The Soft-Collinear Effective Field Theory

Iain W. Stewart
INT, Department of Physics
University of Washington
Box 351550
Seattle, WA 98195

1 Introduction

Understanding the dependence of results on non-perturbative contributions is important for extracting interesting results out of both high and low energy experminents. In general, to understand hadronic uncertainties we need a separation of short $p \sim Q$ and long $p \sim \Lambda_{\rm QCD}$ distance fluctuations. For processes in QCD with momentum transfers $Q \gg \Lambda_{\rm QCD}$, the short distance part is calculable in terms of Wilson coefficients or hard scattering functions. The long distance contributions can be arranged into universal non-perturbative matrix elements which can be extracted from data or calculated on the lattice. This process of separating short and long distance fluctuations is sometimes referred to as factorization.

In B physics a proper understanding of hadronic uncertainties is crucial since the size of typical expansion parameters $\Lambda_{\rm QCD}/m_b$, $\Lambda_{\rm QCD}/m_c$, or $\sqrt{\Lambda_{\rm QCD}/m_b}$ leaves room for power corrections to play a non-negligible role. For many processes Heavy Quark Effective Theory (HQET) provides the conceptual framework for quantifying the factorization between the scales $m_{b,c}$ and $\Lambda_{\rm QCD}$. A well known example is the extraction of the CKM matrix element $|V_{cb}|$ from knowledge about the form factors in $B \to D^*\ell\nu$ decays. At zero recoil the leading result is fixed by Heavy Quark Symmetry [2], and the first power corrections vanish [3]. More recently important progress has been made at reducing the dominant model dependence by computing the matrix elements of $1/m_Q^2$ operators on the Lattice [4]. For inclusive $B \to X_c\ell\nu$ and $B \to X_u\ell\nu$ decays the non-perturbative HQET matrix elements $\overline{\Lambda}$ and λ_1 are actively being extracted from experimental data [5].

Since the B is so heavy, many of its decays produce energetic light hadrons. For these decays the energy of the hadron E_H in the B rest frame is an additional perturbative scale, and HQET alone does not separate the perturbative and non-perturbative information. Examples of such processes include the decays $B \to D\pi$, $B \to \pi\pi$, $B \to K\pi$, the large recoil region in $B \to \pi\ell\nu$, $B \to \rho\ell\nu$, $B \to K^*\gamma$, and $B \to K\ell^+\ell^-$, and the endpoint spectra of the inclusive decays $B \to X_u\ell\nu$ and

Type	Momenta $(+,-,\perp)$	Field Scaling	Operators
collinear	$p^\mu \sim (\lambda^2, 1, \lambda)$	$\xi_{n,p} \sim \lambda$ $(A^+_{n,p},\, A^-_{n,p},\, A^\perp_{n,p}) \sim (\lambda^2,1,\lambda)$	$\overline{\mathcal{P}},\, W_n \sim \lambda^0$ $\mathcal{P}^\mu_\perp \sim \lambda$
soft	$p^\mu \sim (\lambda, \lambda, \lambda)$	$q_{s,p} \sim \lambda^{3/2}$ $A^\mu_{s,p} \sim \lambda$	$S_n \sim \lambda^0$ $\mathcal{P}^\mu \sim \lambda$
usoft	$k^\mu \sim (\lambda^2, \lambda^2, \lambda^2)$	$q_{us} \sim \lambda^3$ $A^\mu_{us} \sim \lambda^2$	$Y_n \sim \lambda^0$

Table 1: Power counting for SCET momenta and fields as well as momentum label operators $(\overline{\mathcal{P}}, \mathcal{P}^\mu_\perp, \mathcal{P}^\mu)$ and collinear and soft Wilson lines induced by integrating out offshell fluctuations $(W,\, S_n)$ and the usoft Wilson line Y_n induced by a collinear field redefinition as described in the text.

$B \to X_s\gamma$. The nature of factorization in these decays shares features in common with many exclusive and inclusive hard scattering processes. Examples are $\gamma^*\gamma \to \pi^0$ at large q^2 and the $x \sim 1$ endpoint region of deep inelastic scattering.

In this talk I discuss an effective field theory that has been developed for processes with energetic hadrons, which is referred to as the Soft-Collinear Effective Theory (SCET) [6, 7, 8, 9]. SCET can be used for both hard scattering processes [10] and B-physics. This theory makes symmetries relevant in the large energy limit explicit at the level of the Lagrangian and operators (such as the reduction of spin structures, helicity constraints, and collinear gauge invariance). Furthermore, SCET has a transparent power counting in $\lambda = \Lambda_{\rm QCD}/Q$ (or $\lambda = \sqrt{\Lambda_{\rm QCD}/Q}$) so that power corrections can be investigated in a systematic way [11, 12, 13]. This includes processes not amenable to an operator product expansion such as exclusive decays. The renormalization group improvement of operators in the effective theory sums single infrared logs, as well as double Sudakov logarithms when they appear. Finally, SCET allows proofs of factorization theorems to be simplified and carried out in a gauge invariant way.

2 Formalism

The factorization of scales in the effective theory is carried out by describing long distance fluctuations with $p^2 \lesssim Q^2\lambda^2$ using effective theory fields, and those with $p^2 \gg Q^2\lambda^2$ by computable short distance Wilson coefficients. Typical processes require collinear fields and in addition either soft or usoft fields. Examples are $B \to X_s\gamma$ which needs collinear and usoft fields, and $B \to D\pi$ which needs collinear and soft fields. This field content is summarized in Table 1 together with the scaling of the

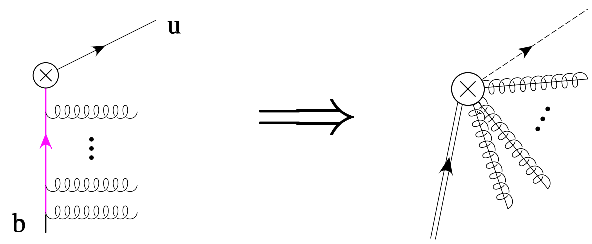

Figure 1: Tree level matching for the leading order heavy-to-light current

momenta and fields with the expansion parameter λ [6, 7]. The momenta scales Q, $Q\lambda$, and $Q\lambda^2$ are separated by making phase redefinitions to pull out the larger momenta, $\phi_n(x) = \sum_p e^{-ip\cdot x}\phi_{n,p}(x)$. Derivatives on the new fields then always pick out the small scale, $\partial^\mu \phi_{n,p}(x) \sim (Q\lambda^2)\phi_{n,p}(x)$, while the large momenta are picked out by introducing label operators, for example $\overline{\mathcal{P}}\,\xi_{n,p} = (\bar{n}{\cdot}p)\,\xi_{n,p}$. Since $\overline{\mathcal{P}} \sim \lambda^0$ in the power counting the hard coefficients $C(\overline{\mathcal{P}})$ are arbitrary functions of this operator [8], which can be determined by matching. More generally we have functions $C(\omega_i)\prod_i \delta(\omega_i - \overline{\mathcal{P}})$ where the delta functions are inserted inside collinear operators in the most general locations allowed by gauge and reparameterization invariance.

Furthermore, there are gluon fields which are order λ^0 in the power counting, namely $\bar{n}\cdot A_{n,q} \sim 1$. Integrating out offshell fluctuations builds up Wilson lines in these fields, such as in the example [7] of matching the full theory heavy-to-light current $\bar{u}\Gamma b$ onto the SCET current, $J_0 = C(\overline{\mathcal{P}})\,\bar{\xi}_{n,p}W\Gamma h_v$. In fact the $\bar{n}\cdot A_{n,q}$ field can be traded for the Wilson line

$$W = \Big[\sum_{\text{perms}} \exp\Big(-\frac{g}{\overline{\mathcal{P}}}\,\bar{n}\cdot A_{n,q}(x)\Big)\Big], \tag{1}$$

since the covariant derivative $i\bar{n}\cdot D_c = \overline{\mathcal{P}} + g\bar{n}\cdot A_{n,q} = W\,\overline{\mathcal{P}}\,W^\dagger$. Soft Wilson lines $S_n[n\cdot A_s]$ are also built up by integrating out offshell fluctuations [9]. Beyond tree level the structure of operators containing factors of W or S_n is protected by collinear and soft gauge transformations [8, 9].

In general we have three types of gluon fields, collinear, soft, and usoft. These are the fields associated with gauge transformations $U(x)$ which have support over collinear, soft, and usoft momenta respectively [9]. The usoft fields A^μ_{us} are dynamical quantum fields which due to their slow variation appear as background fields to the soft and collinear quarks and gluons. For a gauge transformation with support over collinear momenta it is convenient to factor out the large momentum components, $U(x) = \sum_R e^{-iR\cdot x}\,\mathcal{U}_R(x)$. The collinear gluon field then transforms as

$$A^\mu_{n,p} \to \mathcal{U}_Q A^\mu_{n,R}\mathcal{U}^\dagger_{Q+R-p} + \frac{1}{g}\,\mathcal{U}_Q[i\mathcal{D}^\mu\mathcal{U}^\dagger_{Q-p}]\,, \tag{2}$$

where $i\mathcal{D}^\mu = n^\mu \overline{\mathcal{P}}/2 + \mathcal{P}_\perp^\mu + \bar{n}^\mu in\cdot D/2$. On the other hand an usoft gauge transformation has $i\partial^\mu U(x) \sim (Q\lambda^2)U(x)$. For the most general collinear gauge transformation with dependence on all x^μ it is not possible to completely separate usoft and collinear components, so for this case the ultrasoft gluon field also transforms. In fact $\mathcal{U}_R(x)$ itself is an usoft gauge transformation since it no longer has a large phase. A simple mnemonic is that gauge transformations that do not impart large momentum to the original field are usoft. Beyond leading order these more general transformations must be considered to correctly constrain operators as pointed out in Ref. [11]. For example, we have

$$A^\mu_{us} \to \mathcal{U}_R A^\mu_{us} \mathcal{U}_R^\dagger + \mathcal{U}_R \frac{i}{g} \partial_\mu \mathcal{U}_R^\dagger \,. \tag{3}$$

The transformation does not induce large momentum in the ultrasoft field because the large momenta in $\mathcal{U}_R$ and $\mathcal{U}_R^\dagger$ cancel. Gauge invariance constrains the form of the Lagrangian at leading and subleading orders. For the explicit form of the leading order gluon and quark actions $\mathcal{L}_c^{(0)}$ we refer the reader to Ref. [9], for higher order terms in the collinear quark action to Refs. [11, 16], and for the mixed collinear-usoft quark action to Ref. [13].

Besides the constraints from gauge invariance on collinear operators there are addition constraints from the way in which Lorentz invariance is realized in the effective theory. The collinear fields are defined by introducing two auxillary light-like vectors, n and $\bar{n}$, such that $n\cdot\bar{n} = 2$. Naively the presence of these vectors breaks Lorentz invariance. However, in practice Lorentz invariance is restored order by order in the power counting by a reparameterization invariance (RPI) [15]. For the collinear theory the study of RPI was initiated in Ref. [11] and generalized to the three most general classes of allowed transformations in Ref. [16]. For Lagrangians and operators with collinear fields, RPI gives non-trivial constraints between the Wilson coefficients of operators at different orders in the power expansion. In general there is no way of deducing these constraints using only the full theory.

Finally, it is worth discussing why the proof of factorization theorems is simplified by using the effective theory. The factorization between hard and collinear fluctuations or soft and collinear fluctuations is simplified by the fact that it takes place at the level of matching onto the effective theory. The resulting structures are constrained by the symmetries of the low energy theory as already discussed. The factorization between collinear and usoft interactions is simplified by the fact that many cancellations occur in a universal way at the level of the effective Lagrangian. For instance, at lowest order the actions for usoft and collinear particles can be factorized by a simple field redefinition on the collinear fields, $\xi_{n,p} = Y_n \xi_{n,p}^{(0)}$ and $A_{n,p} = Y_n A_{n,p}^{(0)} Y_n^\dagger$, where $Y_n = P\exp[ig\int_{-\infty}^x ds\, n\cdot A_{us}(sn)]$. This transformation moves all leading order usoft interactions from the collinear Lagrangian $\mathcal{L}_c^{(0)}$ into the external operators and currents, where cancellations due to the unitarity of the usoft Wilson line, $Y_n^\dagger Y_n = 1$,

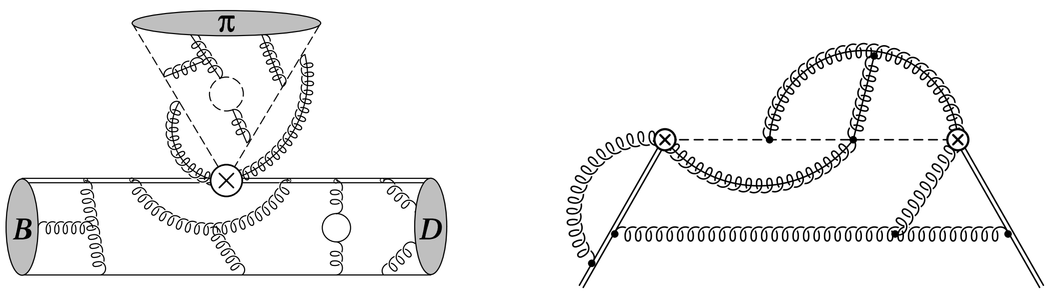

Figure 2: Pictures of how the factorization of interactions occurs in $B \to D\pi$ and $B \to X_s\gamma$.

are more readily seen [9].

3 Results

I briefly discuss two B-physics applications of SCET, the exclusive decay $B \to D\pi$ and the inclusive process $B \to X_s\gamma$.

For the decay $B \to D\pi$ the energy of the outgoing pion in the rest frame of the B is $E_\pi = 2.310\,\mathrm{GeV}$. Since this energy is large it is useful to consider this decay as being in the situation where $Q \gg \Lambda_{\rm QCD}$ with $Q = m_b$, m_c, or E_π. In this limit, SCET has been used to prove the following factorization theorem [14]

$$\langle D^{(*)}\pi|H_w|B\rangle = N F^{B\to D^{(*)}}(0) \int_0^1 d\xi\; T(\xi, Q, \mu)\; \phi_\pi(\xi,\mu) + \dots\,, \tag{4}$$

where the ellipses denote terms that vanish faster than the leading term as $Q \to \infty$. The relevant terms in the electroweak Hamiltonian are $H_W = C_1 O_1 + C_8 O_8$, where $O_1 = \overline{d}_L\gamma^\mu u_L \overline{c}_L\gamma_\mu b_L$ and $O_8 = \overline{d}_L T^A\gamma^\mu u_L \overline{c}_L T^A\gamma_\mu b_L$. Eq. (4) was proposed in Ref. [17], proven to two-loops in Ref. [18], and proven to all orders in α_s in Ref. [14]. The idea behind the proof is shown in Fig. 2. After integrating out offshell fluctuations the leading order interactions involve soft gluons exchanged between quarks in the B and D which build up the $B \to D$ form factor $F^{B\to D}$, and collinear gluons exchanged between quarks in the pion building up the light-cone pion wavefunction $\phi_\pi(x)$. It should be remarked that the effective theory analysis is carried out at an operator level so it does not explicitly rely on perturbation theory.

For the proccess $B \to X_s\gamma$ in the region of photon energies $E_\gamma \gtrsim m_B/2 - \Lambda_{\rm QCD} \simeq 2.2\,\mathrm{GeV}$ the particles in X_s are collimated in a collinear jet with offshellness $p_X^2 \simeq m_B\Lambda_{\rm QCD}$. In this process the B meson is dominated by usoft dynamics and the leading power prediction from factorization is [19]

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dE_\gamma} = H(m_b,\mu)\int_{2E_\gamma - m_b}^{\overline{\Lambda}} dk^+\, S(k^+,\mu)\, J(k^+ + m_b - 2E_\gamma,\mu)\,. \tag{5}$$

The SCET has been used to give a simple direct proof of this result [9]. After the collinear field redefinitions, the usoft interactions rearrange themselves to leave only diagrams such as the one shown in Fig. 2. In Eq. (5) H encodes calculable m_b scale contributions, J encodes $\sqrt{m_b\Lambda_{\rm QCD}}$ scale contributions, and S is the non-perturbative shape function. This factorization formula is required to describe the CLEO data on the photon energy spectrum [20].

To summarize, the soft-collinear effective theory allows factorization proofs to be simplified and formulated at the level of an effective Lagrangian and operators. Essentially the same steps are applied for many exclusive and inclusive processes. Finally, SCET provides us with a new framework for investigating power corrections, and can be used to classify subleading non-perturbative matrix elements.

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