Measuring α using $B \to K^{(*)}\overline{K}^{(*)}$ Decays[1]

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The *raison d'être* for measuring CP violation in the B system is to test the standard model (SM) [1]. As always, the hope is that we will find evidence for the presence of new physics.

There are many signals of new physics, but only a handful are of the "smoking-gun" variety, i.e. they are free of hadronic uncertainties, and hence independent of theoretical input. All of these rely on the measurement of CP-violating rate asymmetries in B decays. They include

- $B_d^0(t) \to \Psi K_S$ vs. $B_d^0(t) \to \phi K_S$. Both of these decay modes probe the CP phase β within the SM.
- $B^\pm \to DK^\pm$ vs. $B_s^0(t) \to D_s^\pm K^\mp$. Similarly, both of these modes can be used to measure γ.
- $B_s^0(t) \to \Psi\phi$. The CP asymmetry for this decay is expected to vanish within the SM (to a good approximation).

In all cases, any deviation from the SM predictions indicates the presence of new physics.

However, these signals have something in common: they are all sensitive to new physics in the $b \to s$ flavour-changing neutral current (FCNC). In the first case, the new physics enters in the $b \to s$ penguin amplitude, while in the last two cases, it affects B_s^0–$\overline{B}_s^0$ mixing.

This then begs the question: are there clean probes of new physics in the $b \to d$ FCNC? However, the answer to this is *no*.

To see this, consider the $b \to d$ penguin amplitude, which can be written as

$$A = P_u \, V_{ub}^* V_{ud} + P_c \, V_{cb}^* V_{cd} + P_t \, V_{tb}^* V_{td} \ , \tag{1}$$

where the P_i ($i = u, c, t$) represent the contributions from the internal i-quark. Due to the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, any one CKM combination can be eliminated in terms of the other two. (Note that, unlike the $b \to s$ penguin amplitude, here all three combinations of CKM matrix elements are of the same order.) There are thus three ways of writing this amplitude:

[1] Talk given by David London

1. $A = (P_u - P_c) V_{ub}^* V_{ud} + (P_t - P_c) V_{tb}^* V_{td}$,
2. $A = (P_c - P_u) V_{cb}^* V_{cd} + (P_t - P_u) V_{tb}^* V_{td}$,
3. $A = (P_u - P_t) V_{ub}^* V_{ud} + (P_c - P_t) V_{cb}^* V_{cd}$.

In the first case, the relative weak phase between the two contributions is α, while in the second and third cases, it is β and γ, respectively. Therefore, there is an ambiguity — called the *CKM ambiguity* — in the definition of the relative weak phase in the $b \to d$ penguin amplitude. Because of this, it is impossible to cleanly extract weak-phase information from $b \to d$ penguins. Thus, in order to test for new physics in the $b \to d$ FCNC, we need a theoretical assumption which will break the CKM ambiguity [2]. This fact will be important in what follows.

Consider the pure $b \to d$ penguin decay $B_d^0 \to K^0 \overline{K}^0$. Its amplitude can be written

$$A = P_u \, V_{ub}^* V_{ud} + P_c \, V_{cb}^* V_{cd} + P_t \, V_{tb}^* V_{td} = \mathcal{P}_{uc} \, e^{i\gamma} \, e^{i\delta_{uc}} + \mathcal{P}_{tc} \, e^{-i\beta} \, e^{i\delta_{tc}} \ . \tag{2}$$

Note that the magnitudes of the CKM combinations $V_{ub}^* V_{ud}$ and $V_{tb}^* V_{td}$ have been absorbed into $\mathcal{P}_{uc}$ and $\mathcal{P}_{tc}$. In this parametrization, there are 4 theoretical parameters: $\mathcal{P}_{uc}$, $\mathcal{P}_{tc}$, $\Delta \equiv \delta_{uc} - \delta_{tc}$, and the CP phase α.

Recall that the time-dependent decay rate for a B_d^0 to decay to a final state f can be written as

$$\Gamma(B_d^0(t) \to f) \sim X + Y \cos \Delta m t - Z_I \sin \Delta m t \ , \tag{3}$$

where

$$X \equiv \frac{|A|^2 + |\overline{A}|^2}{2} \ , \quad Y \equiv \frac{|A|^2 - |\overline{A}|^2}{2} \ , \quad Z_I \equiv \mathrm{Im} \left(e^{-2i\beta} A^* \overline{A} \right) \ . \tag{4}$$

What is important here is that there are only 3 experimental observables. However, as noted above, there are 4 theoretical unknown quantities describing the decay $B_d^0 \to K^0 \overline{K}^0$. Thus, as expected, one cannot extract any of these theoretical parameters. However, it is always possible to express three of the unknowns in terms of the fourth. In particular, a little algebra allows us to write

$$\mathcal{P}_{tc}^2 = \frac{Z_R \cos 2\alpha + Z_I \sin 2\alpha - X}{\cos 2\alpha - 1} \ , \tag{5}$$

where $Z_R \equiv \mathrm{Re} \left(e^{-2i\beta} A^* \overline{A} \right)$. (Note that Z_R is not independent: $Z_R^2 = X^2 - Y^2 - Z_I^2$.)

A similar analysis can be applied to the decay $B_d^0 \to K^* \overline{K}^*$, where K^* represents any excited kaon, such as $K^*(892)$, $K_1(1270)$, etc. In this case we can write

$$\mathcal{P}_{tc}'^{\,2} = \frac{Z_R' \cos 2\alpha + Z_I' \sin 2\alpha - X'}{\cos 2\alpha - 1} \ , \tag{6}$$

where the primed observables correspond to the decay $B_d^0 \to K^* \overline{K}^*$. By combining Eqs. (5) and (6), we obtain

$$\frac{\mathcal{P}_{tc}^2}{\mathcal{P}_{tc}'^{\,2}} = \frac{Z_I \sin 2\alpha + Z_R \cos 2\alpha - X}{Z_I' \sin 2\alpha + Z_R' \cos 2\alpha - X'} ~. \qquad (7)$$

Note that the CKM information in $\mathcal{P}_{tc}$ and $\mathcal{P}_{tc}'$ (the magnitudes of $V_{ub}^* V_{ud}$ and $V_{tb}^* V_{td}$) cancels in the ratio. The key point here is the following: *if we knew the value of $\mathcal{P}_{tc}^2 / \mathcal{P}_{tc}'^{\,2}$, we could extract α.*

We now turn to the analogous processes in the B_s^0 system. Consider $B_s^0 \to K^0 \overline{K}^0$, which is a pure $b \to s$ penguin decay. Its amplitude can be written

$$A^{(s)} = P_u^{(s)} \, V_{ub}^* V_{us} + P_c^{(s)} \, V_{cb}^* V_{cs} + P_t^{(s)} \, V_{tb}^* V_{ts} = \mathcal{P}_{uc}^{(s)} \, e^{i\gamma} \, e^{i\delta_{uc}^{(s)}} + \mathcal{P}_{tc}^{(s)} \, e^{i\delta_{tc}^{(s)}} ~. \qquad (8)$$

The difference between $b \to s$ penguins and $b \to d$ penguins is that, here, $V_{ub}^* V_{us}$ is $O(\lambda^4)$ while $V_{tb}^* V_{ts}$ is $O(\lambda^2)$. Thus, $\mathcal{P}_{uc}^{(s)}$ is negligible compared to $\mathcal{P}_{tc}^{(s)}$, so that the measurement of $B(B_s^0 \to K^0 \overline{K}^0)$ gives $|\mathcal{P}_{tc}^{(s)}|$. Similarly, the measurement of $B(B_s^0 \to K^* \overline{K}^*)$ gives $|\mathcal{P}_{tc}'^{(s)}|$.

We now make the claim that

$$\frac{{\mathcal{P}_{tc}^{(s)}}^2}{{\mathcal{P}_{tc}'^{(s)}}^2} = \frac{\mathcal{P}_{tc}^2}{\mathcal{P}_{tc}'^{\,2}} ~. \qquad (9)$$

Note that the CKM matrix elements cancel in the ratios, so that this is a relation among hadronic parameters. This theoretical assumption breaks the CKM ambiguity. Thus, by combining this relation with that in Eq. (7), one can extract the CP phase α from the pure penguin decays $B_{d,s}^0 \to K^{(*)} \overline{K}^{(*)}$ [3]. A similar method applies to non-CP-conjugate decays of the form $K^0 \overline{K}^*$, $K^* \overline{K}^0$.

Of course, the precision with which α can be obtained depends on the theoretical uncertainty in the above relation. As we will argue below, the theoretical error is small, at most 5% (and may well be even smaller).

Before discussing the theoretical error, we briefly examine some of the experimental considerations in putting this method to use. The branching ratios for decays dominated by $b \to d$ penguins are expected to be $O(10^{-6})$. Combined with the fact that B_s^0 decays are involved, this suggests that this method is most appropriate for hadron colliders. In addition, since the $K^{(*)}$ and $\overline{K}^{(*)}$ mesons can be detected via their decays to charged π's and K's, no π^0 detection is needed – all that is necessary is good K/π separation. This is very important for most hadron colliders. (Of course, if π^0's can actually be detected, this will improve the prospects for using the method.) Finally, it should be possible to trigger on a final-state K^* at hadron colliders, so that final states such as $K^* \overline{K}^0$, $K^0 \overline{K}^*$ and $K^* \overline{K}^*$ will probably be favoured.

The method does have a potential weakness: α is extracted with a 16-fold discrete ambiguity. However, this is not as serious as it appears at first glance. First, the ambiguity can be reduced to 4-fold by considering two different $K^{(*)}\overline{K}^{(*)}$ final states. Examples of these include $K^0\overline{K}^0$ and $K^*\overline{K}^*$, $K^0\overline{K}^*$ and $K^*\overline{K}^0$, or two different helicity states of $K^*\overline{K}^*$. Second, we expect P_u and P_c in $b \to d$ penguins to be at most 50% of P_t. This implies that $\mathcal{P}_{uc}/\mathcal{P}_{tc} < 0.5$ for all decays. By adding this theoretical constraint, the discrete ambiguity can be reduced to 2-fold: α, $\alpha + \pi$.

There are two other methods on the market for cleanly measuring α: (i) the isospin analysis of $B \to \pi\pi$ decays [4], and (ii) the Dalitz-plot analysis of $B \to \rho\pi$ decays [5]. Both of these methods have their problems. The isospin analysis requires the measurement of $B_d^0 \to \pi^0\pi^0$, whose branching ratio may be quite small. And in the Dalitz-plot analysis, one must understand the continuum background to $B \to \rho\pi$ decays with considerable accuracy, and have a correct description of $\rho \to \pi\pi$ decays. Both of these issues may be difficult to resolve. In light of this, the $B \to K^{(*)}\overline{K}^{(*)}$ method could potentially give us the first reasonably clean measurement of α. However, regardless of which is first, a discrepancy in the values of α obtained from these different methods would point clearly to new physics in the $b \to d$ penguin.

We now turn to a brief examination of the theoretical uncertainty in Eq. (9). We claim the equality of the double ratio of matrix elements:

$$\frac{r_t}{r_t^*} \equiv \frac{\left\langle K^0\overline{K}^0 \right| H_d \left| B_d^0 \right\rangle / \left\langle K^0\overline{K}^0 \right| H_s \left| B_s^0 \right\rangle}{\left\langle K^*\overline{K}^* \right| H_d \left| B_d^0 \right\rangle / \left\langle K^*\overline{K}^* \right| H_s \left| B_s^0 \right\rangle} = 1 \ . \tag{10}$$

The two decays in r_t are related by U-spin (i.e. flavour $SU(3)$ symmetry), and similarly for r_t^*. We can therefore write

$$r_t = \frac{\left\langle K^0\overline{K}^0 \right| H_d \left| B_d^0 \right\rangle}{\left\langle K^0\overline{K}^0 \right| H_s \left| B_s^0 \right\rangle} = 1 + C_{SU(3)} \ , \quad r_t^* = \frac{\left\langle K^*\overline{K}^* \right| H_d \left| B_d^0 \right\rangle}{\left\langle K^*\overline{K}^* \right| H_s \left| B_s^0 \right\rangle} = 1 + C_{SU(3)}^* \ , \tag{11}$$

where $C_{SU(3)}$ and $C_{SU(3)}^*$ are both expected to be $\sim 25\%$ (i.e. the typical size of $SU(3)$-breaking effects). Thus,

$$\frac{r_t}{r_t^*} = 1 + (C_{SU(3)} - C_{SU(3)}^*) \ . \tag{12}$$

Now, apart from $SU(3)$, there is no symmetry limit in which $(C_{SU(3)} - C_{SU(3)}^*) \to 0$, so that one might guess that the $SU(3)$ corrections to r_t/r_t^* are also $\sim 25\%$. However, as we argue below, we expect significant cancellations between $C_{SU(3)}$ and $C_{SU(3)}^*$.

At the quark level, the $SU(3)$ breaking vanishes in the limit $m_b \to \infty$, so that the hamiltonians describing the decays $B_d^0 \to K^0\overline{K}^0$ and $B_s^0 \to K^0\overline{K}^0$ are equal to $O(\Delta M_B/M_B) \simeq 2\%$. Writing

$$\begin{aligned} r_t &= \left\langle K^0\overline{K}^0 \right| H_d \left| B_d^0 \right\rangle / \left\langle K^0\overline{K}^0 \right| H_s \left| B_s^0 \right\rangle \\ &= \left\langle K^0\overline{K}^0 \right| H_d \left| B_d^0 \right\rangle / \left\langle K^0\overline{K}^0 \right| U^\dagger H_d U \left| B_s^0 \right\rangle \ , \end{aligned} \tag{13}$$

we see that there are two main sources of $SU(3)$-breaking corrections: (i) "final-state" corrections, $U\left|K^0\overline{K}^0\right\rangle \neq \left|K^0\overline{K}^0\right\rangle$, and (ii) "initial-state" corrections, $U\left|B_s^0\right\rangle \neq \left|B_d^0\right\rangle$.

The key observation here is that the sources of $SU(3)$ breaking in r_t^* are very similar to those in r_t: $U\left|K^*\overline{K}^*\right\rangle \neq \left|K^*\overline{K}^*\right\rangle$ and $U\left|B_s^0\right\rangle \neq \left|B_d^0\right\rangle$. It is therefore not unreasonable to expect sizeable cancellations between $C_{SU(3)}$ and $C_{SU(3)}^*$ in Eq. (12), and indeed this is what is found in model calculations. Such calculations suggest that both the final-state and initial-state corrections are at the level of 1–2%. Furthermore, this can be tested experimentally. For the final-state corrections, one needs to measure the kaon light-cone distribution at the scale of m_b, while information about the size of initial-state corrections can be obtained from measurements of the $D, D_s \to K, K^*$ form factors. For all the details concerning the size of the theoretical uncertainty, as well as the experimental tests, we refer the reader to Ref. [3].

To summarize, we have presented a new method for obtaining the CP phase α via measurements of $B_{d,s}^0 \to K^{(*)}\overline{K}^{(*)}$ decays. This method is particularly appropriate for hadron colliders since some of the branching ratios are small $[O(10^{-6})]$, and since B_s^0 decays are involved. Furthermore, the final-state particles can be detected via their decays to charged particles only; no π^0 detection is needed. By comparing the value of α extracted from this method with that obtained in $B \to \pi\pi$ or $B \to \rho\pi$ decays, one can detect the presence of new physics in the $b \to d$ penguin amplitude. The method does require theoretical input. However, model calculations suggest that the theoretical uncertainty is at most 5%, and might well be even smaller. Furthermore, these estimates can be tested experimentally. The method is therefore quite clean.

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