Neutrino oscillation tomography

or

What could be learned about the Earth's interior from neutrino oscillations in matter?

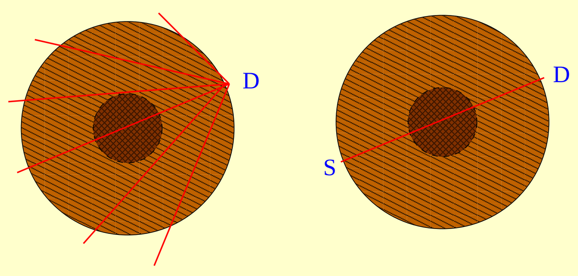
Tommy Ohlsson and Walter Winter

Technische Universität München

Phys. Lett. B512 (2001) 357 (hep-ph/0105293) and hep-ph/0111247

Neutrino tomography

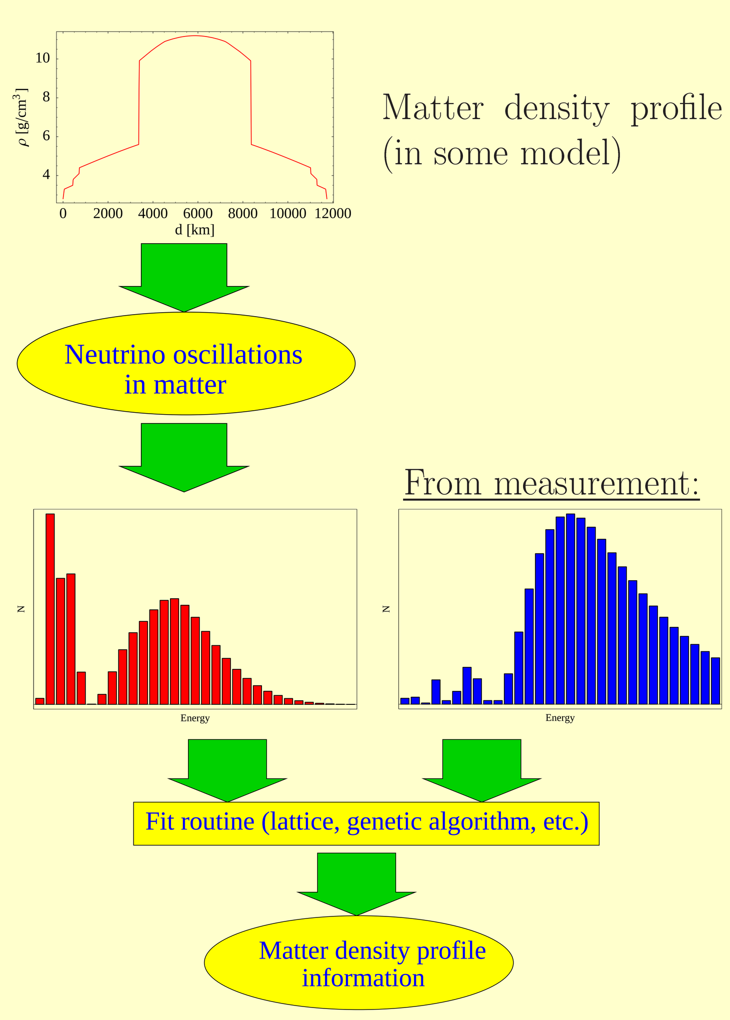
	Neutrino *absorption* tomography	Neutrino *oscillation* tomography



Principle	Neutrino absorption in matter	MSW effect
Baselines	Many	One
Energies	$\geq$ TeV	$\sim 0.1 - 50\,\mathrm{GeV}$
Sources	Cosmic	Superbeams, Neutrino Factories
Analogy	X-ray tomography	Interference experiments
	De Rújula et al. (1983) and others	Ermilova et al. (1988) and others

Principles

- Short or long baseline experimental setup
- Oscillation parameters assumed to be measured with high precision: Δm^2_{32}, Δm^2_{21}, θ_{13}, θ_{12}, θ_{23}, δ_{CP}



The evolution operator method

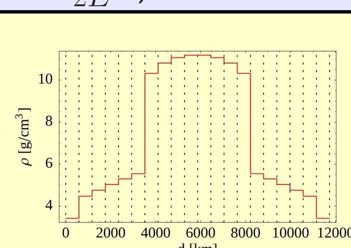
Propagation in a layer of length x_j and constant density ρ_j described by *evolution operator*

$$\mathcal{U}(x_j,\rho_j) = e^{-i\mathcal{H}(\rho_j)x_j},$$

where $\mathcal{H}(\rho_j)$ is the Hamiltonian in constant matter density

$$\mathcal{H}(\rho_j) = U \begin{pmatrix} 0 & 0 & \\ 0 & \frac{\Delta m^2_{21}}{2E} & 0 \\ 0 & 0 & \frac{\Delta m^2_{31}}{2E} \end{pmatrix} U^\dagger + \begin{pmatrix} A(\rho_j) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

For an arbitrary matter density (step) profile

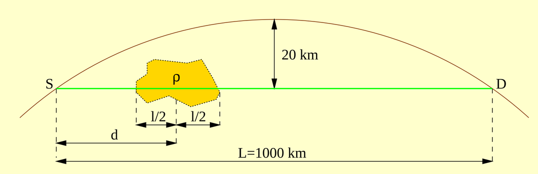


$$P_{\alpha\beta} = \left|\langle\nu_\beta|\mathcal{U}(x_n,\rho_n)\ldots\mathcal{U}(x_1,\rho_1)|\nu_\alpha\rangle\right|^2.$$

Note that in general $[\mathcal{U}(x_i,\rho_i),\mathcal{U}(x_j,\rho_j)] \neq 0$ for $i \neq j$
$\rightarrow$ Additional information by interference effects compared to neutrino absorption tomography

The search for large cavities in the Earth's mantle

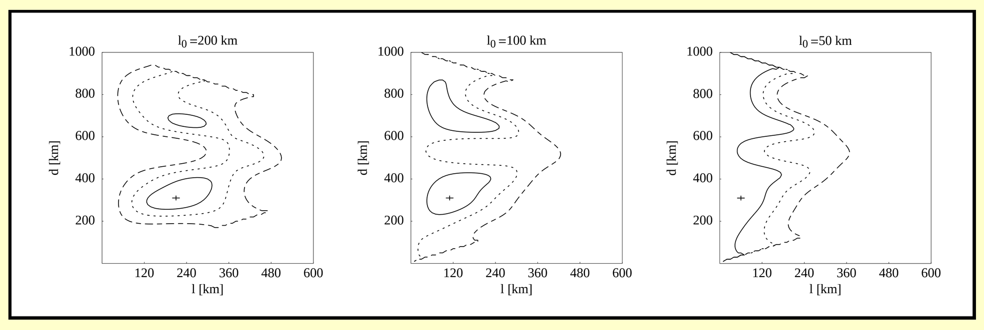
Experimental setup: Low energy superbeam 500 MeV, $\nu_\mu \rightarrow \nu_e$



In this scenario:

- Cavity centered at d_0 with length l_0
- Maximum depth 20 km
- Average depth 13 km
- Average matter density: $2.9\,\mathrm{g/cm^3}$
- Matter density in the cavity: $\rho = 1.0\,\mathrm{g/cm^3}$

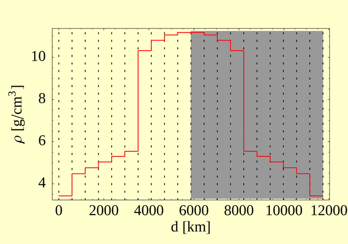
For a cavity centered at $d_0 = 300\,\mathrm{km}$:



The cavity can be seen on the 3σ-level for $l_0 = 200\,\mathrm{km}$, 1σ-level for $l_0 = 100\,\mathrm{km}$ and not at all for $l_0 = 50\,\mathrm{km}$.

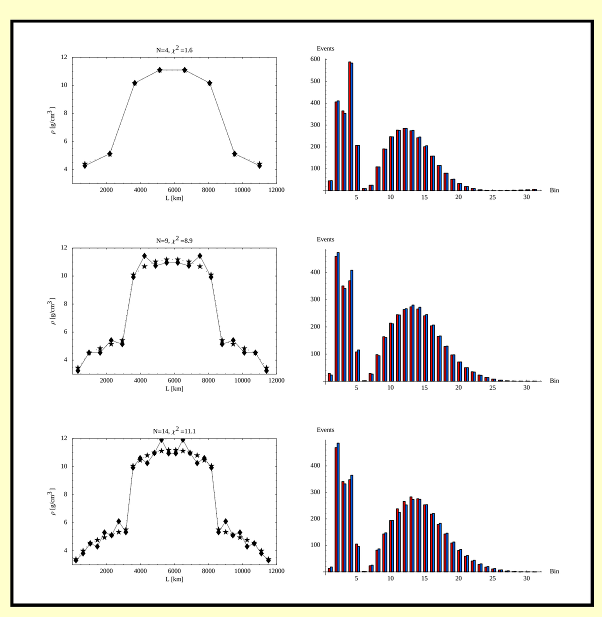
Reconstruction of the Earth's matter density profile

Experimental setup: 20 GeV neutrino factory, baseline $L = 11700\,\mathrm{km}$, $\nu_\mu \rightarrow \nu_e$
Symmetric Earth matter density profile with $2N$ layers assumed, which means that the dimension of the parameter space is N.



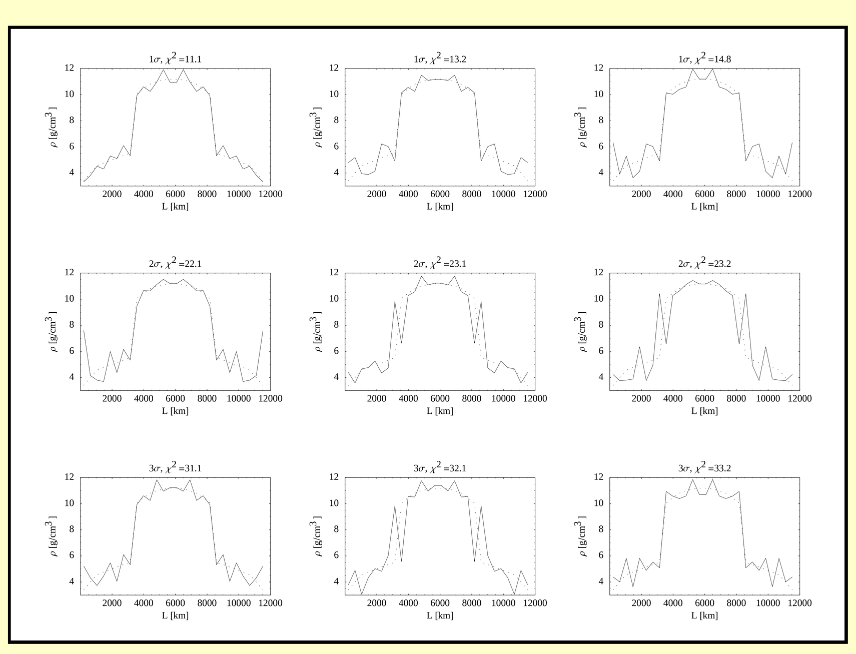
Use genetic algorithm with many trial runs to find matter density profiles fitting the measured energy spectrum.

Best results of genetic algorithm fits ($N = 4, 9, 14$):



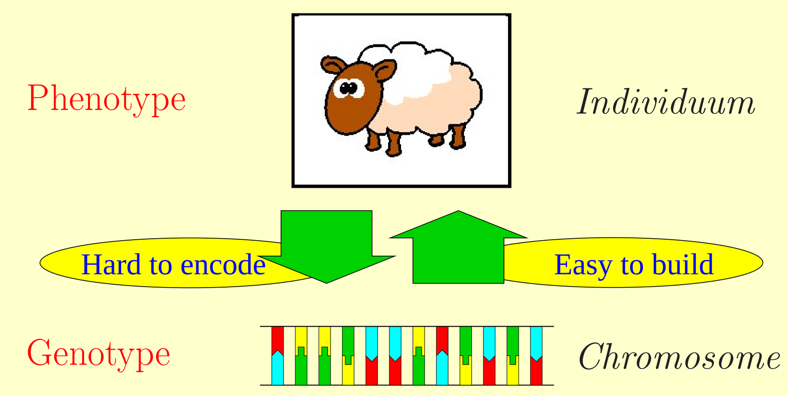
All results are within the 1σ level. Therefore a preciser measurement is not possible and small fluctuations in mantle and core are not resolvable.

Examples for matter density profiles close to the 1σ (first row), 2σ (second row), and 3σ (third row) contours:

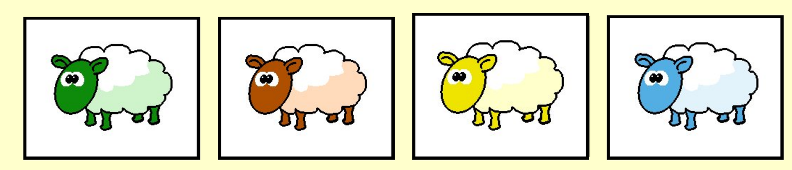


Genetic algorithms ...

... for searching the high-dimensional parameter space



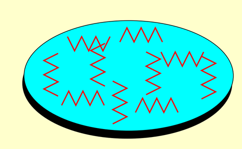
Fitness function: Survival probability of an individuum in its environment



Sample fitness	0.5	0.9	0.3	0.8

Can be easily calculated for phenotype, but not for genotype
$\rightarrow$ Build individuum from chromosome first, then calculate the survival probability

The genetic algorithm



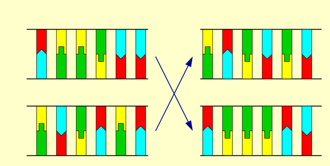
Generate initial generation of chromosomes by random

Reproductive plan for the next generation:

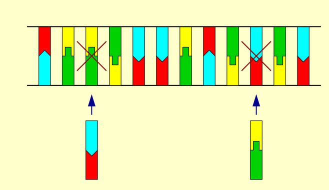
Create individuum from each chromosome and calculate its fitness value

Selection Select two parent chromosomes by random with a probability proportional to the fitness value

Crossover Cross the two parent chromosomes at a random position

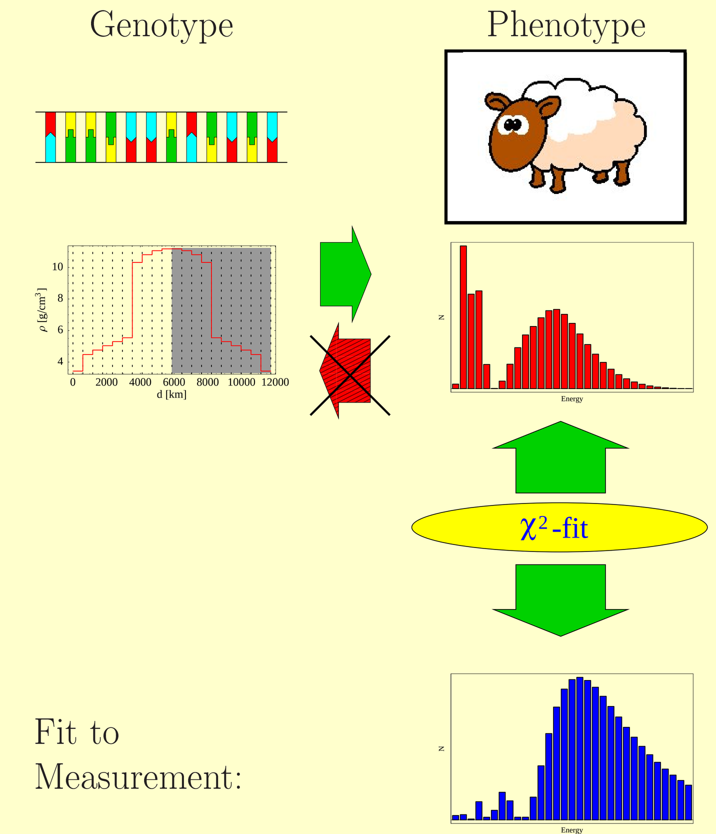


Mutation Mutate each position with a certain probability



$\rightarrow$ Repeat until new generation is completely populated

Application to the reconstruction problem



Genetic algorithms vs. lattice methods

Genetic algorithms	Lattice methods
Polynomial running time $\mathcal{O}(N^k)$	Exponential running time $\mathcal{O}(e^N)$
Finds optima also in between lattice points	May miss minima in between lattice points
May miss regions in parameter space if not enough trial runs	Searches parameter space systematically