Recent results from the QCD factorization approach to non-leptonic B decays

M. Beneke
Institut für Theoretische Physik E
RWTH Aachen
Sommerfeldstr. 28
D-52074 Aachen, GERMANY

1 Introduction

In this talk I report recent results on hadronic B decays obtained with the QCD factorization approach. In the first part I update the fit of the Wolfenstein parameters $(\overline{\rho},\overline{\eta})$ to CP-averaged $B\to\pi\pi,\pi K$ branching fractions performed in [1] to account for the new experimental data, and give the correlation between the time-dependent and direct CP asymmetry in $B_d\to\pi^+\pi^-$ decay, also based on the calculation of [1]. In the second part I present an investigation of B decays into final states containing an η or η' meson [2]. I discuss the new theoretical issues that arise for flavour singlet mesons and present preliminary numerical results that appear to reproduce the pattern of experimental data reasonably well.

The QCD factorization approach [3] uses heavy quark expansion methods ($m_b\gg\Lambda_{\rm QCD}$) and soft-collinear factorization (particle energies $\gg\Lambda_{\rm QCD}$) to compute the matrix elements $\langle f|O_i|\overline{B}\rangle$ relevant to hadronic B decays in an expansion in $1/m_b$ and α_s. Only the leading term in $1/m_b$ assumes a simple form. The basic formula is

$$\begin{aligned}\langle M_1M_2|O_i|\overline{B}\rangle \;=\; & F^{B\to M_1}(0)\int_0^1 du\,T^I(u)\Phi_{M_2}(u)\\ & +\int d\xi du dv\,T^{II}(\xi,u,v)\,\Phi_B(\xi)\Phi_{M_1}(v)\Phi_{M_2}(u),\end{aligned}\tag{1}$$

where $F^{B\to M_1}$ is a (non-perturbative) form factor, Φ_{M_i} and Φ_B are light-cone distribution amplitudes and $T^{I,II}$ are perturbatively calculable hard scattering kernels.

Although not strictly proven to all orders in perturbation theory, the formula is presumed to be valid when both final state mesons are light. (M_1 is the meson that picks up the spectator quark from the B meson.) The formula shows that there is no long-distance interaction between the constituents of the meson M_2 and the (BM_1) system at leading order in $1/m_b$. This is the precise meaning of factorization. A summary of results that have been obtained in the QCD factorization approach is given in [4].

Factorization is not expected to hold at subleading order in $1/m_b$. Attempts to compute subleading power corrections to hard spectator-scattering in perturbation theory usually result in infrared divergences, which signal the breakdown of factorization. Some power corrections related to scalar currents are enhanced by factors such as $m_\pi^2/((m_u + m_d)\Lambda_{\rm QCD})$ [3]. At least these effects should be estimated and included into the error budget. All weak annihilation contributions belong to this class of effects.

2 CP-averaged $B \to \pi\pi, \pi K$ branching fractions

The possibility to determine the CP-violating angle γ by comparing the calculation of branching fractions into $\pi\pi$ and πK final states with the corresponding data has been investigated in detail [1] (see also [5]). The branching fractions for the modes $B^+ \to \pi^+\pi^0$ and $B^+ \to \pi^+ K^0$, which depend only on a single weak phase to very good approximation, are well described by the theory. This demonstrates that the magnitude of the tree and penguin amplitude is obtained correctly, where for the penguin amplitude the 1-loop radiative correction is important to reach this conclusion. There is, however, a relatively large normalization uncertainty for the πK final states, which are sensitive to weak annihilation and the strange quark mass through the scalar penguin amplitude. This uncertainty can be partially eliminated by taking ratios of branching fractions. The agreement is less good for branching fractions with significant interference of tree and penguin amplitudes, if γ is assumed to take values around 55° as favoured by indirect constraints.

In [1] a fit of the Wolfenstein parameters $(\overline{\rho}, \overline{\eta})$ to the six measured CP-averaged $B \to \pi\pi, \pi K$ branching fractions has been performed. The result of this fit is shown in the left panel of Figure 1. (The details of the fit procedure can be found in [1]). We now repeat this fit with the new world averages as presented at this conference [6], see Table 1. There are no dramatic changes in the data since spring 2001, but the small shifts of the various branching fractions (for instance, in the final state $\pi^0 K^0$) all work towards better agreement with the theoretical calculation, resulting in an improved fit. (The best fits with theory parameters in the allowed ranges have $\chi^2 \approx 0.5$.) On the theoretical side we changed the allowed values of the strange quark mass and the B meson decay constant to $[75, 125]\,\text{MeV}$ (from $[85, 135]\,\text{MeV}$)

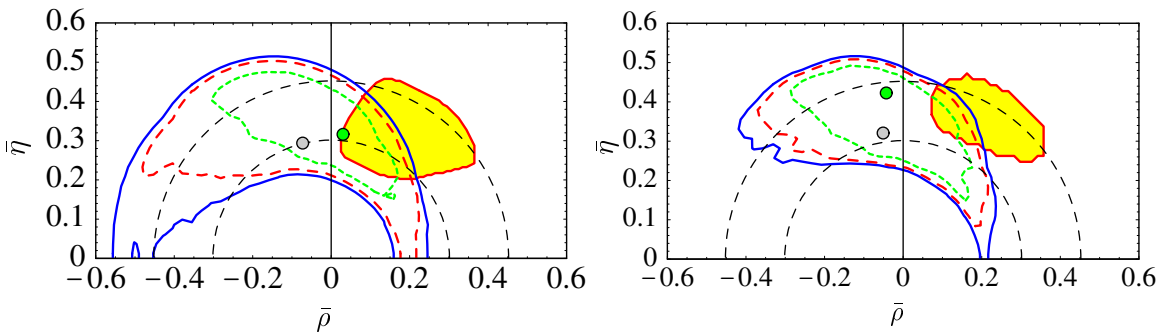

Figure 1: 95% (solid), 90% (dashed) and 68% (short-dashed) confidence level contours in the $(\overline{\rho}, \overline{\eta})$ plane obtained from a global fit to the CP averaged $B \to \pi K, \pi\pi$ branching fractions, using the scanning method as described in [7]. The darker dot shows the overall best fit, whereas the lighter dot indicates the best fit for the default hadronic parameter set. The left panel reproduces the fit of [1] reflecting the status of spring 2001; the right panel summarizes the current results. The light-shaded region indicates the region preferred by the standard global fit [7], excluding (including) the direct measurement of $\sin(2\beta)$ in the left (right) panel.

Decay Mode	Exp. Average	Default fit	Fit2
$B^0 \to \pi^+\pi^-$	5.15 ± 0.61	5.12	5.24
$B^\pm \to \pi^\pm\pi^0$	4.88 ± 1.06	5.00	4.57
$B^0 \to \pi^0\pi^0$	–	0.78	0.94
$B^0 \to \pi^\mp K^\pm$	18.56 ± 1.08	17.99	18.47
$B^\pm \to \pi^0 K^\pm$	11.49 ± 1.26	12.07	11.83
$B^\pm \to \pi^\pm K^0$	17.93 ± 1.70	15.65	17.88
$B^0 \to \pi^0 K^0$	8.82 ± 2.20	5.55	6.87

Table 1: CP-averaged $B \to \pi\pi, \pi K$ branching fractions (in units of 10^{-6}): data vs. results from the fit. The default fit to $(\overline{\rho}, \overline{\eta})$ (returning $|V_{ub}/V_{cb}| = 0.085$, $\gamma = 116^\circ$ with $\chi^2 = 4.5$) refers to the default theory parameter set as explained in the text. "Fit2" (returning $|V_{ub}/V_{cb}| = 0.079$, $\gamma = 97^\circ$, $\chi^2 = 1.0$) refers to a fit without annihilation contributions and chirally enhanced spectator corrections but with $m_s = 80\,\text{MeV}$ [100 MeV], $\lambda_B = 200\,\text{MeV}$ [350 MeV] and $R_{\pi K} = 0.8$ [0.9] (default values in square brackets). The experimental average is based on $9\,\text{fb}^{-1}$ from CLEO, $29.1\,\text{fb}^{-1}$ from Belle and $55.6\,\text{fb}^{-1}$ from Babar [6].

and $[170, 230]\,\text{MeV}$ (from $[140, 220]\,\text{MeV}$), respectively, to account for a change in the theoretically favoured ranges. The result of the current fit is shown in the right panel of Figure 1. The last two columns of Table 1 give the fitted branching fractions for the default theory parameter set (corresponding to the parameters used in [1] and the central values of the new ranges for m_s and f_B) and a second set, where all annihilation effects and chirally enhanced spectator interactions are switched off. The second set therefore shows that very good fits can also be obtained without these theoretically uncertain power-suppressed effects. While a large range of values of γ remains compatible with data, and the result of the fit is consistent with the standard fit based on meson mixing and $|V_{ub}|$, it shows a preference for γ near 90°, or, for smaller γ, smaller $|V_{ub}|$.

3 CP asymmetries in $B_d \to \pi^+\pi^-$ decay

The QCD factorization approach allows us to interpret directly the mixing-induced and direct CP asymmetry in $B_d \to \pi^+\pi^-$ decay without resort to other decay modes, since the tree and penguin amplitudes are both computed. The time-dependent asymmetry is defined by

$$\begin{aligned} A_{\rm CP}^{\pi\pi}(t) &= \frac{\text{Br}(B^0(t) \to \pi^+\pi^-) - \text{Br}(\overline{B}^0(t) \to \pi^+\pi^-)}{\text{Br}(B^0(t) \to \pi^+\pi^-) + \text{Br}(\overline{B}^0(t) \to \pi^+\pi^-)} \\ &= -S_{\pi\pi}\sin(\Delta m_B\, t) + C_{\pi\pi}\cos(\Delta m_B\, t), \end{aligned} \tag{2}$$

where $S_{\pi\pi} = \sin(2\alpha)$, if the penguin amplitude were zero, and $C_{\pi\pi}$ is the direct CP asymmetry. (This convention is related to those used by Babar and Belle by $S_{\pi\pi} = S_{\pi\pi}^{\rm Babar} = S_{\pi\pi}^{\rm Belle}$ and $C_{\pi\pi} = C_{\pi\pi}^{\rm Babar} = -A_{\pi\pi}^{\rm Belle}$.)

In [1] we have shown how a measurement of $S_{\pi\pi}$ translates into a determination of $\sin(2\alpha)$ and results in a stringent constraint in the $(\overline{\rho}, \overline{\eta})$ plane. In view of the new measurements of $S_{\pi\pi}$ and $C_{\pi\pi}$ from Babar [8] and Belle [9] (summarized in Table 2), it is also interesting to exhibit the correlation between the two observables. This is shown in Figure 2, where the $B\overline{B}$ mixing phase has been fixed such that $\sin(2\beta) = 0.78$. We assume this phase to be experimentally given and do not require that $B\overline{B}$ mixing is described by the Standard Model. Each closed curve is then

Experiment	$S_{\pi\pi}$	$C_{\pi\pi}$
Babar	$-0.01 \pm 0.37 \pm 0.17$	$-0.02 \pm 0.29 \pm 0.09$
Belle	$-1.21^{+0.38+0.16}_{-0.27-0.13}$	$-0.94^{+0.31}_{-0.25} \pm 0.09$

Table 2: CP asymmetries in $B_d \to \pi^+\pi^-$ from Babar and Belle [8, 9].

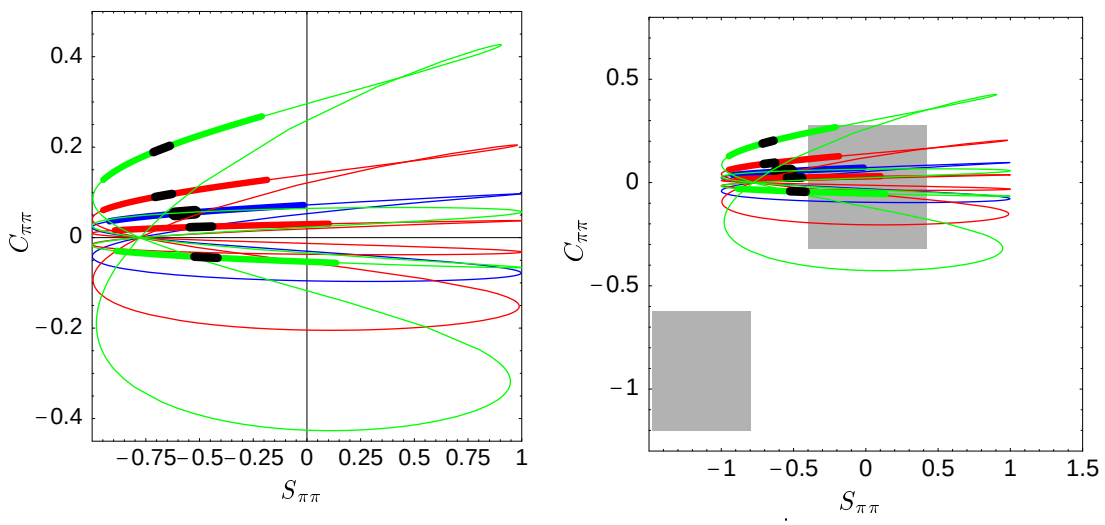

Figure 2: Predicted correlation between mixing-induced and direct CP asymmetry in $B_d \to \pi^+\pi^-$ decay. See text for explanation of the different curves. The right plot is a scaled version of the left plot, and includes the Belle result (upper left) and Babar result (center) with their 1σ errors (gray rectangles). (Note, however, that the physical range is $|S_{\pi\pi}|^2 + |C_{\pi\pi}|^2 \leq 1$.)

generated by specifying the theory input and letting γ vary from 0 to 360°. The central (dark) curve refers to the calculation of P/T, the penguin-to-tree ratio, with the default theory parameter set, the two neighboring (lighter) curves refer to P/T plus-minus its theoretical error without the error from weak annihilation (but including the one from $|V_{ub}|$), and the final (lightest) curves also include the error from weak annihilation. The black part on each curve marks the point $\gamma = 60°$; the fat line segment marks the range $[40°, 80°]$ favoured by the standard unitarity triangle fit with larger γ to the right of the black part. We can see from this that within the 1σ errors only the Babar result is compatible with the theoretical calculation, and it favours γ somewhat larger than the standard unitarity triangle fit, but consistent with the CP-averaged branching fraction data discussed above.

4 Final states containing $\eta^{(\prime)}$

These final states are interesting because the available data exhibit an interesting pattern in $\Delta S = 1$ decays (all branching fractions in units of 10^{-6}):

$$\mathrm{Br}(K\eta') \sim 70 \gg \mathrm{Br}(K\eta) \sim 5\,(?)$$

$$\mathrm{Br}(K^*\eta') \text{ not observed} < (?)\ \mathrm{Br}(K^*\eta) \sim 20 \tag{3}$$

These results are difficult to account for in the naive factorization approach [10] (see also Table 3).

The calculation of the flavour-singlet decay amplitude in QCD factorization involves several aspects specific to singlet mesons [2]. One of them is related to η-η' mixing for which we use the scheme advocated in [11], which amounts to assuming that mixing is described by a single mixing angle common to all matrix elements in the so-called quark-flavour basis. The other aspects are related to the pure gluon content of singlet mesons, which leads to the following new effects:

1. The CKM-enhanced $b \to c\bar{c}s$ transition can contribute to singlet mesons by closing the charm lines and attaching two gluons to them. This effect amounts to assigning a charm decay constant to $\eta^{(\prime)}$, which by explicit calculation of the diagrams is found to be

$$f_P^c = -\frac{m_P^2}{12 m_c^2}\,\frac{f_P^q}{\sqrt{2}} \approx -2.5\,\mathrm{MeV}\,[\eta'],\ -1\,\mathrm{MeV}\,[\eta], \tag{4}$$

   in agreement with [12]. Here f_P^q is the up-quark decay constant and the result is obtained for $m_c \gg \Lambda_{\mathrm{QCD}}$. Note the absence of any factors of α_s.

2. The singlet meson can be produced in a two-gluon state, where one gluon originates from a penguin $b \to s$ transition and the other from the spectator quark. We find a leading-power contribution from the configuration where the second gluon is soft, which implies that factorization breaks down in the conventional sense. Despite this, this non-perturbative contribution can be parameterized by a non-local $B \to K^{(*)}$ form factor, which introduces one new non-perturbative parameter. This effect is proportional to $\alpha_s C_{8g}$, where C_{8g} is the Wilson coefficient of the chromomagnetic dipole operator, and can amount to several percent of the amplitude for $\eta^{(\prime)}$ mesons.

3. There exists a singlet annihilation amplitude which is not power-suppressed in m_b, where two gluons radiate from the spectator quark and form an $\eta^{(\prime)}$ meson.

In Table 3 we present our (still preliminary) results for the CP-averaged $B \to K^{(*)}(\eta^{(\prime)}, \pi^0)$ branching fractions, not including effects 2. and 3. discussed above. We also used $|V_{ub}/V_{cb}| = 0.09$ and $\gamma = 60^\circ$ for the numerical evaluation. The dominant theoretical error is from the strange quark mass and weak annihilation and is correlated among the decay modes displayed in the Table.

Despite the shortcomings of the above analysis (some of which we hope to rectify in the final publication [2]), we see that the QCD factorization calculation appears to reproduce the pattern of the data reasonably well within the uncertainties of the calculation, which are very large for some of the decay modes. The basic features of

Mode	Naive Fact.	QCD Fact.	Exp. average
$B^- \to K^- \eta'$	13	47^{+40}_{-19}	75.1 ± 6.2
$\overline{B}^0 \to \overline{K}^0 \eta'$	14	47^{+38}_{-19}	61.0 ± 12.5
$B^- \to K^- \eta$	0.7	$1.3^{+1.4}_{-0.8}$	5.3 ± 1.8
$\overline{B}^0 \to \overline{K}^0 \eta$	0.1	$0.5^{+1.0}_{-0.5}$	< 9.3
$B^- \to K^- \pi^0$	4.4	$9.4^{+7.3}_{-3.4}$	11.5 ± 1.3
$\overline{B}^0 \to \overline{K}^0 \pi^0$	2.2	$6.4^{+6.1}_{-2.8}$	8.8 ± 2.2
$B^- \to K^{*-} \eta'$	2.9	$3.3^{+8.7}_{-3.3}$	< 35
$\overline{B}^0 \to \overline{K}^{*0} \eta'$	1.6	$2.1^{+7.4}_{-2.1}$	< 24
$B^- \to K^{*-} \eta$	3.8	$9.3^{+16.6}_{-6.1}$	25.4 ± 5.3
$\overline{B}^0 \to \overline{K}^{*0} \eta$	4.3	$10.4^{+17.3}_{-6.5}$	16.4 ± 3.0
$B^- \to K^{*-} \pi^0$	1.7	$3.0^{+4.0}_{-1.4}$	–
$\overline{B}^0 \to \overline{K}^{*0} \pi^0$	0.2	$0.8^{+2.5}_{-0.6}$	–

Table 3: CP-averaged $B \to K^{(*)}(\eta^{(\prime)}, \pi^0)$ branching fractions in units of 10^{-6} in naive factorization and QCD factorization compared to experimental averages. Theoretical results are preliminary as explained in the text. All theoretical errors are strongly correlated.

this pattern can be understood from the structure of the penguin contributions to the decay amplitudes:

$$\begin{aligned} A(\overline{K}\pi^0) &\sim F^{B\to\pi}\,\frac{f_K}{\sqrt{2}}\,\left(a_4^c(\pi K) + r_\chi^K a_6^c(\pi K)\right), \\ A(\overline{K}P) &\sim F^{B\to P}\,\frac{f_K}{\sqrt{2}}\,\left(a_4^c(PK) + r_\chi^K a_6^c(PK)\right) \qquad \text{(I)} \\ &\quad + F^{B\to K}\,\Big(\left(\sqrt{2}f_P^q + f_P^s\right)\,\left(a_3^c(KP) - a_5^c(KP)\right) \qquad \text{(small)} \\ &\quad\quad + f_P^s\,\left(a_4^c(KP) + r_\chi^P a_6^c(KP)\right)\Big), \qquad \text{(II)} \end{aligned} \tag{5}$$

where $P = \eta, \eta'$ and $K = K, K^*$. For the $K\eta'$ decay the two penguin amplitudes I and II add constructively enhancing the branching fraction by a large factor compared to $K\pi^0$. For $K\eta$, on the other hand, the two amplitudes nearly cancel since the strange decay constant in the η satisfies $f_\eta^s/f_K \sim -2/3$. Replacing K by the vector meson K^* again changes the pattern, because the scalar penguin amplitude $r_\chi^{M_2} a_6^c(M_1 M_2)$ changes sign for $M_1 M_2 = K^* P$ (changing the sign of the term II) compared to KP and becomes small for $M_1 M_2 = PK^*$. As a consequence the terms I and II interfere destructively for $K^*\eta'$ but constructively for $K^*\eta$, opposite to the

case of a pseudoscalar kaon. These features are not different from the expectation in naive factorization. However, radiative corrections in QCD factorization enhance the penguin amplitudes significantly and improve the comparison with data. While it appears unlikely that one can obtain an accurate theoretical description of final states with singlet mesons from first principles, the present results clearly demonstrate the relevance of factorization to this class of charmless B decays.

I am grateful to M. Neubert, G. Buchalla and C.T. Sachrajda for collaboration on the topics discussed in this talk. I thank U. Nierste for reading the manuscript.

References

[1] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B **606**, 245 (2001).

[2] M. Beneke and M. Neubert, work in progress.

[3] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999); Nucl. Phys. B **591**, 313 (2000).

[4] M. Beneke, in: Proceedings of the 5th KEK Topical Conference: Frontiers in Flavor Physics (KEKTC5), Tsukuba, Ibaraki, Japan, 20-22 Nov 2001 [hep-ph/0202056].

[5] D. Du, H. Gong, J. Sun, D. Yang and G. Zhu, Phys. Rev. D **65**, 074001 (2002).

[6] R. Bartholdus, these proceedings.

[7] A. Höcker, H. Lacker, S. Laplace and F. Le Diberder, Eur. Phys. J. C **21**, 225 (2001).

[8] P.D. Dauncey [for the Babar Collaboration], these proceedings [hep-ex/0206064].

[9] E. Won [for the Belle Collaboration], these proceedings [hep-ex/0206066].

[10] A. Ali, J. Chay, C. Greub and P. Ko, Phys. Lett. B **424**, 161 (1998).

[11] T. Feldmann, P. Kroll and B. Stech, Phys. Rev. D **58**, 114006 (1998).

[12] M. Franz, M. V. Polyakov and K. Goeke, Phys. Rev. D **62**, 074024 (2000).