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# PQCD approach to exclusive $B$ decays

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## 1 Introduction

The conventional approach to two-body nonleptonic  $B$  meson decays is based on FA [1], in which, however, there exist several serious theoretical drawbacks. For example, FA breaks the scale independence of decay amplitudes. It has been shown that the problem of the scale dependence in FA can be transformed into the one of infrared divergences in the generalized FA [2, 3]. In the PQCD approach [4, 5, 6, 7] the infrared divergences in the corrections to the four-fermion vertices are treated in the presence of the spectator quark [8]. Therefore, the leading-twist  $B$  meson (light meson) wave function  $\phi_B$  ( $\phi_h$ ) can be defined, which absorbs the two-particle reducible infrared divergences on the  $B$  meson (light meson) side. In this treatment the external quarks remain on-shell, and the problem of the scale dependence is resolved without breaking gauge invariance. Following the above reasoning, the  $B \rightarrow hh$  decay amplitudes are written as the convolution,

$$A = \phi_B \otimes H^{(6)} \otimes \phi_{h1} \otimes \phi_{h2} \otimes S, \quad (1)$$

where the six-quark amplitude  $H^{(6)}$  corresponds to the diagrams with a hard gluon emitted from the spectator quark [4, 5, 9], and  $S$  denotes the Sudakov factor.

In PQCD strong phases mainly arise from the annihilation amplitudes, which are almost imaginary [10, 11, 12, 13]. The detailed reason is referred to [14]. The strong phases are large, since they appear at the same order as the factorizable amplitudes. Because of the large imaginary annihilation amplitudes, significant CP asymmetries are expected in the PQCD analyses of two-body nonleptonic  $B$  meson decays, such as  $B \rightarrow K\pi$ ,  $\pi\pi$  [15] and  $B \rightarrow \rho K$ ,  $\omega K$  [16]. The latter modes are especially sensitive to the annihilation contributions. It has been pointed out [17] that contribution from intrinsic charms, one of the higher Fock states of the  $B$  meson bound state, reduces the magnitude but does not flip the sign of the  $B \rightarrow K\pi$  CP asymmetries in the decays. The PQCD predictions with this subleading contribution included is then more consistent with data [18]. It implies that PQCD has caught the correct leading picture of exclusive  $B$  meson decays.

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## 2 Sudakov Effects

In PQCD calculations small parton transverse momenta  $k_T$  are included [19, 20], which smear the end-point singularities from small momentum fractions [21]. The resummation of the resultant double logarithms  $\ln^2(Pb)$ ,  $P$  denotes the dominant light-cone component of a meson momentum, and  $b$  is the variable conjugate to  $k_T$ , leads to a Sudakov form factor  $\exp[-s(P, b)]$ . This factor suppresses the long-distance contributions from the large  $b$  region with  $b \sim 1/\bar{\Lambda}$ , where  $\bar{\Lambda} \equiv M_B - m_b$ ,  $M_B$  being the  $B$  meson mass, represents a soft scale. The suppression renders  $k_T^2$  flowing into the hard amplitudes of order  $k_T^2 \sim O(\bar{\Lambda}M_B)$ . The off-shellness of internal particles then remain of  $O(\bar{\Lambda}M_B)$  even in the end-point region, and the singularities are removed. Since the end-point singularities do not exist [20, 22], the arbitrary cutoffs introduced in QCDF [23, 24] are not necessary. Therefore, factorizable, nonfactorizable and annihilation amplitudes can be estimated in a more consistent way in PQCD than in QCDF.

It is easy to understand the increase of  $k_T^2$  from  $O(\bar{\Lambda}^2)$ , carried by the valence quarks which just come out of the initial meson wave functions, to  $O(\bar{\Lambda}M_B)$ , carried by the quarks which are involved in the hard weak decays. Consider the simple deeply inelastic scattering of a hadron. The transverse momentum  $k_T$  carried by a parton, which just come out of the hadron distribution function, is initially small. After infinite many gluon radiations,  $k_T$  becomes of  $O(Q)$ , when the parton is scattered by the highly virtual photon, where  $Q$  is the large momentum transfer from the photon. The evolution of the hadron distribution function from the low scale to  $Q$  is described by the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation [25]. The mechanism of the DGLAP evolution in DIS is similar to that of the Sudakov evolution in exclusive  $B$  meson decays. The difference is only that the former is the consequence of the single-logarithm resummation, while the latter is the consequence of the double-logarithm resummation.

## 3 Penguin Enhancement

The leading factorizable contributions involve four-quark hard amplitudes in QCDF, but six-quark hard amplitudes in PQCD. This distinction also implies different characteristic scales in the two approaches: the former is characterized by  $m_b$ , while the latter is characterized by the virtuality of internal particles of order  $\sqrt{\bar{\Lambda}M_B} \sim 1.5$  GeV [10, 11, 13]. It has been known that to accommodate the  $B \rightarrow K\pi$  and  $\pi\pi$  data, penguin contributions must be large enough. In QCDF one relies on chiral enhancement by increasing the mass  $m_0$  to a large value  $m_0 \sim 3-4$  GeV [26]. Because of the renormalization-group evolution effect of the Wilson coefficients associated with the QCD penguin operators, the lower hard scale leads to dynamical penguin

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enhancement in PQCD. Whether dynamical enhancement or chiral enhancement is responsible for the large  $B \rightarrow K\pi$  branching ratios can be tested by measuring the  $B \rightarrow \phi K$  modes [14, 27]. In these modes penguin contributions dominate, such that their branching ratios are insensitive to the variation of the unitarity angle  $\phi_3$ . Because the  $\phi$  meson is a vector meson, the mass  $m_0$  is replaced by the physical mass  $M_\phi \sim 1$  GeV, and chiral enhancement does not exist. If the branching ratios of the  $B \rightarrow \phi K$  decays are around  $4 \times 10^{-6}$  [28, 29], chiral enhancement may be essential for the penguin-dominated decay modes. If the branching ratios are around  $10 \times 10^{-6}$  as predicted in PQCD [14, 30], dynamical enhancement may be essential.

## 4 $B \rightarrow D^0\pi^0$ and $B \rightarrow \pi^+\pi^0$

It can be shown that the relative importance of the different topologies of diagrams for the  $B \rightarrow D\pi$  decays is given by

$$\text{emission : nonfactorizable} = 1 : \frac{M_D}{M_B}, \quad (2)$$

which approaches  $1 : \bar{\Lambda}/M_B$  as the  $D$  meson mass  $M_D$  reduces to the pion mass of  $O(\bar{\Lambda})$ . Since the factorizable and nonfactorizable diagrams contribute to the parameters  $a_1$  and  $a_2$  in PQCD, respectively, the ratio  $|a_2|/a_1 \sim 0.3$  is obtained. Moreover, the imaginary nonfactorizable amplitudes determine the relative phase of the factorizable and nonfactorizable contributions, which is about  $60^\circ$ . It has been found that the PQCD predictions for the  $B \rightarrow D\pi$  branching ratios [31],

$$B(D^0\pi^-) \sim 4.8 \times 10^{-3}, \quad B(D^+\pi^-) \sim 3.0 \times 10^{-3}, \quad B(D^0\pi^0) \sim 0.2 \times 10^{-3}, \quad (3)$$

are consistent with the experimental data, including the recently observed one for the  $\bar{B}_d \rightarrow D^{(*)0}\pi^0$  decay [32, 33].

Therefore, it is difficult to explain the large  $B \rightarrow \pi^+\pi^0$  branching ratio around  $7 \times 10^{-6}$  observed by BELLE recently [18]. This large branching ratio implies large  $a_2$  for the  $B \rightarrow \pi\pi$  decays, which is in conflict with the PQCD power counting rules. Final-state-interaction is unlikely to resolve this controversy either, since the upper bound on the  $B \rightarrow K^+K^-$  branching ratio has strongly constrained its effect [34]. Fortunately, the BABAR result for the  $B \rightarrow \pi^+\pi^0$  branching ratio announced at this conference has a central value around  $4 \times 10^{-6}$ , which is consistent with the PQCD prediction.

## 5 Conclusion

I have briefly reviewed the PQCD approach to two-body nonleptonic  $B$  meson decays. The PQCD predictions for the branching ratios and the CP asymmetries of various

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modes with one single parameter, the shape parameter associated with the  $B$  meson wave function, are in agreement with the experimental data [35]. The  $B \rightarrow K\eta'$  data are an exception, which may indicate a significant gluon content of the  $\eta'$  meson [36]. In the future we shall work out the next-to-leading-order and next-to-leading-power corrections to the decay amplitudes.

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