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Deconstruction and New approaches to Electroweak symmetry breaking

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Not “Deconstruction - New approaches to Electroweak symmetry breaking”

“Electroweak symmetry breaking” means “little higgs” models - discovered in the context of deconstruction, but logically unrelated

Two separate threads that came together briefly

I'm going to talk about the little higgs. If I have time at the end, I may say a few words about deconstruction - but I don't expect this to happen.

I've learned about little higgs from many - mostly my former students and their students - Nima Arkani-Hamed and Andy Cohen, Ann Nelson, David Kaplan, Lisa Randall, Martin Schmaltz (ICHEP2002)

Basic motivation for little higgs models - pretty strong circumstantial evidence from the success of the standard model at the level of radiative corrections that the Higgs boson exists with a mass small compared to 1 TeV.

1 - What is it?

2 - Why is it so light?

In more detail:

1 - Is the Higgs a new fundamental scalar or is it built out of other things, and if so, what? This problem we have faced for years with the longitudinal components of the W and Z - or equivalently (by the equivalence theorem) the Goldstone boson of spontaneously broken EW symmetry eaten by the Higgs mechanism.

Indeed, another way of saying why we think the Higgs really exists is that now we are reasonably confident that the longitudinal W and Z at least “look” approximately fundamental up to energies of the order of 1 TeV and the electroweak symmetry then requires that they be part of a multiplet with a Higgs boson.

2 - Why is it so light? The higher we push the scale up to which the Higgs looks fundamental, the more trouble we have with quadratic sensitivity of its mass to physics at the higher scale. This does not mean the GUT scale. We already have problems associated with scales about which we have much more indirect evidence.

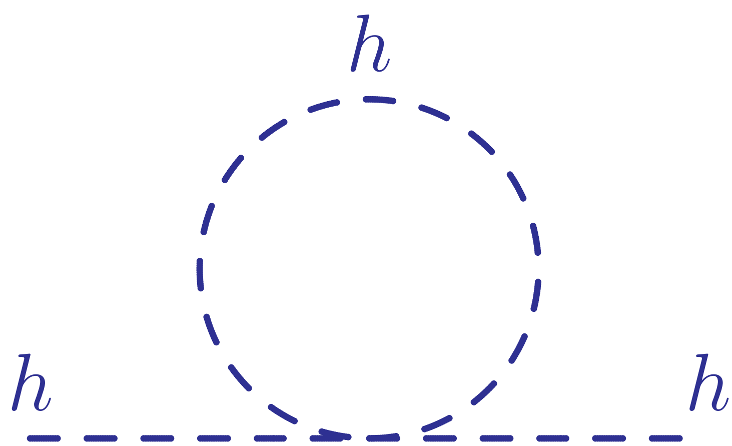
The suppression of Flavor Changing Neutral Current processes strongly suggests that the Higgs looks fundamental well above 1 TeV. What is it that cancels the quadratic divergences in the Higgs mass in radiative corrections that we see if we only include the particles of the standard model?

We want “natural” cancellation of quadratic divergences - not fine tuning! Thus the couplings must be related in diagrams that cancel.

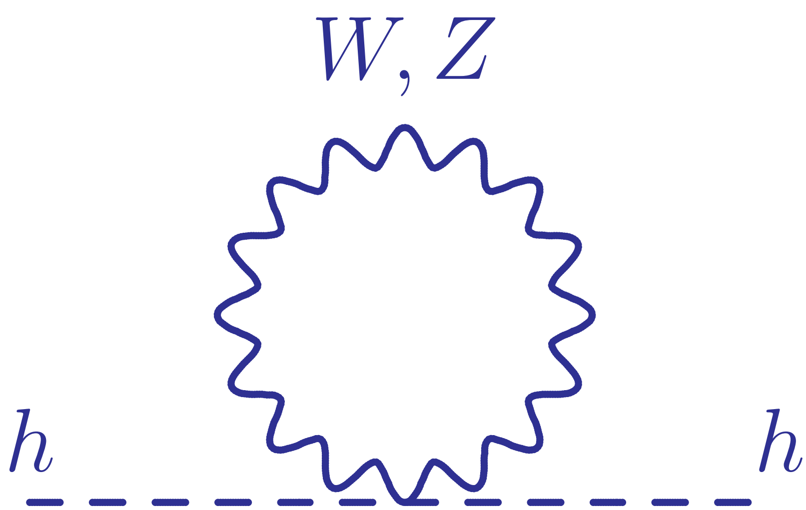
One possibility is SUSY - diagrams with super partners.

If we don’t have SUSY, naturalness becomes a much stronger constraint because the Higgs has three different kinds of couplings.

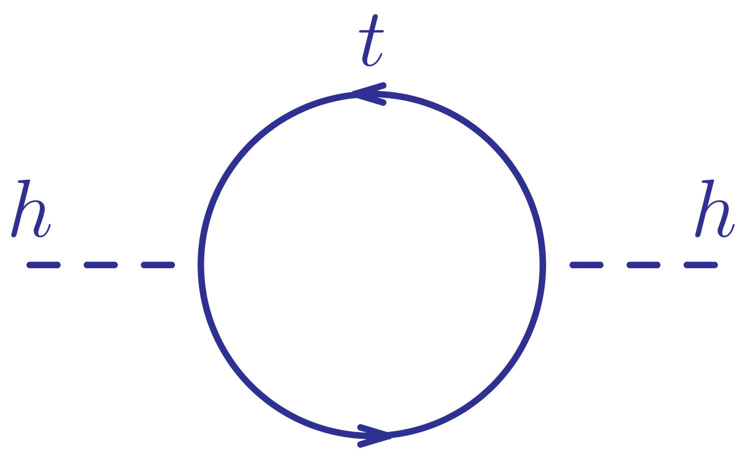
It has to have a λh^4 interaction, which gives rise to a quadratically divergent contribution from scalar loops.



It has to couple to the W and Z, which gives rise to a quadratically divergent contribution from gauge boson loops.



It has to couple to the t, which gives rise to a quadratically divergent contribution from fermion loops.



Thus we would expect a model that addresses these problem to have additional particles of all these types at as scale of not much more than 1 TeV — new scalars, new gauge boson, and new fermions. This sounds like fun!

The idea of little higgs models is that these issues can be addressed in a natural way if the Higgs is a pseudo-Goldstone boson. This is an old idea. I have been looking for such models for nearly 30 years, with very limited success.

So what is the new idea?

Since we need new gauge bosons and new fermions anyway, we will try to arrange their interactions to have symmetries that completely eliminate the mass of the Higgs at the one-loop level. This can be arranged if the new interactions break up into separate sets, each of which treats the Higgs as a Goldstone boson.

Then only when both sets of interactions are involved will one get a quadratically divergent contribution to the mass. This may allow to push the scale at which the Higgs looks fundamental up to the order of 10 TeV without fine tuning.

Sounds easy when explained this way! But I have always found this explanation slightly facile and confusing. So today, I am going to do something that one should never do in a talk. I am going to try to show you how this works in detail. This will make for a talk that is hard to follow in spots. I hope that at the end you will feel that this hard work is worth it.

The $SU(5)/SO(5)$ model - ACKN

- (Arkani-Hamed, Cohen, Katz and Nelson)

One of the simplest and most beautiful little higgs models - I'm going to describe it in some detail because I am so impressed by it.

Though it is not really part of the model, let me show you how the higgs structure might emerge from specific high energy dynamics. This will eventually help me explain what I think is good and what is problematic.

Imagine a high energy theory with an asymptotically free $SO(N)$ gauge group that becomes strongly interacting theory at a scale of order 10 TeV and that includes, among other things, 5 LH fermions transforming like Ns under the $SO(N)$.

In addition there is a much weaker $SU(2)_1 \times U(1)_1 \times SU(2)_2 \times U(1)_2$ gauge group from which will emerge the electroweak $SU(2) \times U(1)$ low energy gauge symmetry.

The 5 Ns transform like $(2,1)+(1,2)+(1,1)$ (in terms of isospins $(1/2,0)+(0,1/2)+(0,0)$) under the two $SU(2)$s, and it is convenient to talk about this structure in a notation with vectors blocked as follows

$$\psi = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} (2,1) \\ (1,1) \\ (1,2) \end{pmatrix}$$

In this notation, matrices look like

$$\begin{pmatrix} 2 \times 2 & 2 \times 1 & 2 \times 2 \\ 1 \times 2 & 1 \times 1 & 1 \times 2 \\ 2 \times 2 & 2 \times 1 & 2 \times 2 \end{pmatrix}$$

In this notation, the weak gauge generators look like this:

$$Q_1^a = \begin{pmatrix} \frac{\sigma_a}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad Q_1' = \begin{pmatrix} q_1 + \frac{1}{2} & 0 & 0 \\ 0 & -q_1 & 0 \\ 0 & 0 & -q_1 \end{pmatrix}$$

$$Q_2^a = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{\sigma_a^*}{2} \end{pmatrix} \qquad Q_2' = \begin{pmatrix} -q_2 & 0 & 0 \\ 0 & -q_2 & 0 \\ 0 & 0 & q_2 - \frac{1}{2} \end{pmatrix}$$

The $U(1)s$ have $SO(N)$ anomalies but who knows what else is happening in the high energy theory - so this may be OK -

set $q_1 = q_2 = 0$ - keeps the algebra simple

$$Q_1^a = \begin{pmatrix} \frac{\sigma_a}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad Q_1' = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$Q_2^a = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{\sigma_a^*}{2} \end{pmatrix} \qquad Q_2' = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix}$$

Condensates and Goldstone bosons

The QCD analogy - familiar story

$$\left[\psi_L \gamma^0 \overline{\psi}_R\right]_{\substack{\text{low}\\\text{energy}}} \propto \Sigma = \exp(i\Pi/f) = \begin{matrix}\text{unitary}\\\text{matrix}\end{matrix}$$

the vacuum "direction" of Σ is determined by quark mass matrix M — potential energy

$$-\,\mathrm{tr}(M\Sigma) \Rightarrow$$

$$\langle\Sigma\rangle = I \quad \text{for} \quad M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

$\Leftrightarrow$ Gell-Mann's $SU(3)$ approximately preserved for light quarks

or

$$SU(3) \times SU(3) \to SU(3)$$

Or - each flavor does its own thing but one doesn't know exactly what the flavors are until the symmetry is explicitly broken.

Other “directions” for condensate are excitations of the Goldstone boson fields

$$\Pi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 - \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 - \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K}^0 & \sqrt{\frac{2}{3}}\eta \end{pmatrix}$$

Hermitian matrix because condensate is unitary.

The Π Goldstone bosons are
quark-antiquark bound states, but they are massless in the absence of explicit symmetry breaking because they are bound by the QCD interactions just as much as the vacuum state itself.

In QCD this is very familiar

$$\Pi \propto \begin{pmatrix} u\overline{u} & u\overline{d} & u\overline{s} \\ d\overline{u} & d\overline{d} & d\overline{s} \\ s\overline{u} & s\overline{d} & s\overline{s} \end{pmatrix}$$

and simple except for some funny business for the π^0 and η due to the anomaly.

A word about scales! f in

$$\Sigma = \exp(i\Pi/f)$$

is f_π - the amplitude for a chiral current to create a Goldstone boson out the vacuum. It is much smaller than the typical mass of a non-Goldstone meson state (like the ρ or the a_1 or whatever) ≈ 1 GeV.

This 1 Gev scale is called (confusingly) Λ - the chiral symmetry breaking scale - not to be confused with Λ_{QCD}. The ratio Λ/f plays a large role in the thinking of little higgsers. We believe that this factor is real, important and simple - except for factors having to do with the numbers of colors and flavors, it is a phase space factor of order $4\pi \approx 10$. This is the difference between 1 TeV and 10 TeV.

In the $SO(N)$ theory, the condensate looks like

$$\left[\psi\gamma^0\psi^T\right]_{\substack{\text{low}\\\text{energy}}} = \Sigma = \begin{array}{c}\text{symmetric}\\\text{matrix}\end{array}$$

Unlike the chiral symmetry breaking condensate in QCD, here there is no difference between LH and RH fields. The condensate is a fermion-fermion condenstate - not fermion-anti-fermion $\rightarrow \Sigma = \Sigma^T$.

Fundamental assumptions - based on QCD analog - is that Σ is also unitary. As in QCD, the vacuum "direction" is determined by symmetry breaking. But for some choice of basis for the fermion fields, $\Sigma = I$ - breaks the $SU(5)$ global symmetry down to $SO(5)$ because under an $SU(5)$ transformation U

$$\Sigma = I \rightarrow U\,\Sigma\,U^T = U\,U^T$$

and only if U is real is $U^T = U^\dagger = U^{-1}$

The low energy excitations of the vacuum are again parametrized by Goldstone boson fields

$$\Sigma = \exp(i\Pi/f) \quad \text{where} \quad \Pi = \Pi^T$$

These Goldstone bosons are fermion-fermion bound states. What exactly does the vacuum look like? How does it fit with the low energy gauge groups?

The result is something I find quite counterintuitive, but ultimately very beautiful, so I am going to show you how it works in a bit of detail - the individual $SU(2) \times U(1)$ break, but the combination of the two is left unbroken.

More QCD analog - the K^+-K^0 and π^+-π^0 mass differences - K^+ is heavier than the K^0 even though the u quark is lighter than the s quark because of photon exchange. Electromagnetism gives no mass at all to the K^0 because it doesn't break the chiral d-s symmetry - just adds to the QCD attraction that forms the condensate and does the same in the K^0 bound state.

But the K^+ and π^+ mass squared get a positive contribution from photon exchange because their quark and antiquarks repel one another and they are less bound. Formally, the photon exchange potential is

$$x\, e^2 f_\pi^2 \left|Q\Sigma - \Sigma Q\right|^2$$

where $x = \mathcal{O}(1) > 0$ and Q is the quark charge matrix

$$Q = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}$$

An interaction of this kinds simply tries to make the condensates neutral to maximize the binding.

It produces a contribution to the Goldstone boson mass squared proportional to the square of the charge. Obviously in QCD this gives equal mass squared to the K^+ and π^+ and nothing to any of the neutral states.

In this case, because the nonzero elements of condensate are all neutral, the condensate we already have from the quark masses minimizes this contribution to the potential as well, and the electromagnetic gauge symmetry is not broken by the vacuum.

In the $SU(5)/SO(5)$ model, there are good reasons to believe that this works in a similar way - but there are important differences:

1 - Now there are lots of charges - we have to sum over each type of charge, multiplying by the coupling constant. The charges of the Goldstone bosons or the entries in the condensate matrix Σ are just the sums of the charges of the fermion constituents.

2 - This time, we don't already know the form of the vacuum from something like the quark masses that we had in QCD. These terms determine the vacuum structure.

3 - Finally, we won't be able to find elements of the condensate matrix that preserves all the symmetries — some will get spontaneously broken.

So for example

$$Q_1^a = \begin{pmatrix} \frac{\sigma_a}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad Q_1' = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$Q_2^a = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{\sigma_a^*}{2} \end{pmatrix} \qquad Q_2' = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix}$$

gives charge squared of the form

$$Q_1' = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \left(\frac{1}{2} + \frac{1}{2}\right)^2 & \left(\frac{1}{2} + 0\right)^2 & \left(\frac{1}{2} + 0\right)^2 \\ \left(0 + \frac{1}{2}\right)^2 & 0 & 0 \\ \left(0 + \frac{1}{2}\right)^2 & 0 & 0 \end{pmatrix}$$

Similarly

$$Q_2' = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \longrightarrow \begin{pmatrix} 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 1 \end{pmatrix}$$

For the $SU(2)$ gauge groups, we want the sum of the squares of the components, which gives $i(i+1)$ for the representation. Under $SU(2)_1 \times SU(2)_2$, the various parts of the condensate matrix have isospin

$$\begin{pmatrix} (1,0) & (1/2,0) & (1/2,1/2) \\ (1/2,0) & (0,0) & (0,1/2) \\ (1/2,1/2) & (0,1/2) & (0,1) \end{pmatrix}$$

Notice that in

$$\begin{pmatrix} (1,0) & (1/2,0) & (1/2,1/2) \\ (1/2,0) & (0,0) & (0,1/2) \\ (1/2,1/2) & (0,1/2) & (0,1) \end{pmatrix}$$

there is no $(0,0)$ component because of the symmetry of the matrix.

Thus the $SU(2)$ contributions look like

$$Q_1^a = \begin{pmatrix} \frac{\sigma_a}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & \frac{3}{4} & \frac{3}{4} \\ \frac{3}{4} & 0 & 0 \\ \frac{3}{4} & 0 & 0 \end{pmatrix}$$

$$Q_2^a = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{\sigma_a^T}{2} \end{pmatrix} \longrightarrow \begin{pmatrix} 0 & 0 & \frac{3}{4} \\ 0 & 0 & \frac{3}{4} \\ \frac{3}{4} & \frac{3}{4} & 2 \end{pmatrix}$$

Putting this together gives

$$
g_1^2 \begin{pmatrix} 2 & \frac{3}{4} & \frac{3}{4} \\ \frac{3}{4} & 0 & 0 \\ \frac{3}{4} & 0 & 0 \end{pmatrix} + g_2^2 \begin{pmatrix} 0 & 0 & \frac{3}{4} \\ 0 & 0 & \frac{3}{4} \\ \frac{3}{4} & \frac{3}{4} & 2 \end{pmatrix}
$$

$$
+ {g'_1}^2 \begin{pmatrix} 1 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 \end{pmatrix} + {g'_2}^2 \begin{pmatrix} 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 1 \end{pmatrix}
$$

$$g_1^2 \begin{pmatrix} 2 & \frac{3}{4} & \frac{3}{4} \\ \frac{3}{4} & 0 & 0 \\ \frac{3}{4} & 0 & 0 \end{pmatrix} + g_2^2 \begin{pmatrix} 0 & 0 & \frac{3}{4} \\ 0 & 0 & \frac{3}{4} \\ \frac{3}{4} & \frac{3}{4} & 2 \end{pmatrix}$$

$$+ {g'_1}^2 \begin{pmatrix} 1 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 \end{pmatrix} + {g'_2}^2 \begin{pmatrix} 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 1 \end{pmatrix}$$

$$\begin{pmatrix} ? & 0 & ? \\ 0 & 1 & 0 \\ ? & 0 & ? \end{pmatrix}$$

clearly minimzes
the potential

$$g_1^2 \begin{pmatrix} 2 & \frac{3}{4} & \frac{3}{4} \\ \frac{3}{4} & 0 & 0 \\ \frac{3}{4} & 0 & 0 \end{pmatrix} + g_2^2 \begin{pmatrix} 0 & 0 & \frac{3}{4} \\ 0 & 0 & \frac{3}{4} \\ \frac{3}{4} & \frac{3}{4} & 2 \end{pmatrix}$$

$$+{g_1'}^2 \begin{pmatrix} 1 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 \end{pmatrix} + {g_2'}^2 \begin{pmatrix} 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 1 \end{pmatrix}$$

$$\begin{pmatrix} I & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & I \end{pmatrix} \quad \text{has higher energy than} \quad \begin{pmatrix} 0 & 0 & I \\ 0 & 1 & 0 \\ I & 0 & 0 \end{pmatrix}$$

Both the $SU(2)$ and the $U(1)$ contributions tend to stabilize the vacuum

$$\begin{pmatrix} 0 & 0 & I \\ 0 & 1 & 0 \\ I & 0 & 0 \end{pmatrix}$$

breaks the symmetry down to the diagonal $SU(2) \times U(1)$

$$\begin{pmatrix} (1,0) & (1/2,0) & (1/2,1/2) \\ (1/2,0) & (0,0) & (0,1/2) \\ (1/2,1/2) & (0,1/2) & (0,1) \end{pmatrix}$$

Notice that this completely determines the vacuum up to gauge transformations, so we expect that there are no exact Goldstone bosons left over. But here is the tricky part. Each of the individual weak gauge symmetries preserves an $SU(3)$ global symmetry.

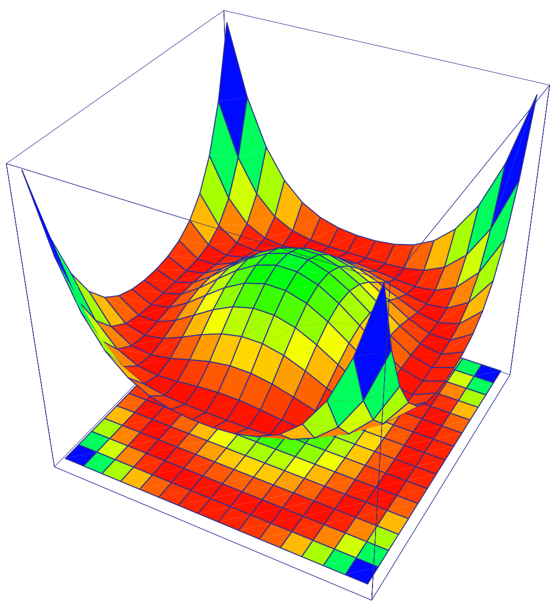
$$Q_1^a = \begin{pmatrix} \frac{\sigma_a}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad Q_1' = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$Q_2^a = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{\sigma_a^*}{2} \end{pmatrix} \qquad Q_2' = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix}$$

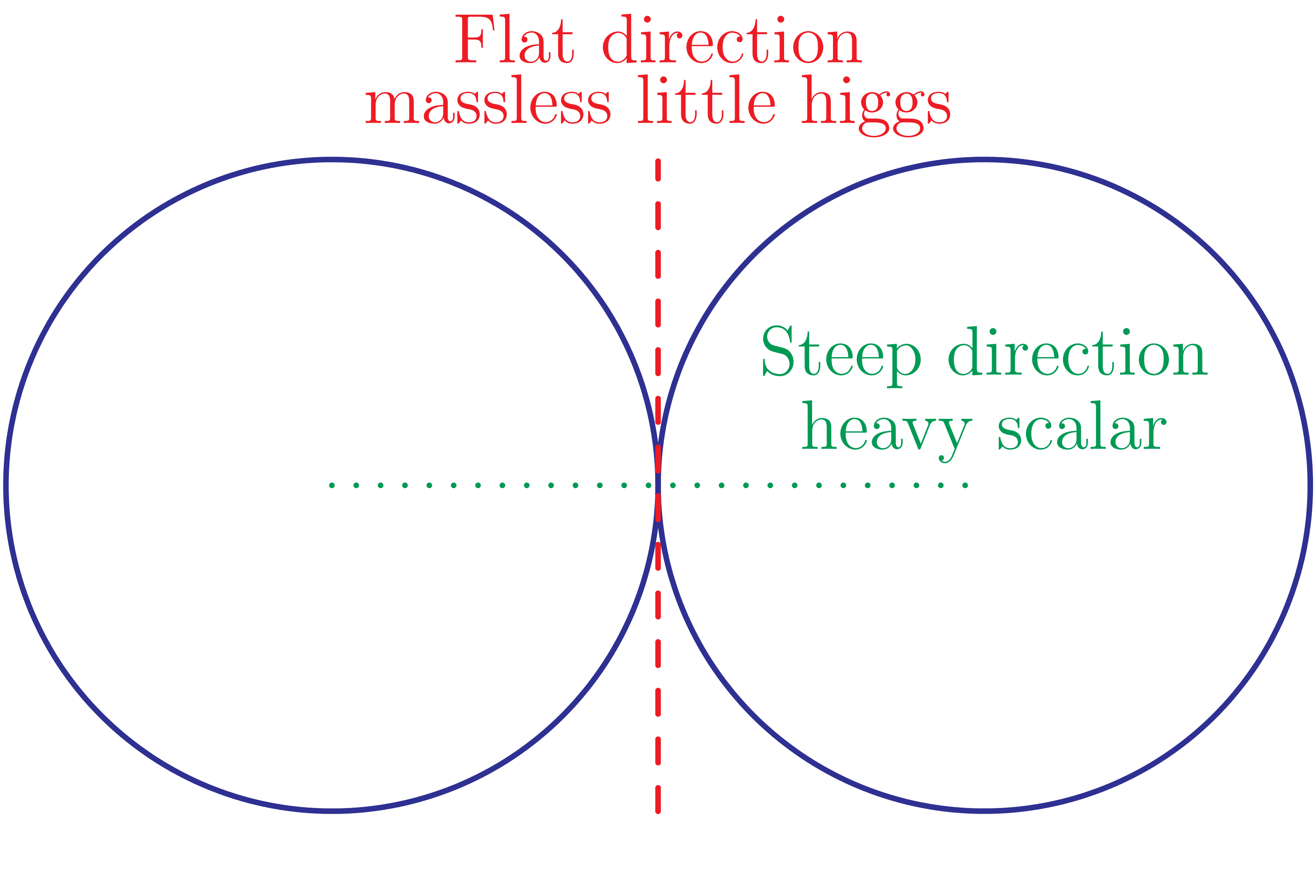
Each of the global $SU(3)$ symmetry contains a generator that behaves like a doublet under the $SU(2)$ symmetry that is left over by the vacuum condensate. And each of these produces the same deformation of the vacuum condensate.

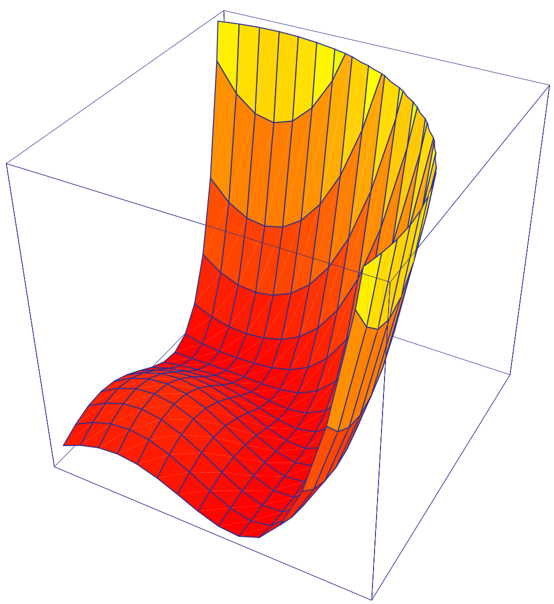
$$\pi_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & h \\ 0 & h^\dagger & 0 \end{pmatrix} \qquad \pi_2 = \begin{pmatrix} 0 & h^T & 0 \\ h^* & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

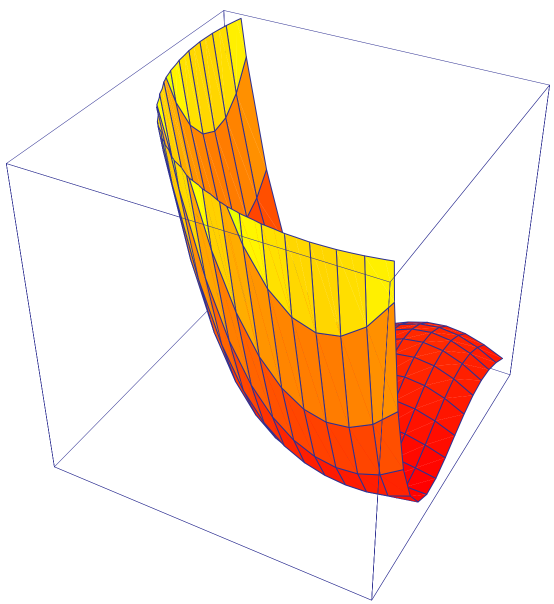
These are generators associated with different spontaneously broken symmetries, but give the same deformation of the vacuum. $h = (h^+, h^0)$ is the little higgs! The potential looks like Kissing Mexican hats!

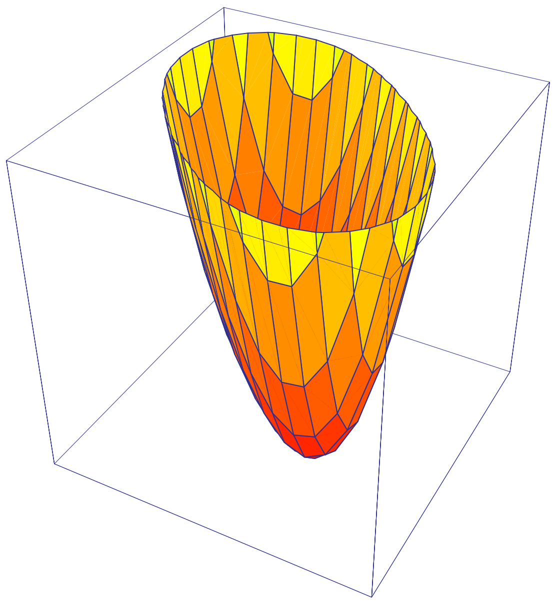


Each $SU(2) \times U(1)$ gauge group contributes a Mexican Hat potential for h - but their flat directions kiss at the true vacuum! The sum produces a quartic potential for h.


Flat direction
massless little higgs
Steep direction
heavy scalar







This gives the usual $\lambda(h^\dagger h)^2$ interaction for the little higgs. I find this really gorgeous!

The quadratic divergences are evidently under total control at one loop, because the masslessness of h follows from a symmetry argument. The cancellations we expected can be seen in detail.

Of course, we need a lot more to turn this into a real model.

We need a negative mass squared for h.

We need a large Yukawa coupling of h to the t quark.

These things better not spoil the cancellation of quadratic divergences at one-loop to the h mass.

But we now have an argument to guide us — as long as we can find interactions with unbroken symmetries that are associated with one of the other of the two kissing Mexican hats, we will not generate a large one-loop mass.

At this point, the details become less interesting, and I am going to switch to philosophy.

Why didn't I find this model long ago, rather than having to learn it from my students recently? Partly stupidity. Partly I didn't know the t quark is so heavy. Partly the kissing Mexican hat (KMH) mechanism is subtle.

But MOSTLY - this represents a slightly different way of thinking about the symmetries. Little higgers talk as if they can impose the global symmetries that maintain the KMH mechanism, but it seems to me that they don't actually mean this. If you must impose a global symmetry, you are actually doing fine tuning.

Of course, you may be able to argue about the size of the fine-tuning required, but this is dangerous and unsatisfying.

Really, all the global symmetries that go inot maintaining the KMH mechanism should be “accidental symmetries” - automatically produced by the hight energy theory.

The $SU(5)$ global symmetry of the ACKN model is such a a symmetry for the high-energy dynamics that I talked about. But one needs much more. As ACKN noted in their paper, you would really like the symmetries of the new fermions to arise in the same way. This may not be impossible, but it is certainly not trivial either.

I look forward to so fabulous fun model building trying to find high-energy extensions that actually realize little higgs models in a natural way.

In the meantime, there is some very challenging phenomenology to worry about. The cancellation of quadratic divergences gives us some information about what new particles to expect at the TeV scale, but at the moment, it is rather vague. We need to find ways of making this more precise.

But at the very least, one can think of the little higgs story as a cautionary tale. It shows, if showing were needed, that we theorist have probably not, even yet, exhausted all the possibilities for spontaneous electroweak symmetry breaking.

Ultimately, it is going to be up to the experimenters to find the way Nature has chosen - and it may be something very different from anything you find in theorists papers.