X-ray Scatter-to-Primary Ratio versus Thickness

Two analytic Models Evaluated Against Monte Carlo Calculations

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Outline

- •Detectors at GE Research
- •Relevance of Scatter/Absorptive Problem To Detectors
- •Experimental Data
- •Two analytic models
- •Comparison to Monte Carlo Calculations
- •Conclusions

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Niskayuna, NY – World Headquarters

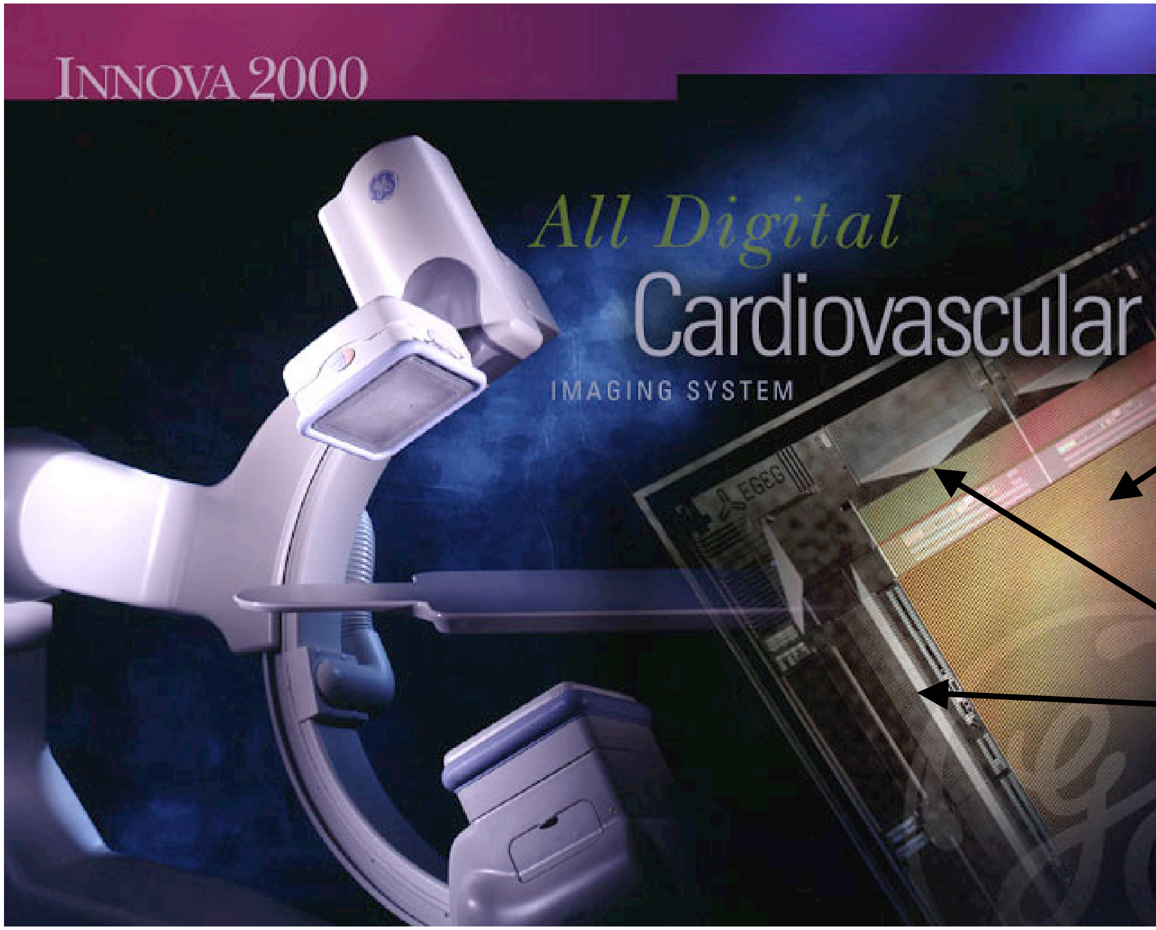
Technical Representation	
Chemistry	**18%**
Mechanical	**17%**
Physics	**9%**
Electrical	**18%**
Computer Sci.	**17%**
All Other	**21%**

Bangalore, India

Shanghai, China

Global Research Organization

a-Si Flat Panel Digital X-ray Imager



CsI Scintillator on Monolithic Array of a-Si Photodiodes and FET Switches

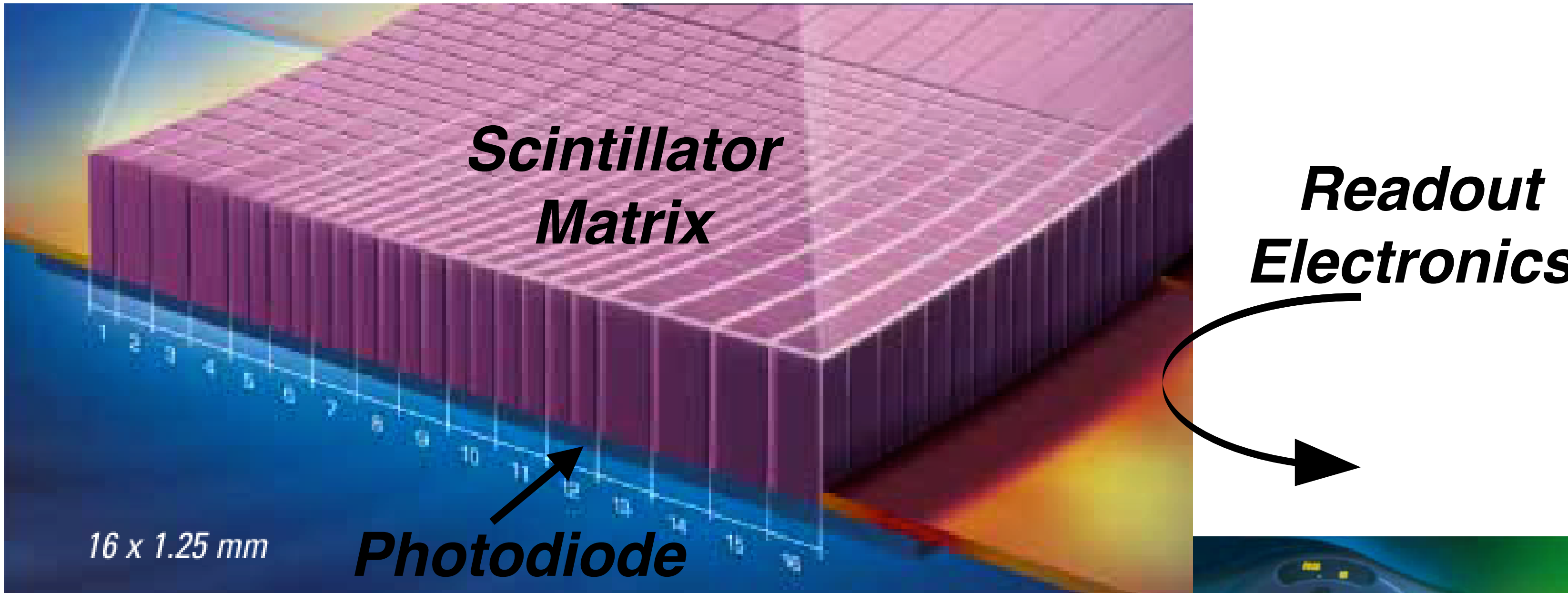
Row & Column Readout Electronics

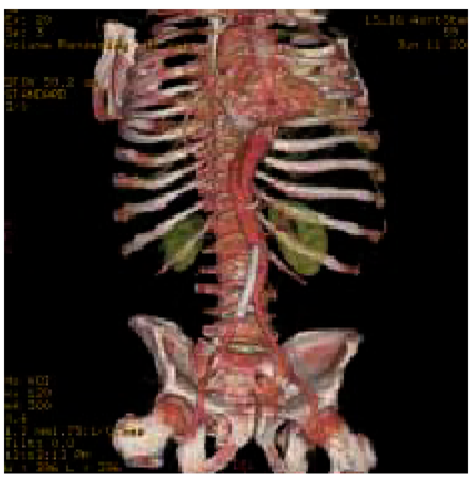
Projected evolution for digital x-ray imaging

- ***higher resolution pixels***
- ***greater patient coverage***
- ***faster scanning***

GE Research

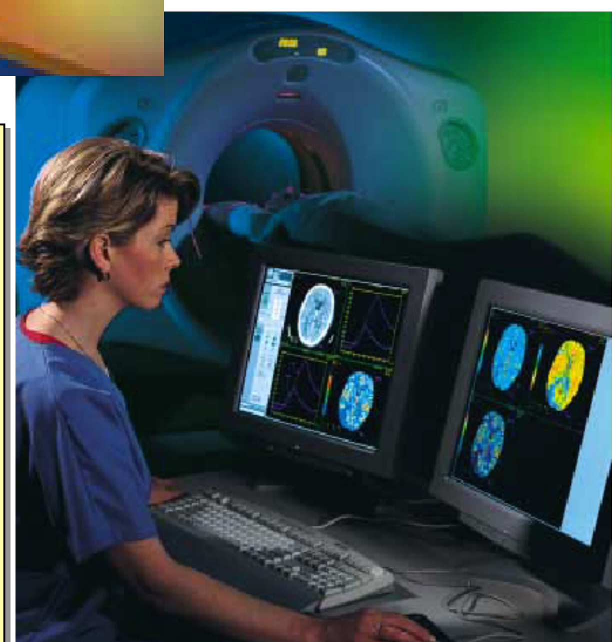
Multi-slice to Volume CT Detectors





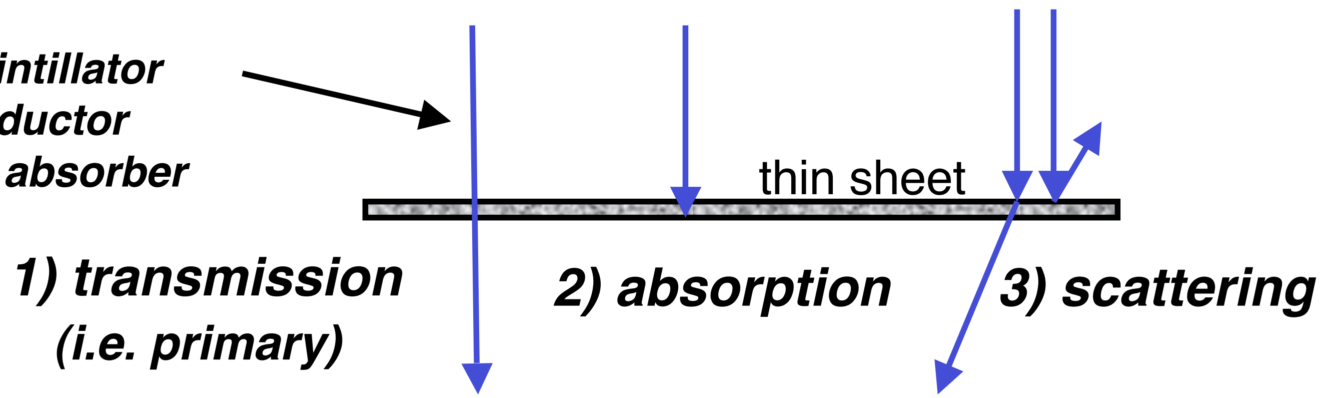
Projected evolution is from multi-slice to volume CT

- ***higher resolution pixels***
- ***much greater patient coverage***
- ***faster scanning speed***

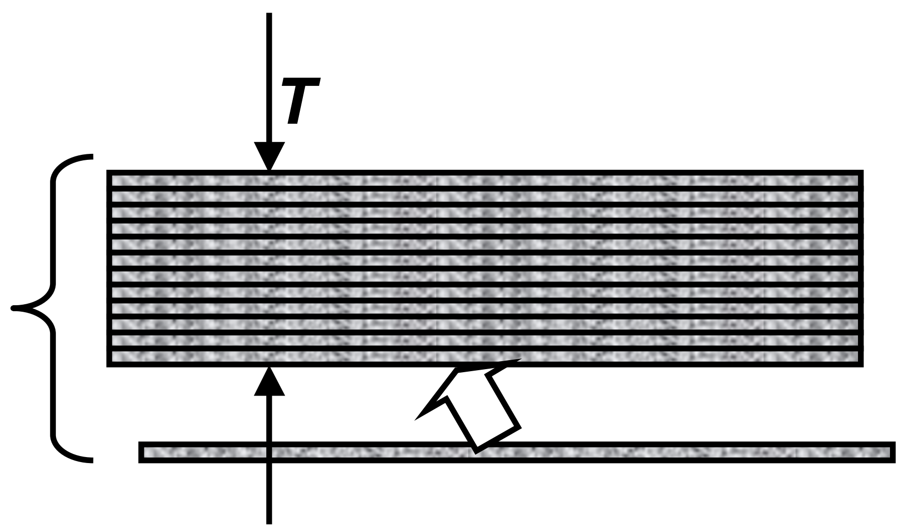


Combined Absorption and Scattering Processes

particle of interest could be
- *x-ray in patient*
- *light photon in scintillator*
- *e/hole in semiconductor*
- *neutrons through absorber*

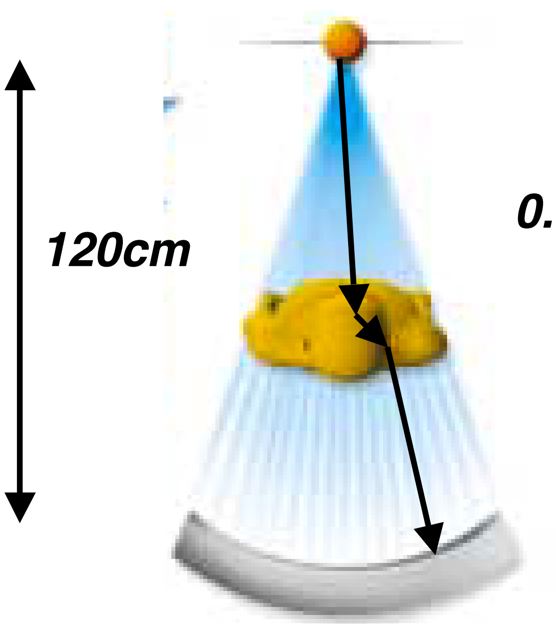


The radiation density profile becomes a remarkably difficult to calculate for non-infinitesimal layers



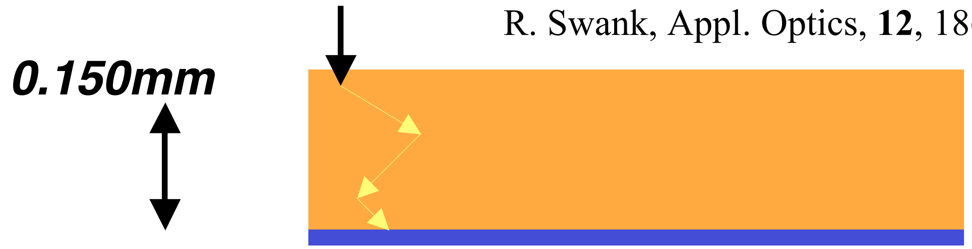
Combined Absorption and Scattering Processes

x-ray in patient

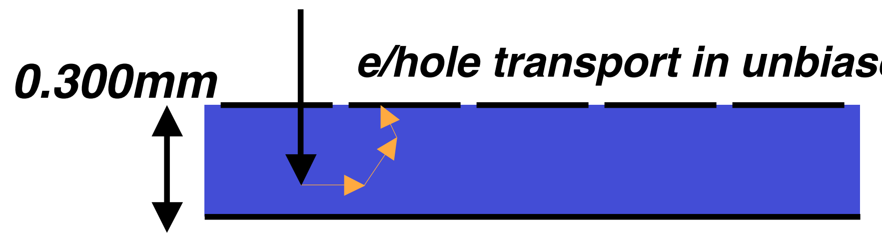


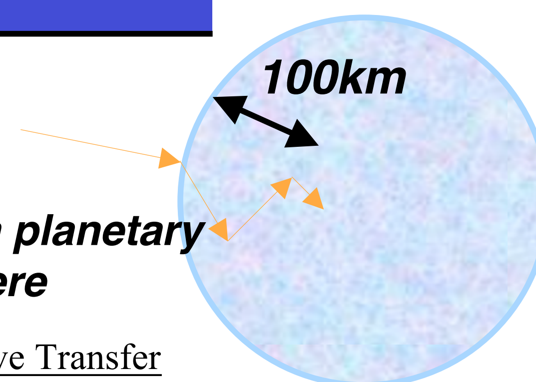
light photon in scintillator

R. Swank, Appl. Optics, **12**, 1865 (1973).



e/hole transport in unbiased semiconductor



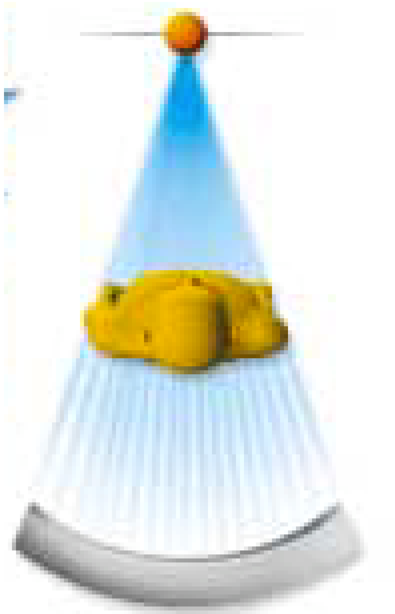


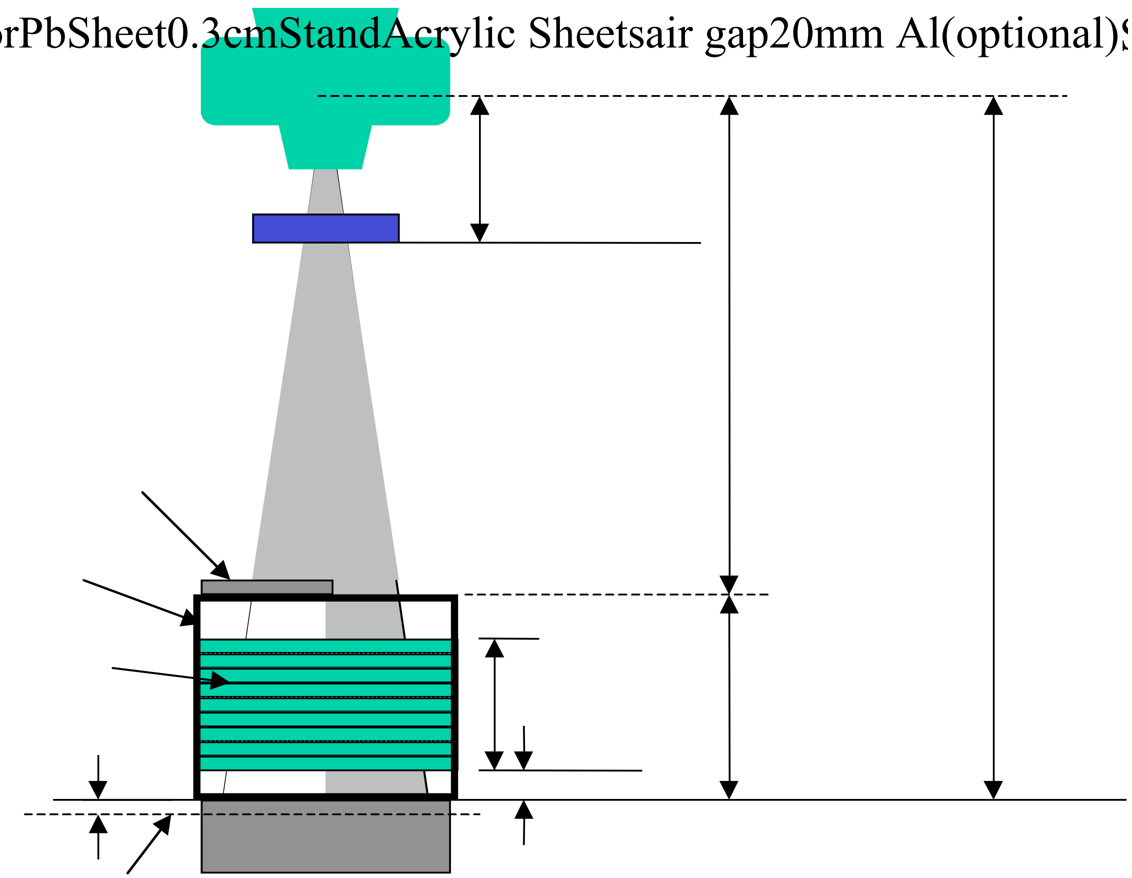
light incident to a planetary atmosphere

S. Chandrasekhar, Radiative Transfer
Dover Pub. 1960

Combined absorption/scatter phenomena is ubiquitous for detectors and imaging.
The albedo = $W=\sigma_{scatter}/\sigma_{total}$ is useful to divide the problem space
Detector studies seek to maximize the gain efficiency and minimize crosstalk

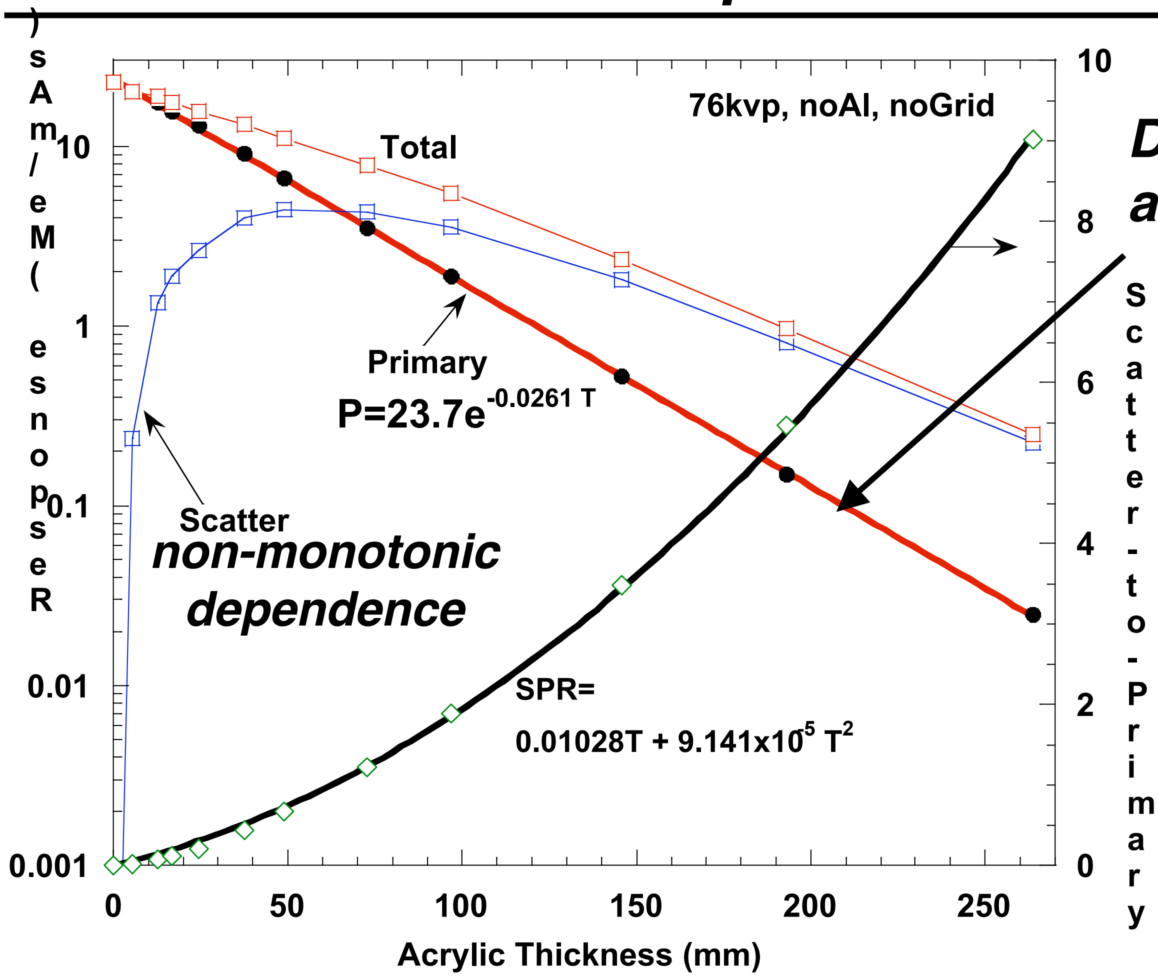
Experimental Study of Scatter





- Measurements with and without Pb Sheet Allows Scatter Measurement
- Experimental Results were simulated with MC calculation using GEANT4

Experimental Study of Scatter



Data shows exponential attenuation of the primary

Scatter is non-monotonic ... it is linearly increasing at small thickness and exponentially decreasing at large thickness

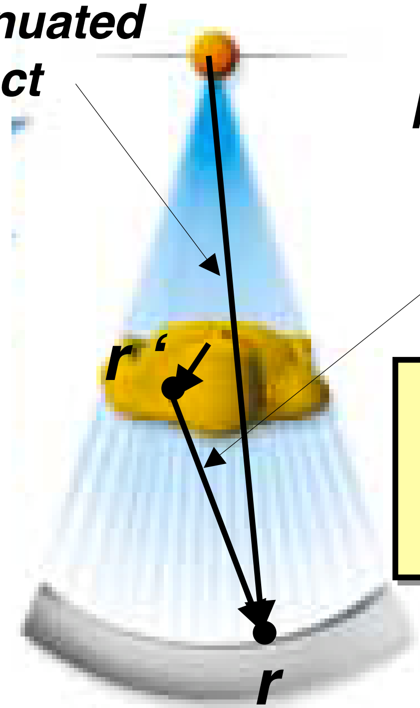
General Formal Solution to the Coupled Scatter/ Absorption Problem is an Integral Equation

S. Chandrasekhar, Radiative Transfer, Dover Pub. 1960

$$I(r,\Omega) = I_{source}(0)\frac{e^{-\tau(r,0)}}{r^2} + \int_V dr' \int_S d\Omega'\, p(\Omega_{r'\to r},\Omega')\, I(r',\Omega')\frac{e^{-\tau(r,r')}}{(r'-r)^2}$$

Primary= source attenuated by object

Scatter from other points in the object



Total Signal is Self-Consistent Sum of Primary from the source and Scatter from all other points

General Solution

$$I(r,\Omega) = I_{source}(0)\frac{e^{-\tau(r,0)}}{r^2} + \int_V dr' \int_S d\Omega'\, p(\Omega_{r'-r},\Omega')\, I(r',\Omega')\frac{e^{-\tau(r,r')}}{(r'-r)^2}$$

conditions of slow variation
(i.e. not valid near surfaces and sources)

A. Ishimaru, Wave Propagation and Scattering in Random Media, Academic Press, Inc. N.Y. 1978.

First Order Solution: Diffusion Equation

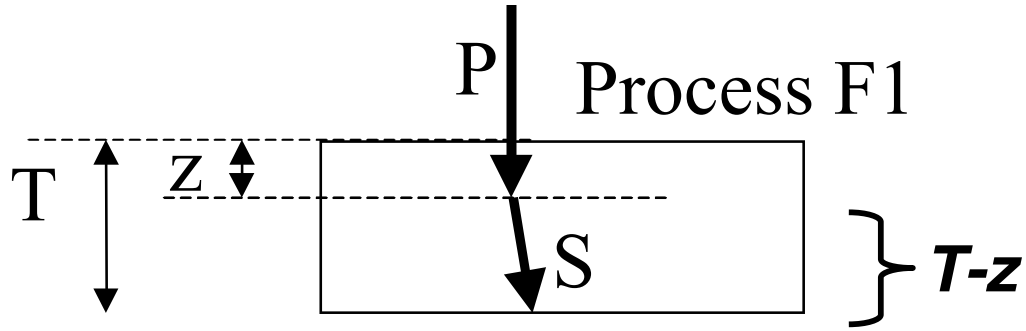
$$\nabla^2\phi(r) + \kappa^2\phi(r) = S(r)$$

reciprocal diffusion length → κ

source function → $S(r)$

Diffusion equation used by Swank to model light propagation in scintillators

Analytic Model 2... after Smith and Kruger



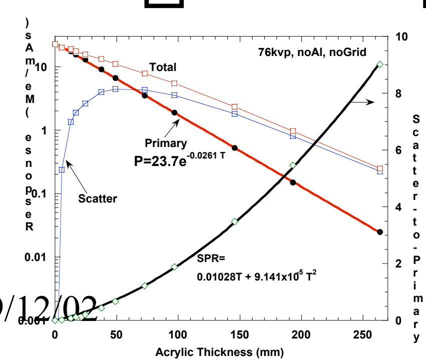
Multiple Scattering is treated by an effective attenuation coefficient for scatter which is less then attenuation coefficient for primary

$\alpha < 1$

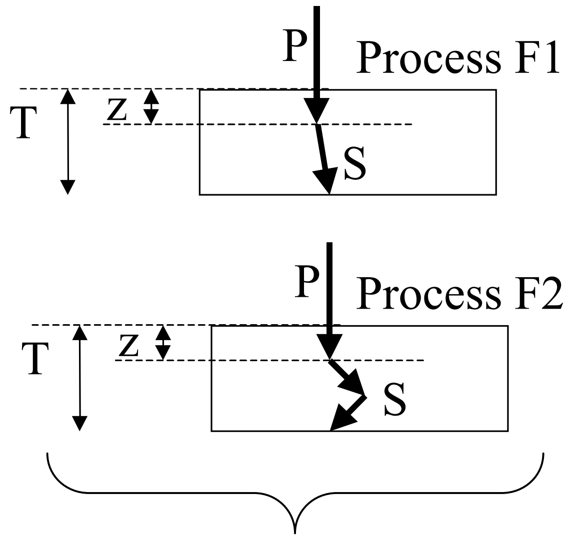
$$S_F = I_o \int_{z=0}^{T} dz \, \frac{\mu_C}{2} \, e^{-\mu_p z} e^{-\alpha \mu_p (T-z)}$$

$$= I_o \left(\frac{W}{2(1-\alpha)} \right) e^{-\alpha \mu_p T} \left(1 - e^{-(1-\alpha) \mu_p T} \right)$$

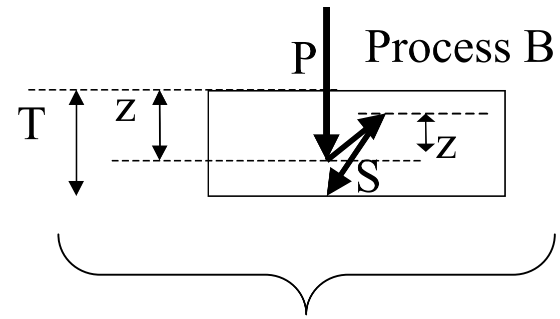
describes the observed non-monotonic dependence



Analytic Model 2... after Smith and Kruger



“1st Forward Scatter”
included by Smith & Kruger



“1st Back-Scatter”
added for more complete treatment

[1] S.W. Smith and R.A. Kruger, “A signal processing model of diagnostic x-ray scatter,” Med. Phys. 13, 831 (1986).

Augment the Method of Smith and Kruger

by including Back-Scattered Processes

Compare Two Analytic Models

Smith and Kruger model extended to include backscatter

$$S_T = I_o \frac{\square W}{(1 \square \square)} e^{\square \square \square_p T} \square \frac{1+(1+c)\square}{1+\square} \square \square$$

$$\square 1 \square \frac{(1+c/2)(1+\square)}{1+(1+c)\square} e^{\square(1 \square \square)\square_p T} + \frac{c}{2} \square \frac{1 \square \square}{1+\square} \square \frac{1+\square}{1+(1+c)\square} e^{\square(1+\square)\square_p T} \square$$

Diffusion equation for slab geometry with perpendicular incidence.

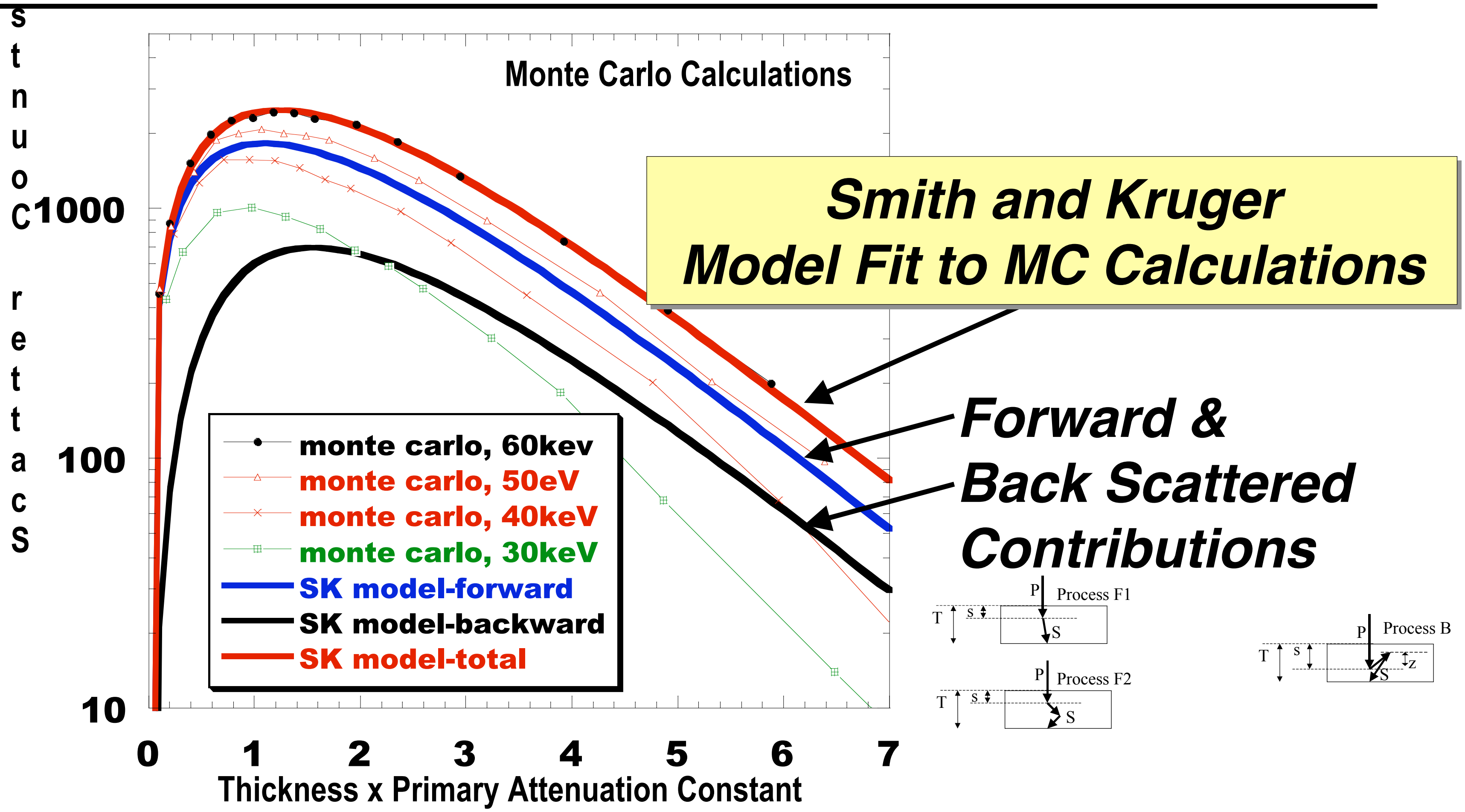
$$S_T = I_o \frac{W/2}{(1 \square \square)} e^{\square \square \square_p T} \square \frac{4\square(1+\square\square)}{(1+\square)^2 (1+\square)} \square \square$$

$$\square 1 \square \frac{(1+\square)(1+\square)}{2(1+\square\square)} e^{\square(1 \square \square)\square_p T} + \frac{1}{2} \square \frac{1 \square \square}{1+\square} \square (1 \square \square) e^{\square(1+\square)\square_p T} \square$$

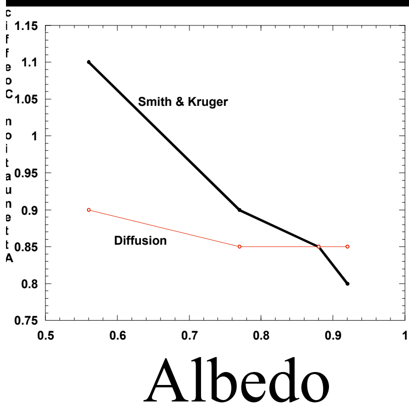
R.K. Swank, "Calculation of Modulation Transfer Function of X-ray Fluorescent Screens,“ (equation 23 was used), Appl. Optics, 12, 1865 (1973)

What is striking about this comparison is the similar exponential dependences obtained from such different approaches
The parameter □ appears is both expressions and represents the relative attenuation of scatter compared to primary

Compare Monte Carlo to Analytic Model 2



kev	Monte Carlo Params				1D model Fits		Diffusion Model Fits			
	?? photoelec	?? compton	?? total	albedo	?	c	?	?	?? expected	?? expected
30	0.1410	0.183	0.324	0.56	1.1	0	0.9	0.2	1.14	0.847222
40	0.0543	0.184	0.238	0.77	0.9	0.0	0.85	0.4	0.83	1.157773
50	0.0265	0.187	0.213	0.88	0.85	0.5	0.85	1.0	0.61	1.31338
60	0.0151	0.181	0.196	0.92	0.8	1.0	0.85	3.0	0.48	1.384439



Final fit parameters

Conclusion

- ***Fits to the analytic model to the MC calculation relates parameters in the analytic model to cross-section for scatter and absorption***
- ***Models can be used in studies of scintillator and photodiode performance***