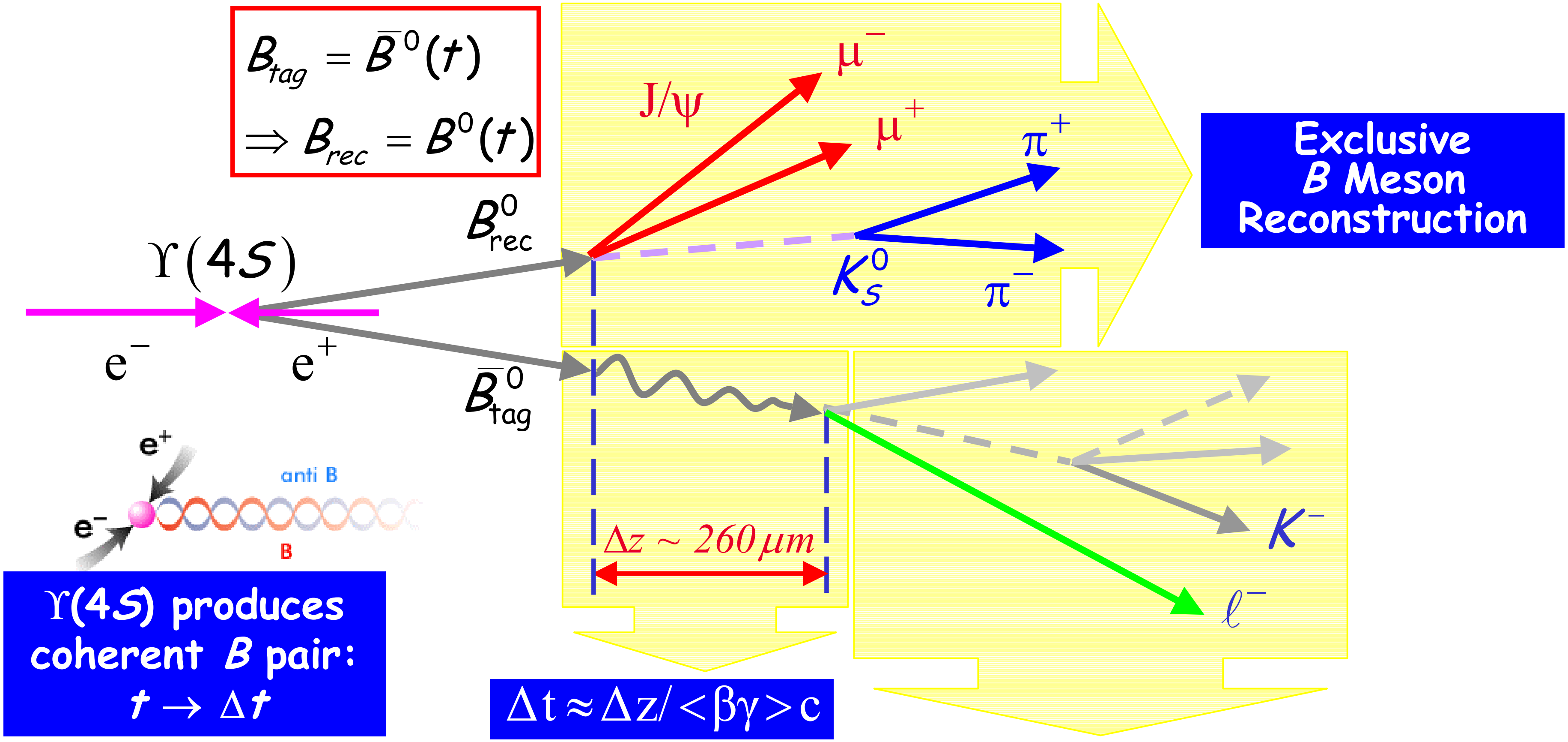
Lecture 2: Time-Dependent Measurements with Flavor and CP Samples

- *Reconstruction of B meson samples*
- *Lifetimes and B^0 oscillation measurements with flavor eigenstate decay*

Experimental Technique for B Factories



***B*-Flavor Tagging**

Time-integrated asymmetries are zero

$B^0_{rec} = B^0_{flav}$ (flavor eigenstates) → lifetime, mixing analyses

$B^0_{rec} = B^0_{CP}$ (CP eigenstates) → CP analysis

Main Variables for B Reconstruction

For exclusive B reconstruction, two nearly uncorrelated* kinematic variables are used:

$$\Delta E = E_B^* - E_{beam}^*$$

Signal at $\Delta E \sim 0$

"Energy-substituted mass"

$$m_{ES} = \sqrt{\left(E_{beam}^*\right)^2 - \left(\mathbf{p}_B^*\right)^2}$$

Signal at $m_{ES} \sim m_B$

$(E_B^*, \mathbf{p}_B^*), E_{beam}^*$ B candidate (energy, 3-momentum) and beam energy in $\Upsilon(4S)$ frame

Resolutions

$$\sigma_{\Delta E}^2 = \sigma_{beam}^2 + \sigma_E^2 \sim \sigma_E^2$$

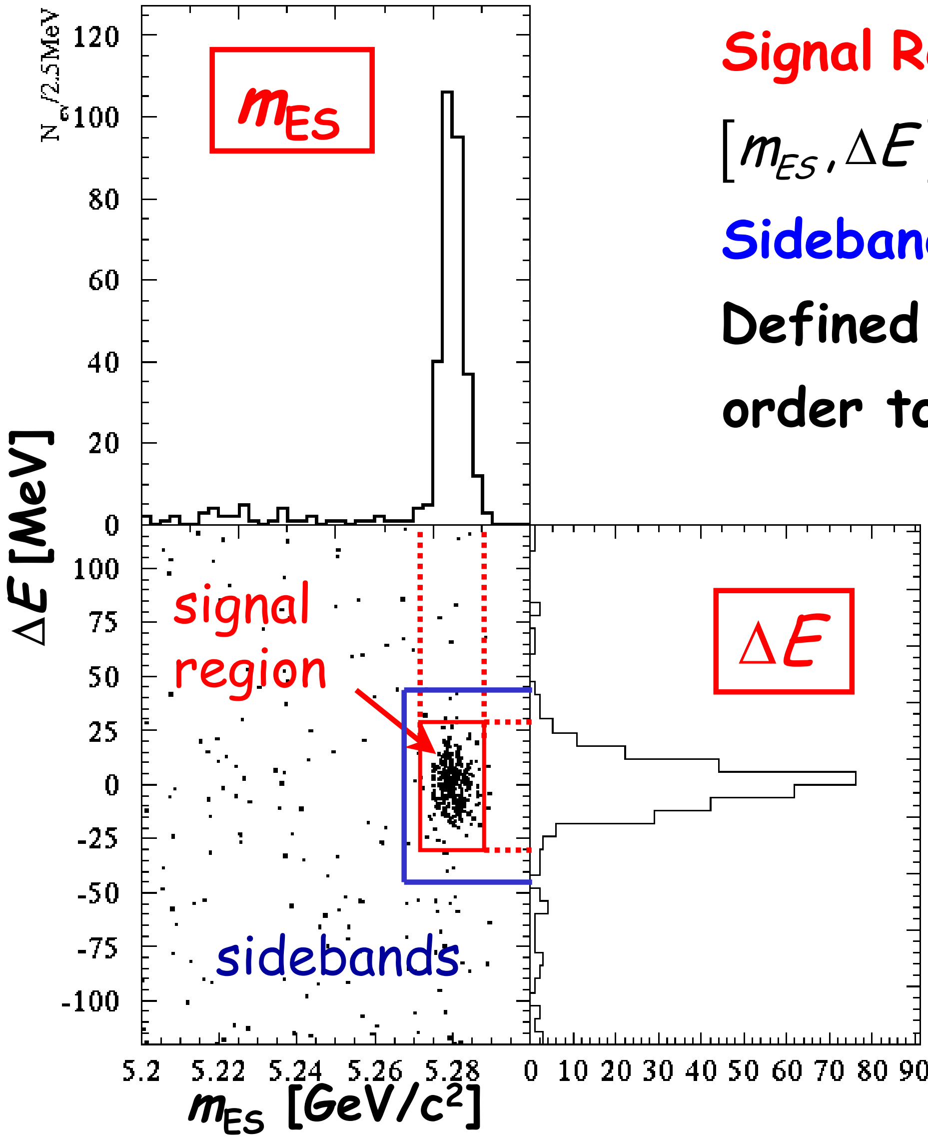
$$\sigma_{\Delta E} \sim 10 - 40 \text{ MeV}$$

$$\sigma_{m_{ES}}^2 = \sigma_{beam}^2 + \left[\frac{p}{m_B}\right]^2 \sigma_p^2 \sim \sigma_{beam}^2$$

$$\sigma_{m_{ES}} \sim 2.6 \text{ MeV}/c^2$$

* If σ_E were zero, the variables would be fully correlated; however, σ_E is typically at least 5 times larger than σ_{beam} and so dominates ΔE

Example for Hadronic B Decays



Signal Region :

$$[m_{ES}, \Delta E] = [m_B \pm 3\sigma_{m_{ES}}, 0 \pm 3\sigma_{\Delta E}]$$

Sideband Region :

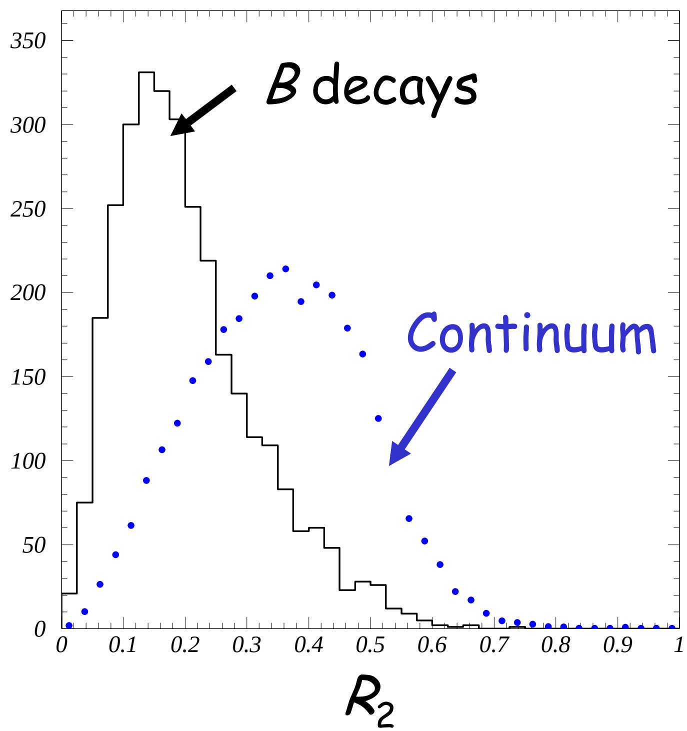
Defined outside signal region in order to estimate backgrounds

$B^0 \rightarrow J/\psi K_S$

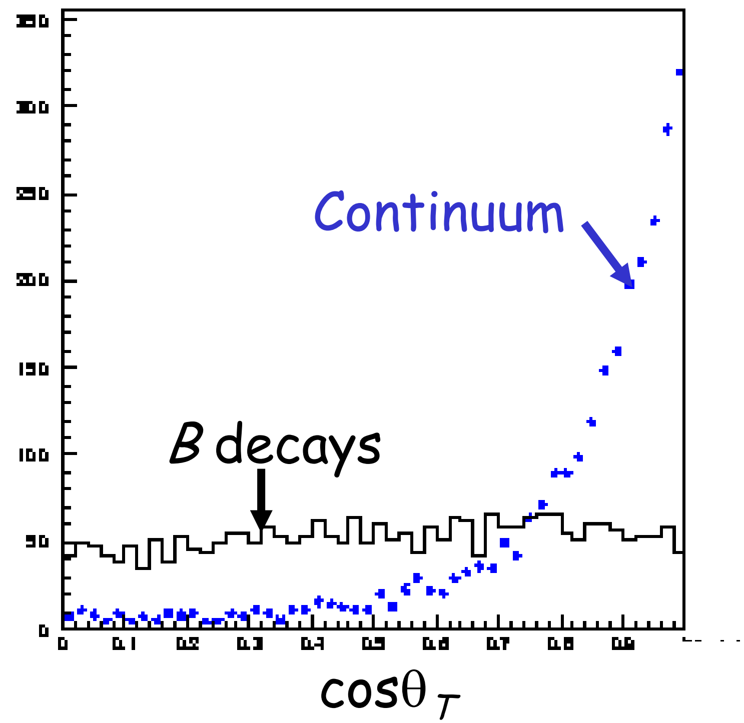
Continuum Background Suppression

- *Separate 2-jet continuum from spherical BB events via event shape variables*

Ratio of second-to-zero[th] order Fox-Wolfram moments (R_2):

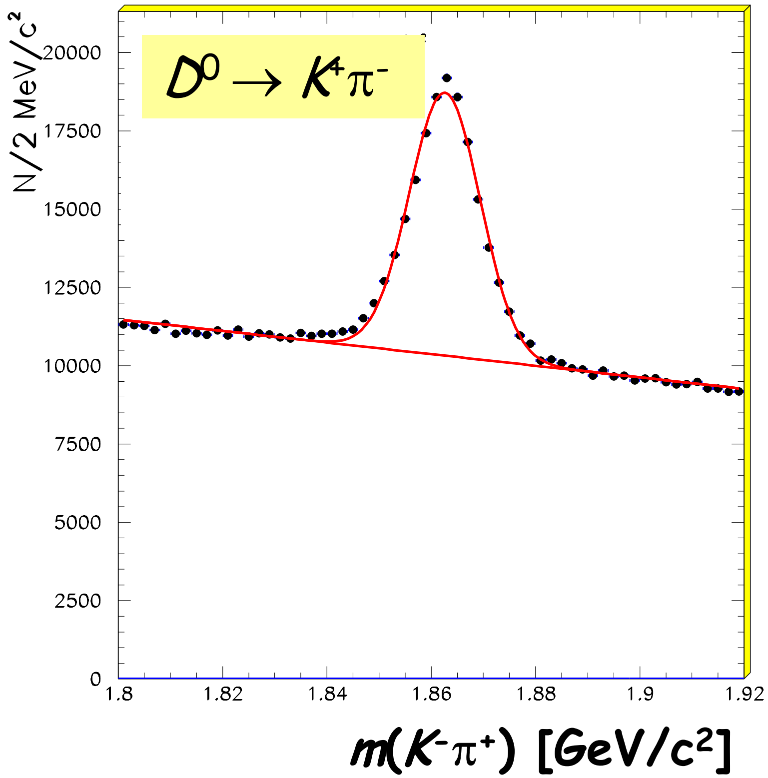


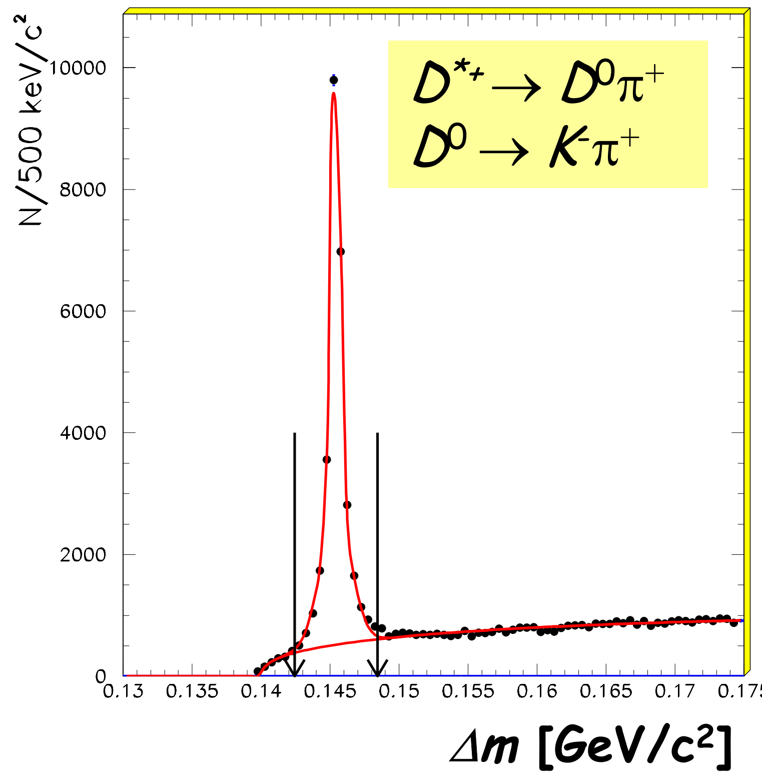
Angle of thrust axis of "rest of the event" wrt *B* candidate direction (θ_T)



Inclusive Open Charm States

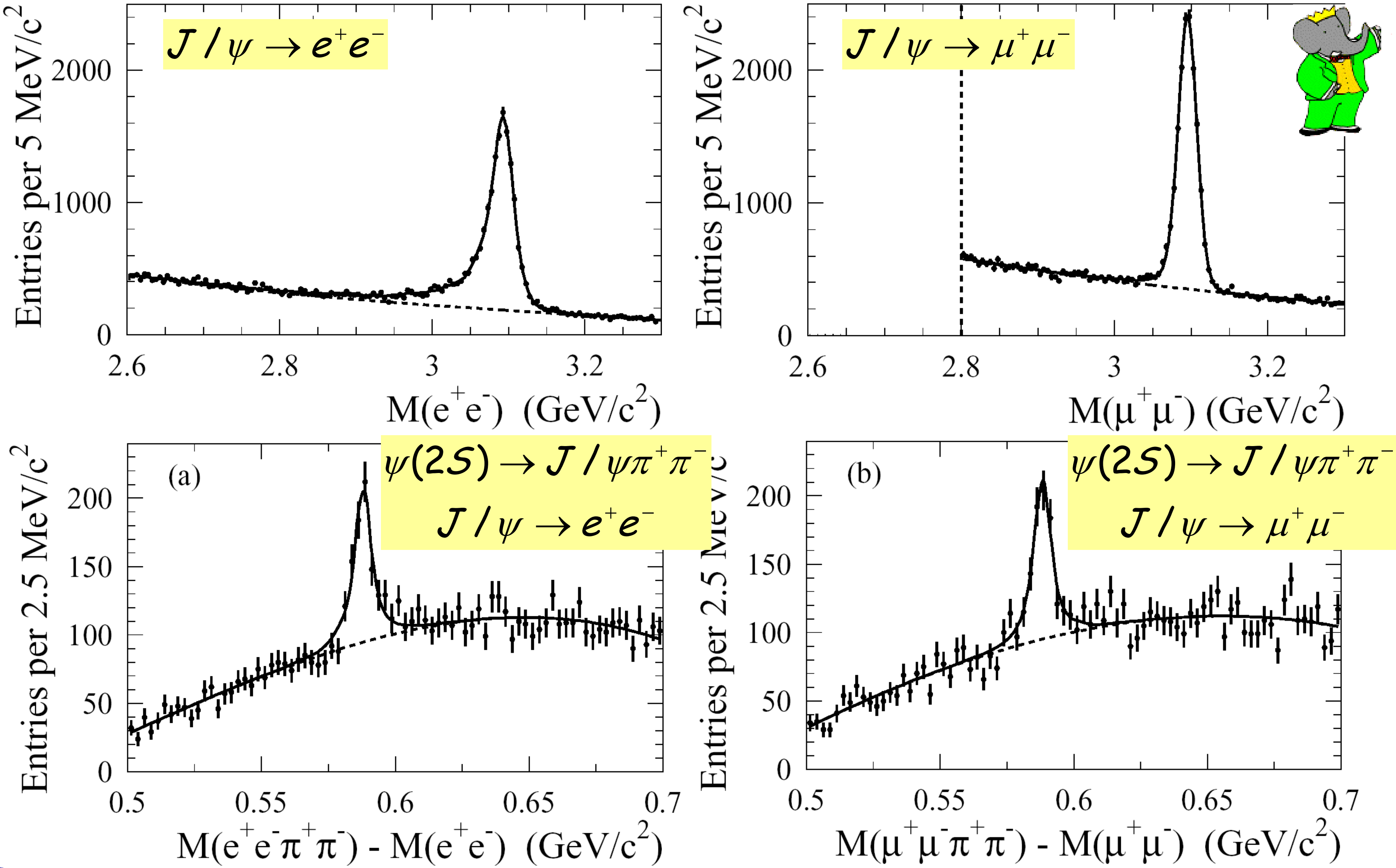
- *Select intermediate mesons using either mass or mass difference:*





- *After selection, candidates are constrained to nominal masses*

Inclusive Charmonium Signals



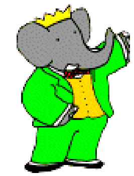
Flavor Eigenstate Neutral B Sample

Charm decay modes $\sum BF(\bar{D}^0) \sim 28\%$ $\sum BF(D^-) \sim 12\%$

Self-Tagging Modes

$D^{*-} \to \bar{D}^0\pi^-, \bar{D}^0 \to K^+\pi^-, K^+\rho^-, K^+\pi^+\pi^-\pi^-, K_S^0\pi^+\pi^-$

$D^- \to K^-\pi^+\pi^+, K_S^0\pi^-$

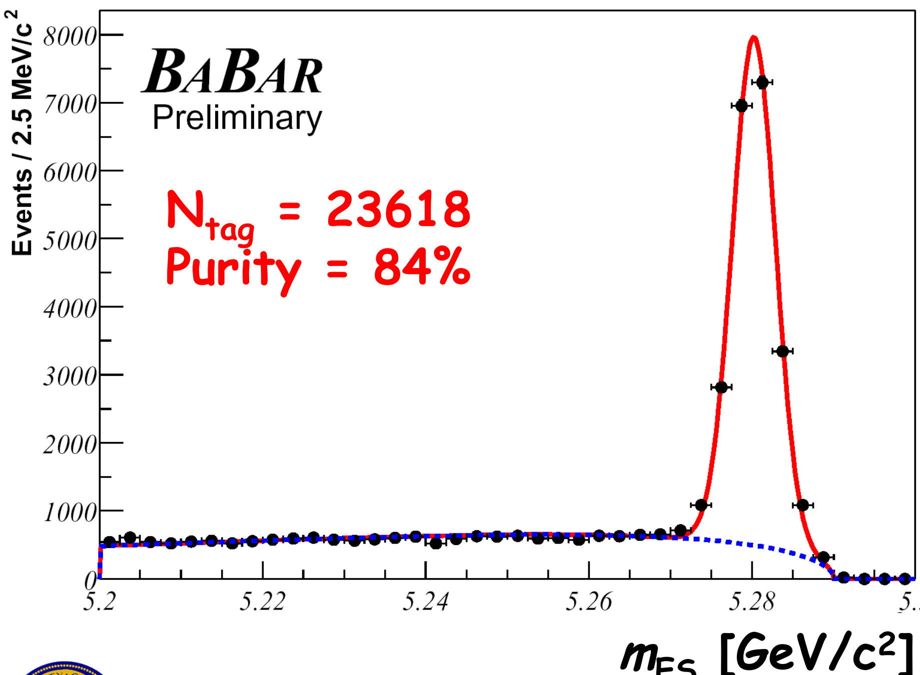


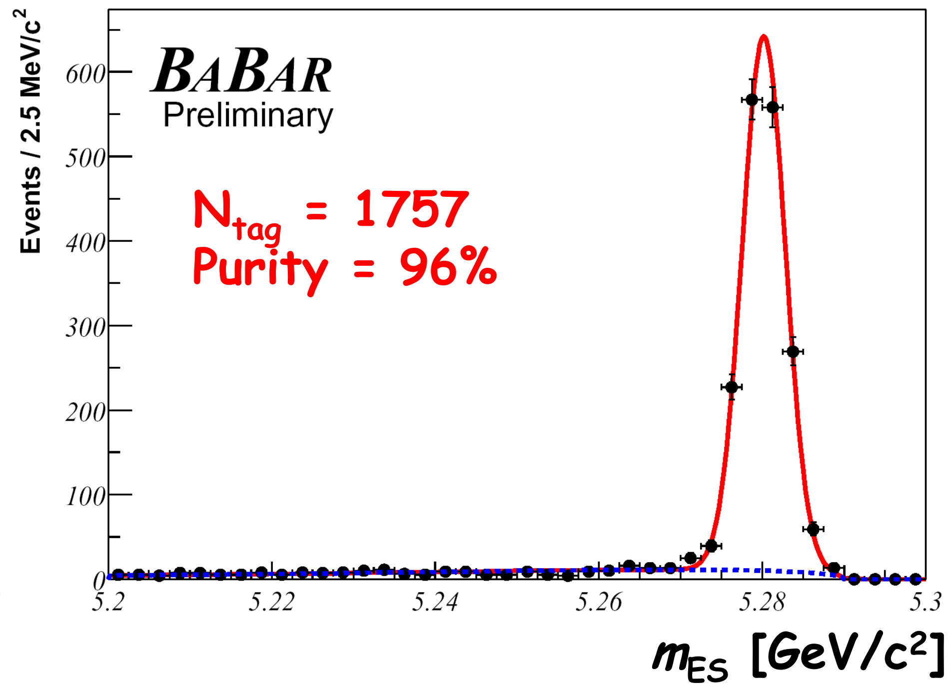
BABAR
$81.3\ fb^{-1}$

B decay modes $\sum BF(B^0) \sim 4.1\%$

$B^0 \to D^{(*)-}h^+ (h^+ = \pi^+, \rho^+, a_1^+)$

$B^0 \to J/\psi K^{*0} (\to K^+\pi^-)$



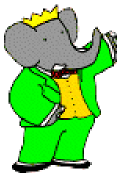


Flavor Eigenstate Charged B Sample

Charm decay modes $\sum BF(\bar{D}^0) \sim 28\%$

Self-Tagging Modes

$\bar{D}^{*0} \to \bar{D}^0\pi^0, \bar{D}^0 \to K^+\pi^-, K^+\rho^-, K^+\pi^+\pi^-\pi^-, K_S^0\pi^+\pi^-$



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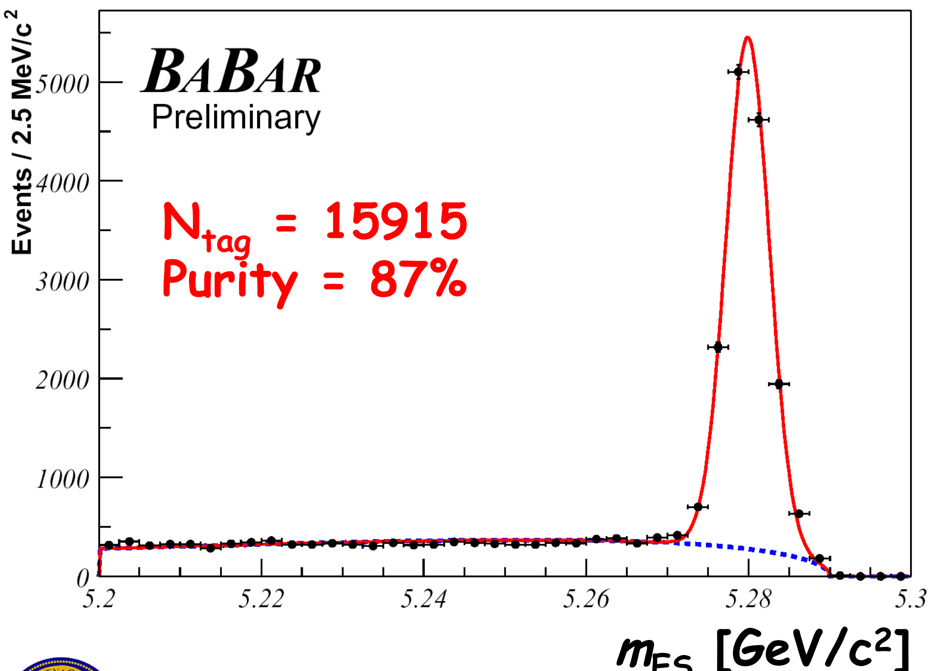
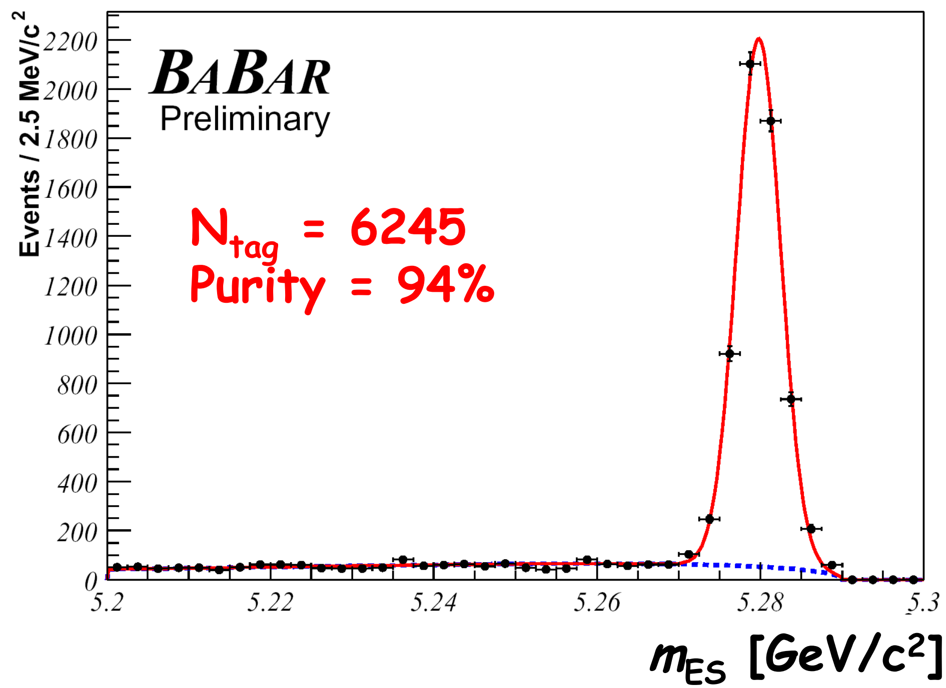
B decay modes $\sum BF(B^0) \sim 1.2\%$

$B^+ \to \bar{D}^{(*)0}\pi^+$

$B^+ \to J/\psi K^{(*)+}, \psi(2S)K^+, \chi_{c1}K^+, \eta_c K^+$





Golden Sample: $(c\bar{c})K_S$ CP Eigenstates

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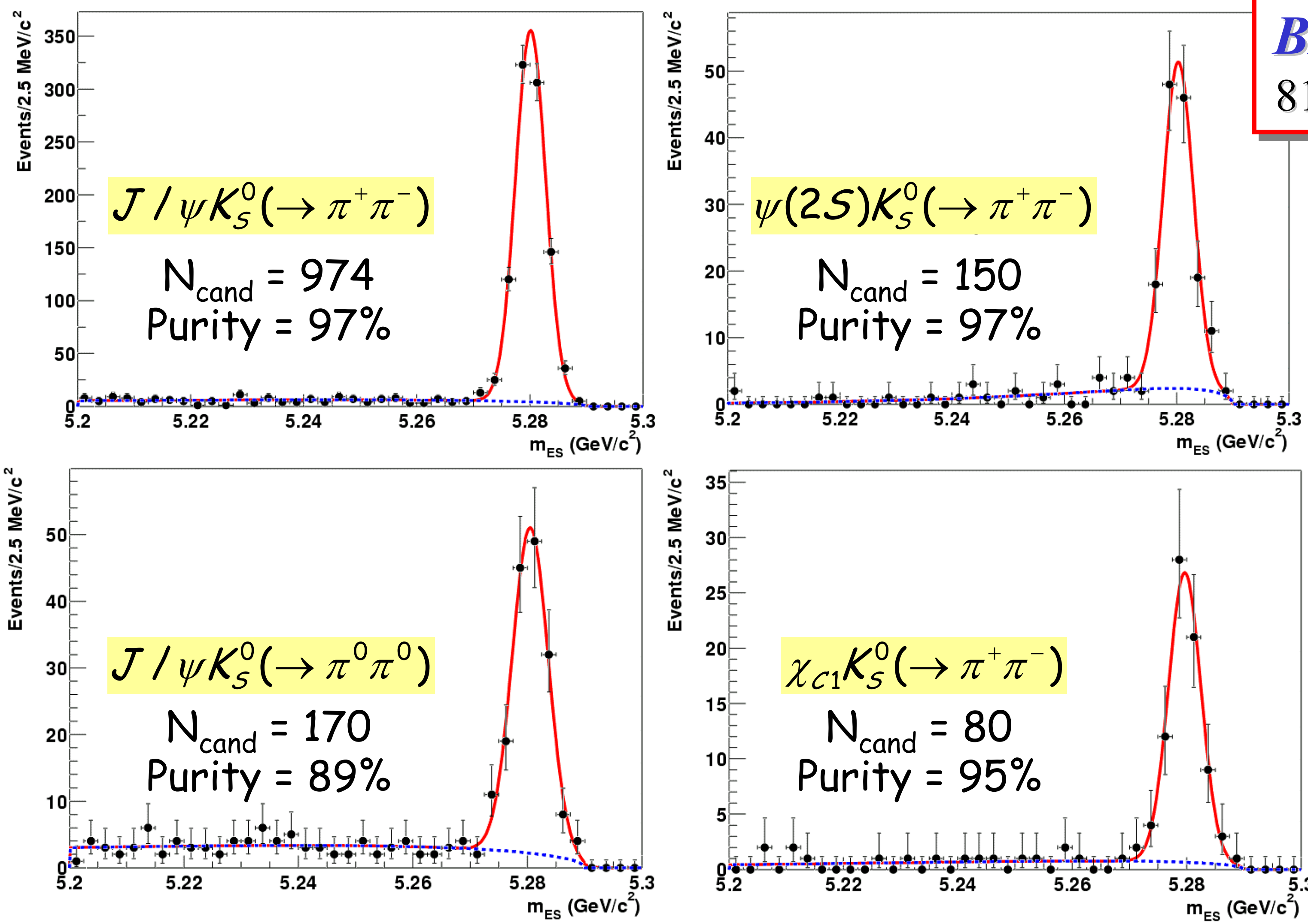
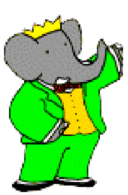


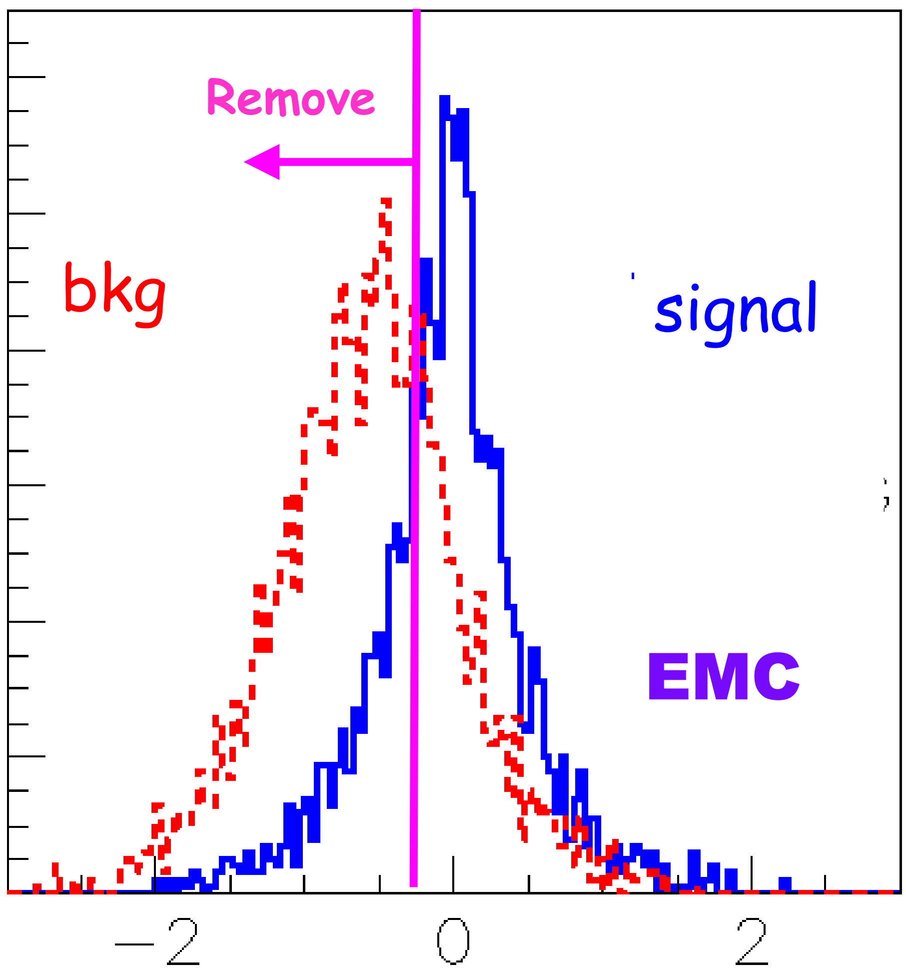
Candidates & purity for m_{ES} > 5.27 GeV/c^2

Selecting candidates for $B \to J/\psi K_L$

- ***K_L detected by nuclear interactions in EMC or IFR***
  - EMC neutral clusters with energy between 0.2 and 2.0 GeV
    - Veto clusters forming π^0s with any other photon ($E_\gamma > 30$ MeV)
    - Remove clusters ($E > 1$ GeV) containing two distinct bumps
  - IFR neutral clusters are 2 or more RPC layers that are unmatched to any projected charged track
  - Only able to determine angle of K_L wrt interaction point, not energy
- ***B candidates formed from mass-constrained $J/\psi \to l^+l^-$ and K_L candidates***
  - Since there should be missing momentum along K_L direction, cut on difference between observed and expected
  - Use cuts on $J/\psi \to l^+l^-$ helicity angle ($\sin^2\theta_h$ for signal) and B candidate polar angle ($\sin^2\theta$ wrt to z-axis)



Final Candidate Selection



measured – expected missing p_t [GeV/c]

- *Apply constraint of known m_B mass & K_L direction $\vec{d}_{K_L}$ to determine momentum p_{KL}*

$$m_B^2 = \left(E_{J/\psi} + \sqrt{m_{K_L}^2 + p_{K_L}^2}\right)^2 - \left(\vec{p}_{J/\psi} + p_{K_L}\vec{d}_{K_L}\right)^2$$

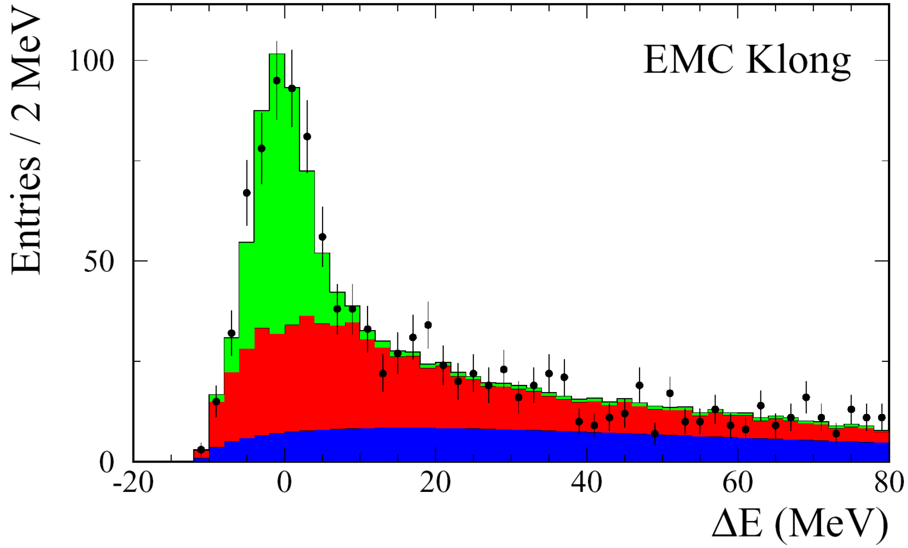
- *Search for signal in the one remaining variable expressed as:*

$$\Delta E = E^*_{J/\psi} + E^*_{K_L} - E^*_{beam}$$

BELLE

$$p^*_B = \left|\mathbf{p}^*_{J/\psi} + \mathbf{p}^*_{K_L}\right|$$

CP Eigenstate Sample: $B \to J/\psi K_L$



Resolution of about 10 MeV for ΔE after m_B constraint

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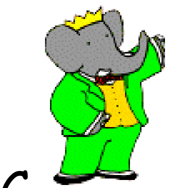
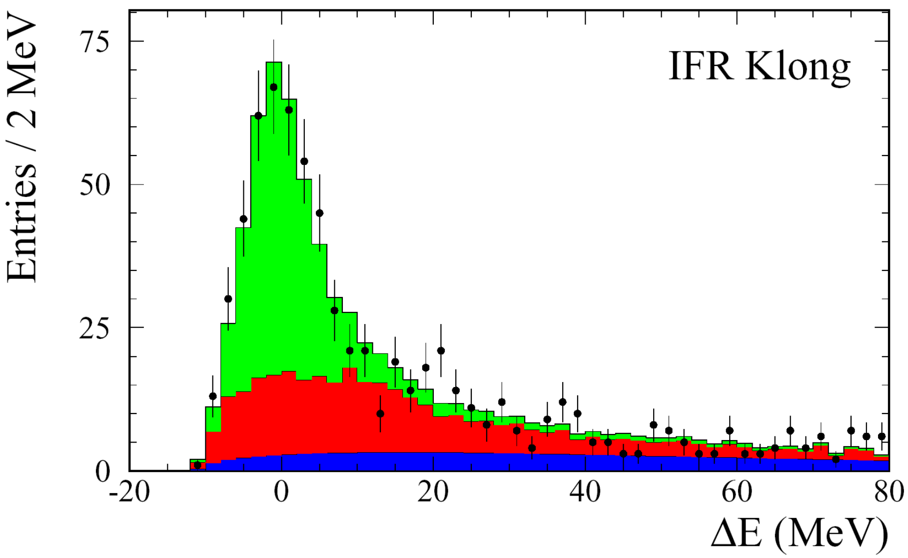
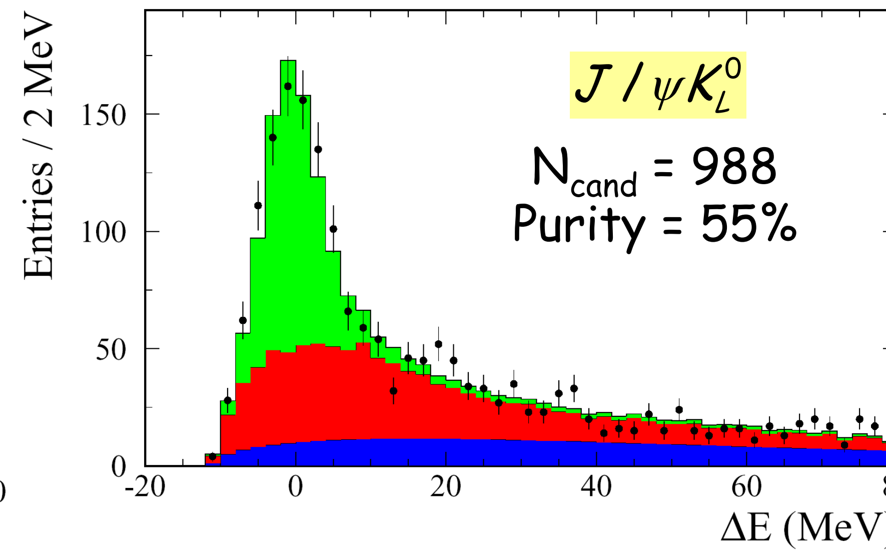


Signal

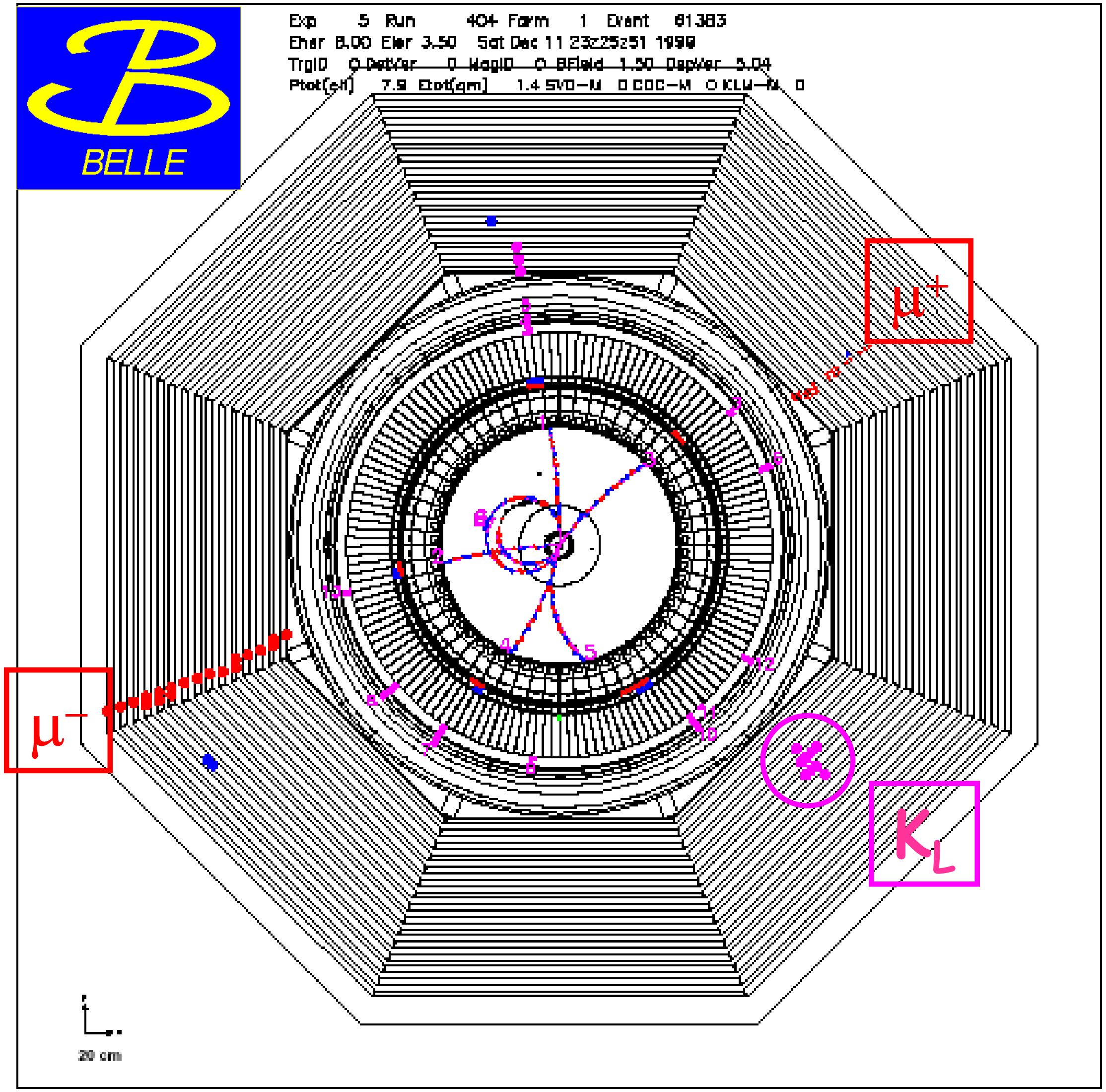
J/ψ Bkg — Estimated with MC

Fake J/ψ Bkg — Estimated with sidebands

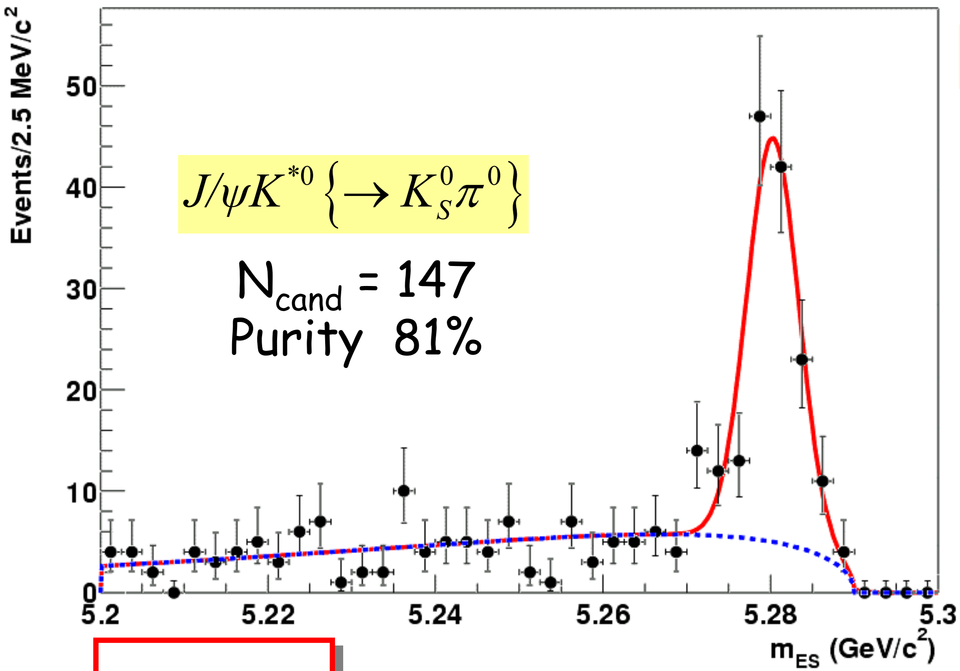




Sample $B^0 \to J/\psi\ K_L$ Event



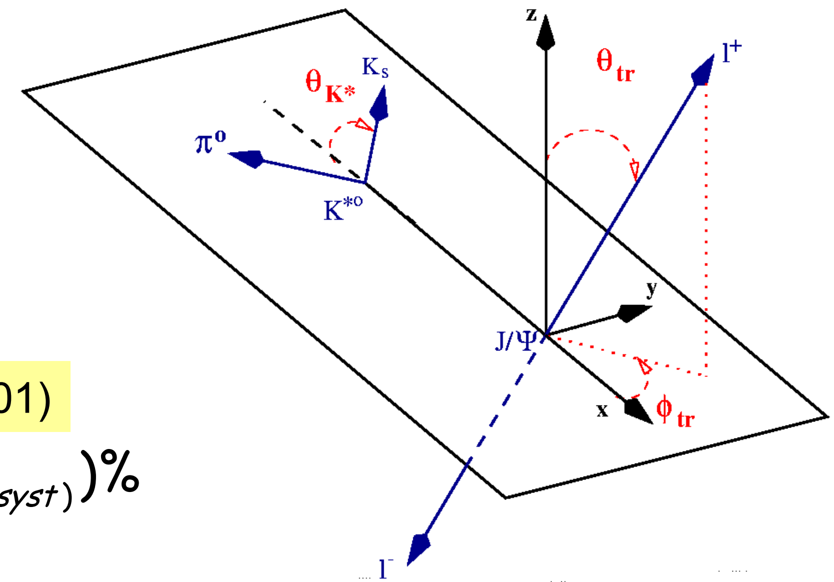
One More Mode for CP Sample



$P \to VV$: mixture of *CP* states

$D_\perp = 1 - 2 \cdot R_\perp,$

$R_\perp \equiv |A_\perp|^2$ fraction of *CP*-odd



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BABAR PRL **87**, 241801 (2001)

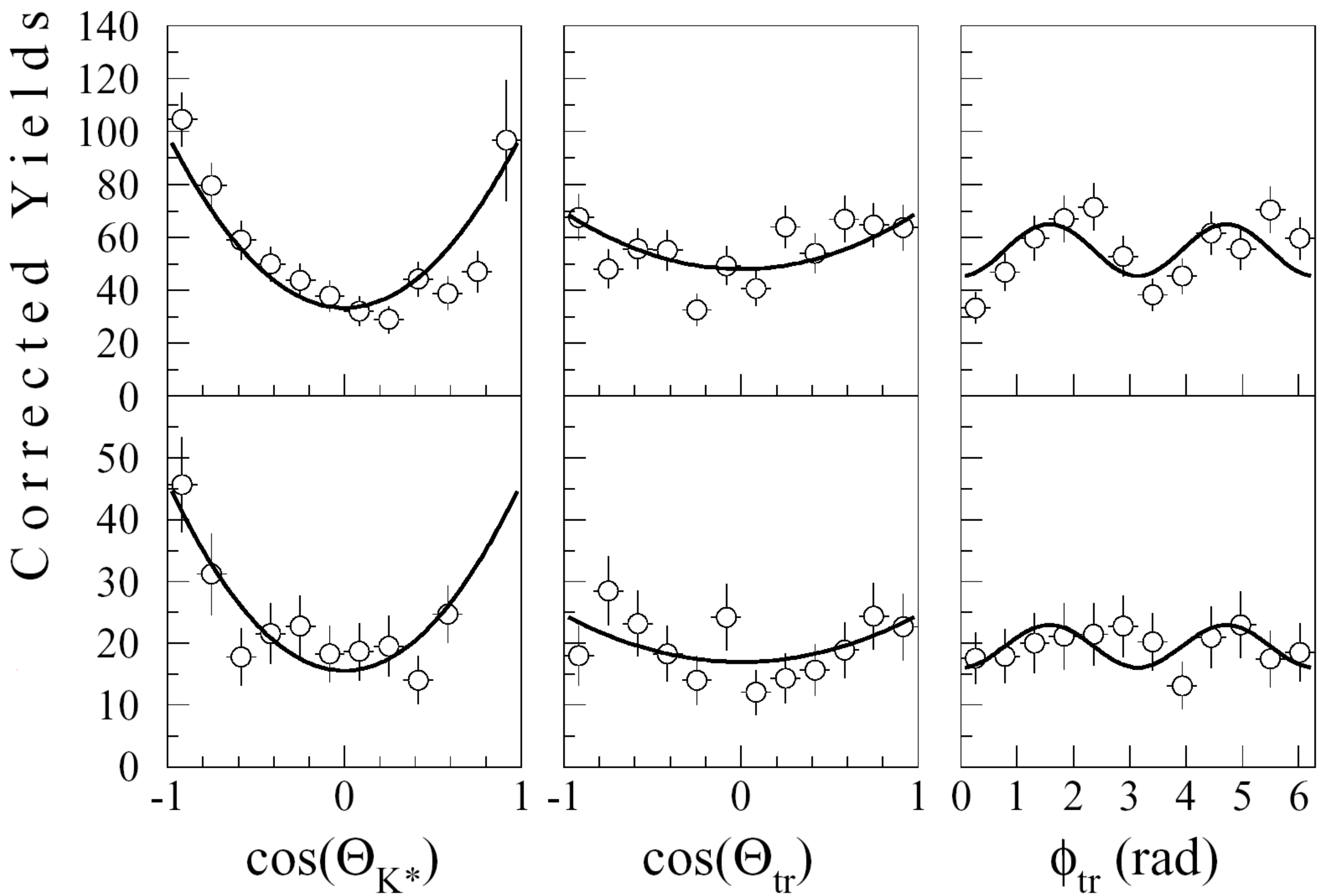
$R_\perp = (16.0 \pm 3.2_{(stat)} \pm 1.4_{(syst)})\%$

BELLE hep-ex/0205021

$R_\perp = (19.1 \pm 2.3_{(stat)} \pm 2.6_{(syst)})\%$

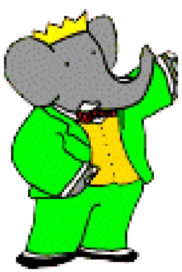
Conclude: J/ψ mostly *CP* even

Angular Analysis at BABAR



K^* modes without π^0

K^* modes with π^0



Projections of fits for amplitudes and rescattering phases:

$$|A_0|^2, |A_\parallel|^2, |A_\perp|^2, \varphi_\perp, \varphi_\parallel$$

Time-Dependent Analysis Strategies

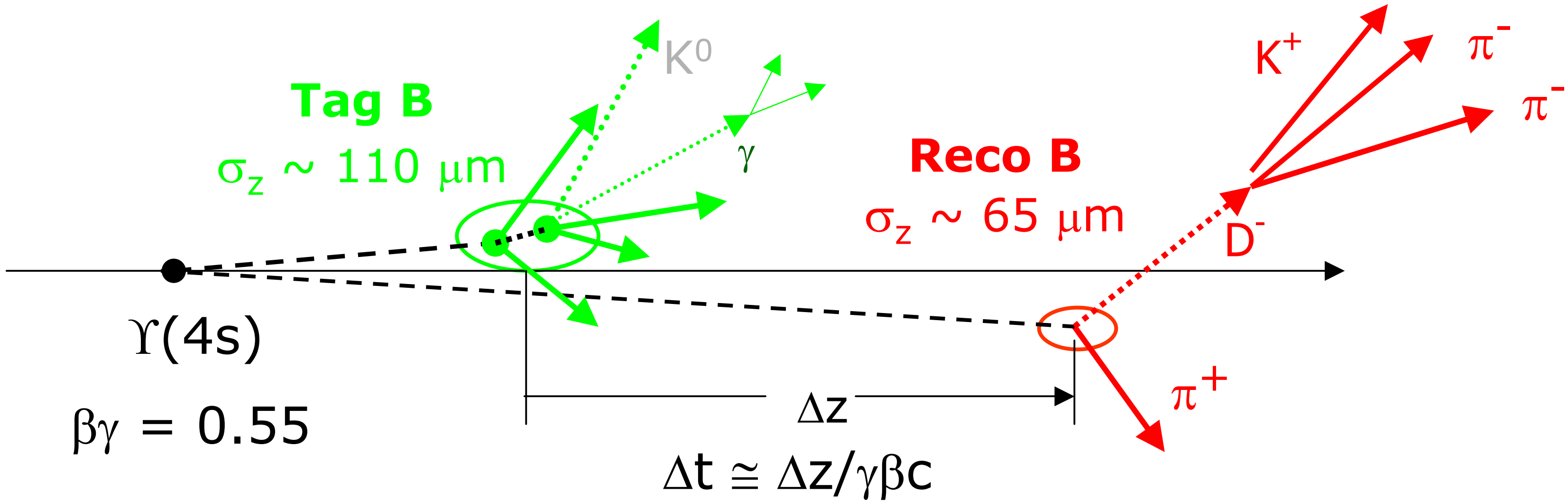
Factorize the analysis into building blocks

Measurements	Analysis Ingredient
B^+ / B^0 Lifetimes	a) Reconstruction of B mesons in flavor eigenstates b) B vertex reconstruction
$B^0\bar{B}^0$ Mixing	c) Flavor Tagging + a + b
CP Asymmetries	d) Reconstruction of neutral B mesons in CP eigenstates + a + b + c

Higher precision (↑)

Increasing complexity (↓)

Measurement of B^0 and B^+ Lifetime



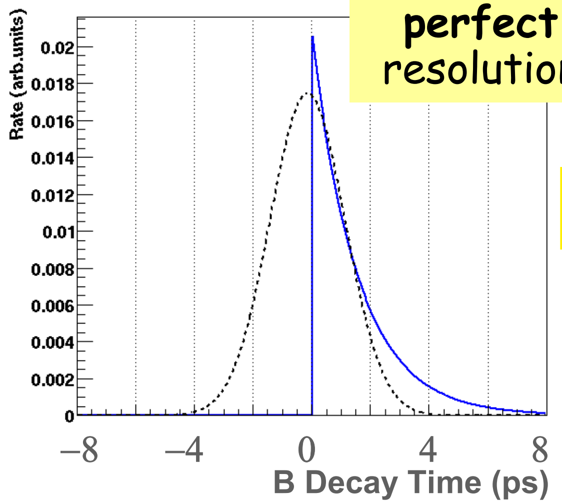
3. Reconstruct inclusively the vertex of the "other" B meson (B_{TAG})

1. Fully reconstruct one B meson in flavor eigenstate (B_{REC})
2. Reconstruct the decay vertex

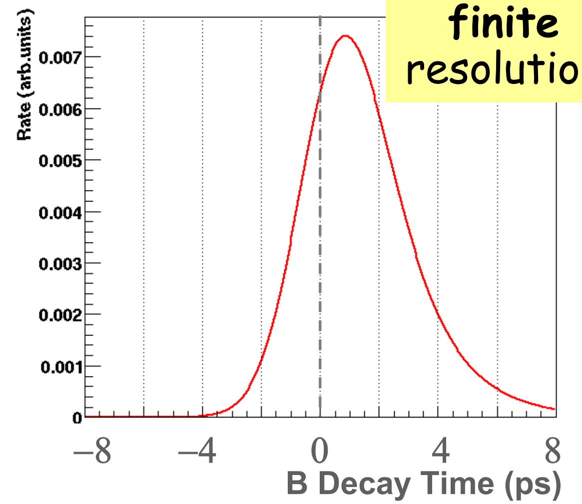
4. Compute the proper time difference Δt
5. Fit the Δt spectra

B-Lifetimes: Time Distributions

LEP/CDF



perfect resolution

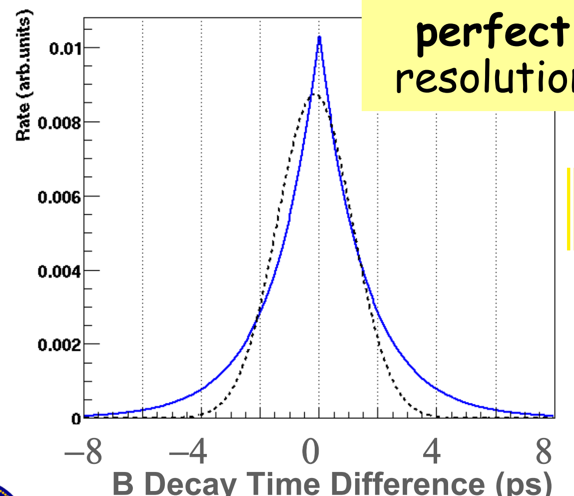


finite resolution

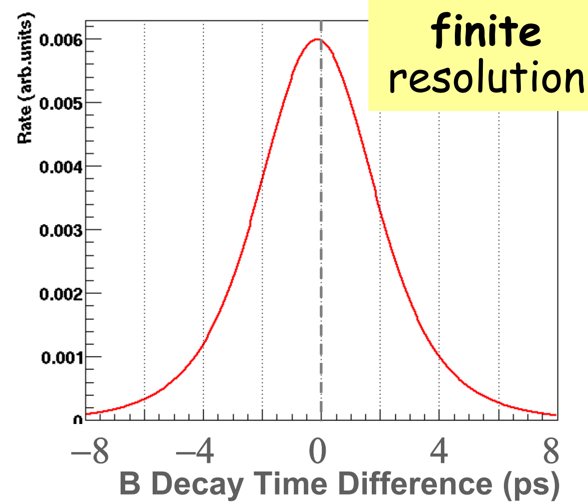
Production point of *B* meson is known with good accuracy

Control of the resolution function at negative times

B Factories



perfect resolution



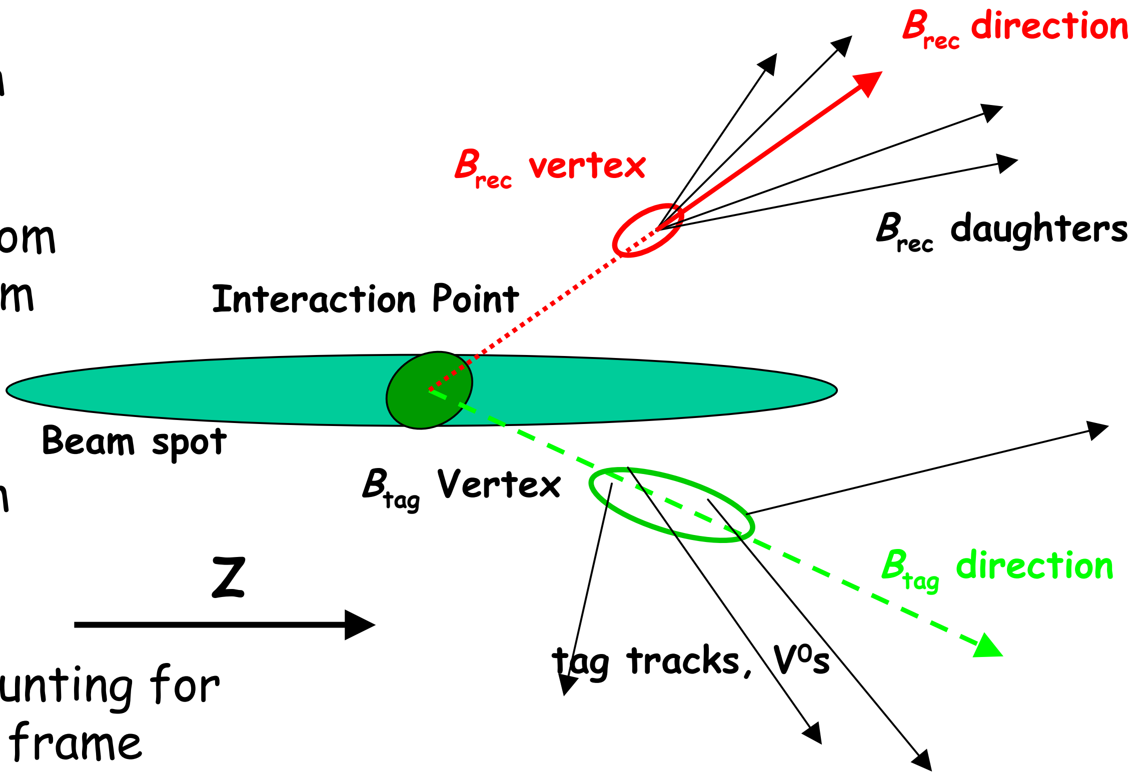
finite resolution

Proper time difference obtained from distance Δz between the two *B* vertices

Fit the parameters of the resolution function together with the lifetime

Vertex and Δz Reconstruction

1. Reconstruct B_{rec} vertex from B_{rec} daughters
2. Reconstruct B_{tag} direction from B_{rec} vertex & momentum, beam spot, and $\Upsilon(4S)$ momentum = pseudotrack
3. Reconstruct B_{tag} vertex from pseudotrack plus consistent set of tag tracks
4. Convert from Δz to Δt, accounting for (small) B momentum in $\Upsilon(4S)$ frame



Result: High efficiency (97%) and $\sigma(\Delta z)_{rms} \sim 180\mu m$ versus $\langle|\Delta z|\rangle \sim \beta\gamma c\tau = 260\mu m$

Conversion from Δz to Δt

Proper time difference: $\Delta t = t_{rec} - t_{tag} = m_B \left[\frac{z_{rec}}{p_{z,rec}} - \frac{z_{tag}}{p_{z,tag}} \right]$

where t_{rec} and t_{tag} are in different frames

Boost Approximation Neglect p_B^*, take $p_{z,rec} = p_{z,tag} \approx \langle p_{\Upsilon(4S),z} \rangle$: $\Delta t = \frac{\Delta z}{\beta\gamma c}$

Since one B is fully reconstructed and two B mesons are correlated:

$$\Delta z = \beta\gamma\gamma^*_{rec} c(t_{rec} - t_{tag}) + \gamma\beta^*_{rec}\gamma^*_{rec} \cos\theta^*_{rec} c(t_{rec} + t_{tag})$$

Improved Boost Approximation Since $\langle \cos\theta^*_{rec} \rangle = 0$, $\Delta t = \frac{\Delta z}{\beta\gamma\gamma^*_{rec} c}$ **0.2% effect**

Do not know $(t_{rec} + t_{tag})$, but can compute Δt average event-by-event:

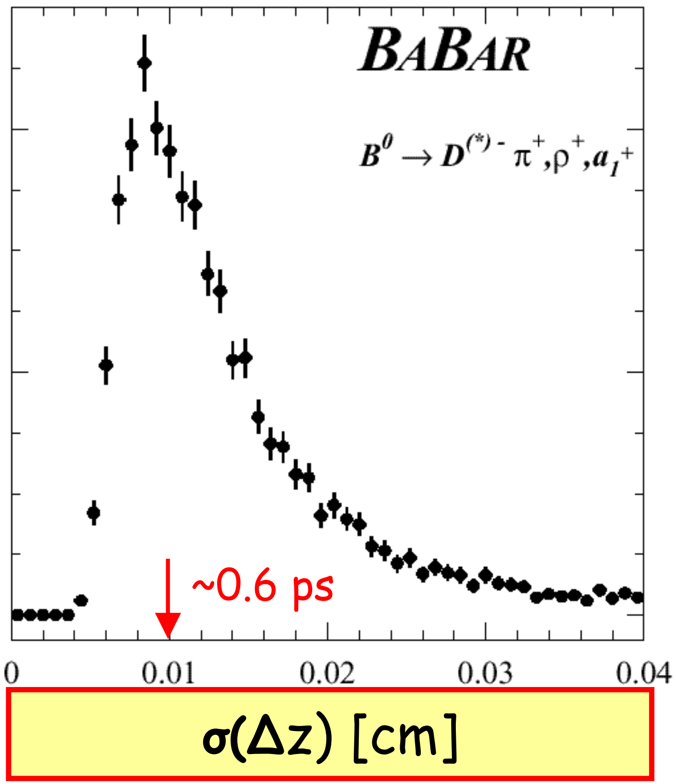
$$\langle t_{rec} + t_{tag} \rangle_{\Delta t} = \tau_B + |\Delta t|$$

Average τ_B Approximation $\Delta z = \beta\gamma\gamma^*_{rec} c\Delta t + \gamma\beta^*_{rec}\gamma^*_{rec} \cos\theta^*_{rec} c(\tau_{B^0} + |\Delta t|)$

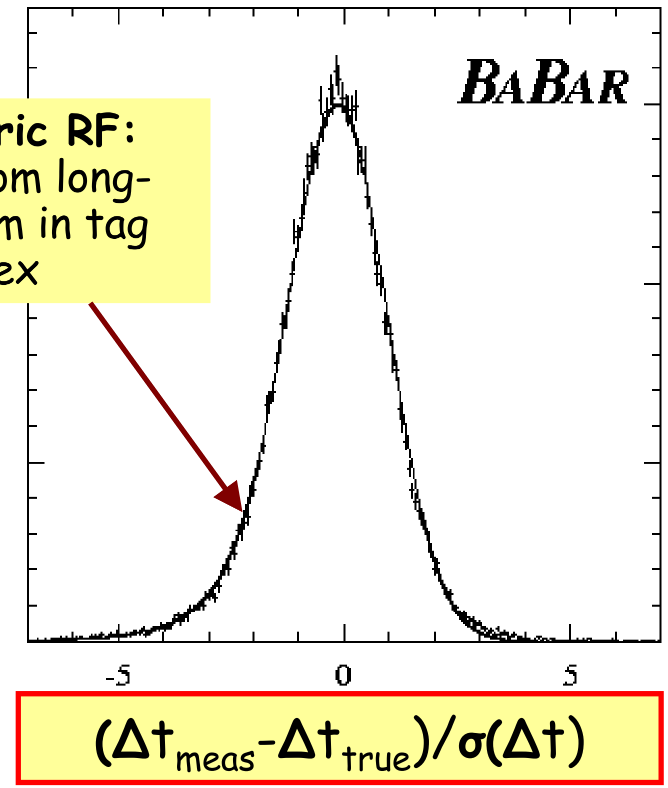
Improves resolution by 5% in quadrature

Actual Δt Signal Resolution Function

Event-by-event σ(Δt) from vertex errors



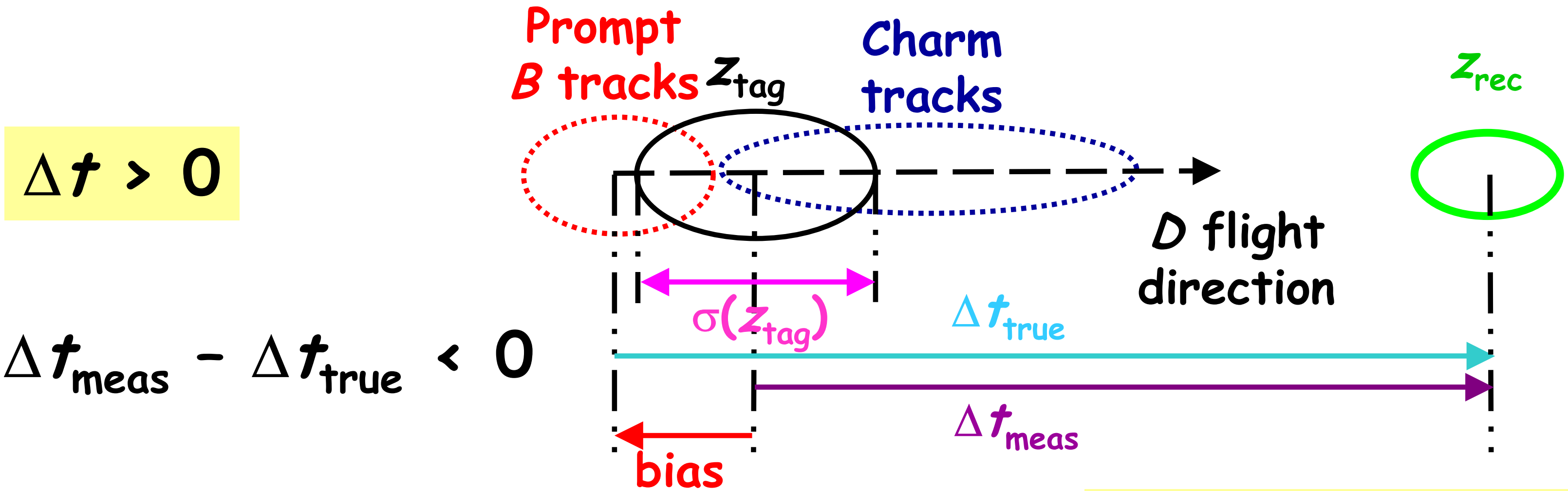
Signal MC: Δt resolution function



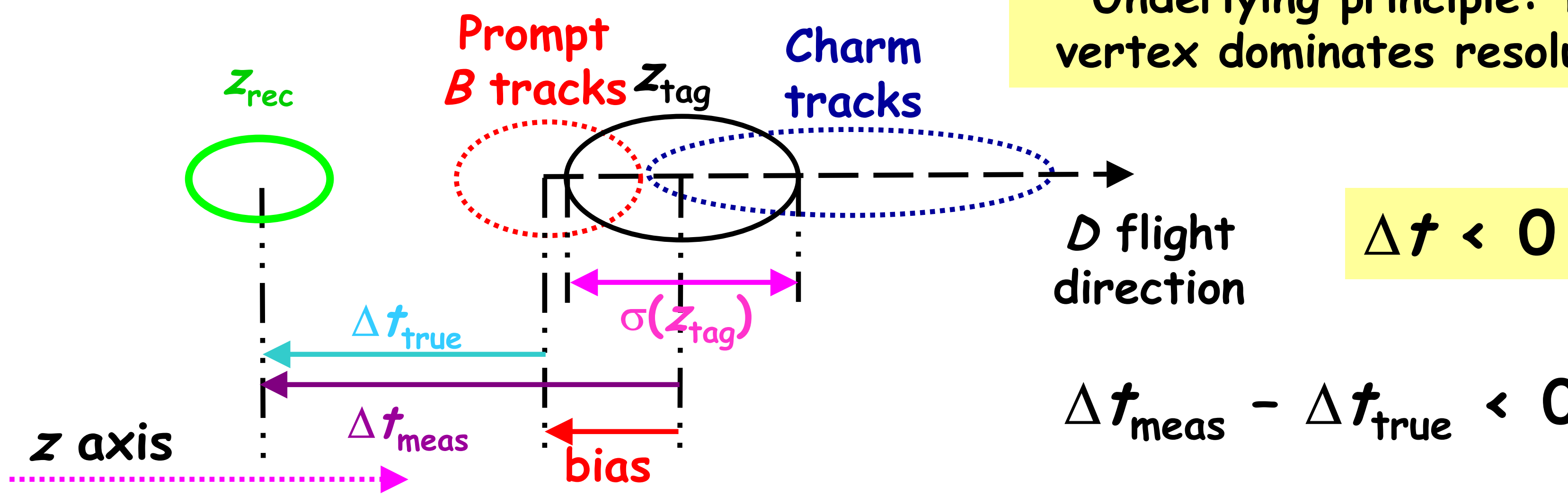
Asymmetric RF:
Tracks from long-lived charm in tag vertex

Empirical Models with parameters fit to data: Gaussian convolved with an exponential [lifetime] or Triple Gaussian [mixing, CP]

Effect of Charm Tracks on Δt



Underlying principle: tag vertex dominates resolution



Gaussian-Exponential Δt Resolution Model

Motivated by inclusion of charm decay products in determination of B_{tag} vertex, creating a small bias in the mean residual Δt distribution

$$R(\delta t;\hat{a}) = f \times G(0, S\sigma_{\Delta t}) + (1-f)\times G(0, S\sigma_{\Delta t}) \otimes E(\kappa\sigma_{\Delta t})$$

where $\delta t = \Delta t - \Delta t_{true}$

Core Gaussian

Gaussian convolved with exponential

Event-by-event uncertainties from vertex fits $\sigma(\Delta t)$

Parameters: $\hat{a} = \{0.69 \pm 0.07, 1.21 \pm 0.07, 1.04 \pm 0.24\}$

***f* = fraction in core Gaussian component**
***S* = scale factor for estimated event-by-event errors**
κ = effective lifetime for charm bias component

Outlier component (measurement error not applicable) added explicitly to PDF used in likelihood fit with one additional free parameter $f_{outlier}$

$$f_{sig,outlier} = 0.2^{+0.2}_{-0.3}\%, f_{bk,outlier} = 0.2^{+0.2}_{-0.3}\%$$

Method for Extracting Lifetime

Global unbinned maximum likelihood technique

- Includes probability-density functions (PDFs) for signal & backgrounds
- Incorporates model for Δt resolution function for signal & background

Primary advantages:

- **Incorporates all correlations between parameters describing dataset**
- **Extracts maximum statistical precision for desired result**

Cautions:

- Need to build reasonable model that incorporates physical correlations
- Need to thoroughly test the model with Monte Carlo simulation to verify complete understanding

Likelihood Function for Lifetime Fits

Signal model:

$$H_{sig,i}(\Delta t_i, \sigma_{\Delta t,i}; \tau_B, \hat{a}) = \frac{e^{-|\Delta t_i|/\tau_B}}{2\tau_B} \otimes R(\Delta t_i; \hat{a})$$

Two single-sided exponentials convolved with signal Gaussian resolution function

$$H_{bk,i}(\Delta t_i, \sigma_{\Delta t,i}; f_{bk,0}, \tau_{bk}, \hat{b}) = \left[f_{bk,0}\,\delta(\Delta t_i) + (1 - f_{bk,0}) \frac{e^{-|\Delta t_i|/\tau_{bk}}}{2\tau_{bk}} \right] \otimes R(\Delta t_i; \hat{b})$$

Prompt and lifetime components convolved with separate background Gaussian resolution function

Outlier model:

$$O_{outlier,i}(\Delta t_i) = G(0, 10\ ps)$$

Gaussian with zero mean and fixed 10 ps width

Probability Density Function (PDF):

$$P_i = p_{sig,i}\left[(1 - f_{sig,outlier})H_{sig,i} + f_{sig,outlier}O_{outlier,i}\right] + (1 - p_{sig,i})\left[(1 - f_{bk,outlier})H_{bk,i} + f_{bk,outlier}O_{outlier,i}\right]$$

Assign probabilities for individual events to be signal ($p_{sig,i}$) or background (1 - $p_{sig,i}$), based on observed m_{ES} value and a separate global fit to the m_{ES} distribution for the sample

Likelihood Function

$$\ln L = \sum_{i0} \ln P_{i0}(\Delta t_i, \sigma_{\Delta t,i}, p_{sig,i}; \tau_{B^0}, \hat{a}, \hat{b}_0, f^0_{sig,outlier}, f^0_{bk,outlier}, f^0_{bk,0}, \tau^0_{bk}) + \sum_{i+} \ln P_{i+}(\Delta t_i, \sigma_{\Delta t,i}, p_{sig,i}; \tau_{B^+}, \hat{a}, \hat{b}_+, f^+_{sig,outlier}, f^+_{bk,outlier}, f^+_{bk,0}, \tau^+_{bk})$$

Likelihood Function for Lifetime Fits

Signal model: Two single-sided exponentials convolved with signal Gaussian resolution function

Background model: Prompt and lifetime components convolved with separate background Gaussian resolution function

Outlier model: Gaussian with zero mean and fixed 10 ps width

Probability Density Function (PDF): Assign probabilities for individual events to be signal ($p_{sig,i}$) or background ($1- p_{sig,i}$), based on observed m_{ES} value and a separate global fit to the m_{ES} distribution for the sample

Likelihood Function: Sum PDFs for charged and neutral samples for a combined fit with a total of 19 free parameters

Likelihood Function for Lifetime Fits

Signal model: $H_{sig,i}(\Delta t_i, \sigma_{\Delta t,i}; \tau_B, \hat{a}) = \frac{e^{-|\Delta t_i|/\tau_B}}{2\tau_B} \otimes R(\Delta t_i; \hat{a})$

$$H_{bk,i}(\Delta t_i, \sigma_{\Delta t,i}; f_{bk,0}, \tau_{bk}, \hat{b}) = \left[f_{bk,0}\delta(\Delta t_i) + (1 - f_{bk,0}) \frac{e^{-|\Delta t_i|/\tau_{bk}}}{2\tau_{bk}} \right] \otimes R(\Delta t_i; \hat{b})$$

Outlier model: $O_{outlier,i}(\Delta t_i) = G(0, 10\ \text{ps})$

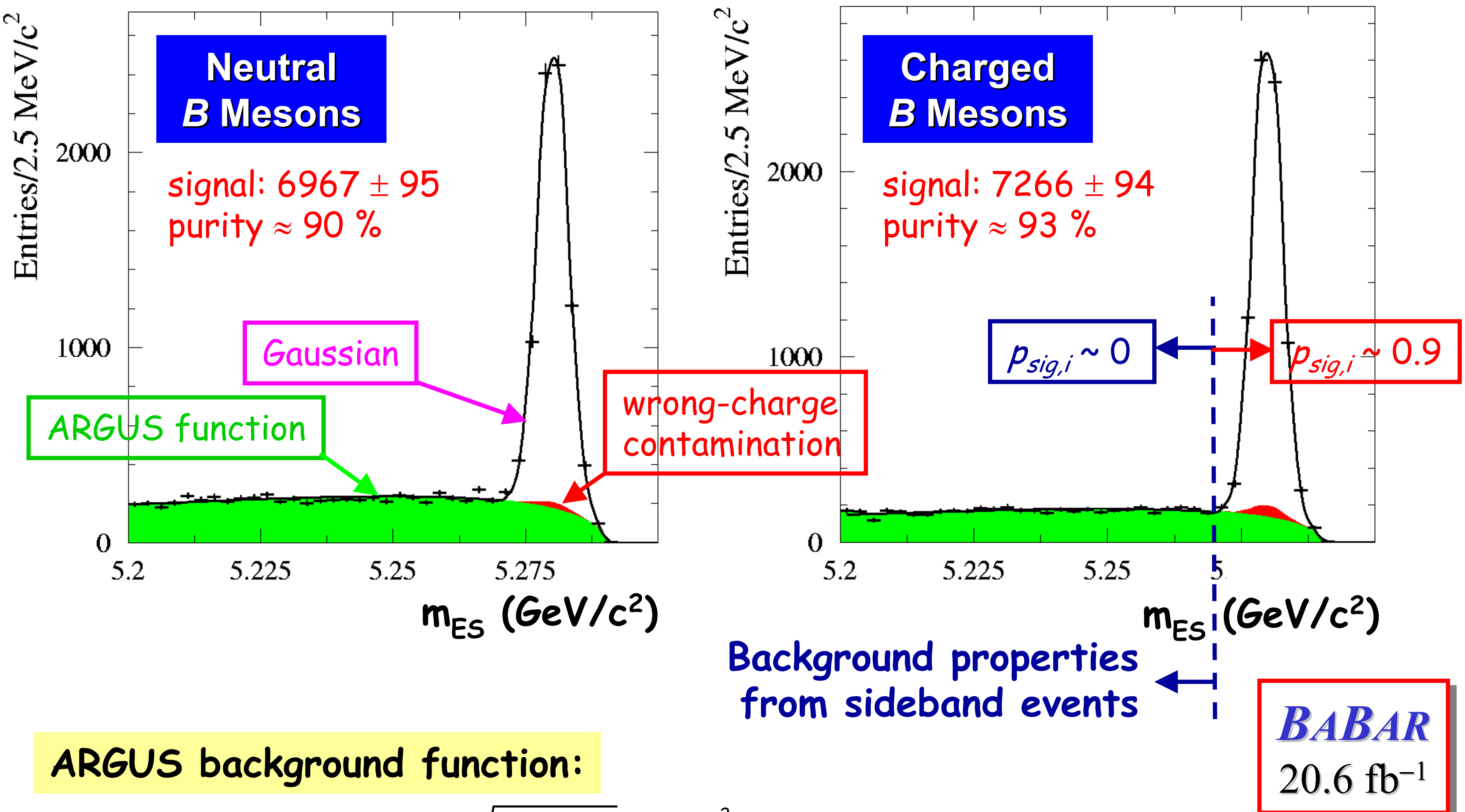
Probability Density Function (PDF):

$$P_i = p_{sig,i}\left[(1 - f_{sig,outlier})H_{sig,i} + f_{sig,outlier}O_{outlier,i}\right]$$
$$(1 - p_{sig,i})\left[(1 - f_{bk,outlier})H_{bk,i} + f_{bk,outlier}O_{outlier,i}\right]$$

Likelihood Function

$$\ln L = \sum_{i0} \ln P_{i0}(\Delta t_i, \sigma_{\Delta t,i}, p_{sig,i}; \tau_{B^0}, \hat{a}, \hat{b}_0, f^0_{sig,outlier}, f^0_{bk,outlier}, f^0_{bk,0}, \tau^0_{bk}) +$$
$$\sum_{i+} \ln P_{i+}(\Delta t_i, \sigma_{\Delta t,i}, p_{sig,i}; \tau_{B^+}, \hat{a}, \hat{b}_+, f^+_{sig,outlier}, f^+_{bk,outlier}, f^+_{bk,0}, \tau^+_{bk})$$

Signal and Background Probabilities



ARGUS background function:

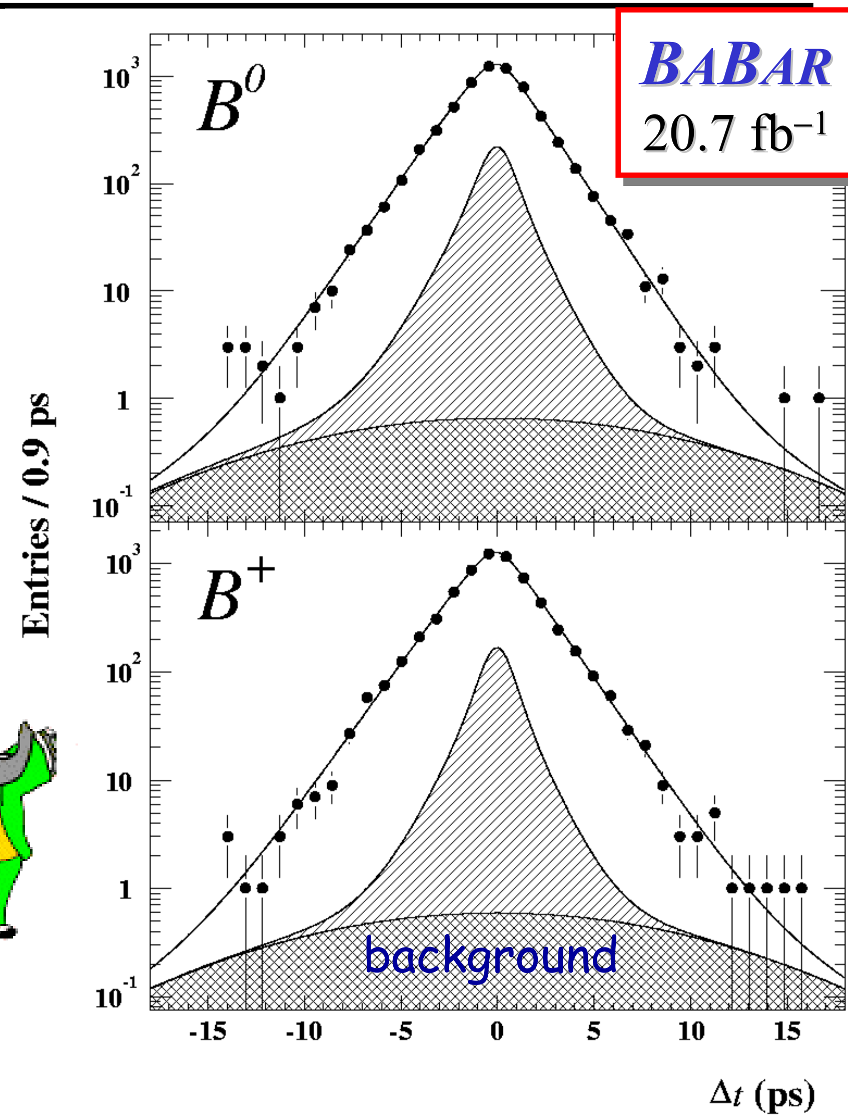
$$A(m_{ES}; m_0, \xi) = A_B m_{ES} \sqrt{1 - x_{ES}^2}\, e^{\xi(1 - x_{ES}^2)}, \text{ where } x_{ES} = m_{ES} / m_0$$

B-Lifetime Measurements

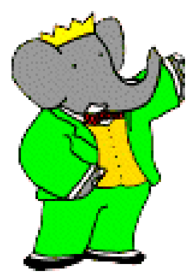
$$\tau_{B^0} = 1.546 \pm 0.032 \pm 0.022 \text{ ps}$$
$$\tau_{B^+} = 1.673 \pm 0.032 \pm 0.023 \text{ ps}$$
$$\tau_{B^+} / \tau_{B^0} = 1.082 \pm 0.026 \pm 0.012$$

BABAR PRL **87**, 201803 (2001)

(error PDG2000 ~ 0.03 ps, stat+syst)



- *Good agreement with previous lifetime measurements*
- *Excellent control of the time resolution function (parameterization, tails)*



Proof of principle for time-dependent analysis at *B* Factories

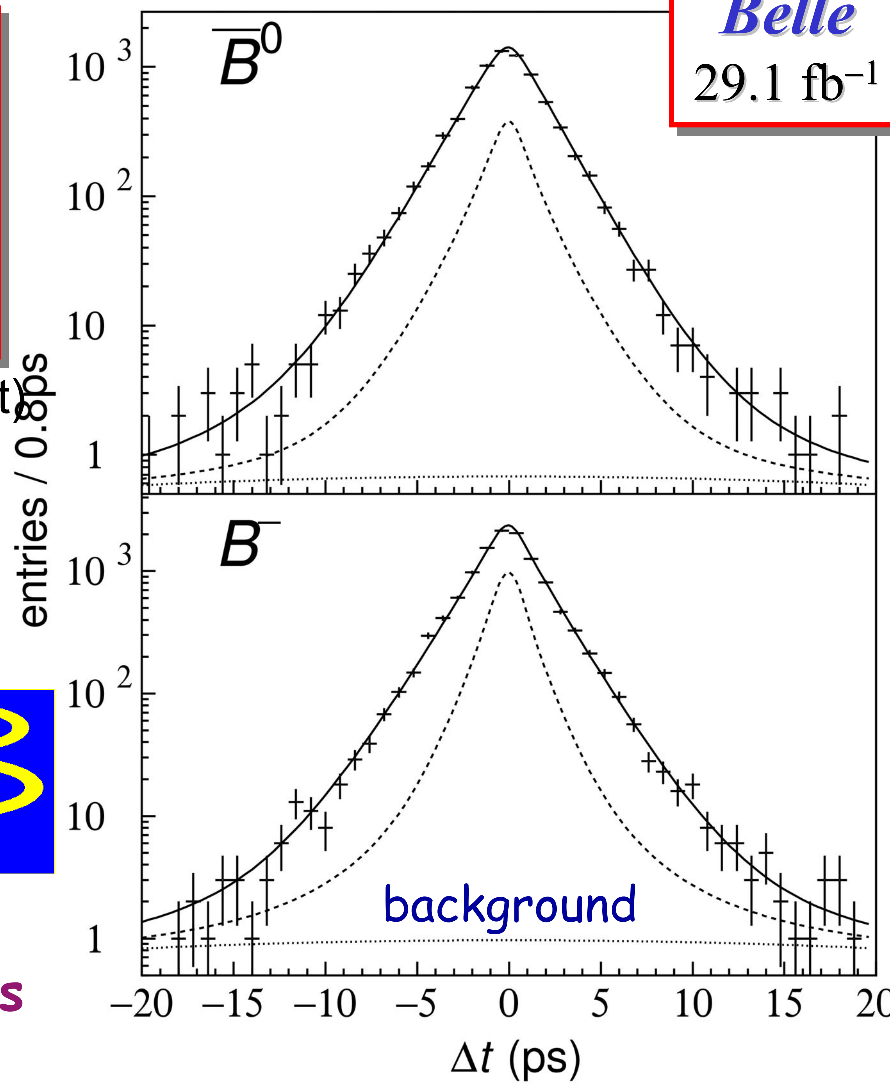
B-Lifetime Measurements

$$\tau_{B^0} = 1.554 \pm 0.030 \pm 0.019 \text{ ps}$$
$$\tau_{B^+} = 1.695 \pm 0.026 \pm 0.015 \text{ ps}$$
$$\tau_{B^+} / \tau_{B^0} = 1.091 \pm 0.023 \pm 0.014$$

Belle PRL **88**, 171801 (2002)

(error PDG2000 ~ 0.03 ps, stat+syst)

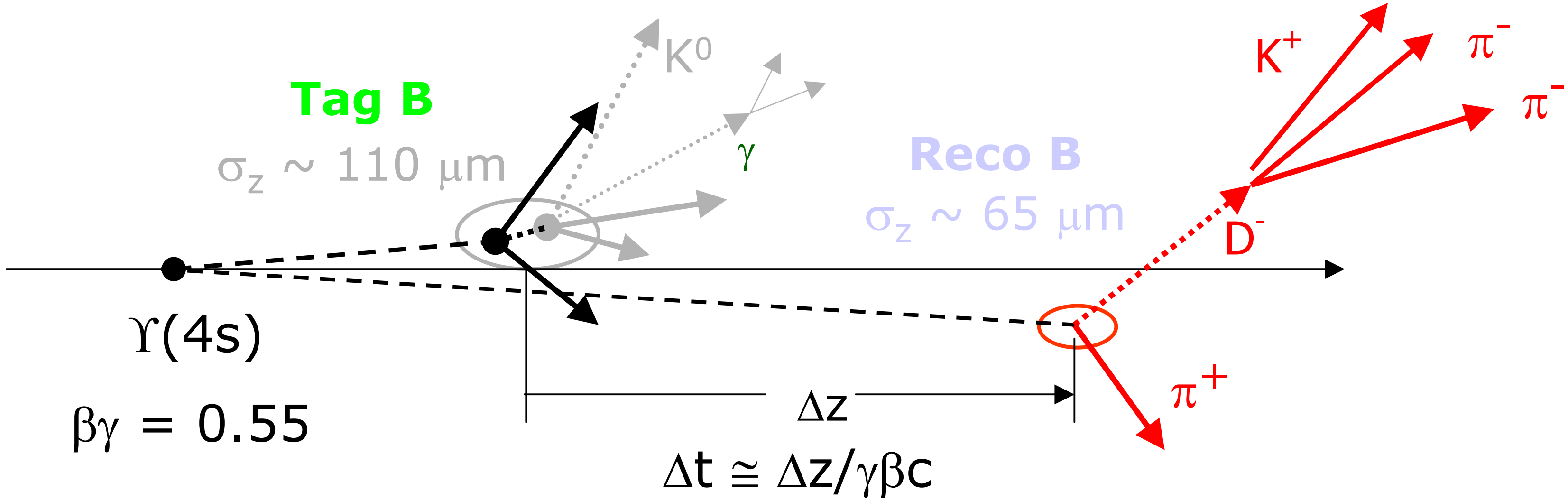
Belle
29.1 fb^{-1}



- *Good agreement with previous lifetime measurements*
- *Excellent control of the time resolution function (parameterization, tails)*

Proof of principle for time-dependent analysis at *B* Factories

Measurement of $B^0\overline{B^0}$ Mixing



3. Reconstruct Inclusively the vertex of the "other" *B* meson (B_{tag}) ✓
4. Determine the flavor of B_{tag} to separate Mixed and Unmixed events

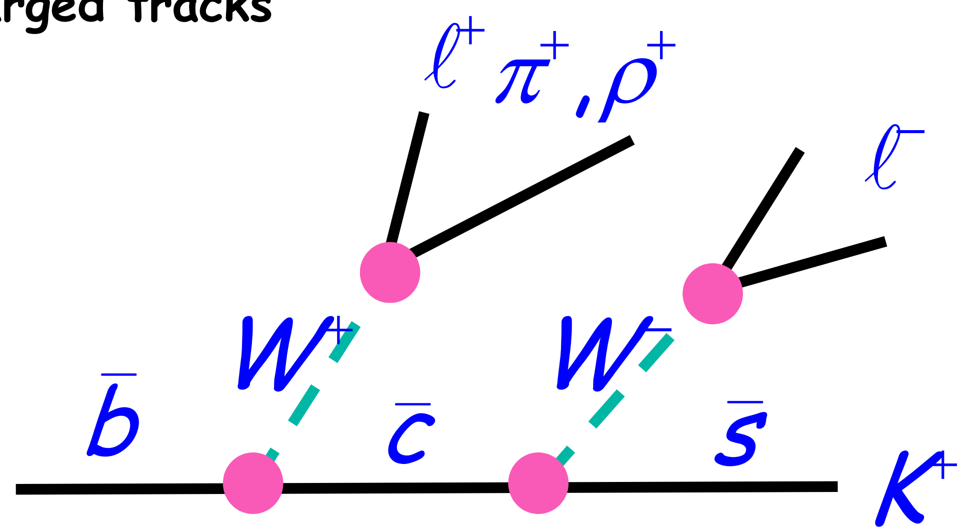
1. Fully reconstruct one *B* meson in flavor eigenstate (B_{rec}) ✓
2. Reconstruct the decay vertex ✓

5. compute the proper time difference Δt ✓
6. Fit the Δt spectra of mixed and unmixed events

Methods for B Flavor Tagging

Many different physics processes can be used

Primary lepton	$B^0 \to D^{*-}\ell^+\nu$
Secondary lepton	$B^0 \to D^-\pi^+, D^- \to K^{*+}\ell^-\bar{\nu}$
Kaon(s)	$B^0 \to \bar{D}X, \bar{D} \to K^+X$
Soft pions from D^* decays	$B^0 \to D^{*-}X^+, D^{*-} \to \bar{D}^0\pi_s^-$
Fast charged tracks	



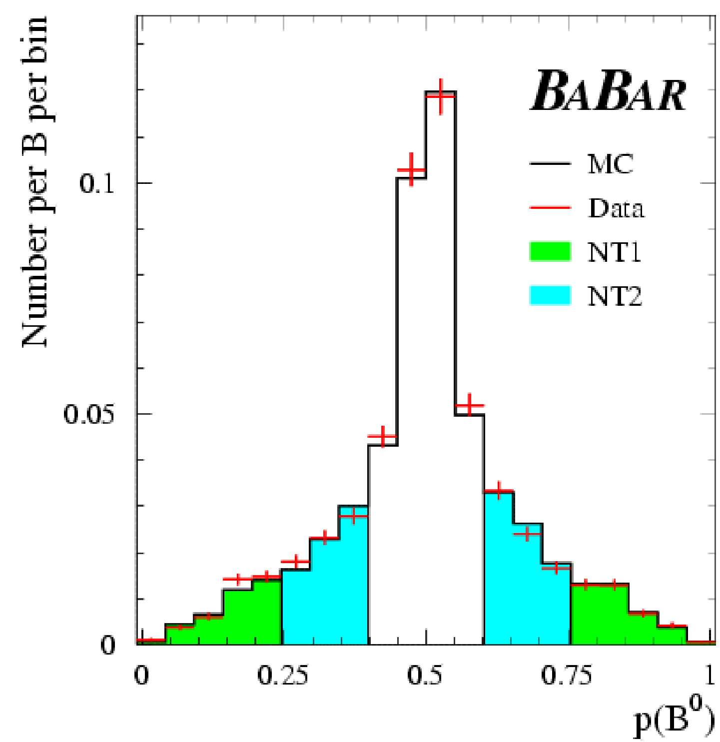
Flavor Tagging for Mixing Study

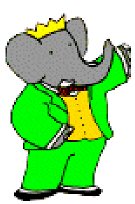
- *Use charge correlations with decay products to define two physics categories*
  - "Lepton" $B^0 \to D^{*-}\ell^+\nu$
  - "Kaon" $B^0 \to \bar{D}X, \bar{D} \to K^+X$

$p_\ell^* > 1.0[1.1]$ GeV/c for $e[\mu]$

No conflicting Lepton tag, $\sum_{kaons} q_i \neq 0$

- *Multivariable techniques used to combine PID, kinematic variables, correlations, event information*
  - e.g., primary lepton without PID, soft pions from D* decays
    $B^0 \to D^{*-}X^+, D^{*-} \to \bar{D}^0\pi_s^-$
  - Multivariable analysis with neural network techniques: "NT1", "NT2" categories

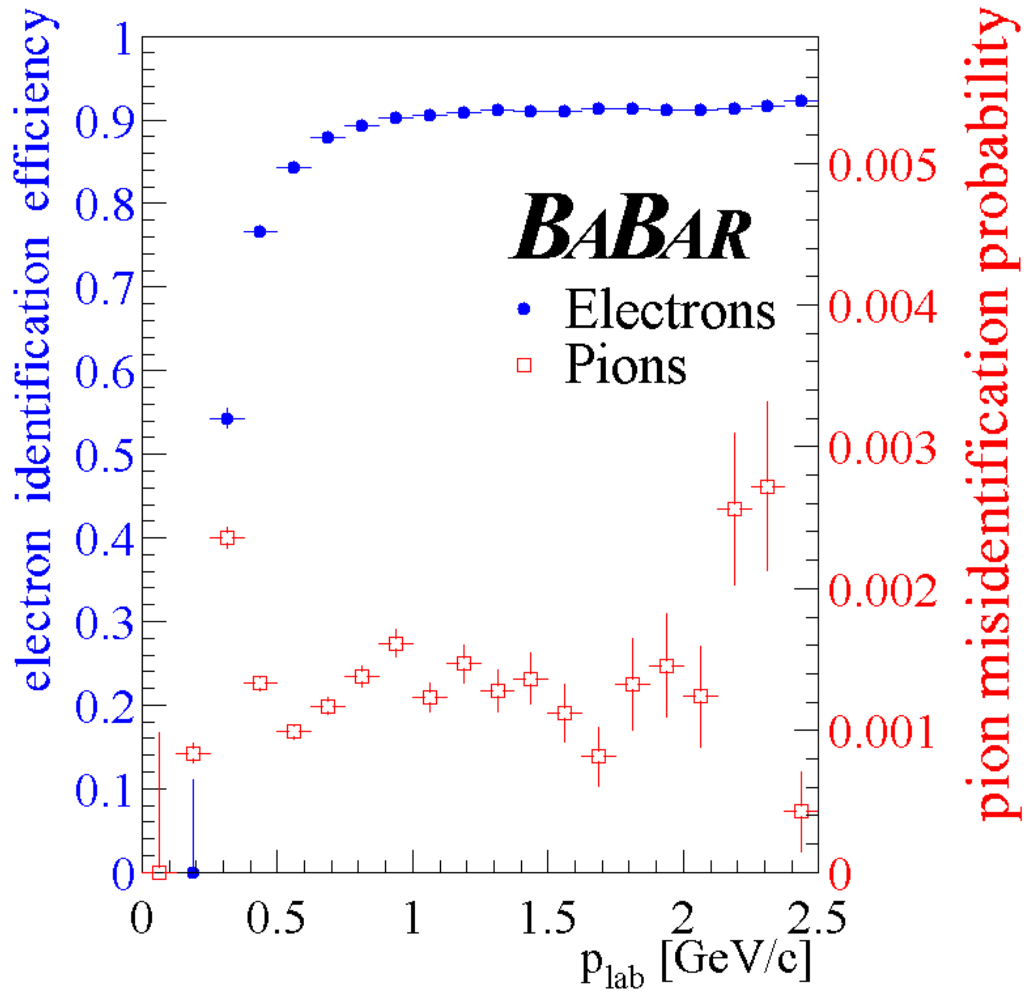


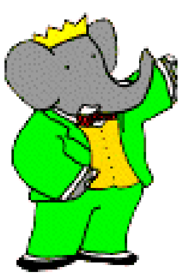


Electron ID at BABAR

- Match track to EMC cluster
- $0.89 < E_{EMC}/p < 1.2$
- EM shower shape requirements
- DCH dE/dx and DIRC Cherenkov angle consistent with electron hypothesis

eff e=91%, π misid=0.13%

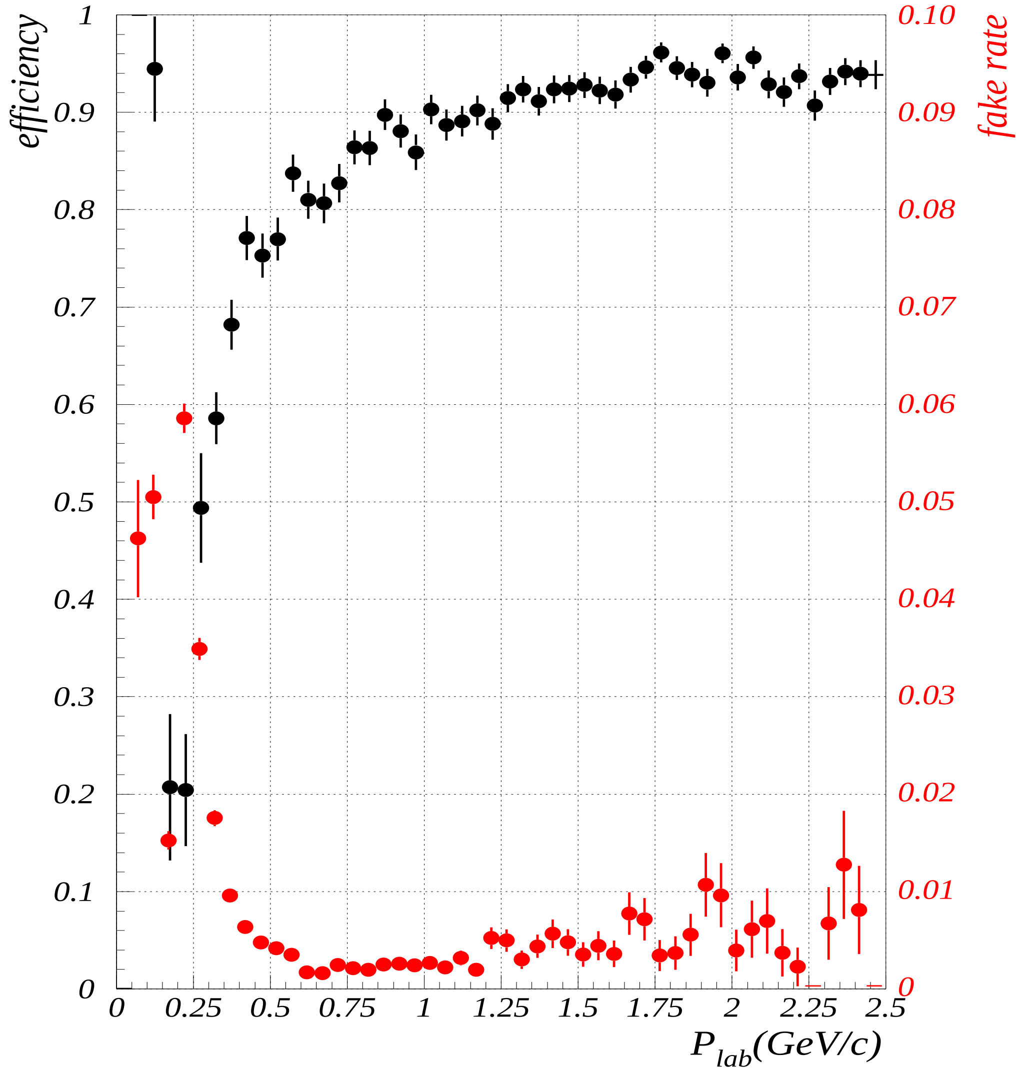




Electron ID at Belle

- Match track to ECL cluster
- E_{ECL}/p ratio requirement
- EM shower shape requirements, track-cluster matching
- DCH dE/dx, TOF, ACC consistent with electron hypothesis

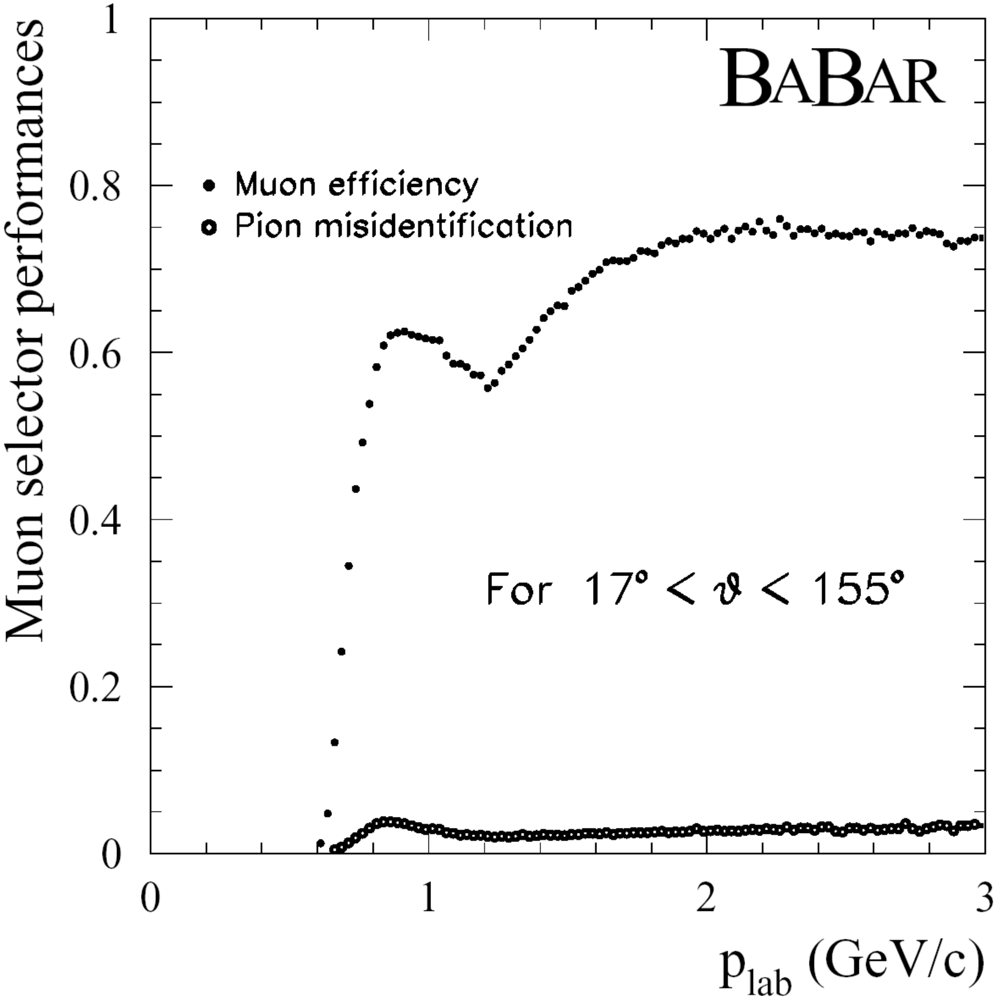
eff e=94%, π misid=0.5%

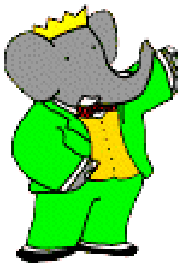


Muon ID at BABAR

- # int. lengths in IFR >2.2
- Difference in measured and expected int. length <1
- Match bet'n extrapolated track and IFR hits
- Requirements on average and spread of # of IFR hits per layer

eff μ=75%, π misid=2.5%

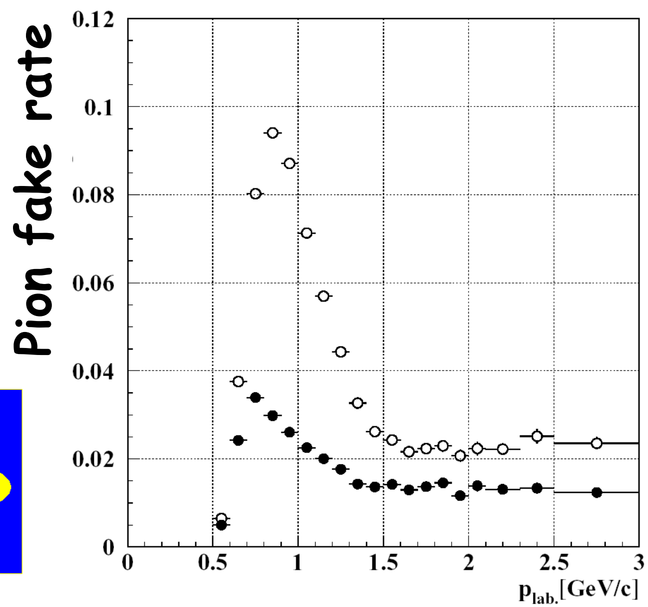




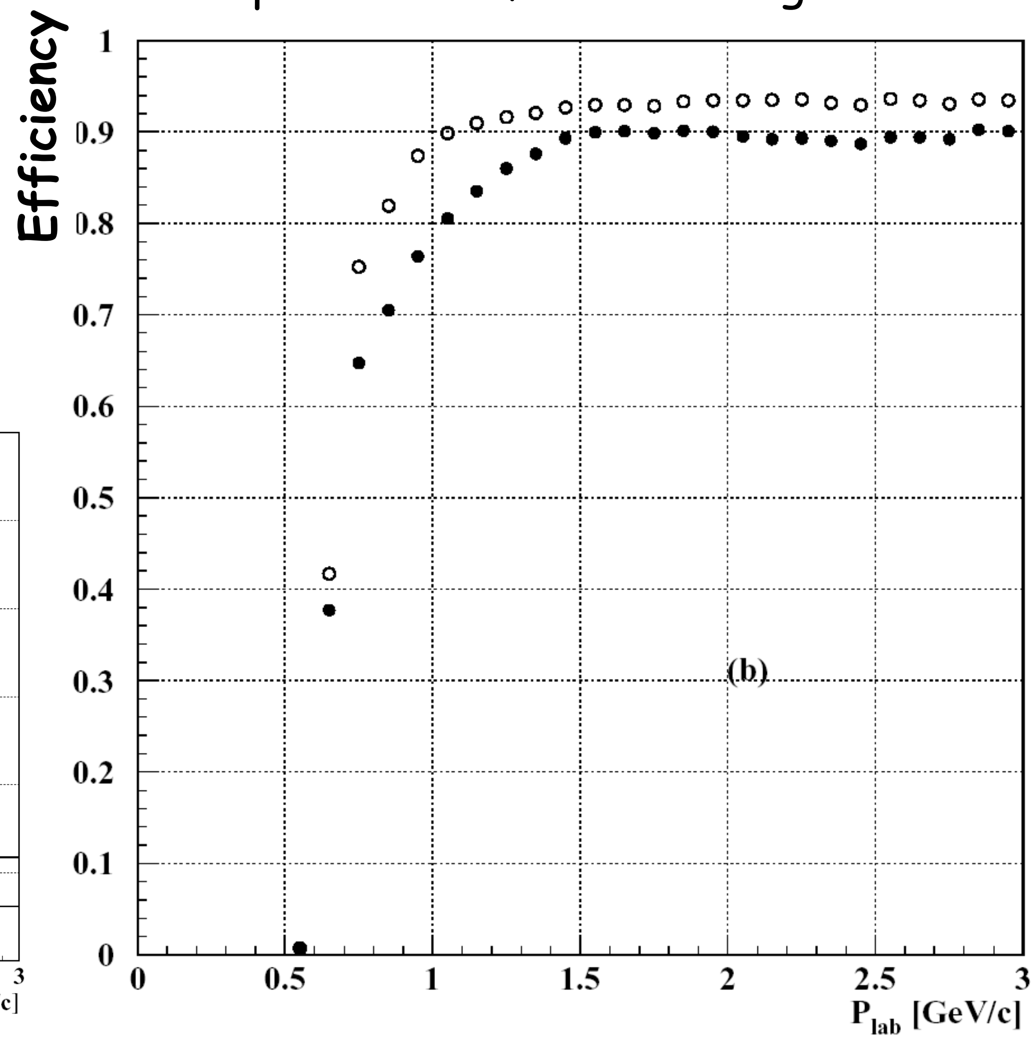
Muon ID at Belle

- Difference in measured and expected range
- Match bet'n extrapolated track and KLM hits
- Likelihood based selection

eff μ=90%, π misid=1%



Open = Loose; Closed = Tight



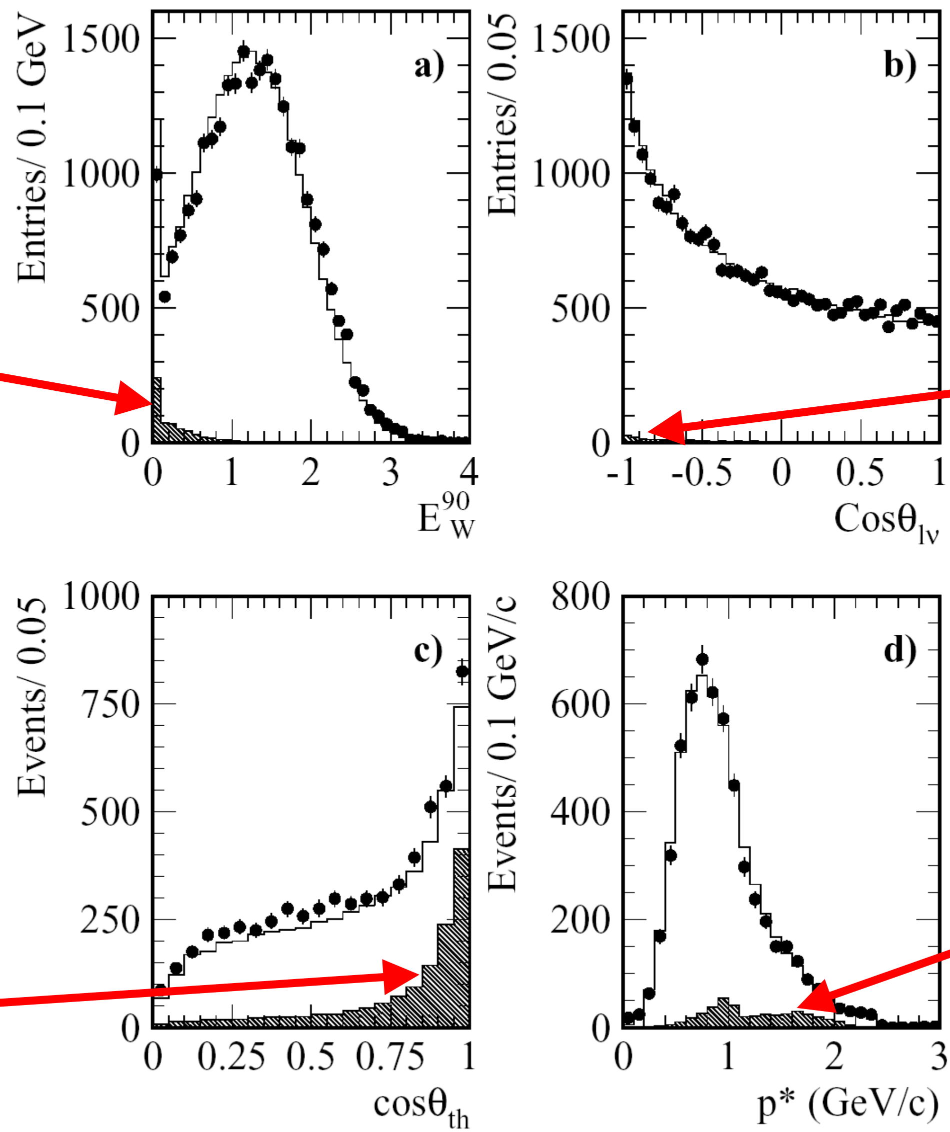
Some Inputs to NN Tagger

Sum of energy within 90° of estimated *W* direction

$B^0 \to D^{*-}\ell^+\nu$

Opening angle between input track and missing momentum vector

$B^0 \to D^{*-}\ell^+\nu$



Opening angle between input track and thrust axis of recoil *B*

$B^0 \to D^{*-}X^+,$

$D^{*-} \to \bar{D}^0\pi_s^-$

CMS momentum of input track

$B^0 \to D^{*-}\ell^+\nu$

Flavor Tagging Performance in Data

The large sample of fully reconstructed events provides the precise determination of the tagging parameters required in the *CP* fit

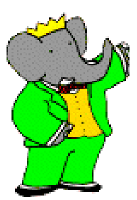
Tagging category	Fraction of tagged events ε (%)	Wrong tag fraction w (%)	Mistag fraction difference Δw (%)	$Q = \varepsilon(1-2w)^2$ (%)
Lepton	10.9 ± 0.3	9.0 ± 1.4	0.9 ± 2.2	7.4 ± 0.5
Kaon	35.8 ± 1.0	17.6 ± 1.0	-1.9 ± 1.5	15.0 ± 0.9
NT1	7.7 ± 0.2	22.0 ± 2.1	5.6 ± 3.2	2.5 ± 0.4
NT2	13.8 ± 0.3	35.1 ± 1.9	-5.9 ± 2.7	1.2 ± 0.3
ALL	68.4 ± 0.7			26.1 ± 1.2

Highest "efficiency"

Smallest mistag fraction

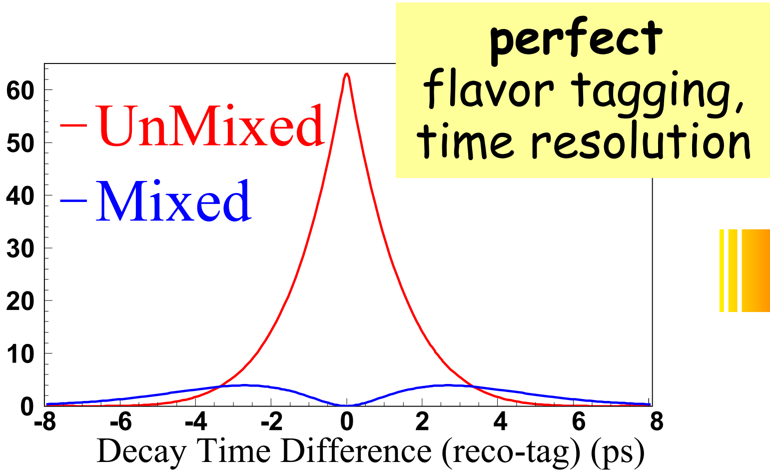
Error on sin2β and Δm_d depend on the "quality factor" Q approx. as:

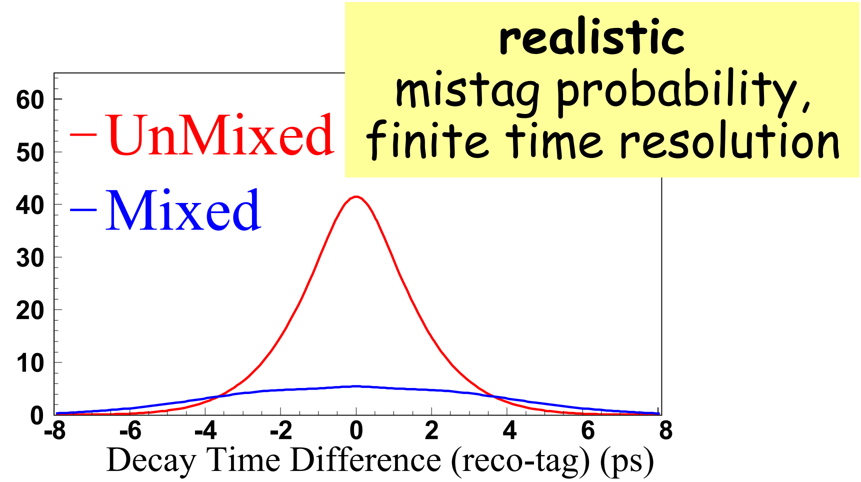
$$\sigma(\sin 2\beta) \sim \frac{1}{\sqrt{Q}}$$



BABAR
$29.7\ \text{fb}^{-1}$

B-Mixing Analysis: Time Distributions





$$f_{mixing,\pm}(\Delta t) = \left\{ \frac{e^{-|\Delta t|/\tau_B}}{4\tau_B} \left(1 \pm (1-2w)\cos\Delta m_d \Delta t\right) \right\} \otimes R(\Delta t)$$

"$f_{mixing,+}$" $\Leftrightarrow$ unmixed $(B^0_{flav}\bar{B}^0_{tag}$ or $\bar{B}^0_{flav}B^0_{tag})$

"$f_{mixing,-}$" $\Leftrightarrow$ mixed $(B^0_{flav}B^0_{tag}$ or $\bar{B}^0_{flav}\bar{B}^0_{tag})$

ω is the flavor mistag probability

$R(\Delta t)$ is the time resolution function

Triple-Gaussian Δt Resolution Model

$$R(\delta t;\hat{a}) = f_{core} \times G(\mu_{core},\sigma_{core}) + f_{tail} \times G(\mu_{tail},\sigma_{tail}) + f_{outlier} \times G(\mu_{outlier},\sigma_{outlier})$$

Core Gaussian

Tail Gaussian

Outlier Gaussian

$$f_{core} = (1 - f_{tail} - f_{outlier})$$

Bias
$$\begin{cases} \mu_{core} = b_{core,c}\sigma_{\Delta t} \\ \mu_{tail} = b_{tail}\sigma_{\Delta t} \\ \mu_{outlier} = 0 \end{cases}$$

Widths
$$\begin{cases} \sigma_{core} = S_{core}\sigma_{\Delta t} \\ \sigma_{tail} = S_{tail}\sigma_{\Delta t} \\ \sigma_{outlier} = 8\text{ ps} \end{cases}$$

Parameters:

$$\hat{a} = \{f_{tail}, f_{outlier}, S_{core}, b_{core,c}, b_{tail}\}$$

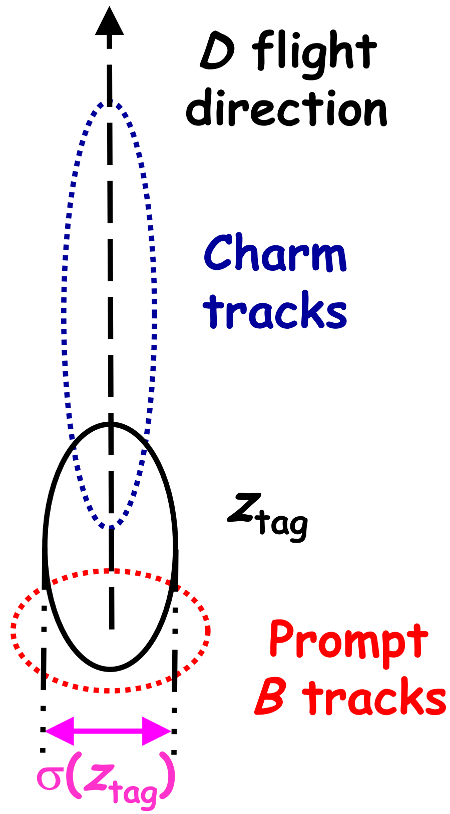
f* = *fractions in tail and outlier

S* = *scale factor for estimated event-by-event errors

b* = *bias factor due to inclusion of charm products in tag vertex

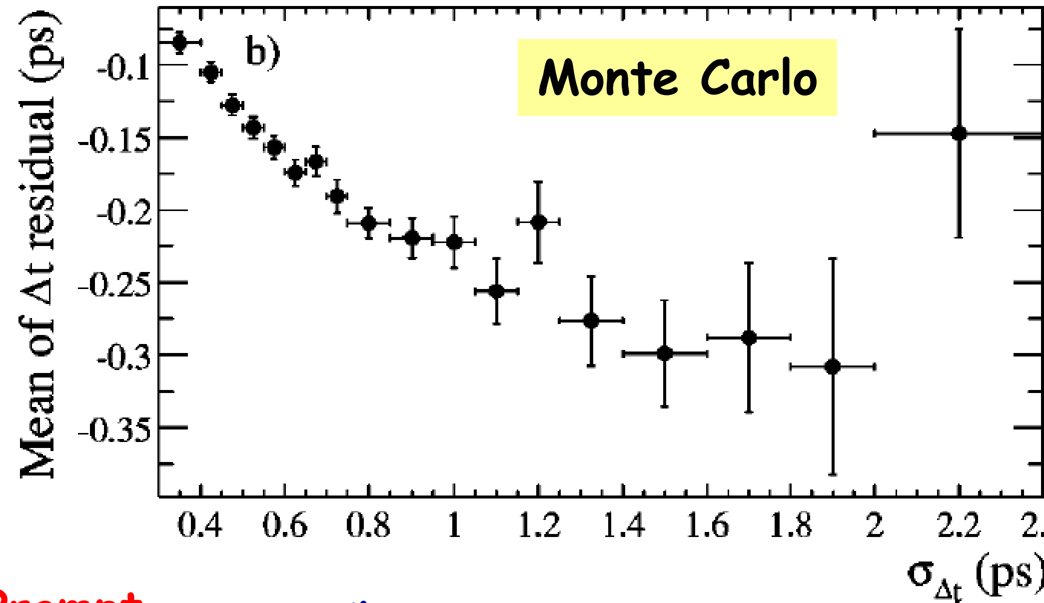
Event-by-event uncertainties from vertex fits $\sigma(\Delta t)$

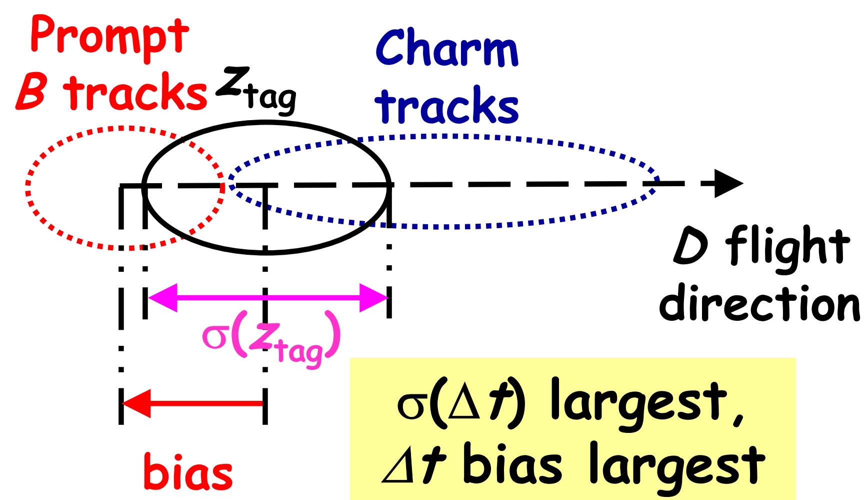
Correlation: σ(Δt) and residual Δt bias



σ(Δ*t*) smallest,
Δ*t* bias zero

z axis



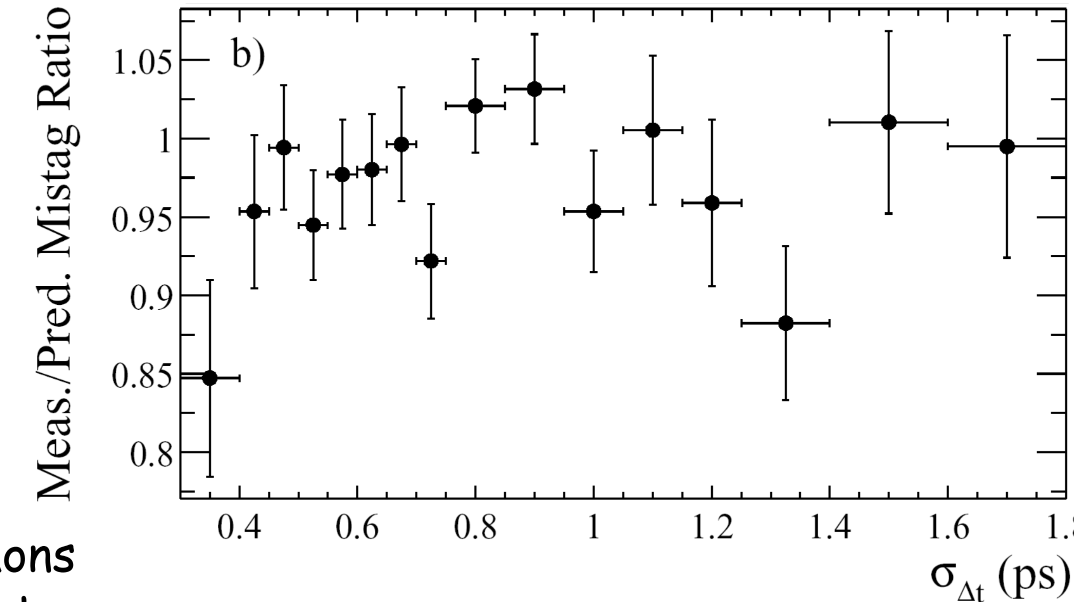


σ(Δ*t*) largest,
Δ*t* bias largest

Yet another correlation!

Mystery:

Tests of likelihood fit with full Monte Carlo shows bias of $+0.007 \pm 0.003$ for Δm_d (about 40% of statistical error!)



Mistag fraction versus vertex error for kaon tags

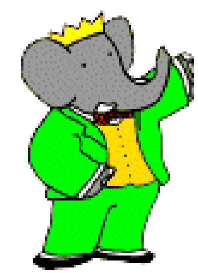
Resolved!

p_t spectrum of wrong-sign kaons softer than those in correct-tag processes

mistag rate scaled by $\sqrt{\sum p_t^2}$

But: $\sigma(\Delta t) \propto 1/\sum p_t^2$

Systematic difference in p_t leads to correlation



Fit Results for Signal Resolution Parameters

	Run 1	Run 2
S_{core}	1.37 ± 0.09	1.18 ± 0.11
b_{core} lepton	0.06 ± 0.13	-0.04 ± 0.16
b_{core} kaon	-0.22 ± 0.08	-0.25 ± 0.09
b_{core} NT1	-0.07 ± 0.15	-0.45 ± 0.21
b_{core} NT2	-0.46 ± 0.12	-0.20 ± 0.16
b_{tail}	-5.0 ± 4.2	-7.5 ± 2.4
f_{tail}	0.014 ± 0.020	0.015 ± 0.010
$f_{outlier}$	0.008 ± 0.004	0.000 ± 0.014

Non-zero bias for kaons

Additions to the Likelihood Function

1. Allow for difference in B^0 vs $\bar{B}^0$ mistag rates

8 parameters

$$\langle w \rangle = \frac{1}{2}(w + \bar{w}) \quad \Delta w = (w - \bar{w})$$

$$D = 1 - 2w \quad \bar{D} = 1 - 2\bar{w}$$

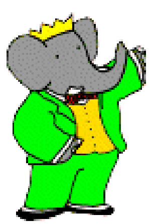
$$\langle D \rangle = \frac{1}{2}(D + \bar{D}) \quad \Delta D = D - \bar{D}$$

PDFs now depend on tag state as well

$$f_{mixing,\pm,tag=B^0}(\Delta t) \propto \left\{ \left(1 + \frac{1}{2}\Delta D\right) \pm \langle D \rangle \cos \Delta m_d \Delta t \right\}$$

$$f_{mixing,\pm,tag=\bar{B}^0}(\Delta t) \propto \left\{ \left(1 - \frac{1}{2}\Delta D\right) \pm \langle D \rangle \cos \Delta m_d \Delta t \right\}$$

2. Allow for prompt and non-prompt background components



Adds fractions for each tagging category, effective lifetime for non-prompt component, and "effective dilutions" for both

4+1+8 parameters

Blind Analyses

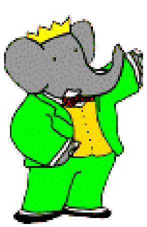
- ***Analyses were done "blind" to eliminate possible experimenters' bias***
    - In general, measurements of a quantity "X" are done with likelihood fits – blinding achieved by replacing "X" with "X+R" in likelihood fits
        - R is drawn from a Gaussian with a width a few times the expected error
        - Random number sequence is "seeded" with a "blinding string"
        - The reported statistical error is unaffected
        - Allows all systematic studies to be done while still blind

BELLE

Δm_d Likelihood Fit

Combined unbinned maximum likelihood fit to Δt spectra of mixed and unmixed events in the B flavor sample

Fit Parameters	#	Main Sample
Δm_d	1	Signal
Mistag fractions for B^0 and $\bar{B}^0$ tags	8	Signal
Signal resolution function	2x8 *	Signal
Description of background Δt	5+8	Sidebands
Background Δt resolution	2x3 *	Sidebands
B lifetime from PDG 2002	0	τ_B = 1.548 ps
Total parameters	44	



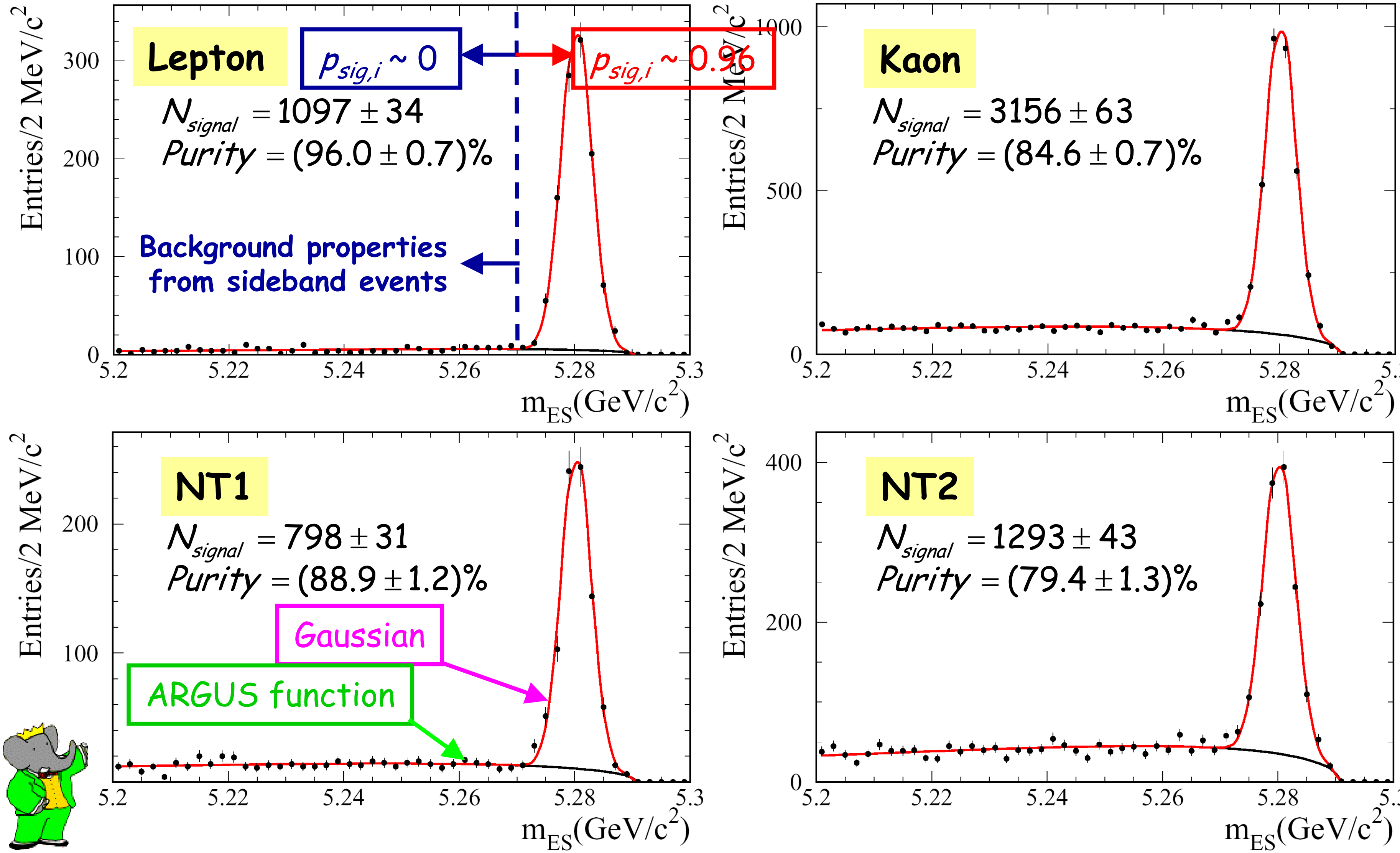
* 2 running periods

✓ All Δt parameters extracted from data
✓ Correct estimate of the error and correlations

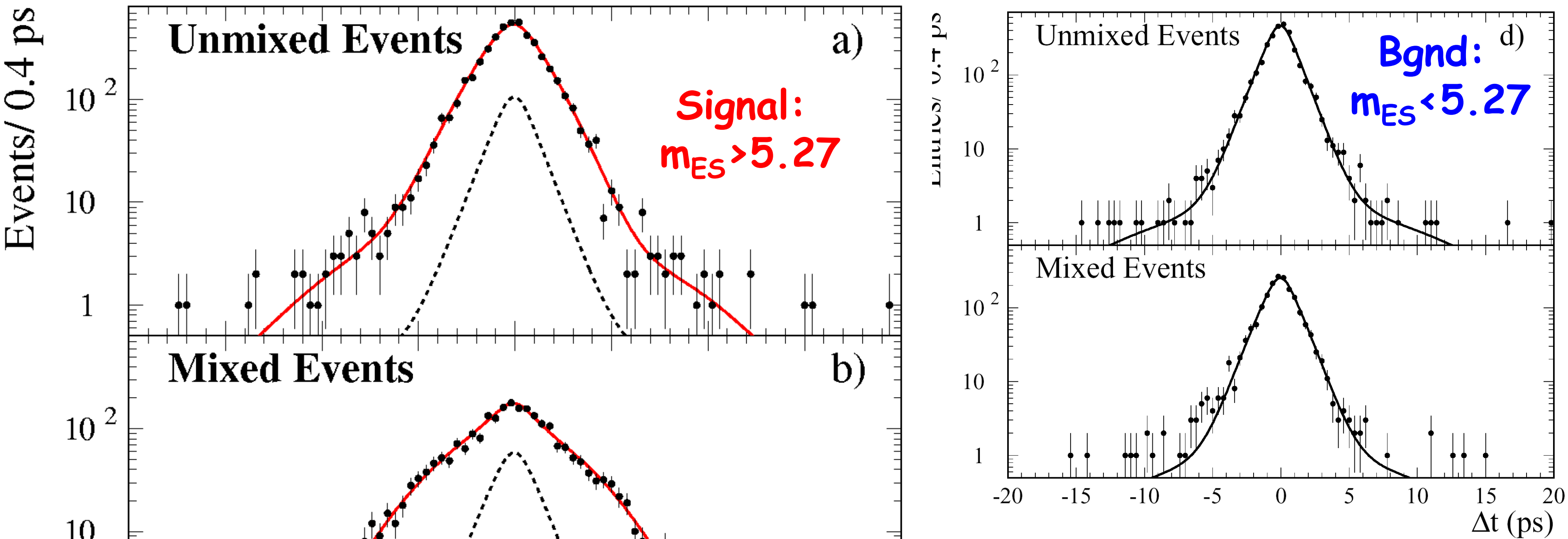
Likelihood Function for Mixing Fits

Signal model: 1+2x8+8 parameters	PDF with Δm_d [1] for mixed and unmixed events convolved with triple Gaussian signal resolution function [8] for 2 periods of alignment. Dilutions and dilution differences [8] are also incorporated.
Background model: 5+2x3+8 parameters	Prompt and lifetime components [5] for mixed and unmixed samples convolved with a common background double Gaussian resolution function [3] for 2 periods of alignment. Separate dilutions and dilution differences incorporated [8].
Outlier model:	Incorporated into resolution functions as Gaussian, with zero mean and fixed 8 ps width
Probability Density Function (PDF):	Assign probabilities for individual events to be signal ($p_{sig,i}$) or background ($1 - p_{sig,i}$), based on observed m_{ES} value and a separate global fit to the m_{ES} distribution for the sample
Likelihood Function:	Sum PDFs for mixed and unmixed samples for a combined fit with a total of 44 free parameters

Final Tagged Mixing Sample



Mixing with Hadronic Sample



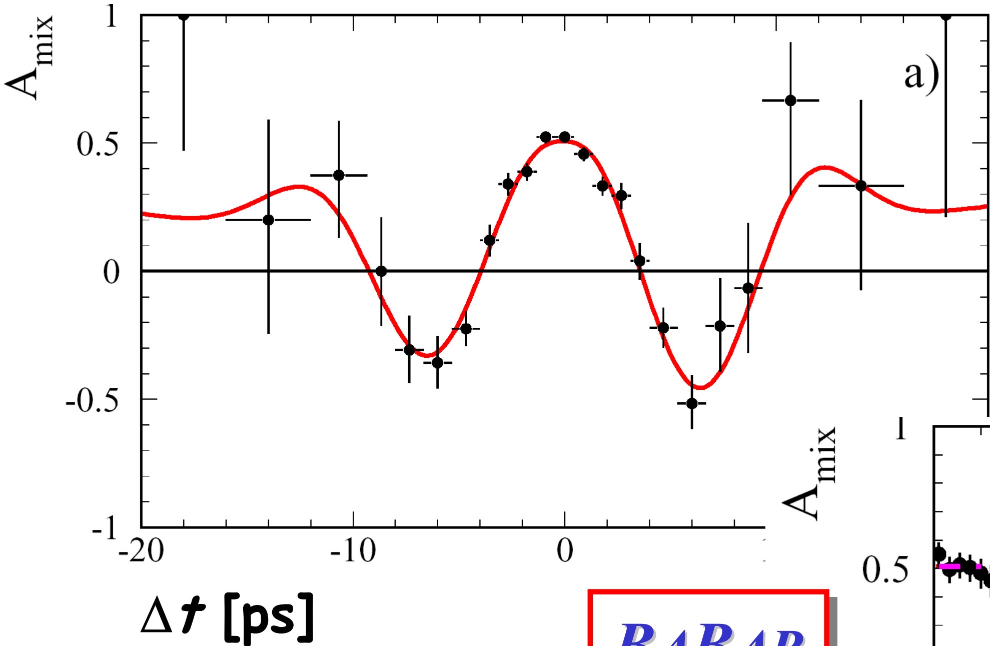
BABAR
29.7 fb^{-1}

Precision measurement consistent with world average

$$\Delta m_d = (0.516 \pm 0.016_{(stat)} \pm 0.010_{(syst)})\ \ ps^{-1}$$

BABAR PRL **88**, 221802 (2002)

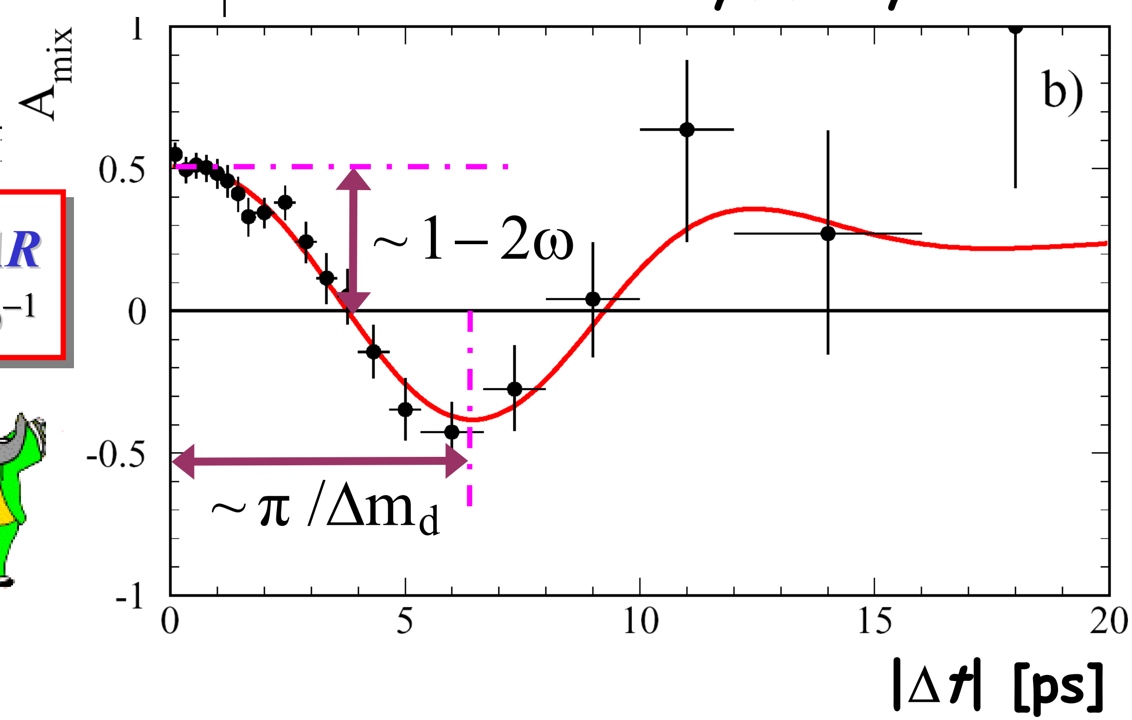
Mixing Asymmetry with Hadronic Sample



Unfolded raw asymmetry

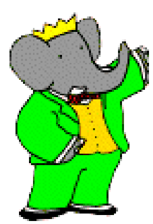
$$A_{mixing}(\Delta t) \approx (1-2w)\cos \Delta m_{B_d} \Delta t$$

Folded raw asymmetry



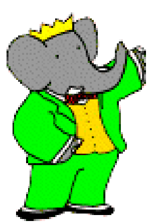
BABAR

29.7 fb^{-1}



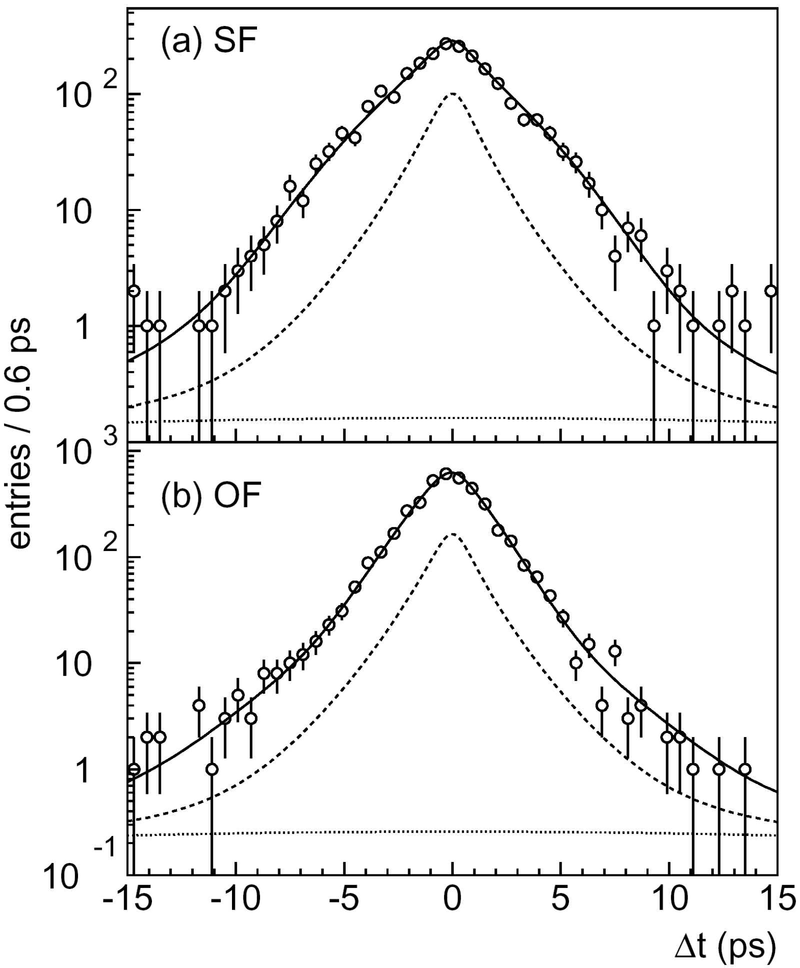
Systematic Errors on Δm_d

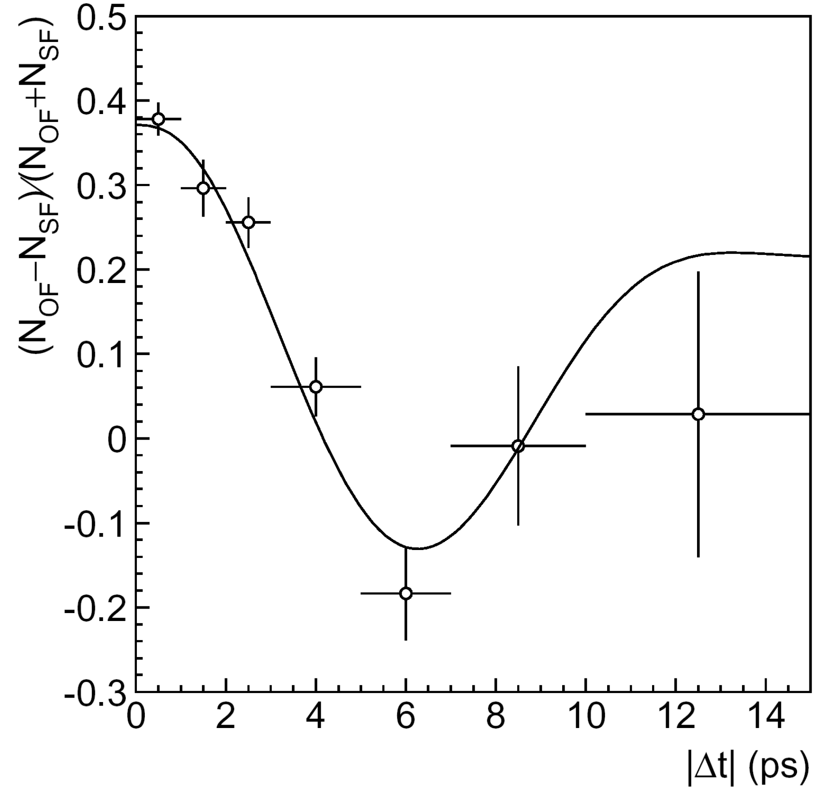
	$\sigma[\Delta m_d]$
Description of background events	**0.004**
Background fractions, sideband extrapolation	
Background Δt structure and resolution	
Peaking B+ background	
Δt resolution and detector effects	**0.005**
Silicon detector residual misalignment	
Δt resolution model	
$\sigma(\Delta t)$ requirement	
Fit bias correction and MC statistics	**0.003**
Fixed lifetime from PDG2000 *	**0.006**
Total	**0.010**



*** Now improved with PDG2002**

Mixing in Hadronic Modes at Belle

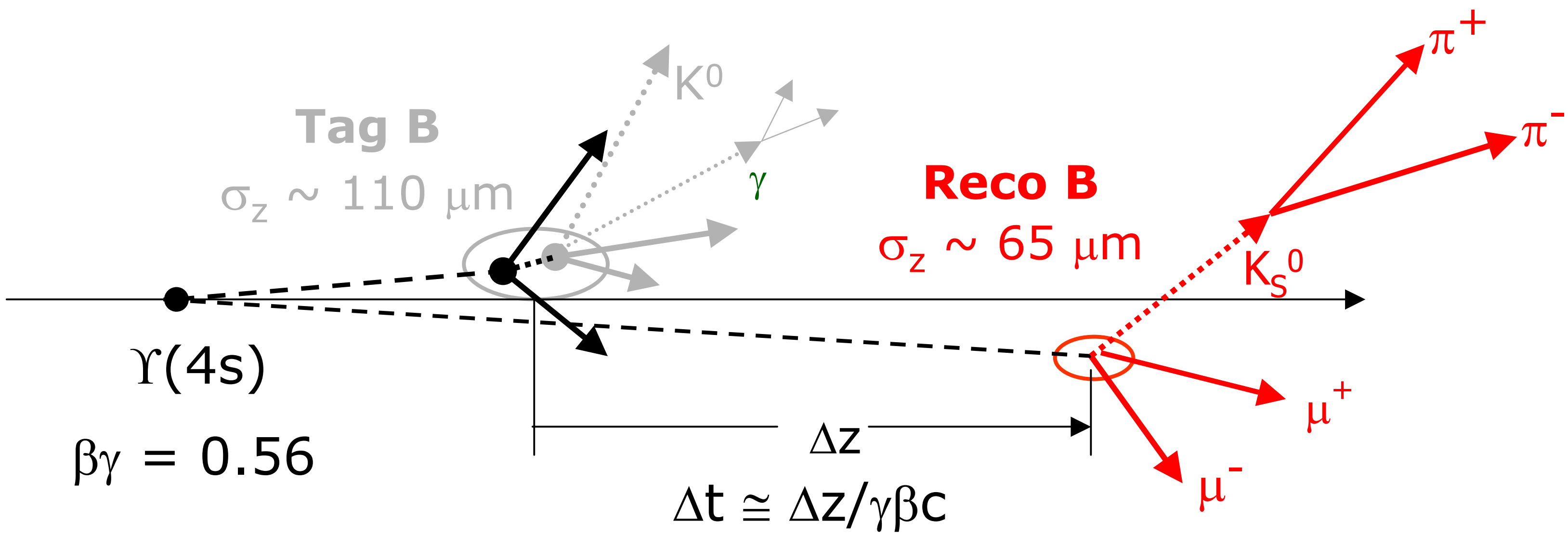




BELLE
$29.1\ fb^{-1}$

$$\Delta m_d = (0.528 \pm 0.017_{(stat)} \pm 0.011_{(syst)})\ \ ps^{-1}$$

Measurement of sin2β



1. Fully reconstruct one *B* meson in *CP* eigenstate (B_{rec})
2. Reconstruct the decay vertex ✓

3. Reconstruct Inclusively the vertex of the "other" *B* meson (B_{tag}) ✓
4. Determine the flavor of B_{tag} to separate B^0 and $\overline{B}^0$ ✓

5. compute the proper time difference Δt ✓
6. Fit the Δt spectra of B^0 and $\overline{B}^0$ tagged events

Time-Dependent CP Asymmetries

Time-dependence of
B^0-$\bar{B}^0$ mixing

$$A_{mixing}(\Delta t) = \frac{N(unmixed) - N(mixed)}{N(unmixed) + N(mixed)} \approx (1-2w)\cos\Delta m_d \Delta t$$

Time-dependence of
CP-violating asymmetry in
$B^0_{CP} \to J/\psi K^0_S$

(Assuming no confusion of B_{rec} state)

$$A_{CP}(\Delta t) = \frac{N(B_{tag} = B^0) - N(B_{tag} = \bar{B}^0)}{N(B_{tag} = B^0) + N(B_{tag} = \bar{B}^0)} \approx (1-2w)\sin 2\beta \sin\Delta m_d \Delta t$$

Use the large statistics B_{flav} data sample to determine the **mistag probabilities** and the parameters of the **time-resolution function**

Summary

- ***Precision measurements of lifetimes and B0 oscillations have been performed at the B Factories***
  - Require the development of all techniques for *B* reconstruction, determination of vertex separation, tagging of the recoil *B* state at decay, unbinned maximum likelihood fitting and validation procedures
  - Results are in excellent agreement with previous results and represent some of the single most-precise measurements available

$\Delta m_d = 0.516 \pm 0.016\ (\text{stat}) \pm 0.010\ (\text{syst})\ \text{ps}^{-1}$

BABAR PRL **88**, 221802 (2002)

$\Delta m_d = 0.528 \pm 0.017\ (\text{stat}) \pm 0.011\ (\text{syst})\ \text{ps}^{-1}$

BELLE preprint-02/020, hep-ex/0207022, to appear in PLB

- ***Tomorrow***
  - Apply tools to measurement of *CP* violation in neutral *B* decays

Bibliography: Lecture 2

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