Flavour-Symmetry Strategies to Extract γ

Robert Fleischer
DESY Hamburg, Theory Group

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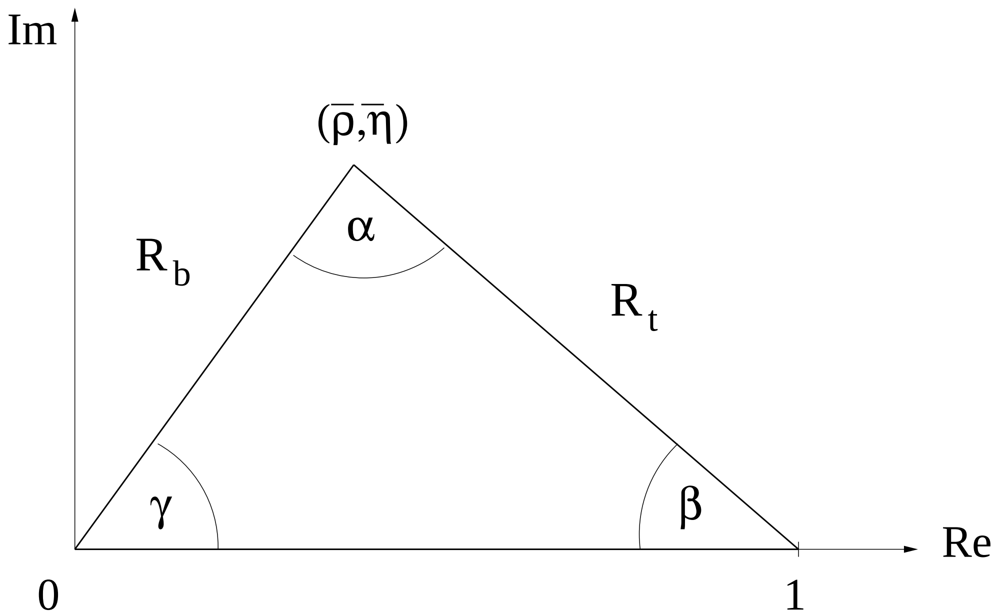
- Setting the Stage
- Isospin + $SU(3)$ + Dynamics: γ from $B \to \pi K, \pi\pi$
- U-Spin Strategies:
  - Focus on the following systems:
    - $B_d \to \pi^+\pi^-$, $B_s \to K^+K^-$
    - $B_{(s)} \to \pi K$.
- Conclusions

Setting the Stage

Preliminaries

- Central target of CP-B studies: $\Rightarrow$ UT



$$\overline{\rho} \equiv \left(1 - \lambda^2/2\right) \rho, \quad \overline{\eta} \equiv \left(1 - \lambda^2/2\right) \eta$$

- Particularly interesting element for tests of KM picture:

Direct determination of γ

$\longrightarrow$ Comparison between different approaches.

$\longrightarrow$ Comparison with UT fits, yielding $\gamma \sim 60^\circ$.

Key Problem in Determination of γ

- Wolfenstein (LO): $\Rightarrow$ CKM elements real, apart from

$$\boxed{V_{td} = |V_{td}|e^{-i\beta} \quad \text{and} \quad V_{ub} = |V_{ub}|e^{-i\gamma}.}$$

- Sensitivity to γ due to interference effects between different CKM amplitudes in non-leptonic B decays:

$$\text{CKM unitarity} \;\Rightarrow\; \left\{ \begin{array}{l} A(\overline{B} \to \overline{f}) = |A_1|e^{i\delta_1} + e^{-i\gamma}|A_2|e^{i\delta_2} \\ A(B \to f) = |A_1|e^{i\delta_1} + e^{+i\gamma}|A_2|e^{i\delta_2}, \end{array} \right.$$

where γ enters through V_{ub} and $|A_{1,2}|e^{i\delta_{1,2}}$ CP-conserving "strong" amplitudes ($\longrightarrow$ hadron dynamics!?):

$\Rightarrow$ "direct" CP asymmetry:

$$\mathcal{A}_{\rm CP} = \frac{|A(B \to f)|^2 - |A(\overline{B} \to \overline{f})|^2}{|A(B \to f)|^2 + |A(\overline{B} \to \overline{f})|^2}$$

$$= \frac{2|A_1||A_2|\sin(\delta_1 - \delta_2)\sin\gamma}{|A_1|^2 + 2|A_1||A_2|\cos(\delta_1 - \delta_2)\cos\gamma + |A_2|^2}.$$

- Goal: extraction of γ from $\mathcal{A}_{\rm CP}$!

- Problem: hadronic uncertainties due to $|A_{1,2}|e^{i\delta_{1,2}}$:

$$|A|e^{i\delta} \sim \sum_k \underbrace{C_k(\mu)}_{\text{pert. QCD}} \times \underbrace{\langle \overline{f}|Q_k(\mu)|\overline{B}\rangle}_{\text{"unknown"}}.$$

Major Approaches to Extract γ

- Try to calculate $\langle \overline{f} | Q_k(\mu) | \overline{B} \rangle$: interesting progress $\longrightarrow$
  - "QCD factorization" [Beneke *et al.*]
  - "PQCD" [Li *et al.*].

  $B \to \pi K, \pi\pi$

- Use decays of neutral B_d- or B_s-mesons:

  $\Rightarrow$ Interference effects due to B_q^0–$\overline{B_q^0}$ mixing!

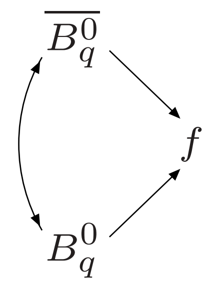
  - Fortunate cases, where hadronic matrix elements cancel:

    * $B_d \to D^{(*)\pm} K^{\mp}$ $\Rightarrow$ $\underbrace{2\beta}_{B_d \to \psi K_S} + \gamma$

      [Dunietz & Sachs ...]

    * $B_s \to D_s^{(*)\pm} K^{(*)\mp}$ $\Rightarrow$ $\underbrace{\phi_s}_{B_s \to \psi\phi} + \gamma$

      [Aleksan, Dunietz & Kayser ...]



- Amplitude relations to eliminate the hadronic uncertainties:
  - Exact relations: [Gronau & Wyler; Dunietz; R.F. & Wyler ...]

    "Tree" decays $B \to KD$ or $B_c \to D_s D$.

  - Flavour symmetries, i.e. isospin, $SU(3)$ or U-spin:

    $B_{(s)} \to \pi\pi, \pi K, KK$ $\longrightarrow$ Focus of this talk!

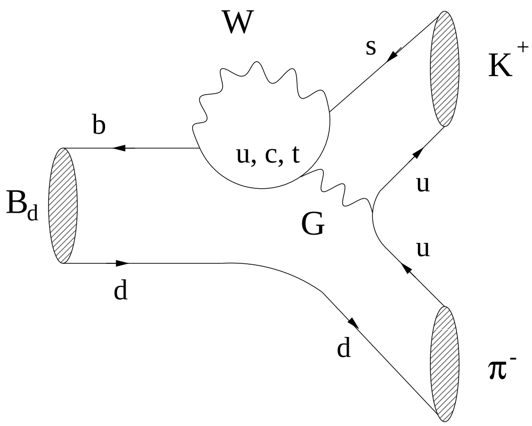
Isospin + $SU(3)$ + Dynamics:

γ from $B \to \pi K, \pi\pi$

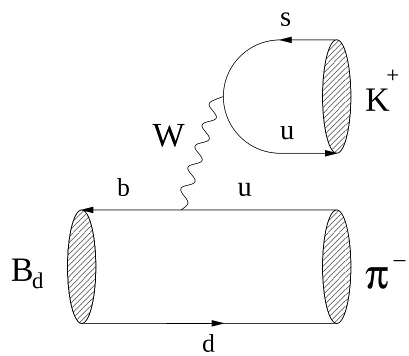
Basic Features of $B \to \pi K$ Decays

- $B \to \pi K$ decays are governed by QCD penguins:

  - Example: $B_d^0 \to \pi^- K^+$



"penguins"



"tree"

  - $|V_{us}V_{ub}^*/(V_{ts}V_{tb}^*)| \approx 0.02 \quad \Rightarrow \quad$ penguins dominate!

- Rôle of EW penguins (large top-quark mass!):

  - $B_d^0 \to \pi^- K^+$, $B^+ \to \pi^+ K^0$:

    contribute in colour-suppressed form and are expected to play a minor rôle: "factorization" $\to \mathcal{O}(1\%)$.

  - $B^+ \to \pi^0 K^+$, $B_d^0 \to \pi^0 K^0$:

    contribute also in colour-allowed form and may compete with tree-diagram-like topologies $\to \mathcal{O}(20\%)$!

- $SU(2)$ isospin relations:

$$\sqrt{2}A(B^+ \to \pi^0 K^+) + A(B^+ \to \pi^+ K^0)$$

$$= \sqrt{2}A(B_d^0 \to \pi^0 K^0) + A(B_d^0 \to \pi^- K^+)$$

$$= -\Big[\underbrace{|T+C|e^{i\delta_{T+C}}e^{i\gamma}}_{\text{Trees}} + \underbrace{(P_{\rm ew} + P_{\rm ew}^{\rm C})}_{\text{EW Penguins}}\Big] \propto \Big[e^{i\gamma} + q_{\rm ew}\Big]\,.$$

  – Amplitude relation with analogous phase structure also for the "mixed" $B^+ \to \pi^+ K^0$, $B_d^0 \to \pi^- K^+$ system.

- Combinations of $B \to \pi K$ decays to probe γ:

  – $B^\pm \to \pi^\pm K$, $B_d \to \pi^\mp K^\pm$ ("mixed")

    [R.F. ('95); R.F. & Mannel ('97); Gronau & Rosner ('98)]

  – $B^\pm \to \pi^\pm K$, $B^\pm \to \pi^0 K^\pm$ ("charged")

    [Gronau, Rosner, London ('94); Neubert, Rosner; Buras, R.F. ('98)]

  – $B_d \to \pi^0 K$, $B_d \to \pi^\mp K^\pm$ ("neutral")

    [Buras & R.F. ('98–'00)]

- Interestingly, already CP-averaged branching ratios may lead to highly non-trivial constraints on γ.

  [R.F. & Mannel ('97); Neubert & Rosner ('98)]

Extracting γ from $B \to \pi K$ Decays

- Key observables:

$$\left\{\begin{array}{c} R \\ A_0 \end{array}\right\} \equiv \left[\frac{\text{BR}(B_d^0 \to \pi^- K^+) \pm \text{BR}(\overline{B_d^0} \to \pi^+ K^-)}{\text{BR}(B^+ \to \pi^+ K^0) + \text{BR}(B^- \to \pi^- \overline{K^0})}\right] \frac{\tau_{B^+}}{\tau_{B_d^0}}$$

$$\left\{\begin{array}{c} R_{\rm c} \\ A_0^{\rm c} \end{array}\right\} \equiv 2\left[\frac{\text{BR}(B^+ \to \pi^0 K^+) \pm \text{BR}(B^- \to \pi^0 K^-)}{\text{BR}(B^+ \to \pi^+ K^0) + \text{BR}(B^- \to \pi^- \overline{K^0})}\right]$$

$$\left\{\begin{array}{c} R_{\rm n} \\ A_0^{\rm n} \end{array}\right\} \equiv \frac{1}{2}\left[\frac{\text{BR}(B_d^0 \to \pi^- K^+) \pm \text{BR}(\overline{B_d^0} \to \pi^+ K^-)}{\text{BR}(B_d^0 \to \pi^0 K^0) + \text{BR}(\overline{B_d^0} \to \pi^0 \overline{K^0})}\right].$$

- Employing the $SU(2)$ flavour symmetry and dynamical assumptions, concerning mainly the smallness of FSI:

$$\boxed{R_{\rm (c,n)},\ A_0^{\rm (c,n)} = \text{functions}\left(q_{\rm (c,n)}, r_{\rm (c,n)}, \delta_{\rm (c,n)}, \gamma\right).}$$

- Here the following variables are involved:
  - $q_{\rm (c,n)}$: ratio of EW penguins to "trees".
  - $r_{\rm (c,n)}$: ratio of "trees" to QCD penguins.
  - $\delta_{\rm (c,n)}$: strong phase between "trees" and QCD penguins.

[Buras & R.F. ('98); alternative parametrization: Neubert ('98)]

- The $q_{(\mathrm{c,n})}$ can be fixed through theoretical arguments:

  - $B^\pm \to \pi^\pm K$, $B_d \to \pi^\mp K^\pm$: $q \approx 0$, as EW penguins contribute only in colour-suppressed form.
    [R.F. ('95); R.F. & Mannel ('97); Gronau & Rosner ('98)]

  - $B^\pm \to \pi^\pm K$, $B^\pm \to \pi^0 K^\pm$: q_c can be fixed through the $SU(3)$ flavour symmetry (no dynamics !).
    [Neubert & Rosner (1998)]

  - $B_d \to \pi^0 K$, $B_d \to \pi^\mp K^\pm$: q_n can also be fixed through the $SU(3)$ flavour symmetry (no dynamics !).
    [Buras & R.F. (1998)]

- The $r_{(\mathrm{c,n})}$ can be fixed as follows:

  - $B^\pm \to \pi^\pm K$, $B_d \to \pi^\mp K^\pm$: r can be fixed using "factorization" or $B_s \to \pi K$ modes.
    [R.F. ('95); Gronau & Rosner ('98,'00); Beneke et al. ('01)]

  - $B^\pm \to \pi^\pm K$, $B^\pm \to \pi^0 K^\pm$: r_c can be fixed from the $B^+ \to \pi^+\pi^0$ branching ratio by using the $SU(3)$ flavour symmetry (no dynamics !).
    [Gronau, Rosner & London (1994)]

  - $B_d \to \pi^0 K$, $B_d \to \pi^\mp K^\pm$: r_n can also be fixed through $SU(3)$ from $B^+ \to \pi^+\pi^0$ (no dynamics !).
    [Buras & R.F. (1998)]

- Uncertainties can be reduced through "QCD factorization".

  [Beneke, Buchalla, Neubert & Sachrajda (2001)]

Comments on Rescattering Effects

- Whereas the determination of q and r as sketched above may be affected by rescattering effects, this is not the case for the $q_{\rm c,n}$ and $r_{\rm c,n}$, since here $SU(3)$ suffices.

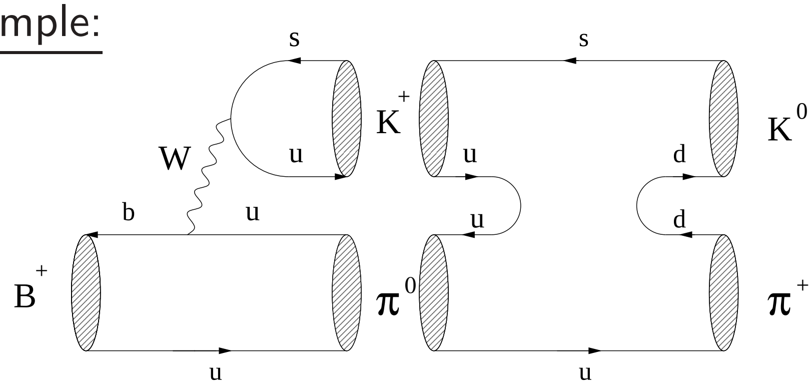
- Nevertheless, we have to assume that $B^+ \to \pi^+ K^0$ or $B_d \to \pi^0 K$ do not involve a CP-violating weak phase:

$$A(B^+ \to \pi^+ K^0) = -\,|\tilde{P}|e^{i\delta_{\tilde{P}}} = A(B^- \to \pi^- \overline{K^0}).$$

- This relation may be affected by rescattering processes:

$$A(B^+ \to \pi^+ K^0) = -\,|\tilde{P}|e^{i\delta_{\tilde{P}}}\left[1 + \underbrace{\rho_{\rm c}\, e^{i\theta_{\rm c}}}_{\propto \lambda^2 R_b}\, e^{i\gamma}\right].$$

  – Example:



- Can be taken into account through additional input, i.e. $SU(3)$ and data on $B^\pm \to K^\pm K$. In the case of the neutral strategy, rescattering processes can be included in an exact manner with the help of $\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to \pi^0 K_{\rm S})$.

- "QCD factorization" is in favour of small effects!

Back to the Determination of γ ...

- Observables:

$$\left.\begin{array}{l} R_{(\mathrm{c,n})}\left(q_{(\mathrm{c,n})}, r_{(\mathrm{c,n})}, \delta_{(\mathrm{c,n})}, \gamma\right) \\ A_0^{(\mathrm{c,n})}\left(q_{(\mathrm{c,n})}, r_{(\mathrm{c,n})}, \delta_{(\mathrm{c,n})}, \gamma\right) \end{array}\right\} \Rightarrow$$

$$\boxed{\delta_{(\mathrm{c,n})} = \delta_{(\mathrm{c,n})}\left(q_{(\mathrm{c,n})}, r_{(\mathrm{c,n})}\right), \quad \gamma = \gamma\left(q_{(\mathrm{c,n})}, r_{(\mathrm{c,n})}\right).}$$

- Interesting constraints on γ already from $R_{(\mathrm{c,n})}$:

  - $\delta_{(\mathrm{c,n})}$ suffers from large hadronic uncertainties!

  - However, we can get rid of $\delta_{(\mathrm{c,n})}$ by keeping it as a "free" variable, yielding minimal and maximal values for $R_{(\mathrm{c,n})}$:

$$\left.R_{(\mathrm{c,n})}^{\mathrm{ext}}\right|_{\delta_{(\mathrm{c,n})}} = \text{function}\left(q_{(\mathrm{c,n})}, r_{(\mathrm{c,n})}, \gamma\right).$$

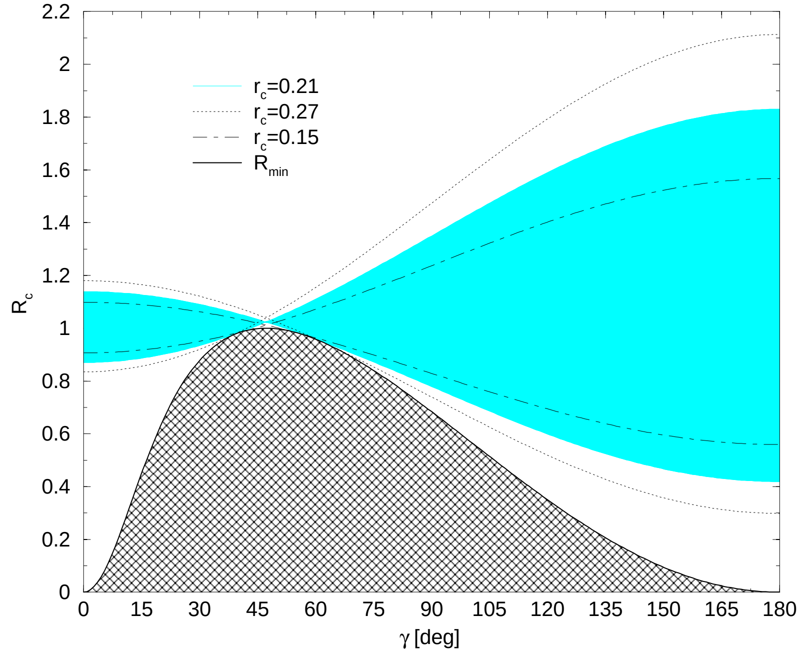
  - Keeping, in addition, $r_{(\mathrm{c,n})}$ as a "free" variable, we obtain another – less restrictive – minimal value for $R_{(\mathrm{c,n})}$:

$$\left.R_{(\mathrm{c,n})}^{\mathrm{min}}\right|_{\delta_{(\mathrm{c,n})}, r_{(\mathrm{c,n})}} = \text{function}\left(q_{(\mathrm{c,n})}, \gamma\right)\sin^2\gamma.$$

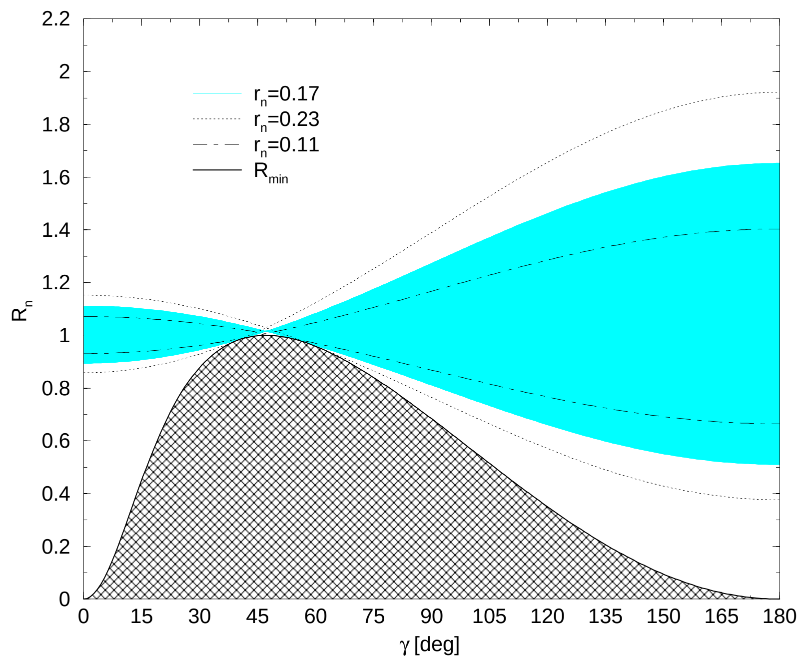
  - These extremal values of $R_{(\mathrm{c,n})}$ imply constraints on γ, as the following cases are excluded:

$$R_{(\mathrm{c,n})}^{\mathrm{exp}} < R_{(\mathrm{c,n})}^{\mathrm{min}}, \quad R_{(\mathrm{c,n})}^{\mathrm{exp}} > R_{(\mathrm{c,n})}^{\mathrm{max}}.$$

- Dependence of extremal values of R_c on γ ($q_c = 0.68$):



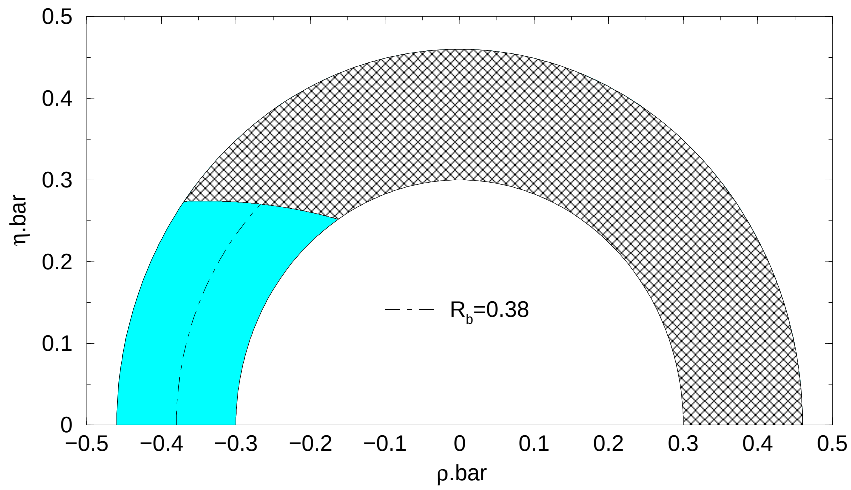
- Dependence of extremal values of R_n on γ ($q_n = 0.68$):



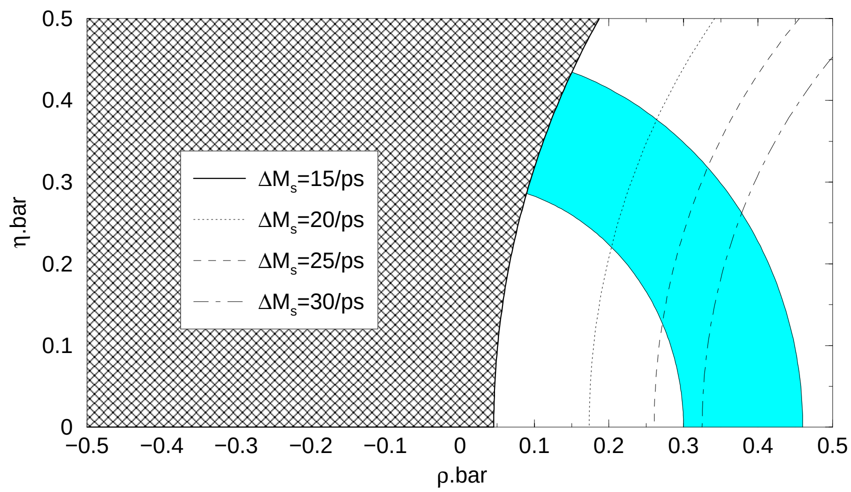
Observable	CLEO ('00)	BaBar ('01)	Belle ('01)
R	1.00 ± 0.30	0.97 ± 0.23	1.50 ± 0.66
R_c	1.27 ± 0.47	1.19 ± 0.35	2.38 ± 1.12
R_n	0.59 ± 0.27	1.02 ± 0.40	0.60 ± 0.29

May Arrive at Puzzling Situation!

- Constraints in the $\overline{\varrho}$–$\overline{\eta}$ plane: note that $q_{\rm c,n} \propto 1/R_b$!
  - Example: $R_{\rm n} = 0.6$, $r_{\rm n} = 0.17 \Rightarrow$



  - Impact of lower bound on ΔM_s: $\Rightarrow \gamma < 90^\circ$!



- In addition to $\gamma > 90^\circ$, as indicated by $R_{\rm c}$ and $R_{\rm n}$, CLEO & Belle may point towards another "puzzle":

$$\cos\delta_{\rm c} > 0 \text{ and } \cos\delta_{\rm n} < 0!$$

[Buras & R.F. (2000)]

Towards Calculations of $B \to \pi K, \pi\pi$

- Interesting theoretical progress:
  - "QCD factorization" [Beneke *et al.*]
  - "PQCD" [Li *et al.*].

- "QCD factorization" formula of the following structure:

$$A(\overline{B} \to M_1 M_2) = \langle M_2 | j_2 | 0 \rangle \langle M_1 | j_1 | \overline{B} \rangle \left[1 + \mathcal{O}(\alpha_s) + \mathcal{O}(\Lambda/m_b)\right]$$

  - $\mathcal{O}(\alpha_s)$ can be calculated in a systematic way.
  - $\mathcal{O}(\Lambda/m_b)$ represent major limitation!

[Beneke, Buchalla, Neubert & Sachrajda (1999–2001)]

- Detailed recent analysis: [Beneke *et al.*, hep-ph/0104110]
  - QCD factorization allows a reduction of the theoretical uncertainties of $r_{\rm c,n}$ and $q_{\rm c,n}$ to the level of

$$\mathcal{O}\left(\frac{1}{N_{\rm C}} \times \frac{m_s - m_d}{\Lambda} \times \frac{\Lambda}{m_b}\right) = \mathcal{O}\left(\frac{1}{N_{\rm C}} \times \frac{m_s - m_d}{m_b}\right).$$

  - Complementary approaches to probe γ, making more extensive use of QCD factorization:
    - Rôle of Λ/m_b corrections: $\longrightarrow$ hot topic:

      "Charming" penguins ... [Ciuchini *et al.*, hep-ph/0104126]

U-Spin Strategies

... employ U-spin-related B decays:

$$d \leftrightarrow s$$

["Prehistory": Dunietz, *Snowmass '93 proceedings*; Lipkin (1997); Buras, R.F. & Mannel (1997); Falk, Kagan, Nir and Petrov (1997); ...]

- $B_{s(d)} \to \psi K_{\rm S}$, $B_{d(s)} \to D^+_{d(s)} D^-_{d(s)}$ or $K^0 \overline{K^0}$: $\Rightarrow \gamma$
  [R.F. (1999)]

- $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$: $\Rightarrow \beta$ and γ
  [R.F. (1999–2000)]

- Strategies employing angular distributions: $\Rightarrow \beta, \gamma, \delta\gamma$
  [R.F. (1999)]

- $B_d \to \pi^\mp K^\pm$ and $B_s \to \pi^\pm K^\mp$ + $B^\pm \to \pi^\pm K$: $\Rightarrow \gamma$
  [Gronau & Rosner (2000); Chiang & Wolfenstein (2000)]

- $B_{s(d)} \to J/\psi\, \eta$: $\Rightarrow \gamma$
  [Skands (2000)]

Extracting β and γ from

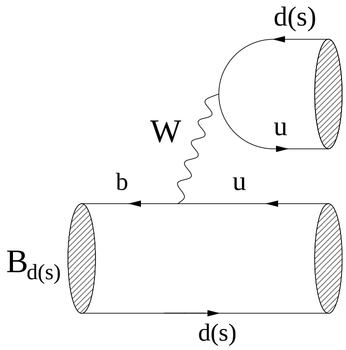
$B_d \to \pi^+\pi^-$

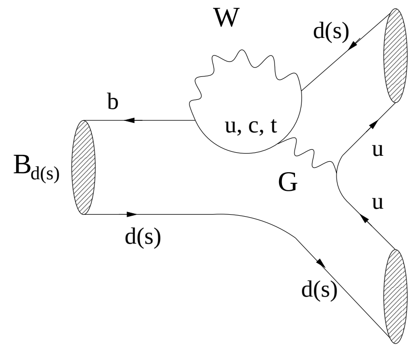
and

$B_s \to K^+K^-$

[R.F., *PLB* **459** (1999) 306; *EPJC* **16** (2000) 87]

The $B_d \to \pi^+\pi^-$, $B_s \to K^+K^-$ System





$\lambda_u^{d(s)} \equiv V_{ud(s)}V_{ub}^*$ $\lambda_j^{d(s)} \equiv V_{jd(s)}V_{jb}^*$ $(j \in \{u, c, t\})$

- Structure of decay amplitudes:

$$A(B_d^0 \to \pi^+\pi^-) = \lambda_u^d\left(A_{\rm tree}^u + A_{\rm pen}^u\right) + \lambda_c^d A_{\rm pen}^c + \lambda_t^d A_{\rm pen}^t$$

$$A(B_s^0 \to K^+K^-) = \lambda_u^s\left(A_{\rm tree}^{u'} + A_{\rm pen}^{u'}\right) + \lambda_c^s A_{\rm pen}^{c'} + \lambda_t^s A_{\rm pen}^{t'}.$$

- Unitarity of CKM matrix: $\lambda_t^q = -\lambda_u^q - \lambda_c^q \quad \Rightarrow$

$$A(B_d^0 \to \pi^+\pi^-) = \mathcal{C}\left[e^{i\gamma} - de^{i\theta}\right]$$

$$A(B_s^0 \to K^+K^-) = \lambda\mathcal{C}'\left[e^{i\gamma} + \left(\frac{1-\lambda^2}{\lambda^2}\right)d'e^{i\theta'}\right]$$

$$de^{i\theta} = \left.\frac{\text{“Pen”}}{\text{“Tree”}}\right|_{B_d\to\pi^+\pi^-}, \quad d'e^{i\theta'} = \left.\frac{\text{“Pen”}}{\text{“Tree”}}\right|_{B_s\to K^+K^-}.$$

[d, d': real “hadronic” numbers; θ, θ': CP-conserving strong phases]

- CP asymmetries:

$$a_{\rm CP}(B_q(t)\to f) = \left[\frac{\mathcal{A}_{\rm CP}^{\rm dir}\cos(\Delta M_q t) + \mathcal{A}_{\rm CP}^{\rm mix}\sin(\Delta M_q t)}{\cosh(\Delta\Gamma_q t/2) - \mathcal{A}_{\Delta\Gamma}\sinh(\Delta\Gamma_q t/2)}\right]$$

- CP-violating observables:

$$\mathcal{A}_{\rm CP}^{\rm dir}(B_d\to\pi^+\pi^-) = \text{function}(d,\theta,\gamma)$$
$$\mathcal{A}_{\rm CP}^{\rm mix}(B_d\to\pi^+\pi^-) = \text{function}(d,\theta,\gamma,\phi_d=2\beta)$$

$$\mathcal{A}_{\rm CP}^{\rm dir}(B_s\to K^+K^-) = \text{function}(d',\theta',\gamma)$$
$$\mathcal{A}_{\rm CP}^{\rm mix}(B_s\to K^+K^-) = \text{function}(d',\theta',\gamma,\underbrace{\phi_s\approx 0}_{B_s\to\psi\phi}).$$

- $B_d\to\pi^+\pi^-$ and $B_s\to K^+K^-$ are related to each other by interchanging all down and strange quarks:

U-spin symmetry $\Rightarrow$ $d=d'$, $\theta=\theta'$.

$\Rightarrow$ 4 observables, depending on 4 unknowns:

d, θ, $\phi_d=2\beta$, γ,

i.e. these quantities can be determined!

- No dynamical assumptions required, only U spin!

Minimal Use of the U-Spin Symmetry

- The use of the U-spin-symmetry arguments can be minimized, if we employ also $\phi_d = 2\beta$ as an input:

  – $\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^+\pi^-)$ and $\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to \pi^+\pi^-)$ allow us then to eliminate the strong phase θ:

$$\Rightarrow \quad \boxed{d = d(\gamma)}$$

  – $\mathcal{A}_{\rm CP}^{\rm dir}(B_s \to K^+K^-)$ and $\mathcal{A}_{\rm CP}^{\rm mix}(B_s \to K^+K^-)$ allow us to eliminate the strong phase θ' in an analogous way:

$$\Rightarrow \quad \boxed{d' = d'(\gamma)}$$

- The corresponding contours in the γ–d and γ–d' planes can be determined in a theoretically clean way!

- γ and d, θ, θ' can now be extracted with the help of

$$\boxed{d' = d}$$

- Example:

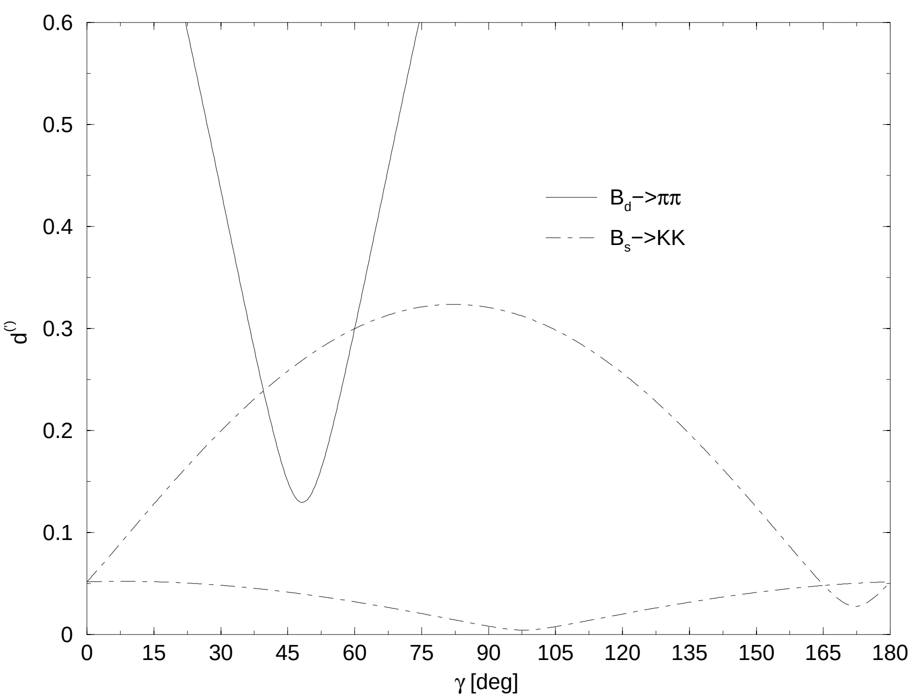
  - Input parameters:

    * negligible B_s^0–$\overline{B_s^0}$ mixing phase, i.e. $\phi_s = 0$
    * $2\beta = 44^\circ$, $\gamma = 60^\circ$, $d = d' = 0.3$, $\theta = \theta' = 210^\circ$

  - Output for the observables:

    * $B_d \to \pi^+\pi^-$: $\mathcal{A}_{\rm CP}^{\rm dir} = +19\%$, $\mathcal{A}_{\rm CP}^{\rm mix} = +62\%$
    * $B_s \to K^+K^-$: $\mathcal{A}_{\rm CP}^{\rm dir} = -17\%$, $\mathcal{A}_{\rm CP}^{\rm mix} = -27\%$.

  - Contours in the γ–d and γ–d' planes:



- Experimental accuracy of $\mathcal{O}(10^\circ)$ and $\mathcal{O}(1^\circ)$ for γ at Tevatron-II and BTeV/LHC, respectively

$\Rightarrow$ **very promising!**

U-spin-breaking Effects

- Interestingly, $d'e^{i\theta'} = de^{i\theta}$ does not depend on decay constants and form factors, and is not affected by U-spin-breaking corrections within the "BSS mechanism":

Strengthens confidence into $d'e^{i\theta'} = de^{i\theta}$!

- Moreover, experimental insights:

  – In addition to γ, $d = d'$, also θ, θ' can be determined:

  First consistency check is provided by $\theta' \stackrel{?}{=} \theta$.

  – Moreover, "normalization" factors $|\mathcal{C}|$ and $|\mathcal{C}'|$ can be determined from the CP-averaged branching ratios:

$$\left|\frac{\mathcal{C}'}{\mathcal{C}}\right|_{\rm fact} = \underbrace{\left[\frac{f_K}{f_\pi}\right]}_{\text{decay constants}} \times \underbrace{\left[\frac{F_{B_sK}(M_K^2;0^+)}{F_{B_d\pi}(M_\pi^2;0^+)}\right]}_{\text{form factors}}.$$

  – Another interesting implication of $d'e^{i\theta'} = de^{i\theta}$:

$$\left[\frac{\mathcal{A}_{\rm CP}^{\rm dir}(B_s \to K^+K^-)}{\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^+\pi^-)}\right] = -\left|\frac{\mathcal{C}'}{\mathcal{C}}\right|^2 \left[\frac{\text{BR}(B_d \to \pi^+\pi^-)}{\text{BR}(B_s \to K^+K^-)}\right]\frac{\tau_{B_s}}{\tau_{B_d}}.$$

  – Similar relations between other U-spin-related B decays and further experimental tests ...

  [R.F. (1999); Gronau (2000)]

Some Interesting Constraints

- Useful quantity:

$$\mathcal{K} \equiv \frac{1}{\epsilon}\left|\frac{\mathcal{C}'}{\mathcal{C}}\right|^2 \left[\frac{\text{BR}(B_d\to\pi^+\pi^-)}{\text{BR}(B_s\to K^+K^-)}\right]\frac{\tau_{B_s}}{\tau_{B_d}}$$

  - Parametrizations given above, and $d'e^{i\theta'} = de^{i\theta}$:

$$\Rightarrow \mathcal{K} = \frac{1-2d\cos\theta\cos\gamma+d^2}{\epsilon^2+2\epsilon d\cos\theta\cos\gamma+d^2}, \quad \text{with} \quad \epsilon\equiv\frac{\lambda^2}{1-\lambda^2}.$$

  - Allows us to determine $C\equiv\cos\theta\cos\gamma$ as function of d:

$$-1\leq C\leq +1 \quad\Rightarrow\quad \text{constraints on } d \text{ and } \mathcal{A}_{\text{CP}}^{\text{dir}}!$$

- $B_s\to K^+K^-$ not accessible at $\Upsilon(4S)$ $\Rightarrow$ $B_d\to\pi^\mp K^\pm$:

  - $SU(3)$ flavour symmetry & dynamical assumptions: $\Rightarrow$

$$\mathcal{A}_{\text{CP}}^{\text{dir}}(B_s\to K^+K^-)\approx\mathcal{A}_{\text{CP}}^{\text{dir}}(B_d\to\pi^\mp K^\pm)$$

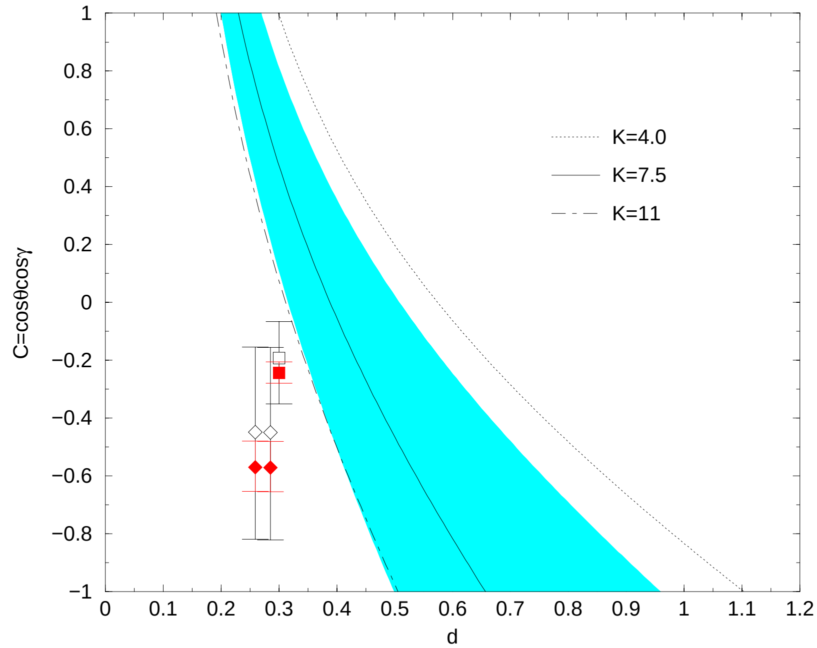
$$\text{BR}(B_s\to K^+K^-)\approx\text{BR}(B_d\to\pi^\mp K^\pm)\,\frac{\tau_{B_s}}{\tau_{B_d}}.$$

  - Determination of $\mathcal{K}$ [data reported in spring 2001]:

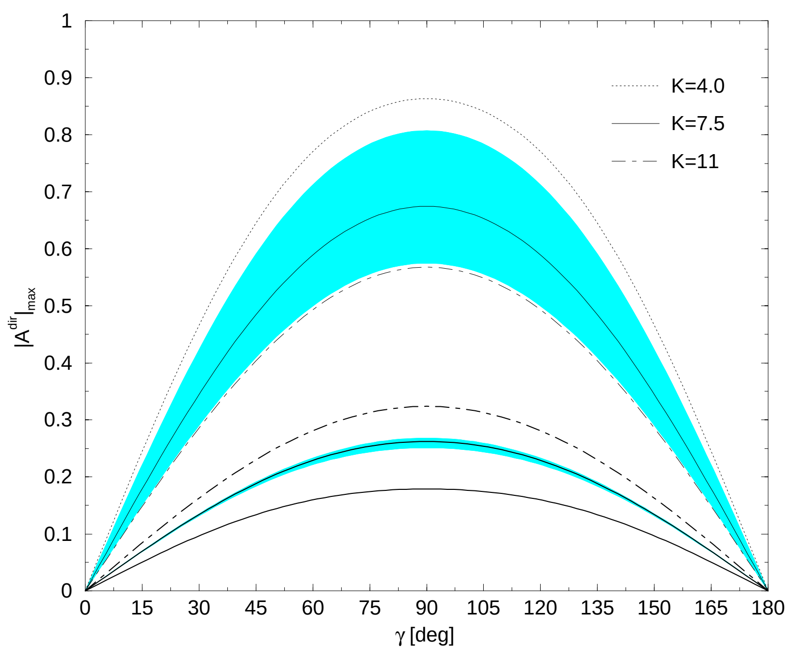
$$\mathcal{K}\approx\frac{1}{\epsilon}\left(\frac{f_K}{f_\pi}\right)^2\left[\frac{\text{BR}(B_d\to\pi^+\pi^-)}{\text{BR}(B_d\to\pi^\mp K^\pm)}\right] = \begin{cases} 7.3\pm2.9 & \text{(CLEO)}\\ 7.2\pm2.3 & \text{(BaBar)}\\ 8.5\pm3.7 & \text{(Belle).}\end{cases}$$

[Details: R.F., *Eur. Phys. J.* **C16** (2000) 87]

- $C = \cos\theta\cos\gamma$ as a function of d:



- The maximal direct CP asymmetries for $B_d \to \pi^+\pi^-$ (upper curves) and $B_s \to K^+K^- \approx B_d \to \pi^\mp K^\pm$:

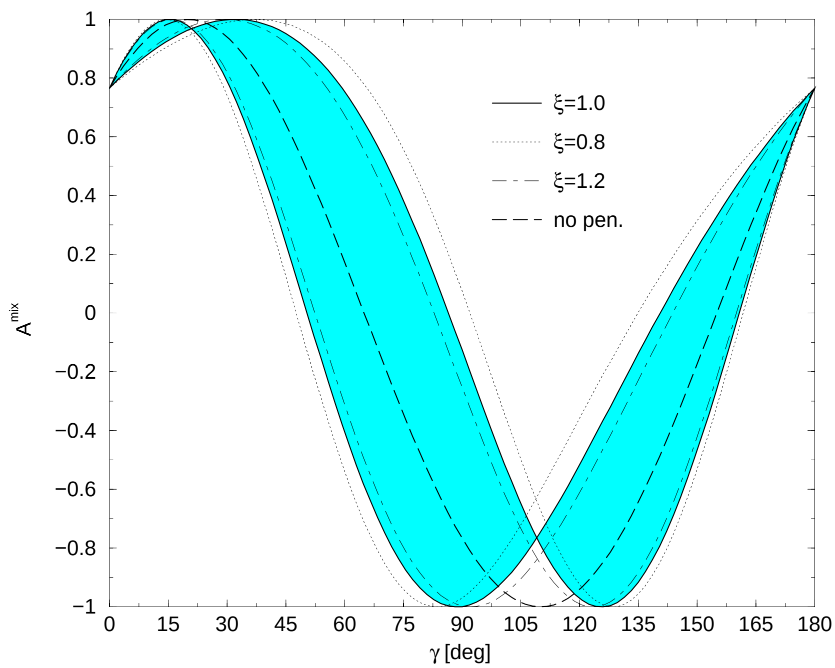


- Shaded regions: $\xi_d \equiv d'/d \in [0.8, 1.2]$ for $\mathcal{K} = 7.5$.

What about $\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to \pi^+\pi^-)$?

- In the following, we assume that ϕ_d has been measured through the "gold-plated" mode $B_d \to J/\psi\, K_{\rm S}$.

- Using $\cos\theta = C/\cos\gamma$ to eliminate θ, extremal values of $\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to \pi^+\pi^-)$ can be obtained as a function of γ:



$\mathcal{K} = 7.5,$

$\phi_d = 50^\circ$

  - For given γ, the allowed range for $\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to \pi^+\pi^-)$ is usually very large.

  - On the other hand, a measurement of $\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to \pi^+\pi^-)$ would imply a rather restricted range for γ!

- If in addition to $\mathcal{K}$ and $\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to \pi^+\pi^-)$ also direct CP violation in $B_d \to \pi^+\pi^-$ or $B_d \to \pi^\mp K^\pm$ is measured, γ and d, θ can be *determined*.

Extraction of γ

from

$B_{(s)} \to \pi K$ Decays

[M. Gronau and J. Rosner, *Phys. Lett.* **B482** (2000) 71]

The $B_{(s)} \to \pi K$ System

- Another interesting U-spin pair:

$$\boxed{B_d^0 \to \pi^- K^+ \text{ and } B_s^0 \to \pi^+ K^-.}$$

- Amplitudes in the strict U-spin limit: $\left[\epsilon = \lambda^2/(1-\lambda^2)\right]$

$$A(B_d^0 \to \pi^- K^+) = -P\left(1 - re^{i\delta}e^{i\gamma}\right)$$

$$A(B_s^0 \to \pi^+ K^-) = P\sqrt{\epsilon}\left(1 + \frac{1}{\epsilon} re^{i\delta}e^{i\gamma}\right).$$

- At first sight: 3 observables, depending on γ, r, δ.
- However, only 2 of them independent!
- Consequently, further information required:
  - Assuming both negligible rescattering effects and colour-suppressed EW penguins, we obtain

    $P = A(B^+ \to \pi^+ K^0) \;\Rightarrow$ 3 independent observables:

$$\left.\begin{array}{l} A_0 = -A_s = 2r\sin\delta\sin\gamma \\ R = 1 - 2r\cos\delta\cos\gamma + r^2 \\ R_s = \epsilon - 2r\cos\delta\cos\gamma + \frac{r^2}{\epsilon} \end{array}\right\} \Rightarrow \boxed{\gamma,\, r,\, \delta\,!}$$

  - Complements "mixed" $B \to \pi K$ approach (see above).

Conclusions

- There are many approaches to extract γ.

- Particularly promising strategies for B experiments:

  – $B \to \pi K, \pi\pi$ strategies: | e^+e^- B-factories |

    * Make use of flavour-symmetry relations and plausible dynamical assumptions.
    * Constraints on γ from CP-averaged branching ratios.
    * Data may point towards $\gamma > 90^\circ$ – in contrast to UT fits – and a puzzling situation for strong phases!?

  – U-Spin strategies: | hadron machines |

    * Several approaches!
    * Particularly promising systems:

$$B_d \to \pi^+\pi^-, B_s \to K^+K^- \quad \& \quad B_{(s)} \to \pi K.$$

- As a "by-product", also insights into hadron dynamics:

  Strong phases & penguin parameters!

- QCD factorization & PQCD: $\Rightarrow$

  reduction of theoretical uncertainties and complementary approaches to probe γ through $B \to \pi K, \pi\pi$ decays!