D^0-$\overline{D^0}$ mixing

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1 Introduction

One of the most important motivations for studies of $D^0-\overline{D}^0$ mixing is the possibility of observing a signal from new physics which can be separated from the one generated by the Standard Model (SM) interactions. The $D^0 - \overline{D}^0$ mixing proceeds extremely slowly, which in the Standard Model is usually attributed to the absence of superheavy quarks destroying GIM cancelations [1]. The low energy effect of new physics particles can be naturally written in terms of a series of local operators of increasing dimension generating $\Delta C = 2$ transitions. These operators, as well as the one loop Standard Model effects, generate contributions to the effective operators that change D^0 state into $\overline{D}^0$ state leading to the mass eigenstates

$$|D_{\frac{1}{2}}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle, \tag{1}$$

where the complex parameters p and q are obtained from diagonalizing the $D^0 - \overline{D}^0$ mass matrix. The mass and width splittings between these eigenstates are parameterized by

$$x \equiv (m_2 - m_1)/\Gamma, \quad y \equiv (\Gamma_2 - \Gamma_1)/(2\Gamma), \tag{2}$$

where $m_{1,2}$ and $\Gamma_{1,2}$ are the masses and widths of $D_{1,2}$ and the mean width and mass are $\Gamma = (\Gamma_1 + \Gamma_2)/2$ and $m = (m_1 + m_2)/2$. Since y is constructed from the decays of D into physical states, it should be dominated by the Standard Model contributions, unless new physics significantly modifies $\Delta C = 1$ interactions.

Presently, experimental information about the $D^0 - \overline{D}^0$ mixing parameters x and y comes from the time-dependent analyses that can roughly be divided into two categories. First, more traditional studies look at the time dependence of $D \to f$ decays, where f is the final state that can be used to tag the flavor of the decayed meson. The most popular is the non-leptonic doubly Cabibbo suppressed decay (DCSD)

$D^0 \to K^+\pi^-$. Time-dependent studies allow one to separate the DCSD from the mixing contribution $D^0 \to \overline{D}^0 \to K^+\pi^-$,

$$\Gamma[D^0(t) \to K^+\pi^-] = e^{-\Gamma t}|A_{K^-\pi^+}|^2 \times \left[R + \sqrt{R}R_m(y'\cos\phi - x'\sin\phi)\Gamma t + \frac{R_m^2}{4}(y^2+x^2)(\Gamma t)^2\right], \quad (3)$$

where R is the ratio of DCS and Cabibbo favored (CF) decay rates and $q/p = R_m e^{i\phi}$. Since x and y are small, the best constraint comes from the linear terms in t that are also *linear* in x and y. A direct extraction of x and y from Eq. (3) is not possible due to unknown relative strong phase δ of DCS and CF amplitudes [2], as $x' = x\cos\delta + y\sin\delta$, $y' = y\cos\delta - x\sin\delta$. This phase can be measured independently [3]. The corresponding formula can also be written for $\overline{D}^0$ decay with $x' \to -x'$ and $R_m \to R_m^{-1}$ [4].

Second, D^0 mixing can be measured by comparing the lifetimes extracted from the analysis of D decays into the CP-even and CP-odd final states. This study is also sensitive to a *linear* function of y via

$$\frac{\tau(D \to K^-\pi^+)}{\tau(D \to K^+K^-)} - 1 = y\cos\phi - x\sin\phi\left[\frac{R_m^2-1}{2}\right]. \quad (4)$$

Time-integrated studies of the semileptonic transitions are sensitive to the *quadratic* form x^2+y^2 and at the moment are not competitive with the analyses discussed above. The construction of a new tau-charm factory at Cornell will introduce other time-independent methods that are sensitive to a linear function of y [5].

The current experimental upper bounds on x and y are on the order of a few times 10^{-2}, and are expected to improve significantly in the coming years. To regard a future discovery of nonzero x or y as a signal for new physics, we would need high confidence that the Standard Model predictions lie well below the present limits. As was recently shown [6], in the Standard Model x and y are generated only at second order in $SU(3)$ breaking,

$$x\,,\, y \sim \sin^2\theta_C \times [SU(3)\ \text{breaking}]^2\,, \quad (5)$$

where θ_C is the Cabibbo angle. Therefore, predicting the Standard Model values of x and y depends crucially on estimating the size of $SU(3)$ breaking. Although y is expected to be determined by Standard Model processes, its value nevertheless affects significantly the sensitivity to new physics of experimental analyses of D mixing [4].

2 Theoretical Expectations

Theoretical predictions of x and y within and beyond the Standard Model span several orders of magnitude (see Ref. [7]). Roughly, there are two approaches, neither of which give very reliable results because m_c is in some sense intermediate between heavy and light. The "inclusive" approach is based on the operator product expansion (OPE). In the $m_c \gg \Lambda$ limit, where Λ is a scale characteristic of the strong interactions, ΔM and $\Delta\Gamma$ can be expanded in terms of matrix elements of local operators [8]. Such calculations yield $x, y < 10^{-3}$. The use of the OPE relies on local quark-hadron duality, and on Λ/m_c being small enough to allow a truncation of the series after the first few terms. The charm mass may not be large enough for these to be good approximations, especially for nonleptonic D decays. An observation of y of order 10^{-2} could be ascribed to a breakdown of the OPE or of duality, but such a large value of y is certainly not a generic prediction of OPE analyses. The "exclusive" approach sums over intermediate hadronic states, which may be modeled or fit to experimental data [9]. Since there are cancellations between states within a given $SU(3)$ multiplet, one needs to know the contribution of each state with high precision. However, the D is not light enough that its decays are dominated by a few final states. In the absence of sufficiently precise data on many decay rates and on strong phases, one is forced to use some assumptions. While most studies find $x, y < 10^{-3}$, Refs. [9] obtain x and y at the 10^{-2} level by arguing that $SU(3)$ violation is of order unity, but the source of the large $SU(3)$ breaking is not made explicit.

In what follows we first prove that $D^0 - \overline{D}^0$ mixing arises only at *second* order in $SU(3)$ breaking effects. The proof is valid when $SU(3)$ violation enters perturbatively. This would not be so, for example, if D transitions were dominated by a single narrow resonance close to threshold [6, 10]. Then we argue that reorganization of "exclusive" calculation by explicitly building $SU(3)$ cancellations into the analysis naturally leads to values of $y \sim 1\%$ if only one source of $SU(3)$ breaking (phase space) is taken into account.

The quantities M_{12} and Γ_{12} which determine x and y depend on matrix elements $\langle \overline{D}^0 | \mathcal{H}_w \mathcal{H}_w | D^0 \rangle$, where $\mathcal{H}_w$ denote the $\Delta C = -1$ part of the weak Hamiltonian. Let D be the field operator that creates a D^0 meson and annihilates a $\overline{D}^0$. Then the matrix element, whose $SU(3)$ flavor group theory properties we will study, may be written as

$$\langle 0 | \, D \, \mathcal{H}_w \mathcal{H}_w \, D \, | 0 \rangle \, . \tag{6}$$

Since the operator D is of the form $\bar{c}u$, it transforms in the fundamental representation of $SU(3)$, which we will represent with a lower index, D_i. We use a convention in which the correspondence between matrix indices and quark flavors is $(1, 2, 3) = (u, d, s)$. The only nonzero element of D_i is $D_1 = 1$. The $\Delta C = -1$ part of the weak Hamiltonian has the flavor structure $(\overline{q}_i c)(\overline{q}_j q_k)$, so its matrix representation is written with a fundamental index and two antifundamentals, H^{ij}_k. This operator is a sum of irreps contained in the product $3 \times \overline{3} \times \overline{3} = \overline{15} + 6 + \overline{3} + \overline{3}$. In the limit in which the third generation is neglected, H^{ij}_k is traceless, so only the $\overline{15}$ and 6 representations appear. That is, the $\Delta C=-1$ part of $\mathcal{H}_w$ may be decomposed as $\frac{1}{2}(\mathcal{O}_{\overline{15}}+\mathcal{O}_6)$, where

$$\begin{aligned}\mathcal{O}_{\overline{15}} &= (\bar{s}c)(\bar{u}d)+(\bar{u}c)(\bar{s}d)+s_1(\bar{d}c)(\bar{u}d)+s_1(\bar{u}c)(\bar{d}d)\\ &\quad - s_1(\bar{s}c)(\bar{u}s)-s_1(\bar{u}c)(\bar{s}s)-s_1^2(\bar{d}c)(\bar{u}s)-s_1^2(\bar{u}c)(\bar{d}s)\,,\\ \mathcal{O}_6 &= (\bar{s}c)(\bar{u}d)-(\bar{u}c)(\bar{s}d)+s_1(\bar{d}c)(\bar{u}d)-s_1(\bar{u}c)(\bar{d}d)\\ &\quad - s_1(\bar{s}c)(\bar{u}s)+s_1(\bar{u}c)(\bar{s}s)-s_1^2(\bar{d}c)(\bar{u}s)+s_1^2(\bar{u}c)(\bar{d}s)\,,\end{aligned} \tag{7}$$

and $s_1=\sin\theta_C$. The matrix representations $H(\overline{15})^{ij}_k$ and $H(6)^{ij}_k$ have nonzero elements

$$\begin{array}{lll} H(\overline{15})^{ij}_k: & H^{13}_2=H^{31}_2=1\,, & H^{12}_2=H^{21}_2=s_1\,,\\ & H^{13}_3=H^{31}_3=-s_1\,, & H^{12}_3=H^{21}_3=-s_1^2\,,\\ H(6)^{ij}_k: & H^{13}_2=-H^{31}_2=1\,, & H^{12}_2=-H^{21}_2=s_1\,,\\ & H^{13}_3=-H^{31}_3=-s_1\,, & H^{12}_3=-H^{21}_3=-s_1^2\,. \end{array} \tag{8}$$

We introduce $SU(3)$ breaking through the quark mass operator $\mathcal{M}$, whose matrix representation is $M^i_j=\mathrm{diag}(m_u,m_d,m_s)$ as being in the adjoint representation to induce $SU(3)$ violating effects. We set $m_u=m_d=0$ and let $m_s\neq 0$ be the only $SU(3)$ violating parameter. All nonzero matrix elements built out of D_i, H^{ij}_k and M^i_j must be $SU(3)$ singlets.

We now prove that $D^0-\overline{D}^0$ mixing arises only at second order in $SU(3)$ violation, by which we mean second order in m_s. First, we note that the pair of D operators is symmetric, and so the product D_iD_j transforms as a 6 under $SU(3)$. Second, the pair of $\mathcal{H}_w$'s is also symmetric, and the product $H^{ij}_kH^{lm}_n$ is in one of the reps which appears in the product

$$\begin{aligned}\left[(\overline{15}+6)\times(\overline{15}+6)\right]_S &= (\overline{15}\times\overline{15})_S+(\overline{15}\times 6)+(6\times 6)_S \\ &= (\overline{60}+\overline{24}+15+15'+\overline{6})+(42+24+15+\overline{6}+3)+(15'+\overline{6})\,.\end{aligned} \tag{9}$$

A direct computation shows that only three of these representations actually appear in the decomposition of $\mathcal{H}_w\mathcal{H}_w$. They are the $\overline{60}$, the 42, and the $15'$ (actually twice, but with the same nonzero elements both times). So we have product operators of the form (the subscript denotes the representation of $SU(3)$)

$$DD=\mathcal{D}_6\,,\qquad \mathcal{H}_w\mathcal{H}_w=\mathcal{O}_{\overline{60}}+\mathcal{O}_{42}+\mathcal{O}_{15'}\,. \tag{10}$$

Since there is no $\overline{6}$ in the decomposition of $\mathcal{H}_w\mathcal{H}_w$, there is no $SU(3)$ singlet which can be made with $\mathcal{D}_6$, and no $SU(3)$ invariant matrix element of the form (6) can be formed. This is the well known result that $D^0-\overline{D}^0$ mixing is *prohibited by* $SU(3)$ *symmetry.* Now consider a single insertion of the $SU(3)$ violating spurion $\mathcal{M}$. The combination $\mathcal{D}_6\mathcal{M}$ transforms as $6\times 8=24+\overline{15}+6+\overline{3}$. There is still no invariant to be made with $\mathcal{H}_w\mathcal{H}_w$, thus $D^0-\overline{D}^0$ mixing is *not induced at first order in* $SU(3)$

breaking. With two insertions of $\mathcal{M}$, it becomes possible to make an $SU(3)$ invariant. The decomposition of $\mathcal{DMM}$ is

$$6 \times (8 \times 8)_S = 6 \times (27 + 8 + 1) = (60 + \overline{42} + 24 + \overline{15} + \overline{15}' + 6) + (24 + \overline{15} + 6 + \overline{3}) + 6\,. \quad (11)$$

There are three elements of the 6×27 part which can give invariants with $\mathcal{H}_w \mathcal{H}_w$. Each invariant yields a contribution to $D^0 - \overline{D}^0$ mixing proportional to $s_1^2 m_s^2$. Thus, $D^0 - \overline{D}^0$ mixing arises only at *second order* in the $SU(3)$ violating parameter m_s.

We now turn to the contributions to y from on-shell final states, which result from every common decay product of D^0 and $\overline{D}^0$. In the $SU(3)$ limit, these contributions cancel when one sums over complete $SU(3)$ multiplets in the final state. The cancellations depend on $SU(3)$ symmetry both in the decay matrix elements and in the final state phase space. While there are $SU(3)$ violating corrections to both of these, it is difficult to compute the $SU(3)$ violation in the matrix elements in a model independent manner. Yet, with some mild assumptions about the momentum dependence of the matrix elements, the $SU(3)$ violation in the phase space depends only on the final particle masses and can be computed. We estimate the contributions to y solely from $SU(3)$ violation in the phase space. We find that this source of $SU(3)$ violation can generate y of the order of a few percent.

The mixing parameter y may be written in terms of the matrix elements for common final states for D^0 and $\overline{D}^0$ decays,

$$y = \frac{1}{\Gamma} \sum_n \int [\text{P.S.}]_n \, \langle \overline{D}^0 | \, \mathcal{H}_w \, | n \rangle \langle n | \, \mathcal{H}_w \, | D^0 \rangle \,, \quad (12)$$

where the sum is over distinct final states n and the integral is over the phase space for state n. Let us now perform the phase space integrals and restrict the sum to final states F which transform within a single $SU(3)$ multiplet R. The result is a contribution to y of the form

$$\frac{1}{\Gamma} \langle \overline{D}^0 | \, \mathcal{H}_w \Big\{ \eta_{CP}(F_R) \sum_{n \in F_R} | n \rangle \rho_n \langle n | \Big\} \mathcal{H}_w \, | D^0 \rangle \,, \quad (13)$$

where ρ_n is the phase space available to the state n, $\eta_{CP} = \pm 1$ [6]. In the $SU(3)$ limit, all the ρ_n are the same for $n \in F_R$, and the quantity in braces above is an $SU(3)$ singlet. Since the ρ_n depend only on the known masses of the particles in the state n, incorporating the true values of ρ_n in the sum is a calculable source of $SU(3)$ breaking.

This method does not lead directly to a calculable contribution to y, because the matrix elements $\langle n | \mathcal{H}_w | D^0 \rangle$ and $\langle \overline{D}^0 | \mathcal{H}_w | n \rangle$ are not known. However, CP symmetry, which in the Standard Model and almost all scenarios of new physics is to an excellent approximation conserved in D decays, relates $\langle \overline{D}^0 | \mathcal{H}_w | n \rangle$ to $\langle D^0 | \mathcal{H}_w | \overline{n} \rangle$. Since $|n\rangle$

and $|\overline{n}\rangle$ are in a common $SU(3)$ multiplet, they are determined by a single effective Hamiltonian. Hence the ratio

$$y_{F,R} = \frac{\sum_{n\in F_R} \langle \overline{D}^0 | \, \mathcal{H}_w | n \rangle \rho_n \langle n | \mathcal{H}_w \, | D^0 \rangle}{\sum_{n\in F_R} \langle D^0 | \, \mathcal{H}_w | n \rangle \rho_n \langle n | \mathcal{H}_w \, | D^0 \rangle} = \frac{\sum_{n\in F_R} \langle \overline{D}^0 | \, \mathcal{H}_w | n \rangle \rho_n \langle n | \mathcal{H}_w \, | D^0 \rangle}{\sum_{n\in F_R} \Gamma(D^0 \to n)} \tag{14}$$

is calculable, and represents the value which y would take if elements of F_R were the only channel open for D^0 decay. To get a true contribution to y, one must scale $y_{F,R}$ to the total branching ratio to all the states in F_R. This is not trivial, since a given physical final state typically decomposes into a sum over more than one multiplet F_R. The numerator of $y_{F,R}$ is of order s_1^2 while the denominator is of order 1, so with large $SU(3)$ breaking in the phase space the natural size of $y_{F,R}$ is 5%. Indeed, there are other $SU(3)$ violating effects, such as in matrix elements and final state interaction phases. Here we assume that there is no cancellation with other sources of $SU(3)$ breaking, or between the various multiplets which occur in D decay, that would reduce our result for y by an order of magnitude. This is equivalent to assuming that the D meson is not heavy enough for duality to enforce such cancellations. Performing the computations of $y_{F,R}$ [6], we see that effects at the level of a few percent are quite generic. Our results are summarized in Table 1. Then, y can be formally constructed from the individual $y_{F,R}$ by weighting them by their D^0 branching ratios,

$$y = \frac{1}{\Gamma} \sum_{F,R} y_{F,R} \Big[\sum_{n\in F_R} \Gamma(D^0 \to n) \Big] \,. \tag{15}$$

However, the data on D decays are neither abundant nor precise enough to disentangle the decays to the various $SU(3)$ multiplets, especially for the three- and four-body final states. Nor have we computed $y_{F,R}$ for all or even most of the available representations. Instead, we can only estimate individual contributions to y by assuming that the representations for which we know $y_{F,R}$ to be typical for final states with a given multiplicity, and then to scale to the total branching ratio to those final states. The total branching ratios of D^0 to two-, three- and four-body final states can be extracted from Ref. [11]. Rounding to the nearest 5% to emphasize the uncertainties in these numbers, we conclude that the branching fractions for PP, $(VV)_{s\text{-wave}}$, $(VV)_{d\text{-wave}}$ and $3P$ approximately amount to 5%, while the branching ratios for PV and $4P$ are of the order of 10% [6].

We observe that there are terms in Eq. (15), like nonresonant $4P$, which could make contributions to y at the level of a percent or larger. There, the rest masses of the final state particles take up most of the available energy, so phase space differences are very important. One can see that y on the order of a few percent is completely natural, and that anything an order of magnitude smaller would require significant cancellations which do not appear naturally in this framework. Cancellations would be expected only if they were enforced by the OPE, or if the charm quark were heavy

Final state representation		$y_{F,R}/s_1^2$	$y_{F,R}$ (%)
PP	8	-0.0038	-0.018
	27	-0.00071	-0.0034
PV	8_A	0.032	0.15
	8_S	0.031	0.15
	10	0.020	0.10
	$\overline{10}$	0.016	0.08
	27	0.04	0.19
$(VV)_{s\text{-wave}}$	8	-0.081	-0.39
	27	-0.061	-0.30
$(VV)_{p\text{-wave}}$	8	-0.10	-0.48
	27	-0.14	-0.70
$(VV)_{d\text{-wave}}$	8	0.51	2.5
	27	0.57	2.8
$(3P)_{s\text{-wave}}$	8	-0.48	-2.3
	27	-0.11	-0.54
$(3P)_{p\text{-wave}}$	8	-1.13	-5.5
	27	-0.07	-0.36
$(3P)_{\text{form-factor}}$	8	-0.44	-2.1
	27	-0.13	-0.64
$4P$	8	3.3	16
	27	2.2	11
	$27'$	1.9	9.2

Table 1: Values of $y_{F,R}$ for some two-, three-, and four-body final states.

enough that the "inclusive" approach were applicable. The hypothesis underlying the present analysis is that this is not the case.

3 Conclusions

We proved that if $SU(3)$ violation may be treated perturbatively, then $D^0 - \overline{D}^0$ mixing in the Standard Model is generated only at second order in $SU(3)$ breaking effects. Within the exclusive approach, we identified an $SU(3)$ breaking effect, $SU(3)$ violation in final state phase space, which can be calculated with minimal model dependence. We found that phase space effects alone provide enough $SU(3)$ violation to induce $y \sim 10^{-2}$. Large effects in y appear for decays close to D threshold, where an analytic expansion in $SU(3)$ violation is no longer possible.

Indeed, some degree of cancellation is possible between different multiplets, as would be expected in the $m_c \to \infty$ limit, or between $SU(3)$ breaking in phase space and in matrix elements. It is not known how effective these cancellations are, and the most reasonable assumption in light of our analysis is that they are not significant enough to result in an order of magnitude suppression of y, as they are not enforced by any symmetry arguments. Therefore, any future discovery of a D meson width difference should not by itself be interpreted as an indication of the breakdown of the Standard Model.

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References

[1] A. Datta, D. Kumbhakar, Z. Phys. C**27**, 515 (1985); A. Petrov, Phys. Rev. D**56**, 1685 (1997).

[2] A. F. Falk, Y. Nir and A. A. Petrov, JHEP **9912**, 019 (1999).

[3] M. Gronau, Y. Grossman and J. L. Rosner, Phys. Lett. B **508**, 37 (2001); J. P. Silva and A. Soffer, Phys. Rev. D **61**, 112001 (2000). E. Golowich and S. Pakvasa, Phys. Lett. B **505**, 94 (2001).

[4] S. Bergmann, Y. Grossman, Z. Ligeti, Y. Nir, A. Petrov, Phys. Lett. B **486**, 418 (2000).

[5] D. Atwood and A. A. Petrov, arXiv:hep-ph/0207165.

[6] A. F. Falk, Y. Grossman, Z. Ligeti and A. A. Petrov, Phys. Rev. D **65**, 054034 (2002).

[7] H. N. Nelson, in *Proc. of the 19th Intl. Symp. on Photon and Lepton Interactions at High Energy LP99* ed. J.A. Jaros and M.E. Peskin, arXiv:hep-ex/9908021.

[8] H. Georgi, Phys. Lett. B297, 353 (1992); T. Ohl, G. Ricciardi and E. Simmons, Nucl. Phys. B403, 605 (1993); I. Bigi and N. Uraltsev, Nucl. Phys. B **592**, 92 (2001), for a recent review see A. A. Petrov, *Proc. of 4th Workshop on Continuous Advances in QCD*, Minneapolis, Minnesota, 12-14 May 2000, arXiv:hep-ph/0009160.

[9] J. Donoghue, E. Golowich, B. Holstein and J. Trampetic, Phys. Rev. D33, 179 (1986); L. Wolfenstein, Phys. Lett. B164, 170 (1985); P. Colangelo, G. Nardulli and N. Paver, Phys. Lett. B242, 71 (1990); T.A. Kaeding, Phys. Lett. B357, 151 (1995).

[10] E. Golowich and A. A. Petrov, Phys. Lett. B **427**, 172 (1998).

[11] D. E. Groom *et al.* [Particle Data Group Collaboration], Eur. Phys. J. C **15**, 1 (2000).