Q. Shafi

KALUZA-KLEIN EXCITATIONS, NEUTRINO OSCILLATIONS AND RARE PROCESSES

(with S. Huber
& C.K. Lee)

B & L Violation (within the SM framework)

These may arise from

(i) Non-perturbative effects; (highly suppressed except for $T \neq 0$)

(ii) Higher dimensional operators (suppressed by powers of $M_{Planck} \sim 10^{18}$ GeV).

From (ii), for instance, one can have

$$\mathcal{O} = qqq\ell \quad (\text{dimension six})$$

$$\Rightarrow \tau_p \gtrsim 10^{44} \text{ yrs.}$$

Thus, discovery of proton decay implies physics beyond the SM.

From dimension five operators

$$\left(\frac{1}{M_P}\right)\underbrace{L\ L}_{\text{lepton doublets}}\ \underbrace{H\ H}_{\text{Higgs doublet}}$$ (Majorana mass for 'Right' ν)

one expects neutrino masses $\lesssim 10^{-5}$ eV.

Thus, discovery of neutrino mass $\gg 10^{-5}$ eV implies the existence of new physics outside the SM. In particular, M_p above should be replaced by $M \lesssim 10^{14}$ GeV if we follow the path of grand unification (& we want $m_\nu \sim 0.1$ eV). (See-saw mechanism)

NEUTRINO OSCILLATIONS

Challenges:

(i) Origin of mass?

(ii) Dirac or Majorana?

(iii) Relations to Quark masses/mixings?

'Bimaximal' mixing versus $\theta_{quark} << 1$;

Third lepton mixing angle small;

Lepton mass ratios not as large as in quark sector;

EXTRA DIMENSION(S)

WHO NEEDS THEM?

- Unification of forces (Kaluza-Klein)

Consider 5 dimensional gravity

metric tensor $\rightarrow g_{AB}$, $A, B = 0, 1, \ldots, 4$

Dimensional reduction to $M_4 \times S^1$:

$$\underset{\substack{\uparrow \\ \text{graviton} \\ (\mu,\nu = 0,\ldots,3)}}{g_{\mu\nu}} \; , \quad \underset{\substack{\uparrow \\ \text{EM field (!)}}}{g_{\mu 4} \sim A_\mu} \, , \quad \underset{\substack{\uparrow \\ \text{scalar}}}{g_{44}}$$

- Electric charge quantized $\left(e \propto \frac{1}{M_{Pl} R_5}\right)$
- Monopoles

Einstein - Hilbert action in 5 dimensions:

$$S \sim \int d^5x \sqrt{-G} \left(\frac{1}{\kappa_5^2} R \right), \quad \kappa_5^2 = 16\pi G_5$$

Suppose extra spatial dimension is S^1

$S^1 : x^5 = x^5 + 2\pi R_5$ (Note: $M^4 \times S^1$ sol^n of vacuum field eq^{ns})

$$G_{MN}(x^\mu, x^5) = \sum_{n=-\infty}^{\infty} \underbrace{G^n_{MN}(x^\mu)}_{\text{KK modes}} \exp\left(\frac{i n x^5}{R_5}\right)$$

Consider $$G^n_{MN}(x^\rho) = \left(\begin{array}{c|c} g^n_{\mu\nu}(x^\rho) & g^n_{\mu 5}(x^\rho) \\ \hline g^n_{5\mu}(x^\rho) & g^n_{55}(x^\rho) \end{array} \right)$$

In terms of 4d we have

n = 0 special ∴ get massless modes

- sym tensors $g^n_{\mu\nu}$
- vector fields $g^n_{\mu 5}$
- scalar fields g^n_{55}

$\Big(0 = p_M p^M = p_\mu p^\mu - (p_5)^2 \Rightarrow m^2 = \frac{n^2}{R_5^2}$ for the n^{th} mode. $\Big)$

Low Energy Effective Theory

$$G_{MN} = \eta_{MN} + \kappa_5 h_{MN}$$

$$h_{MN} = \frac{1}{\sqrt{2\pi R_5}} \left(\begin{array}{c|c} h_{\mu\nu} + \eta_{\mu\nu}\phi & A_\nu \\ \hline A_\mu & 2\phi \end{array} \right)$$

One finds linearized 4d gravity
+ electromagnetism
+ 'massless' scalar ϕ

← Goldstone boson from spontaneous breaking by $M^4 \times S^1$ of classical global scale invarianc

$(\langle\phi\rangle \sim R_5)$

DEGREES OF FREEDOM

- $[h_{MN}]\, 5 \rightarrow 2\,[h_{\mu\nu}] + 2\,[A_\mu] + 1\,[\phi]$
- KK Higgs : At each KK $\overset{\text{mass}}{\wedge}$ level, 2 $h_{\mu\nu}$ 'eat' the A_μ and ϕ KK modes, to become a single massive spin 2

superheavy!! ⟶

- M - Theory

  Presumably 11 dimensional ;

  Low energy limit may be

  11-d supergravity ;

  (graviton, gravitino, A_{MNP})

  Contains all known superstring theories ;

  $\Big\}$ Unification of matter & gauge forces?

- Large extra dimension(s)

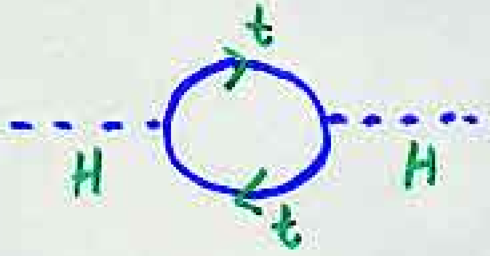
  (New way to approach hierarchy problem)

- Warped Geometry

  (May resolve hierarchy problem without SUSY)

Gauge Hierarchy Problem

$$\underset{\underset{\sim 10^{19}\,\text{GeV}}{\uparrow}}{M_{planck}} \gg \underset{\underset{\sim 10^{2}\,\text{GeV}}{\uparrow}}{M_W}$$

But 

$\delta M_H^2 \sim \Lambda^2 \sim M_{Planck}^2$

⇒ fine tuning needed to generate M_W?

Proposed solutions include:

- SUSY (adds new diagrams, TeV particl
- Extra dimensions

Fermion Masses & Mixings

$m_t \sim 350{,}000\ m_e$

$V_{ud} \sim 0.2 \gg V_{cb} \gg V_{ub}$

$\sin^2 2\theta_{ATM} \simeq 1$

How do we understand these and related hierarchies?

- SUSY does not really help;
  - Supplement with some U(1) family sym?
    (or non-abelian sym) (Frogatt/Nielsen
  - Extra dimension(s)?

Is there a FIFTH dimension ?

MOTIVATION

- Superstrings / M-theory require extra dimensions.
- Gauge Hierarchy Problem

LARGE Extra Dimension(s)

(only felt by gravity)

Arkani- Hamed
Dimopoulos
Dvali
Antoniadis
⋮

Spacetime $= M_4 \times I_n$

$$M_{Planck}^2 = M_f^{n+2} \, V_n$$

$n=2: \quad R \sim mm$

$M_f \sim TeV$

But : hierarchy between $\underset{\substack{\nearrow \\ TeV}}{M_f}$ and $\underset{\substack{\nearrow \\ 10^{-16}\, TeV}}{R^{-1}}$??

WARPED GEOMETRY

$$S = M_5^3 \int d^5x \sqrt{-G}\,(R - \Lambda) + \int d^4x \sqrt{-g}\,(-V_b)$$

↑ 5d gravity + cosm. const

↑ Source term for domain wall at $x_5 = 0$

$(g_{\mu\nu} = G_{MN}(x_5 = 0)\,\delta^M_\mu \delta^N_\nu)$

Ansatz: $ds^2 = e^{2A(x_5)}\eta_{\mu\nu}dx^\mu dx^\nu + dx_5^2$

One finds:

$$6A'^2 + \frac{1}{2}\Lambda = 0$$

$$3A'' + \frac{1}{2}V\delta(x_5) = 0$$

With $\Lambda < 0$, one finds

$$A = \pm k x_5 \;,\quad k = \sqrt{-\Lambda/12}$$

$$3\Delta(A') = -\frac{1}{2}V$$

So

$$A = -k x_5 \;,\quad x_5 > 0$$

$$A = k x_5 \;,\quad x_5 < 0$$

$$V = 12k \;,$$

& we have

$$ds^2 = \underset{\substack{\uparrow \\ \text{peaked at } x_5 = 0}}{e^{-2\overset{\swarrow\ \text{AdS curvature scale}}{k}|x_5|}}\,\eta_{\mu\nu}\,dx^\mu dx^\nu + dx_5^2 \;;$$

Naive reduction to 4 dim by integrating over x_5 yields

$$M_4^2 = M_5^3 \int dx_5\, e^{-2k|x_5|} < \infty$$

Do we obtain 4d gravity?

(on the brane)

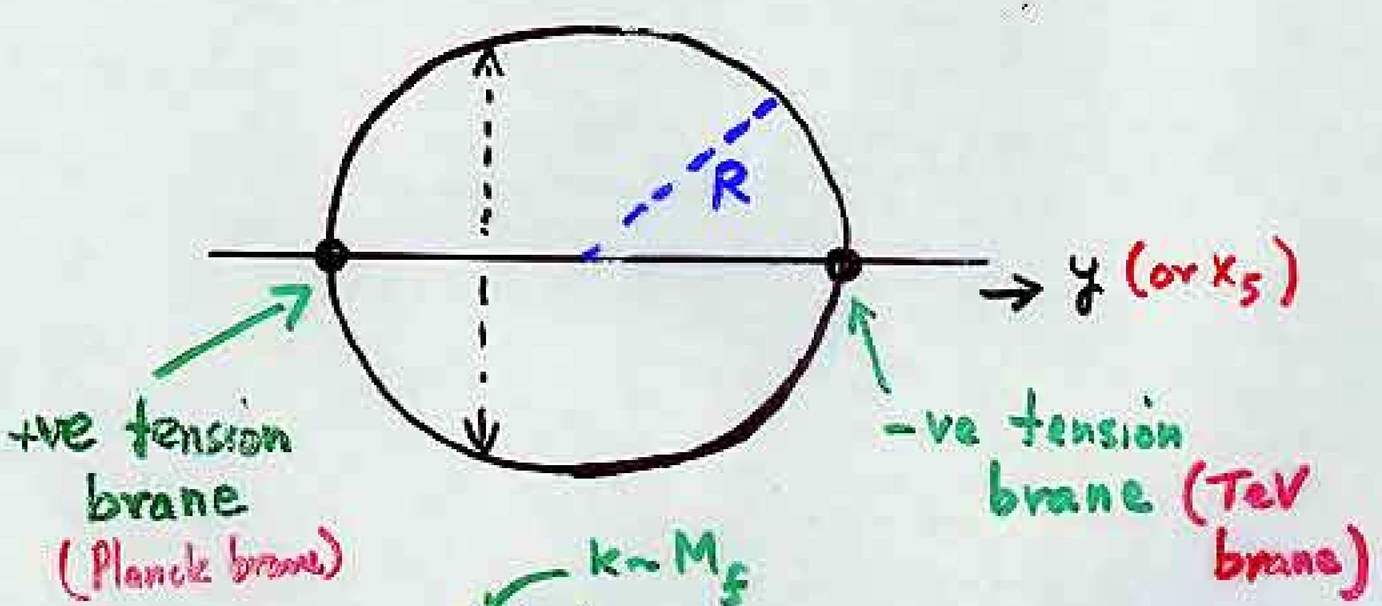
Despite the gapless KK spectrum, observer on the brane effectively sees 4d gravity. This is $\because$ most of the bulk KK modes have wave functions with support far from $x_5 = 0$ $\Rightarrow$ brane fields & localized graviton couple only very weakly to the bulk continuum.

Thus, $V(r) \sim 1/r$ receives small $1/r^3$ corrections. $V \sim \frac{Gm}{r}\left(1 + \left(\frac{1}{k^2 r^2}\right)\right)$

WARPED GEOMETRY

Randall
Sundrum
Gogberashvili
⋮

One extra dimension which corresponds to the orbifold S^1/Z_2



$k \sim M_5$

$$ds^2 = \underbrace{e^{-2k|y|}}_{\text{warp factor}} \eta_{\mu\nu}\, dx^\mu dx^\nu + dy^2$$

$$M^2_{\text{Planck}} = \frac{M_5^3}{k}\left(1 - e^{-2kR\pi}\right)$$

$$v = v_0\, e^{-kR\pi} \sim \text{TeV (!)}$$

VEV in 4d effective theory

VEV in 5 dim

Original Proposal

All SM fields reside on the TeV brane ; only gravity feels the extra dimension.

$\Rightarrow$ hierarchy problem under control!
(scale on TeV-brane ~ TeV)

BUT

Difficulty with non-renormalizabe operators :

$$\frac{1}{M_{PL}^2}\bar{\psi}\psi\bar{\psi}\psi \longrightarrow \frac{1}{(TeV)^2}\bar{\psi}\psi\bar{\psi}\psi$$

→ Rapid p decay
→ Large FCNC
→ Large neutrino masses ($\gg$ eV)

WAY OUT ?

- Symmetries ?
- Permit SM fields to leave the brane ? (Higgs stays on the TeV brane)

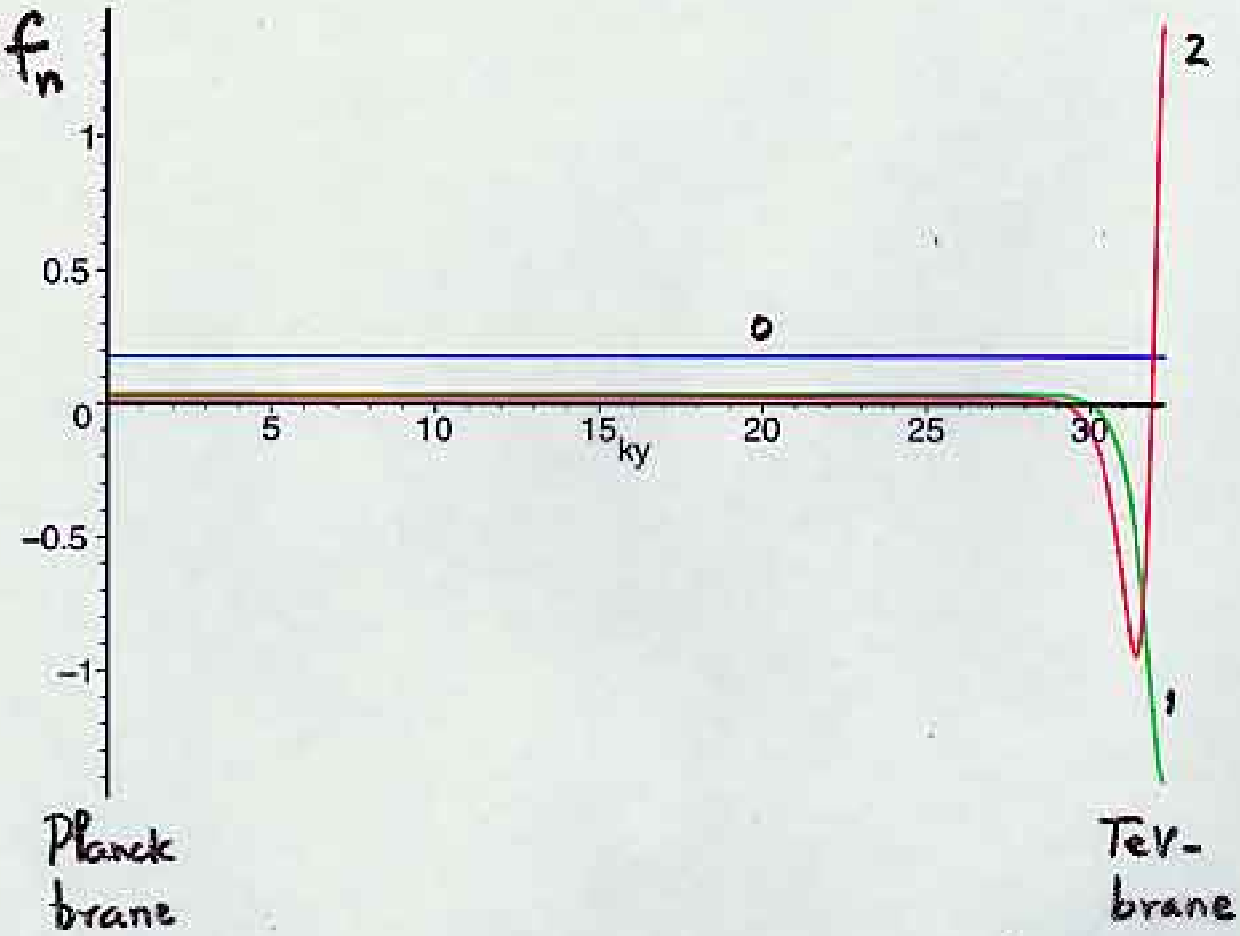
↖ unless SUSY invoked

Gherghetta
Pomarol
Hewett et al
Huber + QS
⋮

5d Bulk field ⟹ Tower of 4d fields

KK ansatz: $\Phi(x^\mu, y) = \sum_{n=0}^{\infty} \Phi^n(x^\mu) f_n(y)$

↗ wave fns from field eqns



(excited states near Tev brane!)

Bounds on KK Gauge Bosons

- Higgs on the TeV-brane

- Zero mode (gauge boson) becomes massive

$\rightarrow$ wave function f_0 becomes y dependent
$(m_0 \neq 0 \Rightarrow f_0 \neq \text{const})$

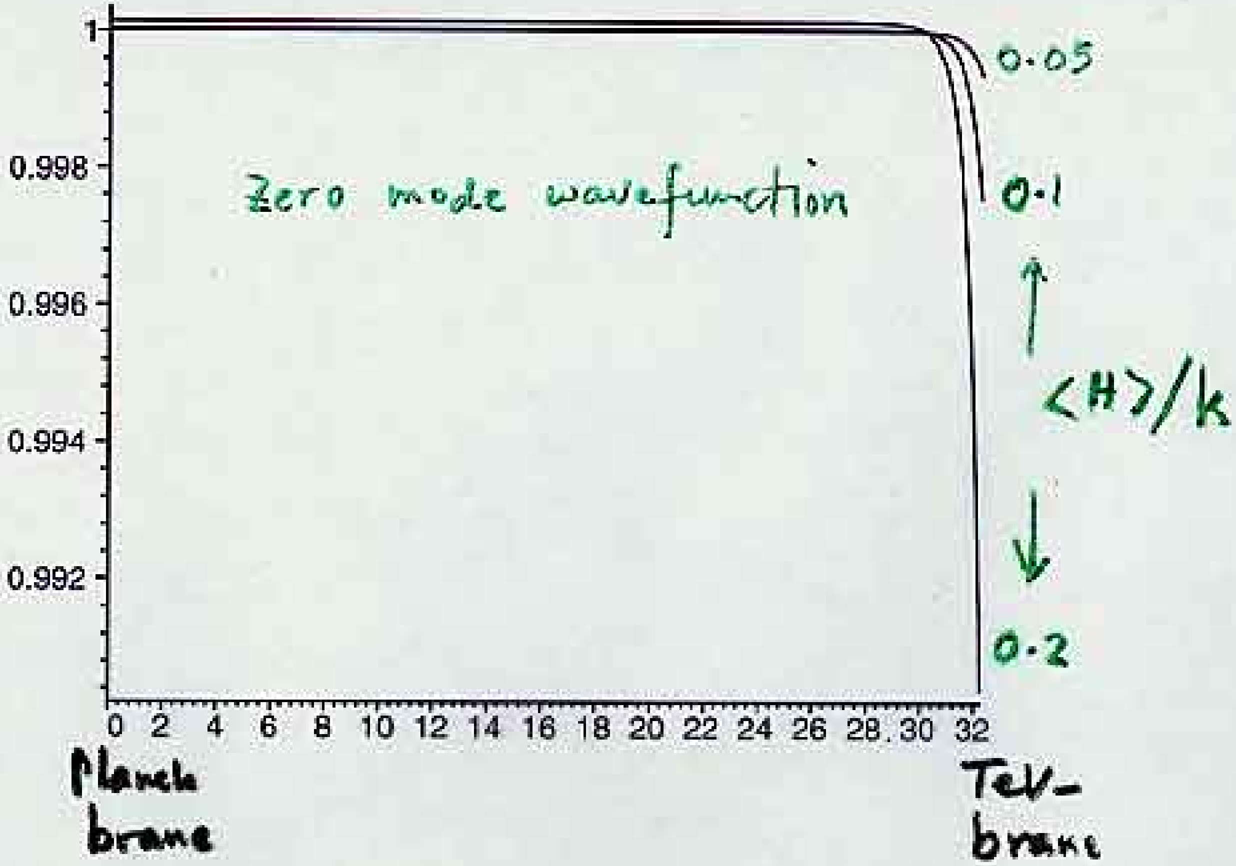
In particular, f_0 reduced at the TeV-brane $\Rightarrow M_Z = m_0 \not\sim g^{(5)}\langle H\rangle$

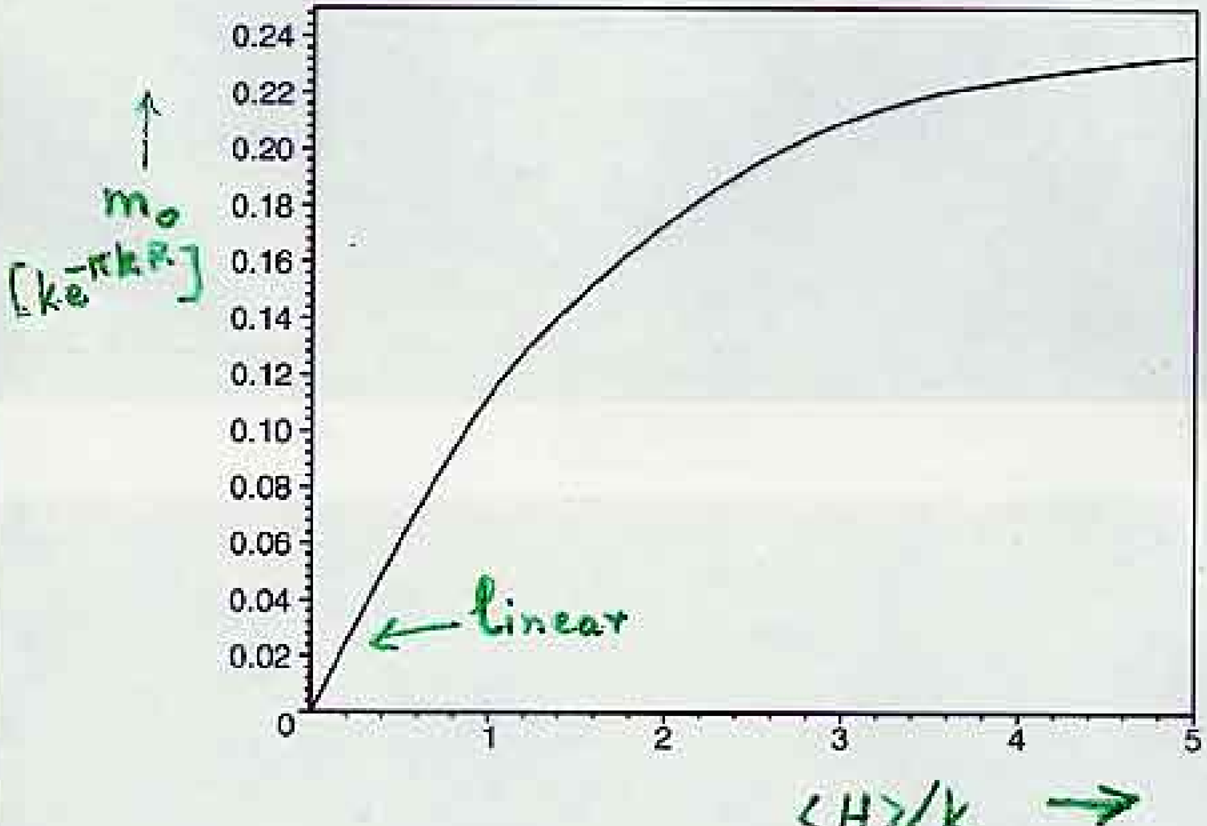
SM relationship between M_Z, M_W and coupling modified (deviations from SM $< 10^{-2}$ $\Rightarrow \langle H\rangle/k \lesssim 10^{-2}$)

$\Rightarrow$ bound on the TeV-brane ${}_{\wedge}^{KK}$ scale

$$m_1 \sim k e^{-\pi k R} \gtrsim 10\,\text{TeV} \quad \left(\langle H\rangle/k \lesssim 10^{-2}\right)$$

$\uparrow$
(could be somewhat smaller)
~ 7 TeV $\leftarrow$ Huber, Lee, Q.S
(7 TeV)


Zero mode wavefunction
1
0.998
0.996
0.994
0.992
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32
0.05
0.1
↑
<H>/k
↓
0.2
Planck brane
TeV-brane


0.24
0.22
0.20
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0
1
2
3
4
5
↑
m_0
$[k e^{-\pi k R}]$
← linear
<H>/k →

SM fermions arise as 'zero' modes of the KK decomposition.

5d Dirac mass $m_\psi = c k \varepsilon(y)$ Grossmann Neubert Huber+05

(Note: $\psi(-y) = \pm \gamma_5 \psi(y)$)

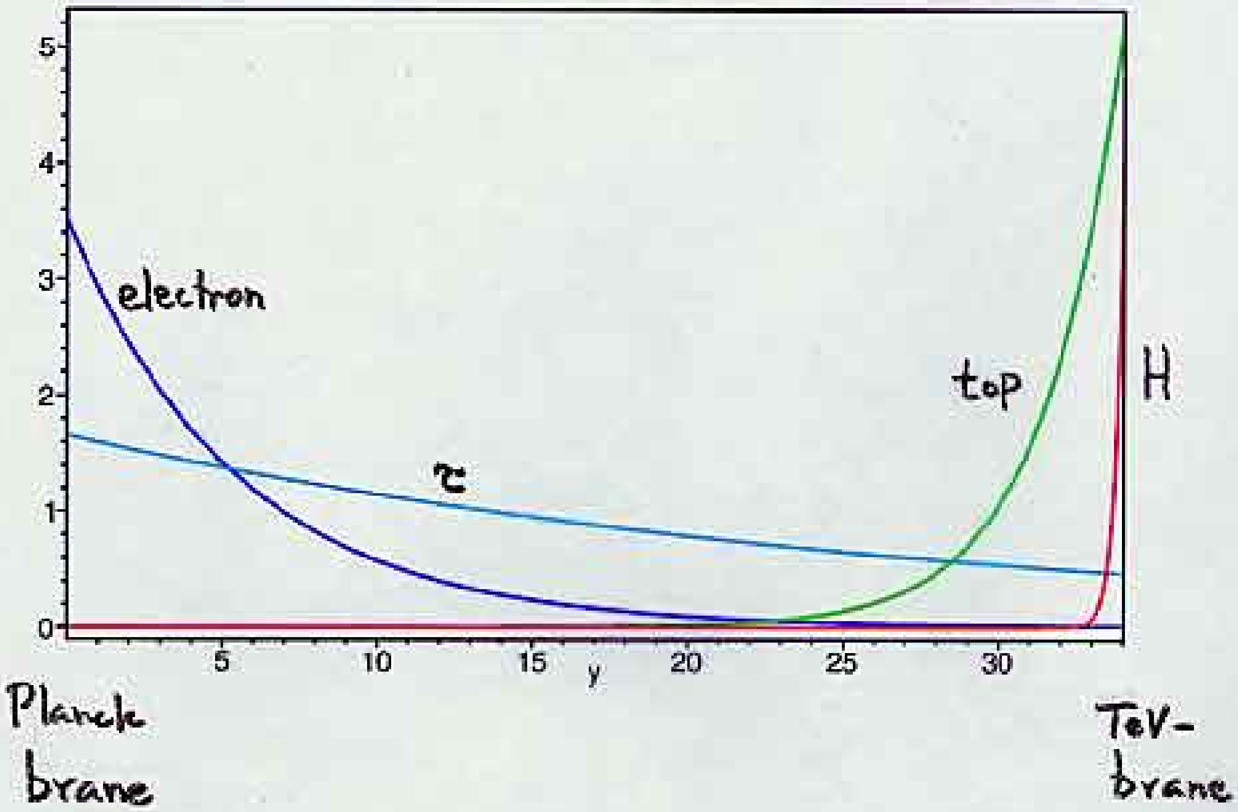
$m_n \sim n \pi k e^{-\pi k R}$ ← excited states

For $c > \frac{1}{2}$: zero mode localized → Planck brane
(exponential)

$c < \frac{1}{2}$: zero mode → TeV-brane

Fermion mass hierarchy: Determined by the overlap between the fermion wave functions and Higgs (on TeV brane)

E.g: $c(e) = 0.68$, $c(\mu) = 0.59$, $c(\tau) = 0.54$



5
4
3
2
1
0
electron
τ
top
H
5
10
15
y
20
25
30
Planck brane
TeV-brane

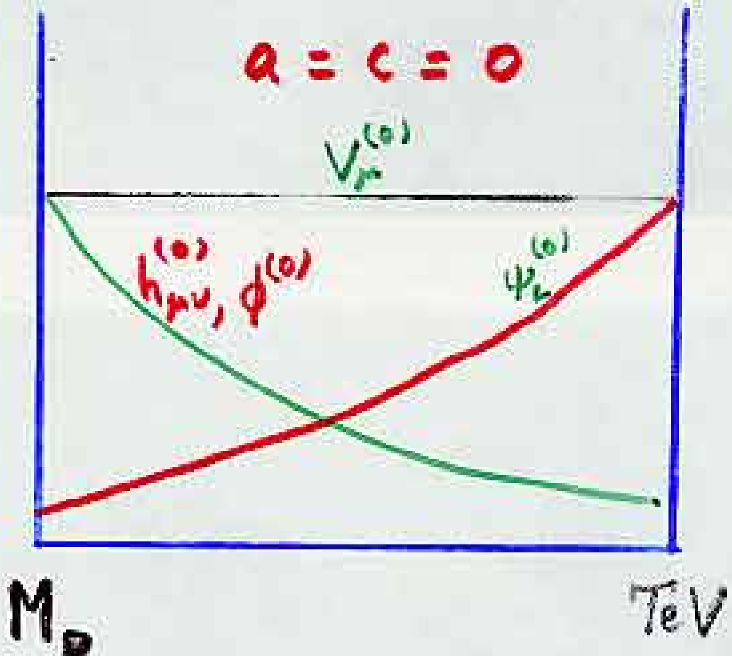
Zero modes

$$\phi^{(0)}(y) \sim e^{(1-\alpha)k|y|}$$ $\left[m_\phi^2 = ak^2 ; \; \alpha = \sqrt{4+a} \right]$

$$\Psi_L^{(0)}(y) \sim e^{(\frac{1}{2}-c)k|y|}$$ $\left[m_\Psi = ck\,\varepsilon(y) \right]$

$V_\mu^{(0)}(y) \sim$ indep. of y

$$h_{\mu\nu}^{(0)}(y) \sim e^{-k|y|}$$



$\phi^{(0)}$, $\Psi_L^{(0)}$ can localise on either brane (depends on a & c)

Deviations from the SM

S. Huber
C. Lee
QS

- gauge boson masses are modified (by mixing with KK states)

- gauge couplings to fermions are modified:
  from $g^{(5)} A_M \bar{\Psi} \gamma_M \Psi$

  $$\longrightarrow \quad g = \frac{g^{(5)}}{(2\pi R)^{3/2}} \int_{-\pi R}^{\pi R} dy \; f_0^c(y)^2 f_0(y)$$

  g depends on the location of the fermion c, and on the shape of f_0

  TeV-brane fermions: $g = \frac{g^{(5)}}{\sqrt{2\pi R}} f_0(\pi R)$

- effective 4 fermion operators from KK gauge boson exchange

Pomarol;
Davoudiasl, Hewett, Rizzo;
Gherghetta, Pomarol

- loops involving KK states

Comparison with the SM

- comparison with precision data on M_W, M_Z, and $\sin^2\theta_W$

- assume: modification of f_0 is main effect
  ignore KK states, check afterwards

- small deviations at tree level
  $\longrightarrow$ change in loops sub-leading
  $\longrightarrow$ compare with SM at tree-level

- the weak mixing angle

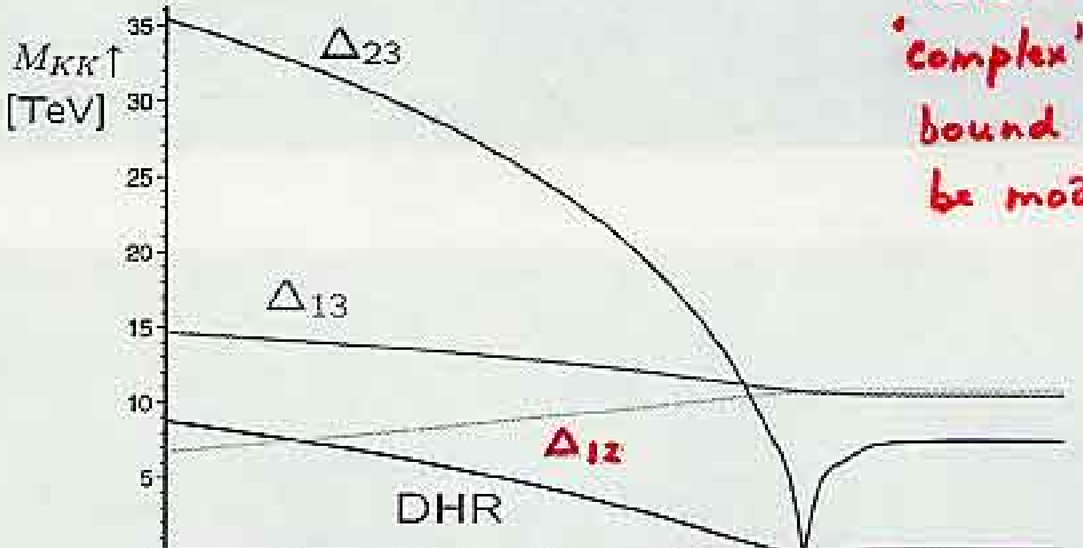
$$\begin{aligned}\cos^2\theta_1 &= \frac{M_W^2}{M_Z^2}\\ \cos^2\theta_2 &= \frac{g_W^2}{g_Z^2}\\ \cos^2\theta_3 &= 1-\frac{g_\gamma^2}{g_W^2}\end{aligned}$$

  SM: $\cos^2\theta_i$ are equivalent (tree-level)
  $\longrightarrow$ $\Delta_{ij} = \cos^2\theta_i - \cos^2\theta_j$ vanish

Constraints on M_{KK}

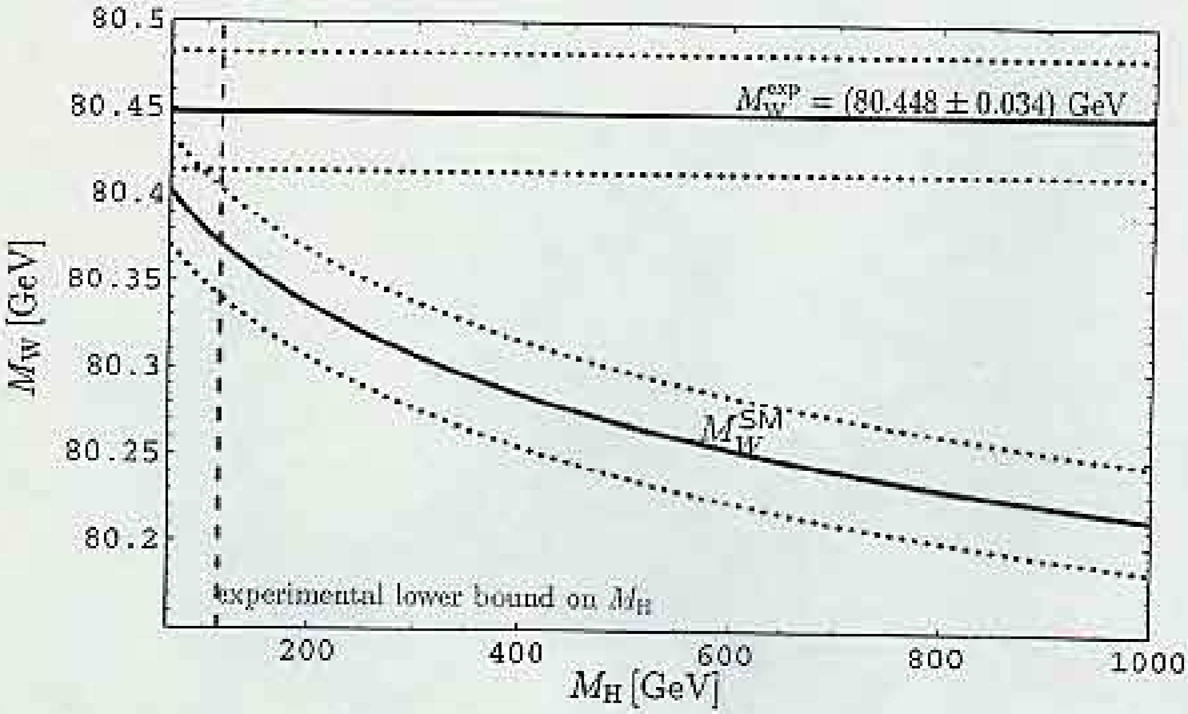
- experimental input:
  $\delta M_W = 50$ MeV, $\delta \sin^2\theta_W = 1.6 \cdot 10^{-4}$
  $\longrightarrow \delta\cos\theta_1 < 1.1 \cdot 10^{-3}$, $\delta\cos\theta_{2,3} < 1.6 \cdot 10^{-4}$
  $\longrightarrow \Delta_{12,13} < 1.2 \cdot 10^{-3}$, $\Delta_{23} < 1.6 \cdot 10^{-4}$

- results:
  $c \gtrsim 1/2 \longrightarrow M_{KK} \gtrsim$ 10 TeV (Δ_{12})
  TeV-brane $\longrightarrow M_{KK} \gtrsim$ 60 TeV (Δ_{23})
  constraint from KK exchange are much weaker (DHR) $\longrightarrow$ negligible



Note: In a more 'complex' scenario the bound on M_{KK} may be modified/weakened

The W mass: SM vs. experiment



Weiglein, hep-ph/0108063

we have $M_{KK} \sim 1/\sqrt{\Delta M_W}$:

exp. 1σ: $\Delta M_W = 34$ MeV $\longrightarrow$ $M_{KK} \gtrsim 12.4$ TeV

$\Delta M_W = 50$ MeV $\longrightarrow$ $M_{KK} \gtrsim 10.2$ TeV

central values: $\Delta M_W = 76$ MeV $\longrightarrow$ $M_{KK} \gtrsim 8.8$ TeV

within error bars: $\Delta M_W = 140$ MeV $\longrightarrow$ $M_{KK} \gtrsim 6.6$ TeV

(with $k/M_{\text{Pl}} = 0.01, M_H \sim 120$ GeV)

compare: LHC limit $M_{KK} \lesssim 7$ TeV

Non-Renormalizable Operators

$$\int d^4x \int dy \sqrt{-g}\ \frac{1}{M_5^3}\ \bar{\psi}_i \psi_j \bar{\psi}_k \psi_\ell$$

$$\rightarrow \int d^4x\ \frac{1}{M^2}\ \bar{\psi}_i^{(0)} \psi_j^{(0)} \bar{\psi}_k^{(0)} \psi_\ell^{(0)}$$

EXAMPLES	Expt	Model
• FCNC $\left(\frac{1}{M^2}(\bar{d}s)^2\right)$	$M \gtrsim 10^6$ GeV	✓
• n-$\bar{n}$ oscillations	$\gtrsim 10^5$ GeV	✓
• p decay	$\gtrsim 10^{15}$ GeV	$\sim 10^{11}$ Ge
• dim 5 ν masses	$\Delta m \lesssim 0.1$ eV	$m_{\nu_\tau} \sim$ ke

Need new ingredient

(e.g. $U(1)_L$?)

Neutrino Oscillations in RS

- 3 right-handed ν's in bulk
- Dirac mass from

$$\int d^4x \int dy \sqrt{-G}\ \frac{\lambda^{(5)}_{ij}}{\sqrt{k}}\ H\ \bar{L}^i\ \Psi^j$$

TeV-brane

'Bulk'

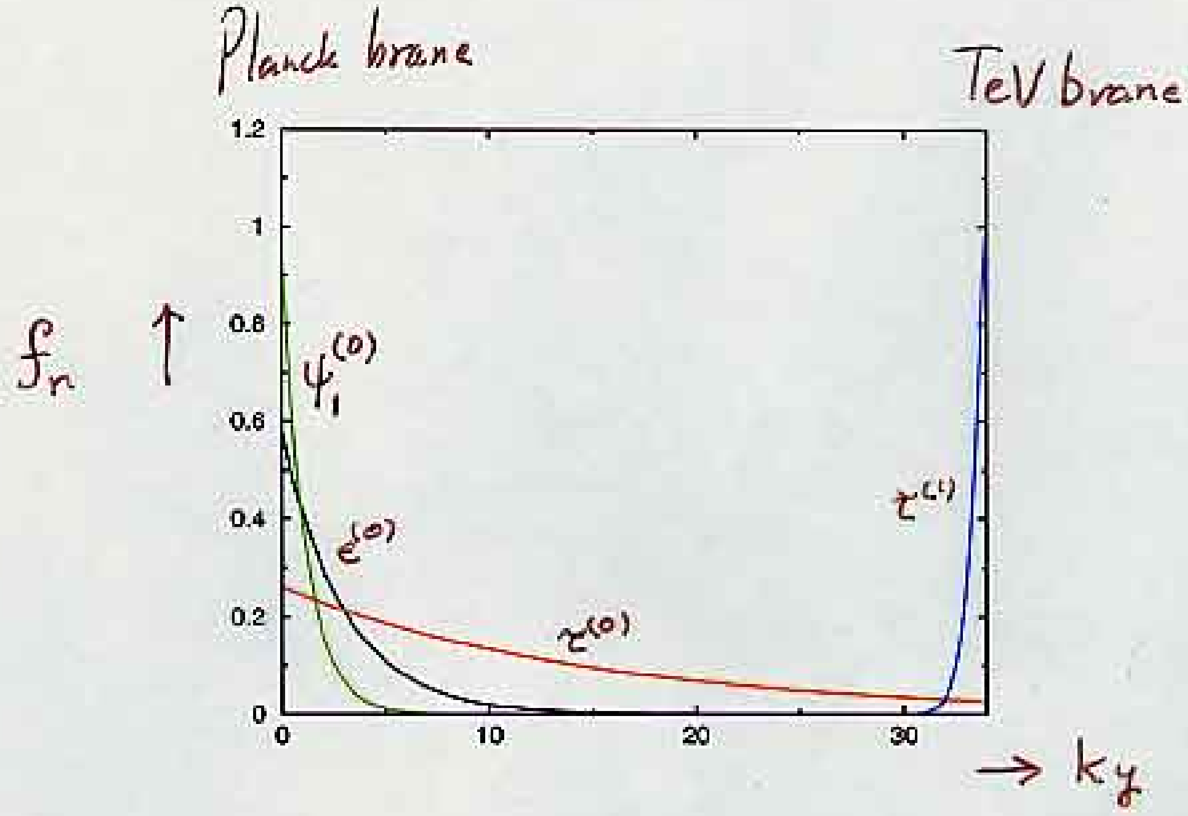
RH-ν near Planck brane

!! (Dirac masses suppressed)

Spectrum (flavor index suppressed)

$$\text{SM } \nu\text{'s : KK-tower} \left\{ \begin{array}{llll} \nu_L^{(0)}, & \nu_L^{(1)}, & \nu_L^{(2)}, & \cdots \\ -, & \nu_R^{(1)}, & \nu_R^{(2)}, & \cdots \end{array} \right.$$

$$\text{Sterile } \nu\text{'s (Right-handed)} \left\{ \begin{array}{llll} -, & \Psi_L^{(1)}, & \Psi_L^{(2)}, & \cdots \\ \Psi_R^{(0)}, & \Psi_R^{(1)}, & \Psi_R^{(2)}, & \cdots \end{array} \right.$$


Planck brane
TeV brane
f_n ↑
$\psi_1^{(0)}$
$e^{(0)}$
$\tau^{(0)}$
$\tau^{(1)}$
→ ky
1.2
1
0.8
0.6
0.4
0.2
0
0
10
20
30

NEUTRINO MIXINGS

- SM neutrinos at different locations

  (e.g. to generate charged lepton masses)

  → 'expect' nearest neighbor-type mixing, small mixing angles;

  - Allow order 10 hierarchy in λ_N to realize large atmospheric mixing small $\angle$ MSW solution;

- SM neutrinos at the same location;

  order unity λ_N

  → bimaximal scenario possible;

  (also LOW & vacuum solutions can be realized)

Consider bimaximal mixing

$$\left.\begin{array}{l} \nu_L:\ C_{e_L} = C_{\mu_L} = C_{\tau_L} = 0.57 \\ (C_{e_R} = 0.79,\ C_{\mu_R} = 0.62,\ C_{\tau_R} = 0.50) \end{array}\right\} \leftarrow \begin{array}{c} \text{Explains} \\ m_\tau \gg m_\mu \gg m_e \end{array}$$

$$\psi_R:\ C_{\psi_1} = 1.43,\ C_{\psi_2} = 1.36,\ C_{\psi_3} = 1.30$$

Large MSW

$$\text{5-d coupling} \rightarrow \lambda_{ij} = \begin{pmatrix} -2.0 & 1.5 & -0.5 \\ -1.8 & -1.1 & 1.9 \\ 0.5 & 1.9 & 1.7 \end{pmatrix}$$

$\Delta m^2_{atm} = 5 \cdot 10^{-3}\ eV^2 \rightarrow$ atm $(\sim m_{\nu_1})$

$\Delta m^2_{sol} = 1.\ 10^{-4}\ eV^2 \rightarrow$ solar $(\sim m_{\nu_2})$

$\sin^2 2\theta_{atm}\quad 0.98 \rightarrow$ atm

$\sin^2 2\theta_{sol} \simeq 0.90 \rightarrow$ solar

$[U^2_{e3} = 0.036]$
close to
exp bound

Neutrino Mass Matrix

$$\begin{array}{c|cccc} & \psi_R^{(0)} & \nu_R^{(1)} & \psi_R^{(1)} & \cdots \\ \hline \nu_L^{(0)} & \langle H \rangle & 0 & \langle H \rangle & \cdots \\ \nu_L^{(1)} & \langle H \rangle & M & \langle H \rangle & \cdots \\ \psi_L^{(1)} & 0 & \langle H \rangle & \underset{\substack{\uparrow \\ \text{KK scale}}}{M} & \ddots \\ \vdots & \vdots & \vdots & & \end{array}$$

For solar ν both small and large mixing angle solutions can be realized, depending on how we choose the 'c' parameters.

MIXING WITH STERILE NEUTRINOS

Neutrino mass matrix : $\lambda_N \nu_L^{(0)} \psi_R^{(1)} \langle H \rangle + \cdots$

→ SM neutrinos mix with KK sterile neutrinos

→ effective weak charge of light ν's reduced:

Effective number of neutrinos

$$n_{eff} = 3 - \delta n, \quad \delta n \le 0.005 \text{ (Z-width)}$$

With bulk SM neutrinos,

$$6 \cdot 10^{-7} \lesssim \delta n \lesssim 2 \cdot 10^{-5}$$

↗ ν_L all at diff location

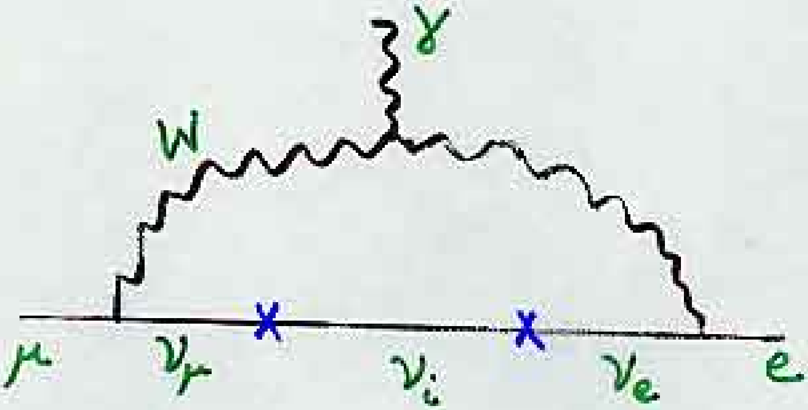
↑ ν_L at same location

Cf: TeV-brane SM neutrinos : δn close to exp bound (Grossman Neubert)

Reason: $\nu_L^{(0)}$ have small overlap with the TeV-brane

LEPTON FLAVOR VIOLATION

E.g. $\mu \to e\gamma$ mediated by KK sterile ν's



$$\Gamma(\mu \to e\gamma) \sim \left| \sum_i U_{ei}\, U_{\mu i}\, \mathcal{F}\left(m_i^2/m_W^2\right) \right|^2$$

- TeV-brane ν's : (m_1 = 10 TeV)

  Br ~ 10^2 × exp bound (~10^{-11})

  Kitano
  (Yukawas ~ unity)

- Bulk neutrinos :

  $10^{-26} \lesssim \mathrm{Br} \lesssim 10^{-15}$ (suppressed ∵ of smaller mixing between SM & sterile ν's)

Rate enhanced if we move
SM ν's closer to the TeV-brane

$n - \bar{n}$ Oscillations

Lepton number conservation → p stable ;
$\uparrow U(1)_L$

$n - \bar{n}$ oscillations possible ;

E.g. $\mathcal{O} \sim \frac{1}{M^5} (u^c d^c d^c)^2$

$(M_{exp} \gtrsim 330 \text{ TeV})$

For bulk quark fields with locations fixed by quark masses & mixings,

$M \sim 800 \text{ TeV}$

$\Rightarrow \tau(n-\bar{n}) \sim 10^2$ beyond exp. bound ;

By reducing quark Yukawa's by ~ 5 & bringing quarks closer to the TeV brane, $\tau(n-\bar{n}) \sim$ exp. bound (FCNC okay).

KK contributions to $(g-2)_\mu$

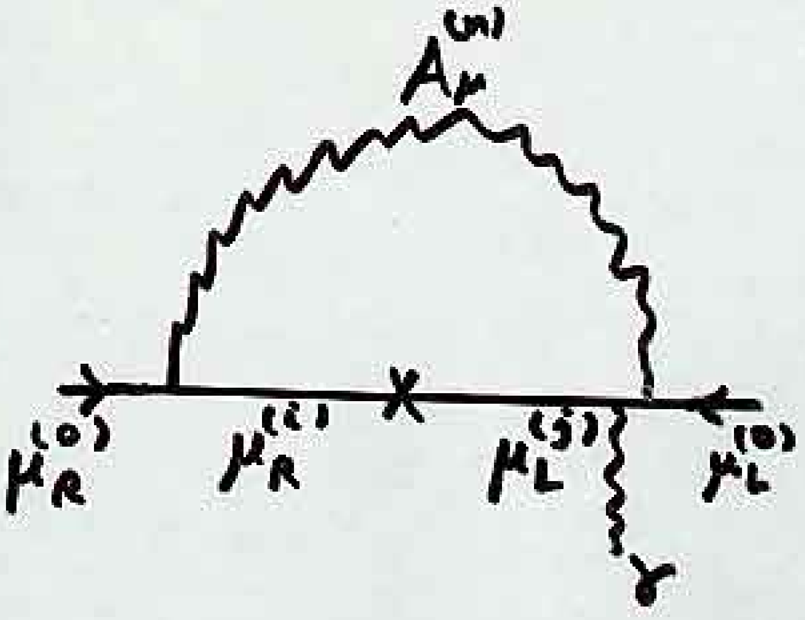
$$c_{\mu_L} = 0.57, \quad c_{\mu_R} = 0.61$$

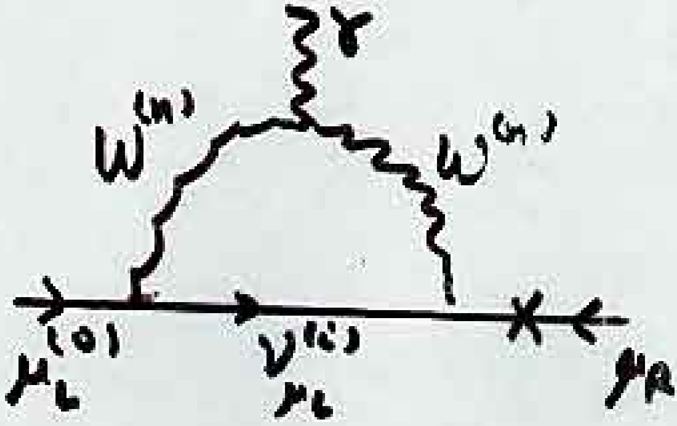
- Gauge boson

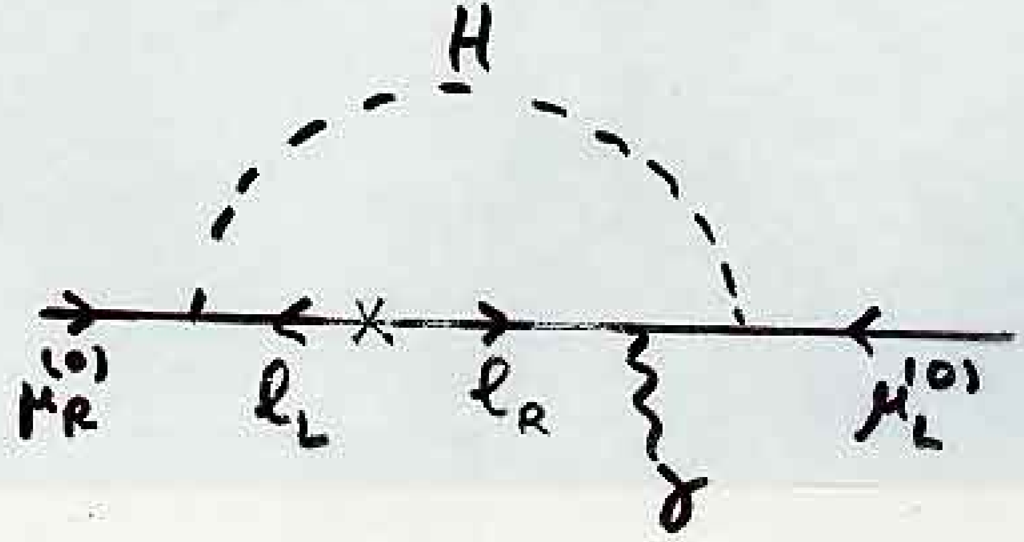
$$\Delta a_\mu^{(A)} = -5 \cdot 10^{-12} \left(\frac{10\,\text{TeV}}{m_{KK}} \right)^2$$

- Higgs

$$\Delta a_\mu^{(H)} = -3 \cdot 10^{-11} \left(\frac{10\,\text{TeV}}{m_{KK}} \right)^2$$


$A_\mu^{(n)}$
$\mu_R^{(0)}$
$\mu_R^{(i)}$
$\mu_L^{(j)}$
$\mu_L^{(0)}$
γ


γ
$W^{(n)}$
$W^{(n)}$
$\mu_L^{(0)}$
$\nu_{\mu_L}^{(i)}$
μ_R


H
$\mu_R^{(0)}$
ℓ_L
ℓ_R
$\mu_L^{(0)}$
γ

SUMMARY

- Bulk SM fields
- Higgs at the TeV-brane

$\Rightarrow$ geometrical interpretation of fermion masses & mixings ; $m_K \gtrsim 10$ TeV ; proton decay not quite sufficiently suppressed ; 'Large' Majorana ν masses ;

- Impose lepton number + right-handed bulk neutrinos

$\Rightarrow$ 'light' Dirac ν's ; → neutrinoless double β decay absent

reproduce correct ν oscillations ;

rare processes ($\mu \to e\gamma$, $n-\bar{n}$) may occur at observable rates;

- SMOKING GUN: KK excitations at LHC?