An Investigation of T-Violating Effects in Neutrino Oscillations in Matter

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WIN 2002, Christchurch, New Zealand

January 21-26, 2002

T-violating Effects

Interplay of fundamental and matter-induced T violation:

T transformation = time reversal transformation

Experimental problem: T-violation cannot be directly experimentally tested, since one cannot change the direction of time.

"Solution": Instead of studying neutrino oscillations "backward" in time, one can study them forward in time, but with initial and final flavors interchanged.

fundamental T violation (intrinsic) = due to non-vanishing $\{\delta_{CP}\}$

matter-induced T violation (extrinsic) = due to interchange of positions of source and detector (asymmetric matter density profile)

T violation in neutrino oscillations:

$$\Delta P^T_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha),$$

where $P(\nu_\alpha \to \nu_\beta)$ is the transition probability for $\nu_\alpha \to \nu_\beta$.

CP and CPT differences:

$$\begin{aligned} \Delta P^{CP}_{\alpha\beta} &\equiv P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) \\ \Delta P^{CPT}_{\alpha\beta} &\equiv P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\beta \to \bar{\nu}_\alpha) \end{aligned}$$

Two neutrino flavors:

There are no T-violating effects!

$P_{e\mu} = P_{\mu e} \quad \Rightarrow \quad \Delta P^T_{e\mu} = 0$

Three neutrino flavors:

- In vacuum:

  CPT invariance $\Rightarrow$ T violation $\Leftrightarrow$ CP violation

- In matter:

  Matter is both CP and CPT-asymmetric, since it consists of particles (electrons and nucleons) and not of their antiparticles or, in general, of unequal numbers of particles and antiparticles.

  – Symmetric matter density profiles:

    Example: Constant matter density profiles

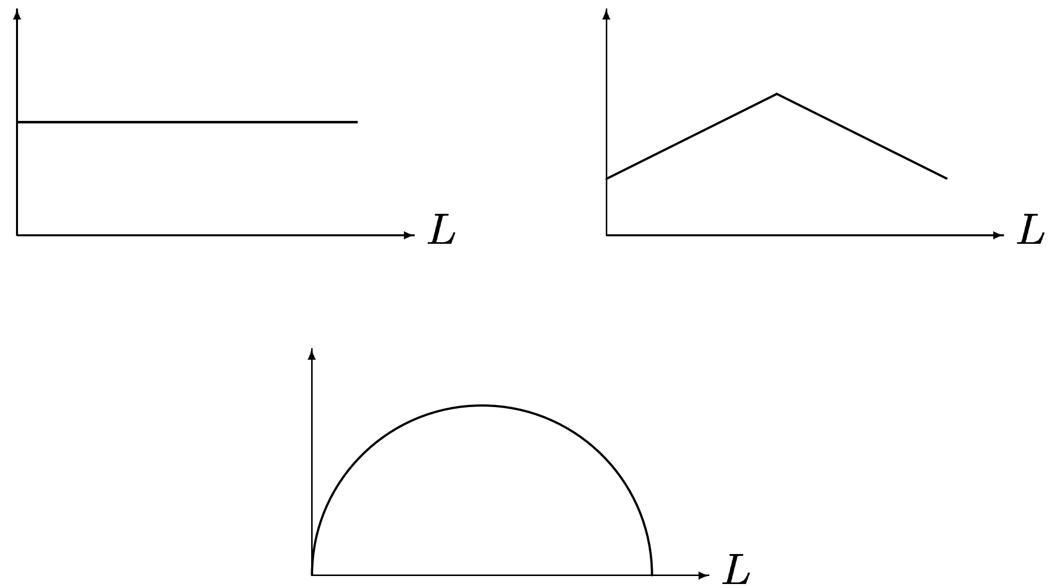
    If $\delta_{CP} = 0$, then $\Delta P^T_{\alpha\beta} = 0$.

  – Asymmetric matter density profiles:

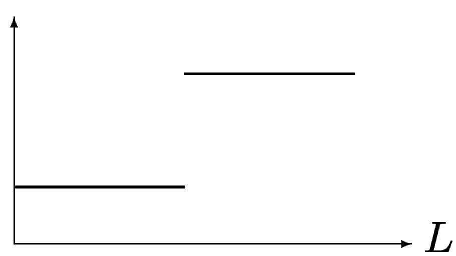
    Example: Step function matter density profiles

Examples:

Symmetric matter density profiles:



Asymmetric matter density profile:



The T-odd probability difference (arbitrary matter density profile): θ_{13} and δ/Δ are small parameters!

$$\begin{aligned}\Delta P^T_{e\mu} &\simeq -2 s_{23} c_{23} \Im \left[\beta^* (A - C^*)\right] \\ &\simeq -2 s_{13} s_{23} c_{23} \left(\Delta - s_{12}^2 \delta\right) \\ &\times \Im \left[e^{-i\delta_{CP}} \beta^* \left(A_a - C_a^*\right)\right]\end{aligned}$$

Here:

$$s_{ij} \equiv \sin\theta_{ij}, \quad c_{ij} \equiv \cos\theta_{ij};^*$$

$$\delta \equiv \frac{\Delta m_{21}^2}{2E_\nu}, \quad \Delta \equiv \frac{\Delta m_{31}^2}{2E_\nu};$$

$$A_a \equiv \alpha \int_{t_0}^{t} \alpha^* f \, dt' + \beta \int_{t_0}^{t} \beta^* f \, dt', \quad C_a \equiv f \int_{t_0}^{t} \alpha f^* \, dt'.$$

$\alpha = \alpha(t, t_0)$ and $\beta = \beta(t, t_0)$ are to be determined from the solutions of the two flavor neutrino problem in the (1,2)-sector and $f = f(t, t_0) = \exp\left\{-i \int_{t_0}^{t} \left(\Delta - \frac{1}{2}\left[\delta + V(t')\right]\right) dt'\right\}$.

In addition: $\Delta P^T_{e\mu} = \Delta P^T_{\mu\tau} = \Delta P^T_{\tau e}$

$\Delta P^T_{e\mu}$ has been calculated for

1. matter consisting of two layers of constant density and
2. an arbitrary matter density profile in the adiabatic approximation.

$^*\theta_{12} = \theta_3, \theta_{13} = \theta_2, \theta_{23} = \theta_1$

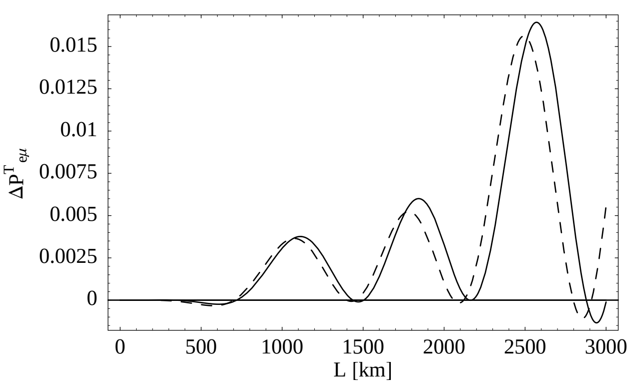
In the low energy regime ($\delta = \Delta m_{21}^2/(2E_\nu) \gtrsim V_{1,2}$):

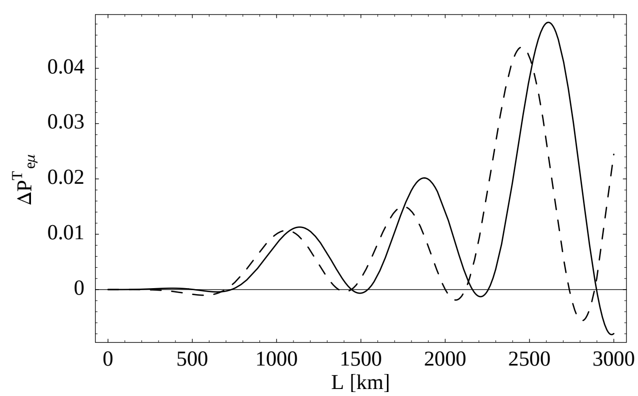
$$\Delta P_{\alpha\beta}^T \simeq \cos\delta_{CP} \cdot 8 \underbrace{s_{12}c_{12}s_{13}s_{23}c_{23}\frac{\sin(2\theta_1 - 2\theta_2)}{\sin 2\theta_{12}}}_{J_{\text{eff}}}$$
$$\times \{s_1 s_2 [Y - \cos(\Delta_1 L_1 + \Delta_2 L_2)]\}$$
$$+ \sin\delta_{CP} \cdot 4 s_{13}s_{23}c_{23}$$
$$\times X_1 [Y - \cos(\Delta_1 L_1 + \Delta_2 L_2)]$$

$\cos\delta_{CP}$ term: matter-induced T violation

$\sin\delta_{CP}$ term: fundamental T violation

$\Delta P_{e\mu}^T = \Delta P_{e\mu}^T(L)$:

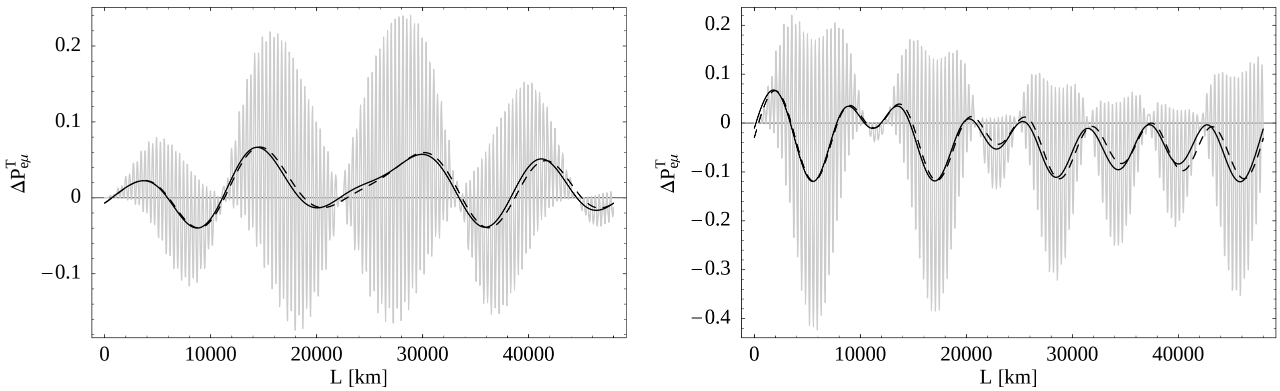




$L_1 = L_2 = L/2; \quad \rho_1 = 1\,\text{g/cm}^3,\ \rho_2 = 3\,\text{g/cm}^3$

solid curve - analytic result   dashed curve - numerical result

$\Delta P_{e\mu}^T = \Delta P_{e\mu}^T(L)$:



$L_1 = L_2 = L/2; \quad \rho_1 = 0, \ \rho_2 = 6.4\,\mathrm{g/cm^3}$

grey curves - analytic results

black solid and dashed curves - result averaged over the fast oscillations of the analytic and numerical calculation, respectively

$\Rightarrow$ Oscillations governed by the large $\Delta m^2_{\mathrm{atm}} = \Delta m^2_{31}$ are very fast!

Left plot: Same parameter values as in Fig. 3a of P.M. Fishbane and P. Kaus, PLB **506**, 275 (2001), `hep-ph/0012088`.

Right plot: Larger values of θ_{13} and Δm^2_{21}.

E. Akhmedov, P. Huber, M. Lindner, and T. Ohlsson, NPB **608**, 394 (2001), `hep-ph/0105029`.

Summary & Conclusions

- ✔ Complex interplay between fundamental T violation and matter-induced T violation!
  In vacuum: T violation correlated to CP violation
  $$\Delta P^{CP}_{\alpha\beta} + \Delta P^{T}_{\bar\alpha\bar\beta} = \Delta P^{CPT}_{\alpha\beta} \equiv 0 \quad \Rightarrow \quad \Delta P^{T}_{\alpha\beta} = -\Delta P^{CP}_{\bar\alpha\bar\beta}$$

- ✔ Approximative analytical formulas for T-odd probability differences $\Delta P^{T}_{\alpha\beta}$ have been derived!
  Arbitrary matter density profile

- ✔ T-violating effects can be considered as a measure of genuine three-flavorness!

- ✔ For terrestrial experiments matter-induced T-violating effects can safely be ignored!

- ✔ Asymmetric matter effects cannot hinder the determination of the fundamental CP and T-violating phase δ_{CP} in LBL exp.!

Reference

☞ E. Akhmedov, P. Huber, M. Lindner, and T. Ohlsson, Nucl. Phys. B **608**, 394 (2001), `hep-ph/0105029`.