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THE HADRONIC CONTRIBUTION TO $(g-2)_\mu$

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ABSTRACT

The precise measurement of the muon magnetic anomaly $(g-2)_\mu$ at BNL constitutes a most sensitive probe of the electroweak sector of the Standard Model, provided the contribution from hadronic vacuum polarization is well enough understood. This talk summarizes the development in the evaluation of the leading order hadronic contributions. Significant improvement has been achieved in a series of analyses which is presented historically in three steps: (1), use of τ spectral functions in addition to e^+e^- cross sections, (2), extended use of perturbative QCD and (3), application of QCD sum rule techniques. The uncertainties, in particular concerning the CVC hypothesis used in step (1), and global quark-hadron duality employed in steps (2) and (3) are discussed. No new analysis results are given in these proceedings, but our previous number for the total Standard Model prediction is updated using the new contribution from hadronic light-by-light scattering.

1 Introduction

Precision measurements of electroweak observables provide powerful tests of the Standard Model[1]. In the last 10 years significant progress has been achieved in this direction owing to the accurate and complete results from the LEP, SLC and TEVATRON colliders. These measurements provided for the first time unique information on vacuum polarization effects in weak boson propagators which allowed the mass of the Higgs boson to be significantly bound. At the other end of the energy scale, the muon magnetic moment is measured with a precision such that electroweak physics can be effectively probed, provided all the contributions from the Standard Model are under control. This review will put its emphasis on the lowest order hadronic vacuum polarization which contributes the dominant uncertainty to the theoretical prediction of $(g-2)_\mu$. As a matter of fact the same physics plays an important role in the analysis of high energy neutral current data through the running of the electromagnetic coupling from $q^2 = 0$ to M_Z^2, relevant for limits on the Higgs mass.

2 The Muonic $(g - 2)$

The muon magnetic anomaly receives contributions from all sectors of the Standard Model,

$$a_\mu(\mathrm{SM}) \equiv \left(\frac{g-2}{2}\right)_\mu = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{weak}} + a_\mu^{\mathrm{had}} \,, \tag{1}$$

the dominant diagrams of which are depicted in Fig. 1. The pure QED contribution, $a_\mu^{\mathrm{QED}} = 116584705.7(2.9) \times 10^{-11}$, has been calculated to fourth order which represents a *tour de force*, so far performed by only one group [3]. The fifth order term has been estimated and found to be small [4]. The weak contribution, $a_\mu^{\mathrm{weak}} = 152(4) \times 10^{-11}$, is known to two-loops [5]. Large logarithms of $\ln(M_W/m_f)$ occur, but can be resummed [6], leading to a robust prediction. The contribution from hadrons stems mainly from vacuum polarization and will be covered in the next section. Its absolute size $\simeq 6800(160)$ is such that it must be known to a precision better than 1% if the experiment is to probe the level of the weak part. The first order correction from hadronic vacuum polarization (Fig. 1) cannot be calculated from first principles since most contributions arise from low-mass states, where quark

[1] After PIC'02 took place, the BNL Muon g-2 Collaboration published (at ICHEP'02) a new measurement, which is compatible with their previous value and which has a two times better precision [1]. As a follow up of this result, my collaborators and me have published a detailed and completely revisited evaluation of the hadronic contributions to the muon magnetic anomaly [2], which is however not the subject of these proceedings, according to the chronological order of this summer's events.

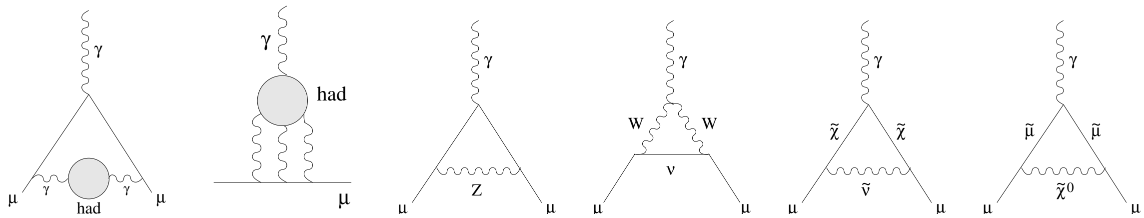

Figure 1: Feynman diagrams corresponding to specific contributions to a_μ: first-order hadronic vacuum polarization, hadronic LBL scattering, first-order weak interaction and possible supersymmetric contributions.

confinement leads to resonances. Fortunately, the result can be expressed as a dispersion integral involving the total cross section for e^+e^- annihilation into hadrons, or alternatively its ratio $R(s)$ to the point-like cross section,

$$a_\mu^{\text{had}} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds\, \frac{K(s)}{s} R(s) \ , \qquad (2)$$

with $K(s) \sim m_\mu/s$ giving a large weight to the small s region. An analog integral occurs for the running of $\alpha(Q^2)$, where $K(s) = (s - Q^2)^{-1}$. A small part ($\sim 1\%$) of the hadronic contribution originates from the so-called LBL scattering (see Fig. 1). These diagrams cannot be treated analogously and must be estimated through specific models for the hadron blob. As a consequence the result is not known accurately and less reliable. Table 2 presents estimates of the three contributions to a_μ in 1995. Obviously, the hadronic piece must be known to better precision before a measurement can witness the effect of the weak interaction or a new physics contribution of similar magnitude, such as Supersymmetry. This is the motivation for an increased effort in the last few years to improve the reliability and the accuracy of the hadronic contribution.

Table 1: The QED, weak and hadronic contributions to a_μ in 1995. The two errors for the hadronic part correspond to vacuum polarization and LBL scattering, respectively.

Source	$10^{11} \times a_\mu$	$10^{11} \times \sigma(a_\mu)$	References
QED	116584705.6	2.9	[3]
Z, W exchange	151	~ 4	[5, 6]
Quarks and Hadrons	~ 6800	$\sim (150 \oplus 40)$	[7]

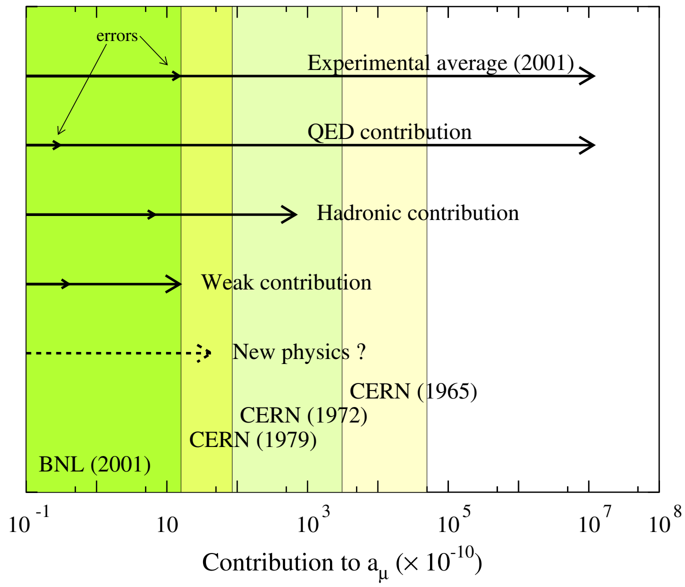

Figure 2: Experimental milestones on the precision of a_μ and the levels of the different contributions and their present uncertainties (depicted by arrows) expected in the Standard Model.

The experimental progress on a_μ is chartered in Fig. 2, together with the levels of the different contributions expected in the Standard Model. While the successive CERN experiments reach enough sensitivity to uncover the expected effect of hadrons, the program underway at BNL (E821) is now at the level of the weak contributions and reaches for a four times smaller sensitivity, thus demanding a corresponding improvement in the accuracy of the hadronic piece.

3 The Precise BNL Result

The new value recently announced by E821 [8] has a precision three times higher than the previous combined CERN and BNL results [9],

$$a_{\mu^+} = 11\ 659\ 202(16)\ \times\ 10^{-10}\ . \tag{3}$$

The quoted uncertainty is dominated by statistics in muon decay counting and the major systematic errors are estimated to 3.5×10^{-10} for the precession frequency and 4.5×10^{-10} for the magnetic field (NMR frequency). The E821 experimenters compare their result to the expected SM value with the hadronic contribution from vacuum polarization taken from Ref. [10],

$$a_{\mu^+}(\mathrm{SM}) = 11\ 659\ 159.6(6.7)\ \times\ 10^{-10}\ . \tag{4}$$

Averaging (3) with previous measurements yields

$$a_\mu(\exp) - a_\mu(\mathrm{SM}) = 43(16)\ \times\ 10^{-10}\ , \tag{5}$$

where the error is dominated by the statistical experimental error (theoretical systematic errors have been added in quadrature). Using the new estimate on the hadronic LBL contribution [11], $a_\mu^{\mathrm{had}}[\mathrm{LBLS}] = (+8.3 \pm 1.2) \times 10^{-10}$ we can rewrite Eq. (4) as

$$a_{\mu^+}(\mathrm{SM}) = 11\ 659\ 176.4(6.4)\ \times\ 10^{-10}\ , \tag{6}$$

so that the 2.6 σ discrepancy[2]. of Eq. (5) reduces to 1.6 σ:

$$a_\mu(\exp) - a_\mu(\mathrm{SM}) = 26(16)\ \times\ 10^{-10}\ , \tag{7}$$

which is not significant.

4 Hadronic Vacuum Polarization for $(g-2)_\mu$ — Improvements in Three Steps

Since 1995 several improvements have been applied to the calculations of hadronic vacuum polarization in order to cope with incomplete or unprecise e^+e^- data. Although QCD predictions were always used at higher energies (> 40 GeV), it became clear that reliable predictions could be made at much lower values. Let me identify the following three steps:

(1) Addition of precise τ data using CVC (see, *e.g.*, [12, 13])

(2) QCD predictions at lower energies (see, *e.g.*, [14, 15, 16, 17])

(3) Constraints from QCD sum rules (see, *e.g.*, [18, 10, 19])

[2] The interpretation of the discrepancy in terms of standard deviations is approximately valid here, since the error is dominated by experimental uncertainties from the a_μ measurement and from the hadronic contribution, where Gaussian Bayesian priors have been used to account for the systematic experimental errors. This is common practise, not to be mixed up with a treatment of theoretical parameters which are not statistically distributed quantities, but whose uncertainties are not of dominance here.

(1) Adding precise τ data under CVC

The Conserved Vector Current (CVC) hypothesis expresses invariance under SU(2) of the electroweak currents. For the problem at hand it relates the isovector vector electromagnetic and the weak hadronic currents, as occurring in e^+e^- annihilation and τ decays. From the point of view of strong interactions this corresponds to a factorization of the hadronic physics: hadrons (quark pairs) are created from the QCD vacuum and the probability to produce hadrons with well-defined quantum numbers at a given mass is expressed through spectral functions. At low energy we expect spectral functions to be dominated by resonances, while QCD should provide a good description at sufficiently high energies. The corresponding energy scale must be determined from experiment. The $I = 1$ vector spectral function $v(s)$ for the two-pion channel is related to the corresponding e^+e^- cross section and τ branching ratio and invariant mass spectrum:

$$v_{\pi^+\pi^-}(s) = \frac{s}{4\pi\alpha^2}\,\sigma(e^+e^- \longrightarrow \pi^+\pi^-)\ , \tag{8}$$

$$v_{\pi^\pm\pi^0}(s) \propto \frac{B_{\pi^\pm\pi^0}}{B_e}\frac{1}{N_{\pi^\pm\pi^0}}\frac{dN_{\pi^\pm\pi^0}}{ds}\frac{m_\tau^2}{(1-s/m_\tau^2)^2(1+2s/m_\tau^2)}\ . \tag{9}$$

Hadronic τ decays represent a clean environment to study hadron dynamics which is in many ways complementary to e^+e^- annihilation:

- τ data have excellent absolute normalization, because the relevant branching ratios have been measured at LEP with high statistics, large acceptance and small non-τ background. On the other hand, the shape of the spectral functions is subject to bin-to-bin corrections from resolution and acceptance effects, which requires to apply an unfolding procedure.

- e^+e^- data have just about the opposite behavior: the point-to-point normalization is excellent since systematic uncertainties are strongly correlated among the measurements. However, the overall normalization is a delicate issue, because of radiative corrections and systematic errors from acceptance and luminosity.

The vector and axial-vector spectral functions have been measured at LEP by ALEPH [20] and OPAL [21]. Detailed QCD studies have been performed by both collaborations.

SU(2) breaking

If the τ data is to be used in the vacuum polarization calculations, that is we identify $v_{\pi^\pm\pi^0}(s)$ with $v_{\pi^+\pi^-}(s)$, it is mandatory to consider in detail the amount of CVC violation [12, 22, 23]. Isospin breaking is expected mainly from electromagnetic effects and it has to be corrected for the calculation of the integral (2). The dominant contribution stems from short distance electroweak radiative corrections to the effective four-fermion coupling $\tau^- \to (d\bar{u})^- \nu_\tau$. It can be absorbed into an overall multiplicative electroweak correction $S_{\rm EW} = 1.0194$ [24], while remaining perturbative electroweak corrections are of order $\alpha^n(m_\tau)\ln^n(M_{\rm Z}/m_\tau)\ 0.3^n$ which is safe to ignore. Sub-leading non-logarithmic short distance corrections have been calculated and found to be small [24]. The electromagnetic $\pi^\pm - \pi^0$ mass splitting affects the measured cross section through phase space corrections. Electromagnetic corrections also affect the pion form factor, in particular the width of the ρ resonance(s): the $\rho - \omega$ mixing, not present in τ decays; the $\pi^\pm - \pi^0$ and $\rho^\pm - \rho^0$ mass splitting; electromagnetic decays. The occurrence of second class currents is expected to be proportional to the mass splitting-squared of the light u, d quarks which is negligible. We observe that most of the effects cancel, so that the net correction applied corresponds to approximately the short-distance radiative correction $S_{\rm EW}$.

The use of τ data improves the precision on the evaluation of $a_\mu^{\rm had}$ by a factor of 1.6 [12].

(2) Replacing poor data by QCD prediction

The data driven analysis [7, 12] shows that in order to improve the precision on the dispersion integral, a more accurate determination of the hadronic cross section between 2 GeV and 10 GeV is needed, where some poorly measured and sparse data points dominate the final error. Indeed, QCD analyses using τ spectral functions [20, 21] revealed the excellent applicability of the Operator Product Expansion (OPE) [25, 26] at the scale of the τ mass, $m_\tau \simeq 1.8$ GeV, and below. The OPE organizes perturbative and nonperturbative contributions to a physical observable by adopting the concept of global quark-hadron duality. Using moments of spectral functions, dimensional nonperturbative operators contributing to the τ hadronic width have been determined experimentally and found to be small. The evolution to lower energy scales proved (to some surprise) the validity of the OPE down to about 1.1 GeV.

An analog analysis based on spectral moments of e^+e^- cross section measurements has been performed in Refs. [15, 27]. The theoretical prediction of these

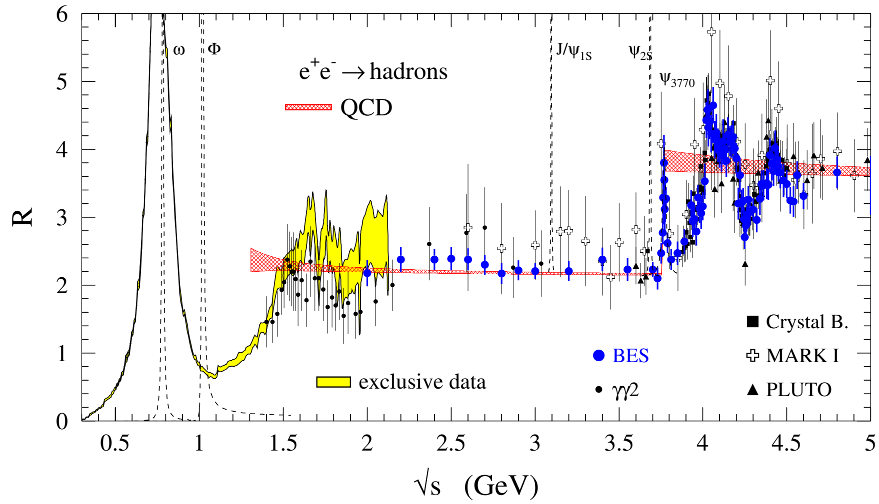

Figure 3: The inclusive hadronic cross section ratio in e^+e^- annihilation versus the c.m. energy $\sqrt{s}$. Shown by the cross-hatched band is the QCD prediction of the continuum contribution. The exclusive e^+e^- cross section measurements at low c.m. energies are taken from DM1, DM2, M2N, M3N, OLYA, CMD, ND and τ data from ALEPH (see [12] for references and more detailed information).

moments and of the total hadronic cross section in e^+e^- annihilation, $R(s_0)$, at a given energy-squared, s_0, involves the Adler D-function [28]. Massless perturbative QCD predictions of D are available [29] to order $(\alpha_s/\pi)^3$. Moreover, two loop quark mass corrections and higher dimensional non-perturbative contributions are taken into account in the calculations. A large number of theoretical uncertainties has been considered. Unknown nonperturbative operators are determined experimentally by means of a combined fit of the theoretical expressions for the moments to data. It results in a very small contribution from the OPE power terms to the lowest moment at the scale of 1.8 GeV, which is in agreement with the findings from the τ analysis. Note that in spite of the implicit assumption of local duality for the theoretical prediction of R, the evaluation of the dispersion integral (2) turns the duality globally, *i.e.*, remaining nonperturbative resonance oscillations are averaged over the integrated energy spectrum.

The available data points together with the theoretical prediction (crossed hatched band) are shown in Fig. 3. Good agreement is found between theory and the newest BES measurements [30], while older data are significantly higher.

The extended application of QCD for R between 1.8 GeV and the $D\bar{D}$ production threshold, as well as from 5 GeV up to infinity [15]. yields a factor of

1.3 improvement on the precision of $a_\mu^{\rm had}$, and a factor of 2.4 better accuracy on $\Delta\alpha_{\rm had}(M_Z^2)$. Similar analyses are reported in Refs. [16, 31].

(3) Improving data with QCD sum rules

It was shown in Refs. [18, 10] that the previous determinations can be further improved by using finite-energy QCD sum rule techniques in order to access theoretically energy regions where perturbative QCD fails locally. In principle, the method uses no additional assumptions beyond those applied in the previous section. The idea is to reduce the data contribution to the dispersion integrals by subtracting analytical functions from the singular integration kernel in Eq. (2), and adding the subtracted part subsequently by using theory only. Two approaches have been applied in Ref. [10]: first, a method based on spectral moments is defined by the identity

$$a^{\rm had}_{\mu,\,[2m_\pi,\,\sqrt{s_0}]} = \int\limits_{4m_\pi^2}^{s_0} ds\, R(s)\left[\frac{\alpha^2 K(s)}{3\pi^2 s} - p_n(s)\right] + \frac{1}{2\pi i}\oint\limits_{|s|=s_0}\frac{ds}{s}\left[P_n(s_0)-P_n(s)\right]D_{uds}(s)\ , \tag{10}$$

with $P_n(s)=\int_0^s dt\, p_n(t)$. The regular functions $p_n(s)$ approximate the kernel $K(s)/s$ in order to reduce the contribution of the non-analytic first integral in Eq. (10), which is evaluated using experimental data. The second integral in Eq. (10) can be calculated theoretically in the framework of the OPE. Another approach [10] involving local quark-hadron duality uses the dispersion relation of the Adler D-function

$$D_f(Q^2) = Q^2\int\limits_{4m_f^2}^{\infty} ds\,\frac{R_f(s)}{(s+Q^2)^2}\ , \tag{11}$$

for space-like $Q^2=-q^2$ and quark flavors f, to approximate the integration kernel. The theoretical errors of both approaches are evaluated in close analogy to the QCD analysis presented in the previous Section. The improvement in accuracy on the dispersion integrals obtained from these constraints is weak for $a_\mu^{\rm had}$ but significant for $\Delta\alpha_{\rm had}(M_Z^2)$.

5 Results

Table 2 shows the experimental and theoretical evaluations of $a_\mu^{\rm had}$ for the different energy regions. We obtain the final results

$$\begin{aligned} a_\mu^{\rm had}[(\alpha/\pi)^2] &= (692.4 \pm 5.6_{\rm exp} \pm 2.6_{\rm theo})\times 10^{-10}\ , \\ a_\mu^{\rm SM} &= (11\,659\,176.4 \pm 5.6_{\rm exp} \pm 3.0_{\rm theo})\times 10^{-10}\ , \end{aligned}$$

Table 2: Contributions to $a_\mu^{\rm had}$ from the different energy regions. The subscripts in the first column give the quark flavors involved in the calculation.

Energy (GeV)	$a_\mu^{\rm had} \times 10^{10}$
$(2m_\pi - 1.8)_{uds}$	$634.3 \pm 5.6_{\rm exp} \pm 2.1_{\rm theo}^{(*)}$
$(1.8 - 3.700)_{uds}$	$33.87 \pm 0.46_{\rm theo}$
$\psi(1S, 2S, 3770)_c + (3.7 - 5)_{udsc}$	$14.31 \pm 0.50_{\rm exp} \pm 0.21_{\rm theo}$
$(5 - 9.3)_{udsc}$	$6.87 \pm 0.11_{\rm theo}$
$(9.3 - 12)_{udscb}$	$1.21 \pm 0.05_{\rm theo}$
$(12 - \infty)_{udscb}$	$1.80 \pm 0.01_{\rm theo}$
$(2m_t - \infty)_t$	≈ 0
$(2m_\pi - \infty)_{udscbt}$	$692.4 \pm 5.6_{\rm exp} \pm 2.6_{\rm theo}$

* The theoretical error accounts for uncertainties concerning the QCD prediction only. Due to the correlated average procedure applied in Ref. [12], uncertainties from CVC and radiative corrections are folded into the systematic part of the experimental error.

dominated by the contribution from the $\rho(770)$ resonance. The total $a_\mu^{\rm SM}$ value contains the contributions from non-leading order hadronic vacuum polarization [32, 12] $a_\mu^{\rm had}[(\alpha/\pi)^3] = (-10.0 \pm 0.6) \times 10^{-10}$, and from hadronic LBL scattering for which the new result from Ref. [11] is used.

6 Conclusions and Perspectives

Much effort has been undertaken during the last years to improve the theoretical predictions on $a_\mu^{\rm had}$. To maintain the sensitivity on interesting physics of the experimental improvements to be expected from BNL, more theoretical work is needed. In particular, a better precision on $a_\mu^{\rm had}$ requires further studies of the following items.

- Radiative corrections in e^+e^- annihilation data
- SU(2) breaking: let me recall that the τ data not only provide precise and in many ways complementary cross section measurements, but they also constitute a powerful cross check. The current $a_\mu^{\rm had}$ evaluation being wrong would require not only the e^+e^- data to have unaccounted systematics, but also that CVC violation is much larger than expected, since the e^+e^- and τ data are mutually (fairly) compatible.

- More experimental information. In particular, complementary e^+e^- measurements from, *e.g.*, new precision experiments, or analyses of radiative events using data from existing e^+e^- factories.

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