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RADIATIVE ϕ DECAYS

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ABSTRACT

Radiative ϕ decays give us an excellent opportunity to study scalar and pseudoscalar mesons below 1 GeV. In this paper, results from different experiments are reviewed and compared.

1 The Puzzle Regarding the Scalar Mesons $f_0(980)$ and $a_0(980)$

Although the existence and properties of the scalar mesons $f_0(980)$ and $a_0(980)$ are well established and have been known for about thirty years, the physical nature of these mesons is still unclear. The first evidence for the f_0 was in the reaction $p\pi \to n\pi\pi$. The $I = J = 0$ elastic $\pi\pi$ cross section shows a dip close to the $K\bar{K}$ threshold. Similarly, in $p\pi \to nK\bar{K}$, a sharp onset of inelasticity indicates the presence of a dynamical structure strongly coupled to $K\bar{K}$. Evidence for strange quark content in the f_0 is also confirmed by J/ψ decays. In fact, the Mark-III, DM2, and BES collaborations measure BR($J/\psi{\to}f_0\phi$)> BR($J/\psi{\to}f_0\omega$). However, the result obtained by the E791 collaboration [1] shows that 56% of the D_s decay into 3π proceeds via f_0, which suggests that f_0 has both $s\bar{s}$ and $n\bar{n}$ components. The total width of the f_0 meson has been measured by several experiments and ranges between 40–100 MeV; moreover, the f_0 is weakly coupled to photons [$\Gamma(f_0 \to \gamma\gamma) \approx 0.3$ KeV]. The first evidence for the a_0 was found in the $\eta\pi$ system in $Kp \to \Sigma\eta\pi$. Today, there are very accurate data from the Crystal Barrel, GAMS, Obelix and E852 collaborations [2, 3]. As in the case of the f_0, the a_0 also has a small width compared to ordinary mesons with the same mass, and a small $\gamma\gamma$ coupling. In short, these two scalar mesons are very close in mass, and have small widths and small $\gamma\gamma$ couplings. The standard interpretations of these states as $q\bar{q}$ mesons is not favored. The total widths are much smaller than the ∼500 MeV expected from the $q\bar{q}$ prediction, the $\gamma\gamma$ partial widths measured are a factor 10–20 smaller than the $q\bar{q}$ prediction, and the $K\bar{K}$ coupling is too large for an OZI-forbidden decay. The theoretical situation is further complicated because there are many scalars below 1.5 GeV. Many different interpretations of these states have been proposed: $K\bar{K}$ molecules [5], four-quark states [4], and, in the case of the f_0, glueballs. Interest in studying light scalar mesons also comes from an old suggestion by Gribov that foresees the existence of a peculiar state with the vacuum quantum numbers and a mass close to the proton mass to explain quark confinement. Most recently, Close and Törnqvist [6] have interpreted these mesons as the Higgs nonet of a hidden U(3) symmetry. From all of the above considerations, it is obviously urgent to clarify the situation. The radiative ϕ decays are very useful for addressing this puzzle. In fact, the absolute rates for $\phi{\to}f_0\gamma$ and $a_0\gamma$ and the mass spectra of the f_0 and a_0 are very sensitive to the nature of these scalar particles, as shown in Tab. 1.

Table 1: Branching ratio predictions for different models.

Channel	$q\bar{q}$	$q\bar{q}q\bar{q}$	$K\bar{K}$
BR($\phi \to f_0\gamma$)	5×10^{-5}	10^{-4}	10^{-5}
BR($\phi \to a_0\gamma$)	10^{-5}	10^{-4}	10^{-5}

2 Radiative ϕ Decays to f_0, a_0

Most of the data on the $\phi \to f_0\gamma$ and $\phi \to a_0\gamma$ decays come from three experiments: KLOE [8, 7] at the Frascati ϕ-factory DAΦNE[9], and SND [12, 10] and CMD-2 [13, 11] at VEPP-2M in Novosibirsk. The published analyses are based on 5.3×10^7 ϕ decays for the KLOE experiment, with large acceptances, and 2×10^7 ϕ decays for the VEPP-2M experiments, with smaller acceptances. The f_0 has been studied via its decay to $\pi^0\pi^0$. The a_0 has been studied via its decay to $\eta\pi^0$, with $\eta \to \gamma\gamma$ and $\eta \to \pi^+\pi^-\pi^0$. For this last channel, only data from the KLOE experiment are available.

2.1 $\phi \to \pi^0\pi^0\gamma$

Two amplitudes contribute to the $\phi \to \pi^0\pi^0\gamma$ final state: $\phi \to S\gamma; S \to \pi^0\pi^0$ and $\phi \to \rho^0\pi^0; \rho^0 \to \pi^0\gamma$, where S is a scalar meson. The main backgrounds come from $e^+e^- \to \omega\pi^0 \to \pi^0\pi^0\gamma$, $\phi \to \eta\pi^0\gamma \to 5\gamma$, $\phi \to \eta\gamma \to 3\pi^0\gamma$ with two undetected photons, and $\phi \to \eta\gamma \to 3\gamma$ with two additional photons from accidentals.

The natural cross sections for the signal and background channels are listed in Tab. 2. The $\omega\pi$ background is the most important, since the background from $\eta\gamma$ into 3- and 7-photon final states is immediately reduced by exploiting the detector hermiticity and the photon timing.

Table 2: Cross sections for signal and background channels for $\phi \to \pi^0\pi^0\gamma$.

Channel	cross section (nb)
$f_0\gamma$	≈ 0.4
$\omega\pi$	≈ 0.5
$\eta\pi\gamma$	≈ 0.1
$\eta\gamma$	≈ 17.0

All three collaborations have performed similar analyses. The selection of $\phi \to \pi^0\pi^0\gamma$ events starts from the five-photon sample. A cut on the total energy in the calorimeter rejects the $\phi \to K_S K_L$ background. The photons are paired to search for a π^0 and a cut on $|M_{\pi\gamma} - M_\omega|$ is performed to veto the $\omega\pi$ background. A kinematic fit requiring the π^0 masses further reduces the background. The numbers of signal events and the mean efficiencies are shown in Tab. 3 for each of the three collaborations. The $\pi\pi$ invariant mass spectra are shown in Figs.1 and 2. A clear peak in the f_0 region is seen.

Table 3: Signal statistics and efficiencies for $\phi \to \pi^0\pi^0\gamma$.

EXPERIMENT	Signal events	Mean efficiency (%)
KLOE	2438 ± 61	~ 40
SND	419 ± 31	~ 20
CMD-2	268 ± 27	~ 12

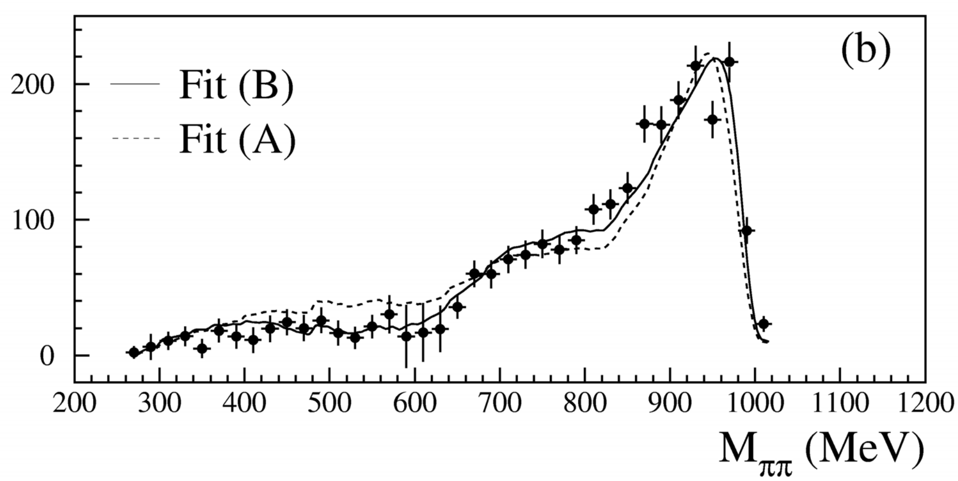

Figure 1: $\pi\pi$ invariant mass spectrum, KLOE experiment.

2.2 Model for Fits to $M_{\pi\pi}$ Spectrum

All the three experiments have fit the mass spectrum using the same model. This spectrum is taken as the sum of $S\gamma$, $\rho\pi$, and interference terms: $f(m) = f_{S\gamma}(m) + f_{\rho\pi}(m) + f_{\rm int}(m)$. For the scalar term, the kaon loop model is used [14]. In this model, the radiative ϕ decays to a scalar proceed through a charged kaon loop, and the scalar term can be written as:

$$f_{S\gamma}(m) = \frac{2\,m^2}{\pi} \frac{\Gamma_{\phi S\gamma}\Gamma_{S\pi^0\pi^0}}{|D_S|^2} \frac{1}{\Gamma_\phi}. \tag{1}$$

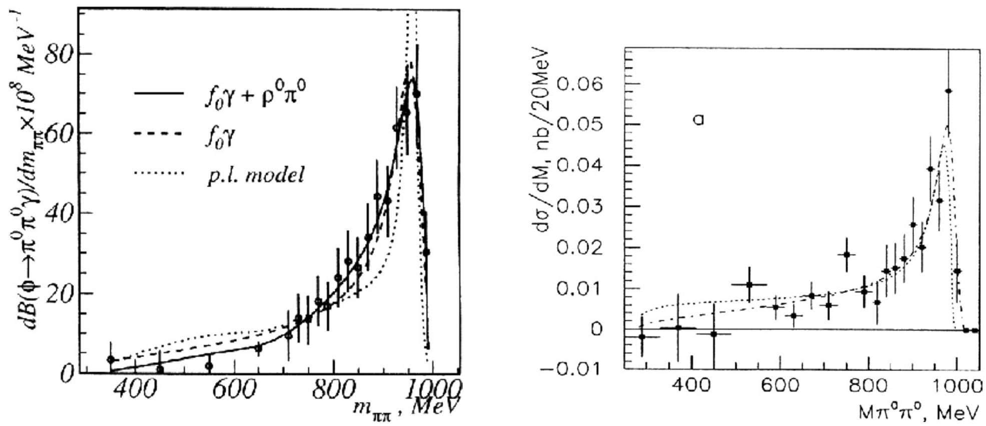

Figure 2: $\pi\pi$ invariant mass spectra, CMD-2 (left) and SND (right) experiments.

$$\Gamma_{\phi f_0 \gamma}(m) = \frac{g^2_{f_0 K^+K^-} g^2_{\phi K^+K^-}}{12\pi} \frac{|g(m)|^2}{M_\phi^2} \left(\frac{M_\phi^2 - m^2}{2M_\phi} \right), \tag{2}$$

where $g_{\phi K^+K^-}$ and $g_{f_0 K^+K^-}$ are the couplings and $g(m)$ is the loop integral function. For the inverse propagator, D_S, the finite width corrections [14] are taken into account. The parameterization of Ref. [15] has been used for the $\rho\pi$ and interference terms. A recent measurement [16, 17] reports the existence of a scalar σ with $M_\sigma = (478^{+24}_{-23} \pm 17)$ MeV and $\Gamma_\sigma = (324^{+42}_{-40} \pm 21)$ MeV. The contribution of this meson [18] has also been included in fits to the mass spectrum, giving different results as discussed below.

The observed mass spectrum, $S_{\rm obs}(M_{\pi\pi})$, is fit by folding the experimental efficiency and resolution into the theoretical shape after properly normalizing for the cross section for ϕ production, $\sigma(\phi)$, and the integrated luminosity, $L_{\rm int}$. The KLOE collaboration has performed a fit including σ and $\rho\pi$ contributions. They find that the data prefer a negligible $\rho\pi$ contribution and a negative interference between the f_0 and σ amplitudes at $M_{\pi\pi} < 700$ MeV. The CMD-2 collaboration has done a similar fit and they find a negligible σ contribution. SND has fit the data considering the f_0 contribution only. It is worth noticing that, due to the lack of statistics, the VEPP-2M experiments do not have much sensitivity in the σ region. The values of the coupling constant obtained by KLOE [8], SND [12], and CMD-2 [13] are listed in Tab. 4, and are compared with the results from the WA102 and E791 collaborations. The coupling constants obtained from ϕ decays agree with each other, and differ from the WA102 result on f_0 production in central pp collisions [19] and from results obtained when the f_0 is produced in $D_s^+ \to \pi^+\pi^-\pi^+$ decays [1], in which g_K is consistent with zero. In the same table, measurements of BR$(\phi \to f_0\gamma)$ for $M_{\pi\pi}>$ 700 MeV are also listed.

Table 4: Comparison of f_0 parameters from different experiments.

	KLOE	SND	CMD-2	WA102	E791
M_{f_0} (MeV)	973 ± 1	969 ± 5	975 ± 7	987 ± 8	977 ± 4
$g^2_{f_0 K^+K^-}/(4\pi)$ (GeV^2)	2.79 ± 0.12	2.47 ± 0.73	1.48 ± 0.32	0.40 ± 0.06	0.02 ± 0.05
$g^2_{f_0 K^+K^-}/g^2_{f_0\pi^+\pi^-}$	4.00 ± 0.14	4.6 ± 0.8	3.61 ± 0.62	1.63 ± 0.46	—
$g_{\phi\sigma\gamma}$	0.060 ± 0.008	—	—	—	—
BR($\phi \to \pi^0\pi^0\gamma$)$\times 10^4$ $M_{\pi\pi}>$ 700 MeV	0.96 ± 0.05	1.03 ± 0.09	0.92 ± 0.09	—	—

2.3 $\phi \to \eta\pi^0\gamma$

Production of the a_0 meson followed by $a_0 \to \eta\pi^0$ dominates the final state $\eta\pi^0\gamma$ in ϕ-decays. A small contribution from $\phi \to \rho^0\pi^0$, $\rho \to \eta\gamma$ is present.

The $\phi \to \eta\pi^0\gamma$ decay is studied by KLOE [7], CMD-2 [11] and SND [10] using the $\eta\to\gamma\gamma$ decay mode; the analysis starts from the same 5γ sample as for the f_0 selection. The main backgrounds come from the $\phi \to \pi^0\pi^0\gamma$ channel, which is dominated by $\phi \to f_0\gamma$, the non-resonant $e^+e^- \to \omega\pi^0$ interaction with $\omega \to \pi^0\gamma$, and $\phi \to \eta\gamma$ with $\eta \to \gamma\gamma$ and $\eta \to \pi^0\pi^0\pi^0$. The numbers of signal events and the mean efficiencies are summarized in Tab.5. The KLOE collaboration also makes

Table 5: Signal statistics and efficiencies for $\phi \to \eta\pi^0\gamma$.

EXPERIMENT	Signal events	Mean efficiency (%)
KLOE	607 ± 36	33
SND	35 ± 6	2.3
CMD-2	80 ± 22	4

use of the $\phi \to \eta\pi^0\gamma$, $\eta \to \pi^-\pi^+\pi^0$ channel. In this case, the lower statistics are compensated for by the fact that there are no backgrounds with the same final state. The KLOE collaboration selects 197 events with an estimated background of 4 ± 4 events and an efficiency of $\approx 19\%$.

The fit to the mass spectrum for the a_0 is performed using the kaon loop model as in the case of the f_0. The $\phi \to \rho\pi^0$, $\rho \to \eta\gamma$ contribution is also considered by the KLOE collaboration. The SND collaboration fits the data assuming only the a_0 contribution. The KLOE collaboration performs a combined fit using the two η decay modes and setting M_{a_0}=984.8 MeV, from Ref. [40]. The free parameters of the fit are the branching ratio for the $\phi \to \rho\pi^0$ contribution and the two coupling constants.

The CMD-2 collaboration does not attempt to fit the mass spectrum due to lack of statistics; only BR($\phi \to \eta\pi^0\gamma$) is quoted. In Fig.3, the mass spectrum from the KLOE experiment is shown for the two η decay channels. The values of the coupling constant obtained by KLOE, SND, and CMD-2 are listed in Tab. 6 and are compared with the results from BNL-E852 [2] and with the various Crystal Barrel results [3]

Table 6: Comparison of a_0 parameters from different experiments.

	KLOE	SND	CMD-2	E852	Crystal Barrel
M_{a_0} (MeV)	984.8 (fixed)	995^{+52}_{-10}	—	991 ± 3	1000 ± 2
$g^2_{a_0 K^+K^-}/(4\pi)$ (GeV^2)	0.40 ± 0.04	$1.4^{+9.4}_{-0.9}$		—	—
$g_{a_0\eta\pi}/g_{f_0 K^+K^-}$	1.35 ± 0.09	0.75 ± 0.52	—	1.05 ± 0.06	$0.93 - 1.07$
BR($\phi \to \eta\pi^0\gamma$)$\times 10^5$	7.4 ± 0.7	8.8 ± 1.7	9.2 ± 2.6	—	—

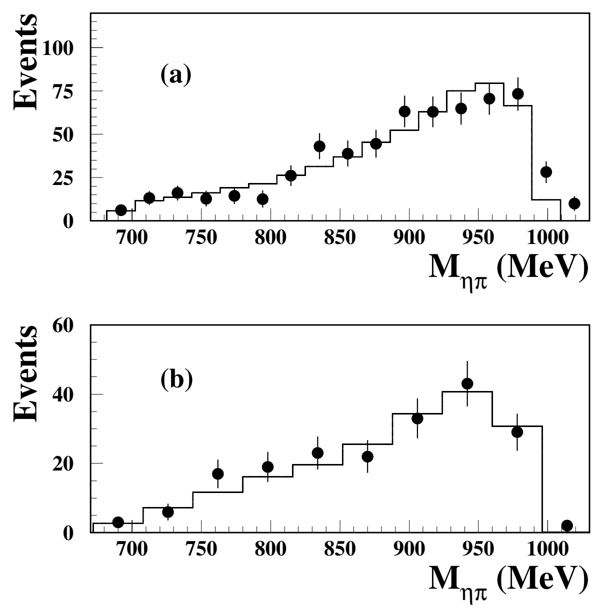

Figure 3: KLOE result for combined fit: comparison of data (exp. points) vs. fit (histogram) for $\phi\to\eta\pi^0\gamma$ with (a) $\eta\to\gamma\gamma$ and (b) $\eta\to\pi^+\pi^-\pi^0$.

2.4 Comparison with Theoretical Expectations

The measured coupling constants for the a_0 and f_0 can be compared with the predictions for the $q\bar{q}$ and $qq\bar{q}\bar{q}$ models. These two models make predictions for the ratio of the coupling constants, R, based on SU(3) algebra assuming no OZI-violation. Qualitative predictions are available for the absolute values of the couplings. Tab. 7 shows these predictions compared with KLOE results. The predictions have been

Table 7: Comparison of KLOE results with predictions from $q\bar{q}$ and $qq\bar{q}\bar{q}$ models. $q\bar{q}$ refers to either $f_0 = (u\bar{u} + d\bar{d})/\sqrt{2}$ (b) or $f_0 = s\bar{s}$ (a). Here, R_{f_0} is $g^2_{f_0 K^+K^-}/g^2_{f_0\pi^+\pi^-}$ and R_{a_0} is $g^2_{a_0 K^+K^-}/g^2_{a_0\eta\pi}$.

Experiment	$g^2_{f_0 K\bar{K}}/4\pi$	$g^2_{a_0 K\bar{K}}/4\pi$	R_{f_0}	R_{a_0}
KLOE	2.79 ± 0.12	0.40 ± 0.04	4.00 ± 0.14	0.55 ± 0.07
$qq\bar{q}\bar{q}$	”superallowed”	“superallowed”	$4 \div 8$	1.2
$q\bar{q}$ (a)	“OZI-allowed”	“OZI-forbidden”	4	0.43
$q\bar{q}$ (b)	“OZI-forbidden”	“OZI-forbidden”	0.5	0.43

evaluated using a value of $\theta_P = -13.7°$ for the pseudoscalar mixing angle. From this comparison, it is evident that:

1. The f_0 parameters are not compatible with the predictions for the $q\bar{q} = u\bar{u}+d\bar{d}$ model. R_{f_0} and $g_{f_0 K\bar{K}}$ are both too large.

2. R_{f_0} is compatible both with $q\bar{q}q\bar{q}$ model parameters and with the expectation for $s\bar{s}$.

3. The a_0 coupling, $g_{a_0 K\bar{K}}$, is too small with respect to predictions for the $q\bar{q}q\bar{q}$ model.

4. R_{a_0} seems to indicate better agreement with $q\bar{q}$ model.

Predictions for the values of the branching ratios have also been made in the framework of the linear sigma model [20, 21] and in the unitarized chiral model approach [22], and show rather good agreement.

3 Pseudoscalar: $\phi \to \eta\gamma$, $\phi \to \eta'\gamma$

Radiative decays of light vector mesons to pseudoscalars have been used as a testing ground since the early days of the quark model [23]. The branching ratio of the decay $\phi \rightarrow \eta'\gamma$ is particularly interesting since its value can probe the $|s\bar{s}\rangle$ and gluonium content of the η' [24]. In particular, the ratio R of the branching ratios for $\phi \rightarrow \eta'\gamma$ and $\phi \rightarrow \eta\gamma$ can be related to the η-η' mixing parameters [25, 26, 27, 28, 29] and determine the mixing angle in the flavor basis φ_P, which has been identified as the best suited parameter for a process-independent description of the mixing. In fact, within the two-mixing-angles scenario, which has emerged from an extended chiral perturbation theory framework [30] as well as from phenomenological analyses [31], it has been demonstrated that the two mixing parameters in the flavor basis are equal, apart from terms which violate the OZI rule [32, 33]. It is therefore safe to use a single mixing angle in this basis. The measurements available to date on BR($\phi \rightarrow \eta'\gamma$) come from the KLOE [34], CMD-2[38], and SND[35] collaborations. The most precise measurements come from the KLOE collaboration.

KLOE has analyzed a sample of 5.3×10^7 ϕ decays, looking for $\phi \rightarrow \eta'\gamma$ via the $\eta' \rightarrow \pi^+\pi^-\eta$ and $\phi \rightarrow \eta\gamma$, $\eta \rightarrow \pi^+\pi^-\pi^0$ decay chains. For both decay chains, the $\pi\pi 3\gamma$ final state is used; therefore, many common systematic effects approximately cancel out in the ratio $R = BR(\phi \rightarrow \eta'\gamma\ )/BR(\phi \rightarrow \eta\gamma\ )$. After the analysis, KLOE finds 128 η' events with an efficiency of $\approx 23\%$ and a background level of 6%. The value $R = (4.7 \pm 0.5 \pm 0.3) \cdot 10^{-3}$ has been obtained.

This value for R can be related directly to the mixing angle in the flavor basis. The Bramon *et al.* [27] and Feldmann [29] parameterizations are used to extract the mixing angle; essentially the same result is obtained using both approaches, i.e., $\varphi_P = (42.2 \pm 1.7)^\circ$, which gives in a mixing angle in the octet-singlet basis $\vartheta_P = (-12.9 \pm 1.7)^\circ$. Moreover, using the value in [40] for BR($\phi \rightarrow \eta\gamma$), the most precise determination of BR($\phi \rightarrow \eta'\gamma$) to date has been extracted (see Fig. 4, left):$BR(\phi \rightarrow \eta'\gamma\ ) = (6.1 \pm 0.6\ (\text{stat.}) \pm 0.4\ (\text{syst.})) \cdot 10^{-5}$

This value for the mixing angle has been obtained neglecting OZI-rule violation and possible contributions from gluonium to the η and η' mesons. Allowing for a gluonium component, [25] we write:

$$
\begin{aligned}
|\eta\rangle &= X_\eta \left|u\bar{u} + d\bar{d}\right\rangle / \sqrt{2} + Y_\eta \left|s\bar{s}\right\rangle + Z_\eta \left|glue\right\rangle , \\
|\eta'\rangle &= X_{\eta'} \left|u\bar{u} + d\bar{d}\right\rangle / \sqrt{2} + Y_{\eta'} \left|s\bar{s}\right\rangle + Z_{\eta'} \left|glue\right\rangle . \qquad (3)
\end{aligned}
$$

A non-zero gluonium component of the η' would correspond to $Z^2_{\eta'} > 0$, or equivalently $X^2_{\eta'} + Y^2_{\eta'} < 1$. The following constraints on $X_{\eta'}$ and $Y_{\eta'}$ can be obtained in a nearly model-independent way [25, 28, 39]:

$$
\frac{\Gamma(\eta' \rightarrow \rho\gamma)}{\Gamma(\omega \rightarrow \pi^0\gamma)} \simeq 3 \left(\frac{m^2_{\eta'} - m^2_\rho}{m^2_\omega - m^2_\pi} \frac{m_\omega}{m_{\eta'}} \right)^3 X^2_{\eta'} \qquad (4)
$$

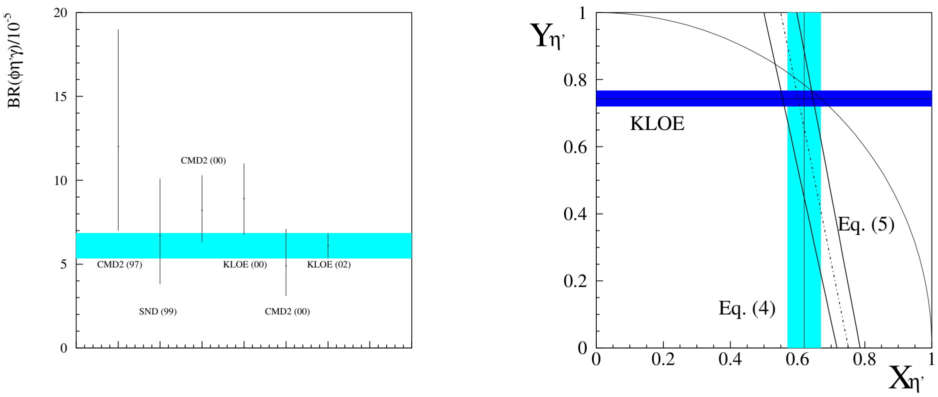

Figure 4: Left: Determinations of the BR($\phi \to \eta' \gamma$) in the literature: CMD2[36, 37, 38]; SND[35]; KLOE[34]. Right: Bounds on $X_{\eta'}$ and $Y_{\eta'}$ from SU(3) calculations and experimental branching fractions. The horizontal band is the KLOE result assuming $Z_{\eta'} = 0$.

and

$$\frac{\Gamma(\eta' \to \gamma\gamma)}{\Gamma(\pi^0 \to \gamma\gamma)} = \frac{1}{9}\left(\frac{m_{\eta'}}{m_{\pi^0}}\right)^3 (5X_{\eta'} + \sqrt{2}Y_{\eta'}\frac{f_\pi}{f_s})^2. \tag{5}$$

The consistency of the assumption of $\eta - \eta'$ mixing without gluonium can be checked as follows: if $Z_{\eta'} = 0$, one has $|Y_{\eta'}| = \cos\varphi_P$. This remains a reasonable approximation if the gluonium component is small. In Fig. 4 (right), the allowed bands corresponding to equations (4) and (5) and the KLOE measurement of $\cos\varphi_P$ are plotted in the $X_{\eta'}$, $Y_{\eta'}$ plane, as well as the circumference $X_{\eta'}^2 + Y_{\eta'}^2 = 1$, corresponding to zero gluonium in the η'. $Z_{\eta'}^2$ is seen to be $0.06^{+0.09}_{-0.06}$, which is compatible with zero within 1σ and consistent with a gluonium fraction of less than 15%.

4 Conclusions

The data coming from radiative ϕ decays are fundamental in clarifying the nature of the scalar mesons. The branching ratios for $\phi \to \pi^0\pi^0\gamma$ and $\phi \to \eta\pi^0\gamma$ and the f_0 and a_0 coupling constants have been measured with the best accuracy by the KLOE collaboration and are in agreement with CMD-2 and SND results. There is still much to do in this field and more data are expected from KLOE and from other experiments (D decays etc). In the pseudoscalar sector there is a new, precise measurement of BR($\phi \to \eta' \gamma$) and the η-η' mixing angle.

References

1. E.M. Aitala *et al*, Phys. Rev. Lett. **86**, 765 (2001).
2. S. Teige *et al*, Phys. Rev. D **59**, 12001 (2001).
3. C. Amsler *et al*, Phys. Lett. B **333**, 277 (1994). D.V .Bugg *et al*,Phys. Rev. D **50**, 4412 (1994).
4. R.L. Jaffe, Phys. Rev. D **15**, 267 (1977).
5. J. Weinstein, N. Isgur, Phys. Rev. Lett. **48**, 659 (1982). N.N. Achasov, V.V. Gubin, V.I. Shevchenko, Phys. Rev. D **56** 203 (1997).
6. N.A. Törnqvist, Hep-ph/0204215 (2002). F.E. Close, N.A. Törnqvist, Hep-ph/0204205 (2002).
7. M. Adinolfi *et al*, Phys. Lett. B**538**, 21-26 (2002).
8. M. Adinolfi *et al*, Phys. Lett. B**537**, 21 (2002).
9. S. Guiducci *et al*,in: Proc. of the 2001 Particle Accelerator Conference ,(ed. P. Lucas S. Webber), 353 (2001), (Chicago, Illinois, USA)
10. M.N. Achasov *et al*, Phys. Lett. B **479** 53 (2000).
11. R.R. Akhmetshin *et al*, Phys. Lett. B **462** 380 (1999).
12. M.N. Achasov *et al*, Phys. Lett. B **485** 349 (2000).
13. R.R. Akhmetshin *et al*, Phys. Lett. B **462** 380 (1999).
14. N.N. Achasov and V.N. Ivanchenko, Nucl. Phys. B **315** 465 (1989).
15. N.N. Achasov and V.V. Gubin, Phys. Rev. D **63** 094007 (2001).
16. E.M. Aitala *et al*, Phys. Rev. Lett. **86** 770 (2001).
17. I. Bediaga,*et al* Hep-ex/020839 (2002).
18. A. Gokalp and O. Yilmaz, Phys. Rev. D **64** 053017 (2001).
19. D. Barberis *et al*, Phys. Lett. B **462** 462 (1999). F.E. Close, A. Kirk, Phys. Lett. B **515** 13 (2001).

20. A. Bramon *et al*, Phys.Lett. B**494**, 221 (2000)

21. A. Bramon, R. Escribano, J. L. Lucio M, M. Napsuciale and G. Pancheri, decays into pi0 pi0 gamma," arXiv:hep-ph/0204339.

22. E.Marco et al., Phys.Lett. B470 (1999) 20.

23. C. Becchi and G. Morpurgo Phys. Rev **140**, B687 (1965).

24. F. E. Close "Pseudoscalar mesons at DAΦNE" in "The DAΦNE phyisics handbook" vol. II (ed. L. Maiani, G. Pancheri and N. Paver, Frascati 1992)

25. J. L. Rosner Phys. Rev. D **27**, 1101 (1983)

26. A. Bramon, R. Escribano and M. D. Scadron Phys. Lett. B **403**, 339 (1997)

27. A. Bramon, R. Escribano and M. D. Scadron Eur. Phys J. C **7**, 271 (1999)

28. A. Bramon, R. Escribano and M. D. Scadron Phys. Lett. B **503**, 271 (2001)

29. T. Feldmann, Int. Jou. Mod. Phys. A **15**, 159 (2000)

30. R. Kaiser and H. Leutwyler, hep-ph/9806336

31. R. Escribano and J.M. Frere, Phys. Lett. B **459**, 288 (1999)

32. Th. Feldmann, P. Kroll, and B. Stech, Phys. Rev. D**58**, (1998)

33. F. De Fazio and M. R. Pennington, JHEP 0007:051 (2000)

34. M. Adinolfi *et al*, Phys. Lett. B**541**, 45-51 (2002).

35. M.N. Achasov *et al*, JETP Lett. **69**, 97 (1999)

36. R.R. Akhmetshin *et al*, Phys. Lett. B **415**, 445 (1997).

37. R.R. Akhmetshin *et al.* , Phys. Lett. B **473**, 337 (2000)

38. R.R. Akhmetshin *et al.*, Phys. Lett. B **494**, 26 (2000)

39. E. Kou, Phys. Rev. D **63**, 054027 (2001)

40. The Particle Data Group (D. Groom *et al.*) Eur. Phys. Jou. **C15** (2000)