Penguin Pollution in $B_d^0 \to \pi\pi$[1]

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A principal decay mode considered for the measurement of the angle α, is $B_d^0(t) \to \pi^+\pi^-$. Unfortunately, this mode suffers from a well-known problem: penguin contributions may be large [2], and their presence will spoil the clean extraction of α. In the presence of penguin amplitudes, the CP asymmetry in $B_d^0(t) \to \pi^+\pi^-$ does not measure $\sin 2\alpha$, but rather some effective ("polluted") angle $2\alpha_{eff}$. We may write the time-dependent rate of $B_d^0(t) \to \pi^+\pi^-$ as,

$$\Gamma(B^0(t) \to \pi^+\pi^-) = e^{-\Gamma t} B^{+-} \left[1 + a_{dir}^{+-} \cos(\Delta m t) - y \sin 2\alpha_{eff} \sin(\Delta m t)\right] ,$$

$$\text{where,} \quad B^{+-} \equiv \frac{1}{2}\left(|A^{+-}|^2 + |\overline{A}^{+-}|^2\right) , \qquad a_{dir}^{+-} \equiv \frac{|A^{+-}|^2 - |\overline{A}^{+-}|^2}{|A^{+-}|^2 + |\overline{A}^{+-}|^2} , \tag{1}$$

A^{+-} and $\overline{A}^{+-}$ are the amplitudes for $B_d^0 \to \pi^+\pi^-$ and $\overline{B}_d^0 \to \pi^+\pi^-$, respectively, and $y \equiv \sqrt{1-(a_{dir}^{+-})^2}$. Writing the time dependent CP asymmetry as,

$$\mathcal{A} = C_{\pi\pi} \cos(\Delta m t) + S_{\pi\pi} \sin(\Delta m t) , \tag{2}$$

we have, $C_{\pi\pi} = a_{dir}^{+-}$ and $S_{\pi\pi} = -y \sin 2\alpha_{eff}$. In Eq. (1), the coefficient of the $\sin(\Delta M t)$ term, probes the relative phase between the A^{+-} and $e^{-2i\beta}\overline{A}^{+-}$ amplitudes, and this phase, $2\alpha_{eff} = 2\alpha$, in absence of penguin contributions.

The problem of penguin pollution can be eliminated with the help of an isospin analysis [3]. By measuring the rates for $B^+ \to \pi^+\pi^0$ and $B_d^0/\overline{B}_d^0 \to \pi^0\pi^0$, in addition to $B_d^0(t) \to \pi^+\pi^-$, α can again be measured cleanly.

However, the isospin analysis requires separate measurement of $BR(B_d^0 \to \pi^0\pi^0)$ and $BR(\overline{B}_d^0 \to \pi^0\pi^0)$, and therefore suffers from potential practical complications: (i)The branching ratio for $B_d^0 \to \pi^0\pi^0$ is expected to be smaller than $B_d^0 \to \pi^+\pi^-$. (ii)The presence of two π^0's in the final state means that the reconstruction efficiency is smaller. (iii)It will be necessary to tag the decaying B_d^0 or $\overline{B}_d^0$ meson, which further reduces the measurement efficiency. Hence, we may only have, an actual measurement or an upper limit, on the sum of the branching ratios. In this case, a full isospin analysis cannot be carried out.

[1]major part of this talk is based on work done in collaboration with Michael Gronau, David London and Rahul Sinha [1].

Question: assuming that we have, at best, only partial knowledge of the sum, $(BR(B_d^0 \to \pi^0\pi^0) + BR(\overline{B}_d^0 \to \pi^0\pi^0))$, can we at least put bounds on the size of penguin pollution? In the presence of penguin amplitudes, the CP asymmetry in $B_d^0(t) \to \pi^+\pi^-$ measures $\sin 2\alpha_{eff}$. Writing $2\alpha_{eff} = 2\alpha + 2\theta$, where 2θ parametrizes the effect of the penguin contributions, the more precise question: is it possible to constrain θ? As demonstrated by Grossman and Quinn(GQ) [4] and later by Charles [5], the answer to this question is yes. They were able to show that $|2\theta|$ can be bounded even if we have only an upper limit on the sum of $BR(B_d^0 \to \pi^0\pi^0)$ and $BR(\overline{B}_d^0 \to \pi^0\pi^0)$:

$$\cos 2\theta \geq \frac{1 - 2B^{00}/B^{+0}}{y} , \qquad \cos 2\theta \geq \frac{1 - 4B^{00}/B^{+-}}{y} , \tag{3}$$

where, B^{00} and B^{+0} are defined analogous to the definition of B^{+-} in Eq. (1). Next question: does a more stringent bound exist? The answer to this is also yes; the most stringent bound possible on $|2\theta|$ was obtained by Gronau, London, Sinha and Sinha [1], by requiring that the two isospin triangles close and have a common base. We note, however, that neither of the bounds in Eq. (3) involves all three charge-averaged decay rates, B^{+-}, B^{+0} and B^{00}. Thus, a condition for the closure of the two isospin triangles is not included in these bounds.

We now present a geometrical derivation of this new bound on $|2\theta|$. We assume that the charge-averaged rates B^{+-} and B^{+0} have been measured, and that we have (at least) an upper bound on B^{00}. The $B \to \pi\pi$ decay amplitudes take the form

$$\frac{1}{\sqrt{2}} A^{+-} = T e^{i\gamma} + P e^{-i\beta} , \qquad A^{00} = C e^{i\gamma} - P e^{-i\beta} , \qquad A^{+0} = (C+T) e^{i\gamma}, \tag{4}$$

where, the complex amplitudes T, C and P, which are sometimes referred to as "tree", "colour-suppressed" and "penguin" amplitudes, include strong phases. Note that we have implicitly imposed the isospin triangle relation,

$$\frac{1}{\sqrt{2}} A^{+-} + A^{00} = A^{+0} . \tag{5}$$

The $\overline{A}$ amplitudes can be obtained from the A amplitudes by reversing the signs of the weak phases. It is convenient to define the new amplitudes $\tilde{A}^{ij} \equiv e^{2i\gamma}\overline{A}^{ij}$. Then, $\tilde{A}^{-0} = A^{+0}$, so that the A and $\tilde{A}$ triangles have a common base. (A tiny electroweak penguin amplitude, forming a very small angle between A^{+0} and $\tilde{A}^{-0}$, will be neglected here.) In the absence of penguin contributions, $\tilde{A}^{+-} = A^{+-}$, thus, the relative phase 2θ between these two amplitudes is due to penguin pollution. Also, the relative phase between the penguin contributions in $\tilde{A}^{00}$ and A^{00} is $2(\beta+\gamma) \sim 2\alpha$. This information is encoded in Fig. 1. Note that the distance between the points X

and Y is $2\ell \equiv 2|P| \sin\alpha$. Now, $|P|$ can be expressed in terms of observables [5], and we can therefore write,

$$\ell = \frac{1}{2}\sqrt{B^{+-}}\sqrt{1 - y\cos 2\theta} \ . \tag{6}$$

Thus, a constraint on ℓ implies a bound on $\cos 2\theta$.

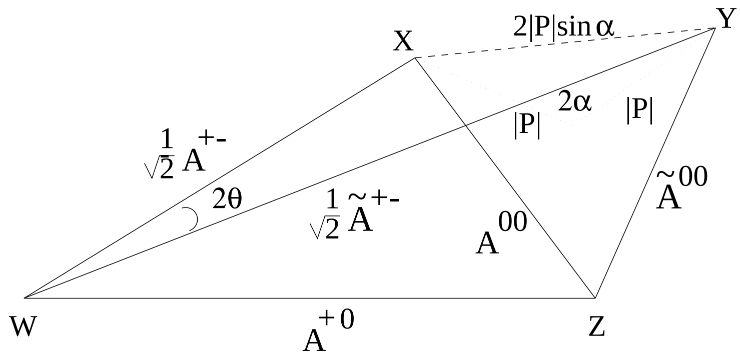

Figure 1: The A and $\tilde{A}$ isospin triangles.

In order to constrain ℓ, we proceed as follows. First, we assign a coordinate system to Fig. 1 such that the origin is at the midpoint of the points X and Y. The points X, Y, W and Z correspond respectively to the coordinates $(+\ell, 0)$, $(-\ell, 0)$, (x_1, y_1) and (x_2, y_2). The goal of the exercise is to find the values of the coordinates (x_1, y_1) and (x_2, y_2). We then note that

$$\begin{array}{ll} B^{+-} = 2(x_1^2 + y_1^2) + 2\ell^2, & B^{+-}a_{dir}^{+-} = -4x_1\ell \ , \\ B^{00} = (x_2^2 + y_2^2) + \ell^2, & B^{+0} = (x_1^2 + y_1^2) + (x_2^2 + y_2^2) - 2x_1x_2 - 2y_1y_2 \ . \end{array} \tag{7}$$

We therefore have four (nonlinear) equations in four unknowns, and we can solve for these coordinates as a function of ℓ. However, we must obtain *only real solutions* for x_2 and y_2, otherwise the triangles do not close. This puts a constraint on ℓ, which in turn, gives the following bound,

$$\cos 2\theta \geq \frac{\left(\frac{1}{2}B^{+-} + B^{+0} - B^{00}\right)^2 - B^{+-}B^{+0}}{B^{+-}B^{+0}y} \ . \tag{8}$$

This is the new lower bound on $\cos 2\theta$ (or upper bound on $|2\theta|$).

The new bound contains the two previous bounds as limiting cases. We can rewrite this lower bound on $\cos 2\theta$ in two alternate forms [1], which involve a sum of two terms. In one form, the first term is simply the GQ bound and in the other, it is the Charles bound. The second terms in both forms are positive definite. Hence, the new bound is stronger than the GQ as well as the Charles bound, and since, all isospin information has been used in obtaining Eq. (8), this is *the most stringent possible bound on* $\cos 2\theta$.

One would also like to know if it is possible to find a lower bound on $|2\theta|$? Unfortunately, the answer is no. This can be seen quite clearly in Fig. 1. Suppose that the two-triangle isospin construction can be made for some nonzero value of 2θ. It is then straightforward to show that one can always rotate A^{+-} and $\tilde{A}^{+-}$ continuously around W towards one another, without changing B^{00}, until they lie on one line corresponding to $\theta = 0$. Thus, without measuring separately $B_d^0 \to \pi^0\pi^0$ and $\overline{B}_d^0 \to \pi^0\pi^0$, one cannot put a lower bound on the penguin pollution parameter.

Using the world average values [7], $BR(B_d^0 \to \pi^+\pi^-) = 5.2 \pm 0.6$ and $BR(B_d^0 \to \pi^+\pi^0) = 4.9 \pm 1.1$ and Babar's value of $BR(B_d^0 \to \pi^0\pi^0) = 0.9^{+0.9+0.8}_{-0.7-0.6}$, our bound yields, $\theta < 57^\circ$ or $\theta > 123^\circ$ at 90% CL, while the GQ bound gives $\theta < 61^\circ$ or $\theta > 119^\circ$ at 90% CL[2]. The Charles bound gives weaker constraints. Note that these values are obtained using zero direct asymmetry, the bounds will be stronger if direct asymmetry is non-vanishing.

The bound on $\cos 2\theta$ in Eq. (8) together with the condition that $\cos 2\theta \leq 1$, leads to a lower limit on B^{00}/B^{+-}. This lower limit, as well as an upper limit on the same quantity, follows directly from the closure of the two isospin triangles, which can be shown to imply that

$$\frac{1}{2} + \frac{B^{+0}}{B^{+-}} - \sqrt{\frac{B^{+0}}{B^{+-}}(1+y)} \leq \frac{B^{00}}{B^{+-}} \leq \frac{1}{2} + \frac{B^{+0}}{B^{+-}} + \sqrt{\frac{B^{+0}}{B^{+-}}(1+y)} \ . \tag{9}$$

The limits are weakest for $y = 1$. Using the central values of the world averages listed above, one finds $0.069 \leq B^{00}/B^{+-} \leq 2.815$, for $a_{dir}^{+-} = 0$; again, a non-zero value of the direct asymmetry will raise the lower limit. This lower limit on B^{00}/B^{+-} is useful, as it will give experimentalists some knowledge of the branching ratios for $B_d^0/\overline{B}_d^0 \to \pi^0\pi^0$, and thus will help to anticipate the feasibility of the full isospin analysis. In addition, since the bound on B^{00}/B^{+-} relies only on the closure of the two triangles, it will hold even in the presence of isospin-violating electroweak-penguin contributions. However, it has been pointed out by Gardner [6] that the triangles will not close in the presence of other isospin-violating effects such as π^0–η, η' mixing. A comparison of the actual branching ratio B^{00} with this bound, may therefore give some information about the size of such isospin-violating effects.

Although no lower limit can be obtained on the penguin-pollution angle $|2\theta|$, we note that a lower bound can be derived for the magnitude of the penguin amplitude P from measurements of $B_d^0(t) \to \pi^+\pi^-$ alone,

$$|P|^2_{min} = \frac{B^{+-}(1-y^2)}{4(1-y\cos 2\alpha_{eff})} \ . \tag{10}$$

[2]We thank Andreas Hoecker and Rainer Bartoldus for help in estimating the numerical values of our bound.

Recently Belle [8] and Babar [9] announced their results for the CP violating asymmetries $C_{\pi\pi}$ and $S_{\pi\pi}$. These asymmetries, may be written [10] in terms of the three parameters $|P/T|$, the strong phase difference of the penguin and tree amplitudes, $\delta = \delta_P - \delta_T$ and the weak phase α. If one assumes a value of $|P/T|$, one can determine α upto discrete ambiguities from the time dependent study of the $B_d^0(t) \to \pi^+\pi^-$ mode alone. There are discrete ambiguities associated with mapping the observables $(S_{\pi\pi}, C_{\pi\pi})$ with the parameters (α, δ). The ambiguities can be resolved by a measurement of $R_{\pi\pi}$, which is the ratio of the flavor-averaged $B_d^0 \to \pi^+\pi^-$ branching ratio to its predicted value due to the tree amplitude alone. If one averages over Belle and Babar asymmetry results, then a larger value of α is favored [10].

Concluding remark: By the end of this summer, with improved statistics ($\approx 100 fb^{-1}$), we hope to have a much clearer understanding about the amount of penguin pollution in $B_d^0 \to \pi^+\pi^-$.

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