X-ray Scatter-to-Primary Ratio versus Thickness
Two analytic Models Evaluated Against Monte Carlo Calculations

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Outline

- Detectors at GE Research
- Relevance of Scatter/Absorptive Problem To Detectors
- Experimental Data
- Two analytic models
- Comparison to Monte Carlo Calculations
- Conclusions
Niskayuna, NY – World Headquarters

Technical Representation
- Chemistry: 18%
- Mechanical: 17%
- Physics: 9%
- Electrical: 18%
- Computer Sci.: 17%
- All Other: 21%

Bangalore, India

Shanghai, China

Global Research Organization
Projected evolution for digital x-ray imaging
• higher resolution pixels
• greater patient coverage
• faster scanning
Projected evolution is from multi-slice to volume CT

- higher resolution pixels
- much greater patient coverage
- faster scanning speed
Combined Absorption and Scattering Processes

particle of interest could be
- x-ray in patient
- light photon in scintillator
- e/hole in semiconductor
- neutrons through absorber

The radiation density profile becomes a remarkably difficult to calculate for non-infinitesimal layers
Combined absorption/scatter phenomena is ubiquitous for detectors and imaging. The albedo $W = \frac{\text{scatter}}{\text{total}}$ is useful to divide the problem space. Detector studies seek to maximize the gain efficiency and minimize crosstalk.
Experimental Study of Scatter

- Measurements with and without Pb Sheet Allows Scatter Measurement
- Experimental Results were simulated with MC calculation using GEANT4
Experimental Study of Scatter

Data shows exponential attenuation of the primary

Scatter is non-monotonic ... it is linearly increasing at small thickness and exponentially decreasing at large thickness
General Formal Solution to the Coupled Scatter/Absorption Problem is an Integral Equation

\[ I(r, \mathbf{r}) = I_{\text{source}}(0) \frac{e^{-\mu(r,0)}}{r^2} + \int_V \int_S d\mathbf{r}' d\mathbf{r} \cdot \rho(\mathbf{r}, \mathbf{r}', \mathbf{r}') I(r', \mathbf{r}') \frac{e^{-\mu(r,r')}}{(r' \cdot r)^2} \]

Primary = source attenuated by object

Scatter from other points in the object

Total Signal is Self-Consistent Sum of Primary from the source and Scatter from all other points
General Solution

\[ I(r, \theta) = I_{source}(0) \frac{e^{-\frac{\Delta (r, 0)}{r^2}}}{r^2} + \int_{\Omega} dr' dr'' \ p(r', r, \theta') \ I(r', \theta') \frac{e^{-\frac{\Delta (r', r)}{r'^2}}}{(r'^2 r^2)} \]

conditions of slow variation
(i.e. not valid near surfaces and sources)


First Order Solution: Diffusion Equation

\[ \Delta^2 \varphi(r) + \Delta^2 \varphi(r) = S(r) \]

reciprocal diffusion length source function

Diffusion equation used by Swank to model light propagation in scintillators
Multiple Scattering is treated by an effective attenuation coefficient for scatter which is less than attenuation coefficient for primary $\mu<1$

\[
S_F = I_o \int_{z=0}^{T} \frac{C}{2} e^{\frac{\mu_p z}{2}} e^{\mu_p (T-z)}
\]

\[
= I_o \frac{W}{2(1 + \mu_p T)} e^{\mu_p T} \left( 1 + e^{(1+\mu_p) \mu_p T} \right)
\]

describes the observed non-monotonic dependence
Augment the Method of Smith and Kruger
by including Back-Scattered Processes

Smith and Kruger model extended to include backscatter

\[ S_T = I_o \frac{W}{(1}\overline{D}) e^{\frac{T}{p T}} \left[ 1 + \frac{(1 + c)}{1 + \overline{D}} \right] \]

\[ \frac{(1 + c/2)(1 + \overline{D})}{1 + (1 + c)\overline{D}} e^{\frac{(1 + \overline{D})}{p T}} \left[ 1 + \frac{c}{2} \left( \frac{1}{1 + \overline{D}} \right) \right] \]

Diffusion equation for slab geometry with perpendicular incidence.

\[ S_T = I_o \frac{W/2}{(1)\overline{D}} e^{\frac{T}{p T}} \left[ \frac{4(1 + \overline{D})}{(1 + \overline{D})^2 (1 + \overline{D})} \right] \]

\[ \frac{(1 + \overline{D})(1 + \overline{D})}{2(1 + \overline{D})} e^{\frac{(1 + \overline{D})}{p T}} \left[ \frac{1}{2} \left( \frac{1}{1 + \overline{D}} \right) \right] e^{\overline{D} T} \]

R.K. Swank, "Calculation of Modulation Transfer Function of X-ray Fluorescent Screens," (equation 23 was used), Appl. Optics, 12, 1865 (1973)

What is striking about this comparison is the similar exponential dependences obtained from such different approaches. The parameter \( \overline{D} \) appears in both expressions and represents the relative attenuation of scatter compared to primary.
Smith and Kruger Model Fit to MC Calculations

Forward & Back Scattered Contributions

Monte Carlo Calculations

- monte carlo, 60kev
- monte carlo, 50eV
- monte carlo, 40keV
- monte carlo, 30keV
- SK model-forward
- SK model-backward
- SK model-total

Smith and Kruger Model Fit to MC Calculations

Compare Monte Carlo to Analytic Model 2

GE Research

9/12/02
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**Conclusion**

- Fits to the analytic model to the MC calculation relates parameters in the analytic model to cross-section for scatter and absorption
- Models can be used in studies of scintillator and photodiode performance