11)$$

Using the experimental upper bound of 1.1×10^{-6} (6.0×10^{-7}), this means that $\tilde{m}_2 > 102$ (119) GeV.

Any contribution to $\tau \rightarrow \mu\gamma$ must also contribute to the muon anomalous magnetic moment (but not vice versa). Here, this contribution is less than about 10^{-10} , which is well below the present experimental accuracy.

4. DIRECT FLAVOR VIOLATION

The new physics scale responsible for the neutrino mass matrix may be only a TeV. This is possible in many models, even in the seesaw case with right-handed neutrinos [9]. Among such models, the most direct connection of the neutrino mass matrix to possible new physics is realized in the triplet Higgs model. The interaction Lagrangian contains

$$f_{ij} \left[\xi^0 \nu_i \nu_j + \xi^+ \left(\frac{\nu_i l_j + l_i \nu_j}{\sqrt{2}} \right) + \xi^{++} l_i l_j \right] \quad (12)$$

which implies $(\mathcal{M}_\nu)_{ij} = 2f_{ij}\langle \xi^0 \rangle$. It has recently been shown [10] that very small $\langle \xi^0 \rangle$ may be obtained naturally, even if m_ξ is only of the order 1 TeV. In that case, the decay $\tau^- \rightarrow l_i^+ l_j^- l_k^-$ may be observable through ξ^{++} exchange. Let the

neutrino mass matrix be given by Eq. (7), with $f_{\mu\tau} = f_{ee} = 0.12$ and $m_\xi = 1$ TeV, then

$$B(\tau^- \rightarrow \mu^+ e^- e^-) \simeq 1.3 \times 10^{-7}, \quad (13)$$

which is below the present experimental bound of 1.5×10^{-6} , and close to the new (preliminary) limit of 2.8×10^{-7} from BELLE. Note that if ξ^{++} could be produced at future colliders, its decay into same-sign charged leptons would map out the neutrino mass matrix, up to an overall scale.

5. GROUP III DECAYS

These decays change L by two units, so they are allowed if m_ν is Majorana. However, if the only $\Delta L = \pm 2$ terms in the Lagrangian come from m_ν , then neutrinoless double beta decay is the only viable experimental signature. If not, then $\tau^- \rightarrow \mu^+ d\bar{u}d\bar{u}$ may have a chance, but it would require something very exotic, such as a heavy neutral Majorana fermion X with four-fermion interactions of the form $(\bar{u}d)(\bar{l}_R X)$.

6. CONCLUSION

Flavor violation in τ decays must occur at some level. There are models of neutrino mass (whether the scale of new physics is 10^{13} GeV or 1 TeV), which predict such decays within reach of future experiments for a reasonable range of parameters.

Afterword

This talk was given on September 11, 2002. One year ago, my former Ph.D. student and colleague D. Ng was on the 80th floor of the North Tower when the first plane hit. He was one of the last people who got out safely before it collapsed.

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