

# CP violation in the lepton sector and neutrino oscillations

LL

Tokyo Metropolitan Univ.

O. YASUDA

## 1. Introduction

- CP violating phases in the lepton sector
- Future long baseline experiments
- $\nu$  factories VS conventional beams

## 2. Possibility of measurements of ~~CP~~ in future long baseline experiments

## 3. Summary

# 1. Introduction

## CP violating phases in the lepton sector

Majorana mass term

$$\mathcal{L}_{Maj} = \nu_L^{T''} C^{-1} m^{(\nu)} \nu_L'' = \underbrace{\nu_L^{T''} \sigma^T}_{\nu_L^{T'}} C^{-1} \text{diag}(m_j^{(\nu)}) \underbrace{\sigma \nu_L''}_{\nu_L'}$$

charged lepton

$$\bar{l}_R'' m^{(l)} l_L'' = \underbrace{\bar{l}_R'' V_R^{(l)T}}_{\bar{l}_R'} \text{diag}(m_j^{(l)}) \underbrace{V_L^{(l)} l_L''}_{l_L'} = \bar{l}_R' e^{i\alpha} e^{i\beta\lambda_3} e^{i\gamma\lambda_8} \text{diag}(m_j^{(l)}) l_L'$$

charged current

$$j_\mu = \bar{l}_L'' \gamma_\mu \nu_L'' = \bar{l}_L' V_L^{(l)} \sigma^T \gamma_\mu \nu_L' \\ = \underbrace{\bar{l}_L' e^{i\alpha} e^{i\beta\lambda_3} e^{i\gamma\lambda_8}}_{\bar{l}_L} \gamma_\mu \underbrace{U e^{i\beta\lambda_3} e^{i\gamma\lambda_8}}_{\nu_L} \nu_L'$$

flavor eigenstate      Majorana phases

$$\underbrace{\nu_L''}_{\text{flavor eigenstate}} = U e^{i\beta\lambda_3} e^{i\gamma\lambda_8} \underbrace{\nu_L'}_{\text{mass eigenstate}}$$

mass eigenstate

$U = U(\theta_{12}, \theta_{13}, \theta_{23}; \delta)$  : Maki-Nakagawa-Sakata  
-Pontecorvo matrix



NB Majorana phases do not affect  $\nu$  oscillation.

$$\textcircled{\text{f}} \quad \nu_{\beta}(L) = \sum_j U_{\beta j} e^{-iE_j L} (U^{-1})_{j\alpha} \nu_{\alpha}(0)$$

$$A(\nu_{\alpha} \rightarrow \nu_{\beta}; L) = (U)_{\beta j} [\text{diag}(e^{-iE_j L})]_{jj} (U^{-1})_{j\alpha}$$

Consider  $\nu$  osc. with different MNSP matrix:

$$\tilde{U} \equiv \underbrace{\text{diag}(e^{i\varphi_{\alpha}})}_P U \underbrace{\text{diag}(e^{i\chi_j})}_Q = PUQ$$

$$\begin{aligned} \tilde{A}(\nu_{\alpha} \rightarrow \nu_{\beta}; L) &= (\tilde{U})_{\beta j} [\text{diag}(e^{-iE_j L})]_{jj} (\tilde{U}^{-1})_{j\alpha} \\ &= (PUQ)_{\beta j} [\text{diag}(e^{-iE_j L})]_{jj} (Q^{-1}U^{-1}P^{-1})_{j\alpha} \\ &= (P)_{\beta\beta} (U)_{\beta j} [\text{diag}(e^{-iE_j L})]_{jj} (U^{-1})_{j\alpha} (P^{-1})_{\alpha\alpha} \\ &= e^{i\varphi_{\beta} - i\varphi_{\alpha}} A(\nu_{\alpha} \rightarrow \nu_{\beta}; L) \end{aligned}$$

$$\begin{aligned} \therefore \tilde{P}(\nu_{\alpha} \rightarrow \nu_{\beta}; L) &= |\tilde{A}(\nu_{\alpha} \rightarrow \nu_{\beta}; L)|^2 \\ &= |A(\nu_{\alpha} \rightarrow \nu_{\beta}; L)|^2 \\ &= P(\nu_{\alpha} \rightarrow \nu_{\beta}; L) \quad // \end{aligned}$$

Bilenky - Hosek - Petcov PL 94B ('80) 495

# Future long baseline experiments

## motivation

Oscillation parameters in  $N_\nu = 3$  framework

$$\left( \underbrace{\Delta m_{21}^2, \theta_{12}}_{\uparrow \nu_\odot} ; \underbrace{|\Delta m_{31}^2|, \theta_{23}}_{\uparrow \nu_{\text{atm}}} ; \boxed{\text{sign}(\Delta m_{31}^2), \theta_{13}, \delta} \right)$$

unknown

next thing to do is to determine :

- $\text{sign}(\Delta m_{31}^2)$   
mass pattern
- |                       |                       |
|-----------------------|-----------------------|
| ————— $m_3^2$         | ————— $m_2^2$         |
| ————— $m_2^2$         | ————— $m_1^2$         |
| ————— $m_1^2$         | ————— $m_3^2$         |
| $\Delta m_{31}^2 > 0$ | $\Delta m_{31}^2 < 0$ |

- $\theta_{13}$   
known bound from CHOOZ  
 $\sin^2 2\theta_{13} < 0.1$

- $\delta$   
CP violation

$N_\nu = 3$  is assumed throughout this talk.



# Near future / ongoing long baseline experiments

accel.	'99 -	K2K	KEK → SK	L = 250 km	$E_\nu \sim 1 \text{ GeV}$
	'04 -	MINOS	FNAL → Sudan	L = 730 km	$E_\nu \sim 10 \text{ GeV}$
	'06(?) -	{ ICARUS OPERA }	CERN → Grand Sasso	L = 730 km	$E_\nu \sim 20 \text{ GeV}$
	'07(?) -	JHF (1st phase)	JAERI → SK	L = 300 km	$E_\nu \sim 1 \text{ GeV}$
			0.77 MW, 22.5 kt		
reactor	'01 -	KAMLAND	Kashiwazaki etc. → Kam.	L ~ 170 km	$E_\nu \sim 4 \text{ MeV}$
	→ nice check for LMA $\nu_0$ sol.				
→ precise measurements of $( \Delta m_{32}^2 , \theta_{23}), \theta_{13} (?)$					

# Far future long baseline experiments

(highly speculative)

JHF (2nd phase) super (conventional) beams	JAERI → HK	L = 300 km	$E_\nu \sim 1 \text{ GeV}$
	4 MW, 1 Mt		
ν factory	? → ?	L = 1000 km - 3000 km	$E_\nu = 20 \text{ GeV} - 50 \text{ GeV}$



Hyper Kamiokande  
↓

- $\text{sign}(\Delta m_{31}^2)$
- $\theta_{13}$
- $\delta$

← final goal in this game



# ν factories VS conventional beams

	conventional	ν factories
production process	$\pi^+ \rightarrow \mu^+ \nu_\mu$ (99%) $\pi^+ \rightarrow e^+ \nu_e \nu_\mu$ (1%)	$\mu^+ \rightarrow e^+ \bar{\nu}_\mu \nu_e$ (50%) $\mu^+ \rightarrow e^+ \bar{\nu}_e \nu_\mu$ (50%)
intensity $\propto \left(\frac{E_{\text{parent}}}{m_{\text{parent}}}\right)^2$ (effect due to boost)	$\left(\frac{E_\pi}{m_\pi}\right)^2 \sim O(1)$ 	$\left(\frac{E_\mu}{m_\mu}\right)^2 \gg 1$ for $E_\mu \sim O(10 \text{ GeV})$ 
how to measure oscillations	$\nu_\mu \xrightarrow{\text{osc.}} \nu_e$ $\nu_e \xrightarrow{\text{interact w/ N}} e^- X$	$\nu_e \xrightarrow{\text{osc.}} \nu_\mu \xrightarrow{\text{interact w/ N}} \mu^- X$ <i>wrong sign μons</i> cf. $\bar{\nu}_\mu \xrightarrow{\text{noosc.}} \bar{\nu}_\mu \xrightarrow{\text{interact w/ N}} \mu^+ X$ <i>right sign μons</i> NB $\bar{\nu}_e N \rightarrow e^+ X$ create showers and charge id is very difficult.
backgrounds	$O(1)\%$ • contamination of $\nu_e$ $f_B \sim O(10^{-2})$ ↑ background fraction • mis identification of $\pi^0$ with $\nu_e$ $\pi^0 \rightarrow \gamma \rightarrow \text{shower}$ (unseen) $\gamma \rightarrow \text{shower}$ (unseen) e-like event	very small in case of iron calorimeter $\nu_\mu N \rightarrow \nu_\mu (\mu^-)$ (unseen) $\nu_\mu N \rightarrow \nu_\mu (\mu^-) \rightarrow \mu^+$ (decay) $f_B \sim O(10^{-5})$ in case of liquid Ar $f_B \sim O(10^{-5})$ in case of water Cherenkov $f_B \lesssim O(10^{-3})$



## 2. Possibility of measurements of $\mathcal{CP}$ in future LBL exp. <sup>17</sup>

### $\mathcal{CP}$ in vacuum

$$\left\{ \begin{array}{l} P(\nu_\alpha \rightarrow \nu_\beta) \\ P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \end{array} \right\} = \delta_{\alpha\beta} - 4 \sum_{j < k} \text{Re}(U_{\beta j} U_{\alpha j}^* U_{\beta k}^* U_{\alpha k}) \sin^2\left(\frac{\Delta E_{jk} L}{2}\right) \\ \pm 2 \sum_{j < k} \text{Im}(U_{\beta j} U_{\alpha j}^* U_{\beta k}^* U_{\alpha k}) \sin(\Delta E_{jk} L)$$

$$P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \\ = 4 \sum_{j < k} \text{Im}(U_{\beta j} U_{\alpha j}^* U_{\beta k}^* U_{\alpha k}) \sin(\Delta E_{jk} L) \\ = 4 \text{Im}(U_{\beta 1} U_{\alpha 1}^* U_{\beta 2}^* U_{\alpha 2}) \left[ \underbrace{\sin(\Delta E_{12} L) + \sin(\Delta E_{23} L) - \sin(\Delta E_{13} L)}_{-4 \sin\left(\frac{\Delta E_{21} L}{2}\right) \sin\left(\frac{\Delta E_{32} L}{2}\right) \sin\left(\frac{\Delta E_{31} L}{2}\right)} \right]$$

↑ unitarity condition

$$\left( \begin{array}{l} \text{Im}(U_{\beta 1} U_{\alpha 1}^* U_{\beta 2}^* U_{\alpha 2}) \\ = \text{Im}(U_{\beta 2} U_{\alpha 2}^* U_{\beta 3}^* U_{\alpha 3}) \\ = -\text{Im}(U_{\beta 1} U_{\alpha 1}^* U_{\beta 3}^* U_{\alpha 3}) \\ = \pm J \end{array} \right)$$

$$J \equiv \frac{c_{13}}{8} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} \sin \delta$$

Jarlskog factor

$$= \pm 16 J \sin\left(\frac{\Delta E_{21} L}{2}\right) \sin\left(\frac{\Delta E_{32} L}{2}\right) \sin\left(\frac{\Delta E_{31} L}{2}\right)$$

signature of CP violation

in vacuum

$$A \equiv \frac{N(\nu_\alpha \rightarrow \nu_\beta) - \overset{\text{compensates}}{\cancel{2}} N(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)}{N(\nu_\alpha \rightarrow \nu_\beta) + \cancel{2} N(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)} \quad \text{is a good}$$

asymmetric factor

in matter

$$A \equiv \sqrt{2} G_F N_e$$

$$P(\nu_\alpha \rightarrow \nu_\beta) = f(\Delta m_{jk}^2, \theta_{jk}, \delta, \boxed{A})$$

$$P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) = f(\Delta m_{jk}^2, \theta_{jk}, -\delta, \boxed{-A})$$

$$P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \propto \sin \delta$$

"A" is not a good factor

in matter

$$\text{for } \nu \quad i \frac{d\psi}{dt} = [U \text{diag}(E_1, E_2, E_3) U^{-1} + \text{diag}(A, 0, 0)] \psi$$

$$\text{for } \bar{\nu} \quad i \frac{d\psi}{dt} = [U^* \text{diag}(E_1, E_2, E_3) U^{*-1} - \text{diag}(A, 0, 0)] \psi$$



NB T violation

Symbolically we can diagonalize

$$U \text{diag}(E_1, E_2, E_3) U^{-1} + \text{diag}(A, 0, 0)$$

$$= U^M \text{diag}(E_1^M, E_2^M, E_3^M) (U^M)^{-1}$$

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{j < k} \text{Re}(U_{\beta j}^M U_{\alpha j}^{M*} U_{\beta k}^{M*} U_{\alpha k}^M) \sin^2\left(\frac{\Delta E_{jk}^M L}{2}\right)$$

$$+ 2 \sum_{j < k} \text{Im}(U_{\beta j}^M U_{\alpha j}^{M*} U_{\beta k}^{M*} U_{\alpha k}^M) \sin(\Delta E_{jk}^M L)$$

$$\Delta E_{jk}^M \equiv E_j^M - E_k^M$$

$$P(\nu_\alpha \rightarrow \nu_\beta) - P(\nu_\beta \rightarrow \nu_\alpha)$$

$$= 4 \sum_{j < k} \text{Im}(U_{\beta j}^M U_{\alpha j}^{M*} U_{\beta k}^{M*} U_{\alpha k}^M) \sin(\Delta E_{jk}^M L)$$

$$= \pm 16 J^M \sin\left(\frac{\Delta E_{21}^M L}{2}\right) \sin\left(\frac{\Delta E_{32}^M L}{2}\right) \sin\left(\frac{\Delta E_{31}^M L}{2}\right)$$

$$J^M \equiv \text{Im}(U_{e1}^M U_{\mu 1}^{M*} U_{e2}^{M*} U_{\mu 2}^M)$$

modified  
Jarlskog  
factor  
in matter

(identity<sup>(\*)</sup>: Naumov, Sov.Phys.JETP 74 ('92))

$$J^M \prod_{j < k} \Delta E_{jk}^M = J \prod_{j < k} \Delta E_{jk}$$

$$= \pm 16 J \frac{\Delta E_{21} \Delta E_{32} \Delta E_{31}}{\Delta E_{21}^M \Delta E_{32}^M \Delta E_{31}^M} \sin\left(\frac{\Delta E_{21}^M L}{2}\right) \sin\left(\frac{\Delta E_{32}^M L}{2}\right) \sin\left(\frac{\Delta E_{31}^M L}{2}\right)$$

$\propto \sin \delta$

$$\chi \neq 0 \iff \sin \delta \neq 0$$

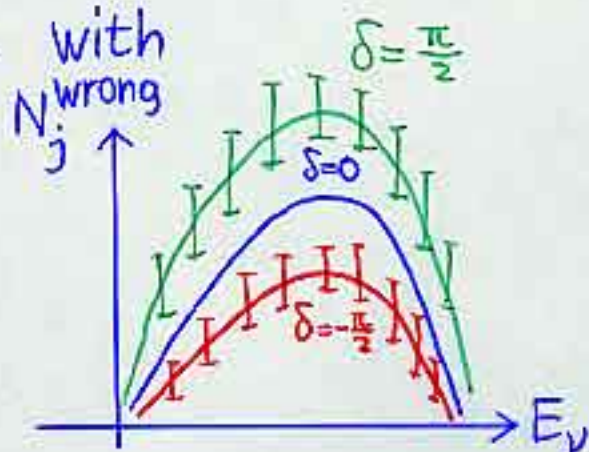
Thus T violation is clean signal of  $\sin \delta \neq 0$   
but it is very hard to measure experimentally.

(\*) An earlier attempt: Krastev - Petcov, PL B205 ('88) 84.  
Rediscovery: Harrison - Scott, PL B476 ('00) 349.



So we consider indirect measurements of CP violation

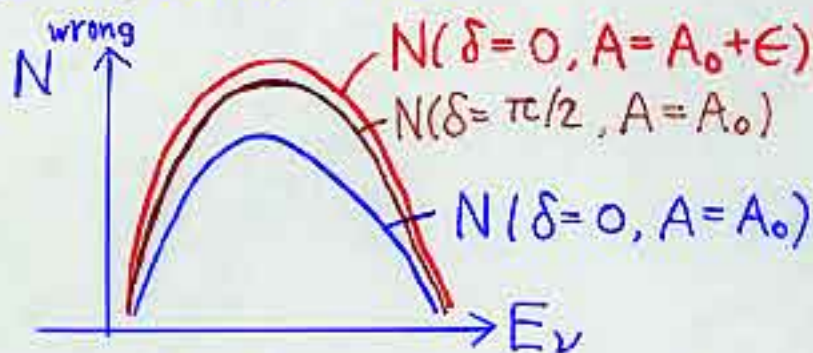
\* Assume three flavor mixing and compare the data with prediction for  $\delta=0$ :



\* Correlation of errors

To measure  $\delta$ , we have to know all other variables with certain precision.

If we had situation like:



then there would be no way to determine  $\delta$ .

$$\Delta\chi^2 \equiv \min_{\bar{\theta}_{jk}, \bar{\Delta m}_{jk}^2, \bar{A}} \sum_i \frac{[N_i(\Delta m_{jk}^2, \theta_{jk}; \delta; A) - N_i(\bar{\Delta m}_{jk}^2, \bar{\theta}_{jk}; \bar{\delta}=0; \bar{A})]^2}{\sigma_i^2}$$

To reject a hypothesis " $\delta=0$ " at  $3\sigma$  CL

$$\Delta\chi^2 \geq \Delta\chi^2(3\sigma \text{ CL})$$

→ We can estimate detector size to reject " $\delta=0$ " at  $3\sigma$  CL.



# Optimization of sensitivity to $\delta$ w.r.t. $(E_\mu, L)$

	$\Delta\theta_{ij}, \Delta m_{ij}^2$	$\Delta A/A$	$f_B$	$\Delta m_{21}^2 / 10^{-5} \text{eV}^2$	$E_{th} / \text{GeV}$	$E_\mu / \text{GeV}$	$L / \text{km}$	
KOS	included	$\pm 10\%$	0	5	1	$\lesssim 6$	600 - 800	
				10	1	$\lesssim 50$	500 - 2000	
				10	4	30 - 50	2800 - 4500	
PY	included	$\pm 10\%$	0	3.2	0.1	$\sim 50$	$\sim 3000$	
						$10^{-5}$	$\sim 20$	$\sim 1000$
						$10^{-3}$	$\sim 15$	$\sim 1000$
						$10^{-3}$	$\sim 10$	$\sim 800$
Huber	included	$\pm 10\%$	0	10	0.1	$\sim 20$	$\sim 2000$	
				3.5		25	1500	

}  $\Delta\chi^2$

Ref: KOS Koike - Ota - Sato, hep-ph/0011387 (revised, to be published in PRD)

FHL Freund - Huber - Lindner, NP B651 (101) 331

PY Pinney - O.Y., PR D64 (101) 093008

Huber private communication

$\approx \Delta\chi^2$  used in the analyses of indirect CP measurements

KOS

$$\Delta\chi^2 \equiv \min_{\theta_{jk}, \Delta m_{jk}^2, \bar{A}} \sum_i \frac{1}{\sigma_i^2} \left[ N_i(\theta_{jk}, \Delta m_{jk}^2, A, \delta) \frac{V_{e \rightarrow \nu_\mu}}{N_i(\theta_{jk}, \Delta m_{jk}^2, A, \delta)} \frac{\bar{V}_{e \rightarrow \bar{\nu}_\mu}}{\bar{V}_{e \rightarrow \bar{\nu}_\mu}} - N_i(\bar{\theta}_{jk}, \overline{\Delta m_{jk}^2}, \bar{A}, 0) \frac{V_{e \rightarrow \nu_\mu}}{N_i(\bar{\theta}_{jk}, \overline{\Delta m_{jk}^2}, \bar{A}, 0)} \frac{\bar{V}_{e \rightarrow \bar{\nu}_\mu}}{\bar{V}_{e \rightarrow \bar{\nu}_\mu}} \right]^2$$

indirect

(FHL, Huber)  
(PY)

$$\Delta\chi^2_{\text{indirect}} \equiv \min_{\theta_{jk}, \Delta m_{jk}^2, \bar{A}} \sum_i \left\{ \frac{[N_i(\theta_{jk}, \Delta m_{jk}^2, A, \delta) - N_i(\theta_{jk}, \Delta m_{jk}^2, A, 0)]^2}{\sigma_i^2} + \frac{[ \quad ]^2}{\sigma_i^2} \frac{V_{e \rightarrow \nu_\mu}}{V_{\mu \rightarrow \nu_\mu}} + \frac{[ \quad ]^2}{\sigma_i^2} \frac{\bar{V}_{e \rightarrow \bar{\nu}_\mu}}{\bar{V}_{\mu \rightarrow \bar{\nu}_\mu}} \right\}$$



koike-Ota-Sato hep-ph/0011387 (revised)  
 optimization with  $\Delta\chi_3^2$

$$\Delta m_{21}^2 = 1 \times 10^{-4} \text{ eV}^2$$

$$\sin^2 2\theta_{13} = 0.04$$

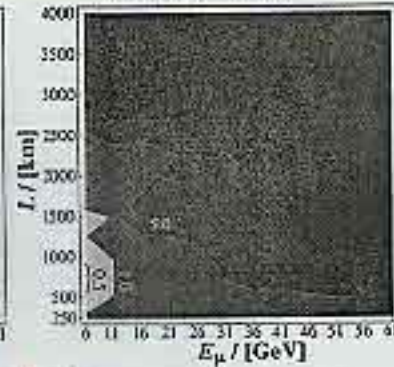
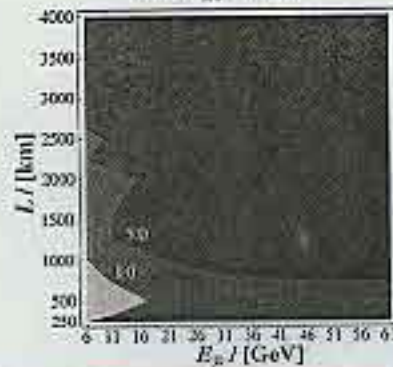
$$\sin^2 2\theta_{13} = 0.01$$

$$\delta m_{21}^2 = 1 \times 10^{-4} \text{ eV}^2$$

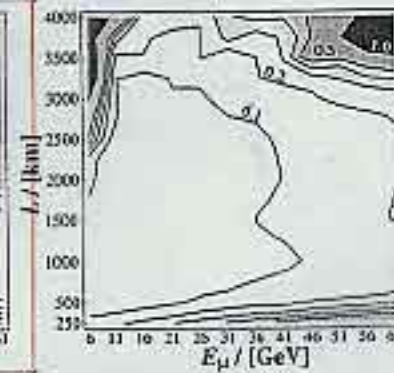
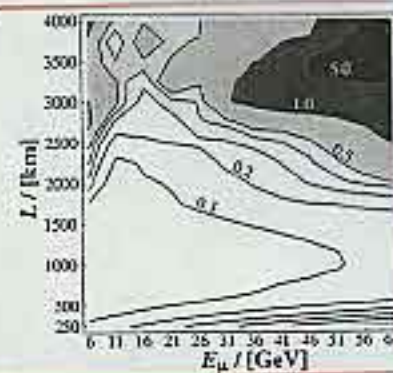
$$\sin \theta_{13} = 0.1$$

$$\sin \theta_{13} = 0.05$$

$$\delta = \frac{\pi}{6}$$



$$\delta = \frac{\pi}{2}$$



$$\delta = \frac{5\pi}{6}$$

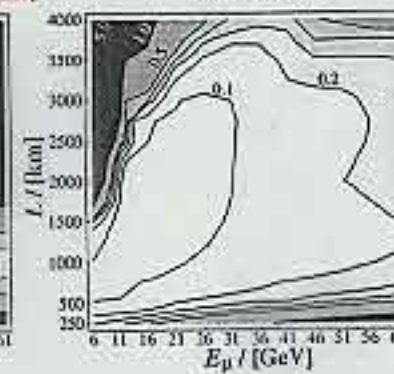
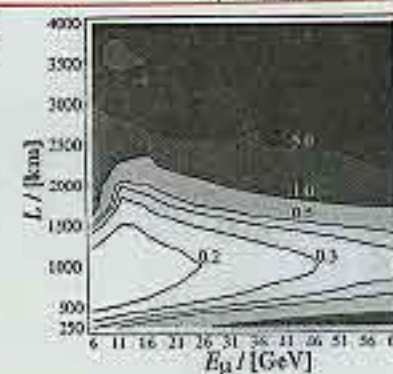


Figure 9: Same as Fig. 8, but for different parameters. All the graphs presented here are for  $\delta m_{21}^2 = 1 \times 10^{-4} \text{ eV}^2$ . The graphs in left column are for  $\sin \theta_{13} = 0.1$  while the ones in right column are for  $\sin \theta_{13} = 0.05$ . The top two graphs are for  $\delta = \pi/6$ , the second two graphs are for  $\delta = \pi/2$ , and the bottom two graphs are for  $\delta = 5\pi/6$ . Parameters not presented here are taken to be same as Fig. 2. The difference of the sensitivity for  $\delta = \pi/6$  and for  $\delta = 5\pi/6$  is due to the difference of matter effect.

Koike-Ota-Sato hep-ph/0011387 (revised)

Optimization with  $\Delta\chi^2$ 

$$\Delta m_{21}^2 = 5 \times 10^{-5} \text{ eV}^2$$

$$\sin^2 2\theta_{13} = 0.04$$

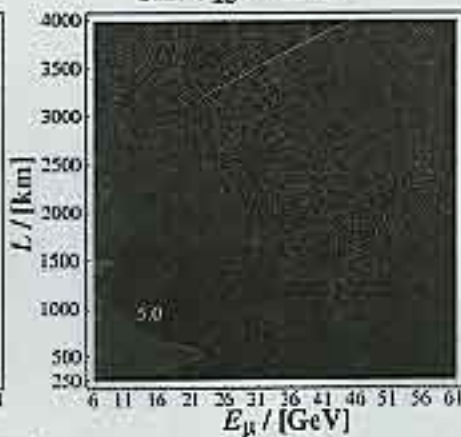
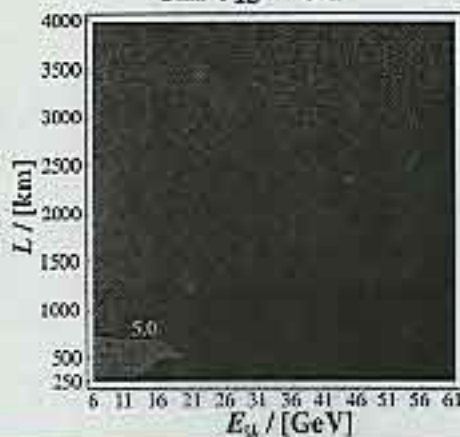
$$\sin^2 2\theta_{13} = 0.01$$

$$\delta m_{21}^2 = 5 \times 10^{-5} \text{ eV}^2$$

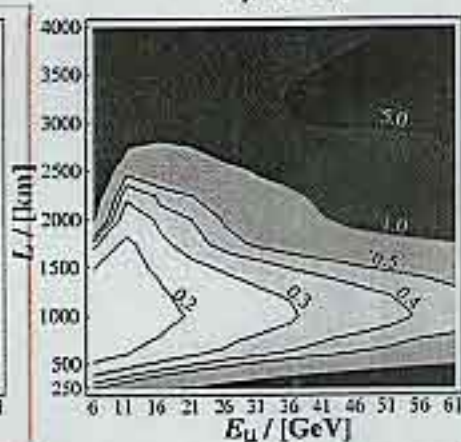
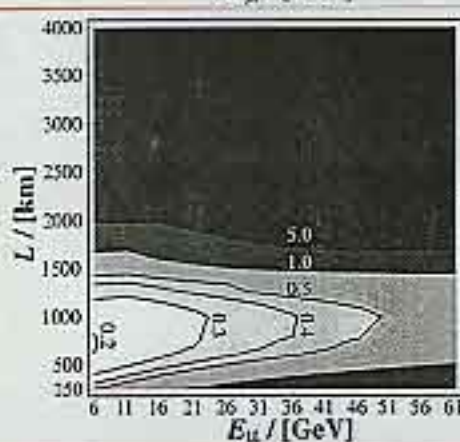
$$\sin \theta_{13} = 0.1$$

$$\sin \theta_{13} = 0.05$$

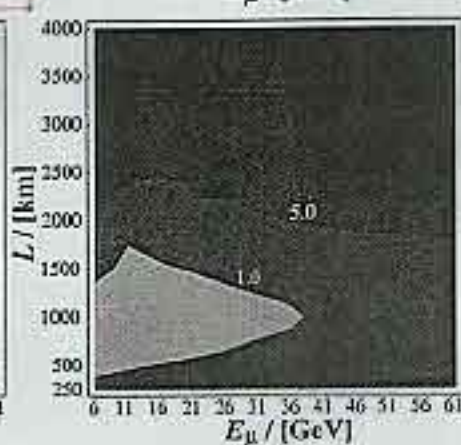
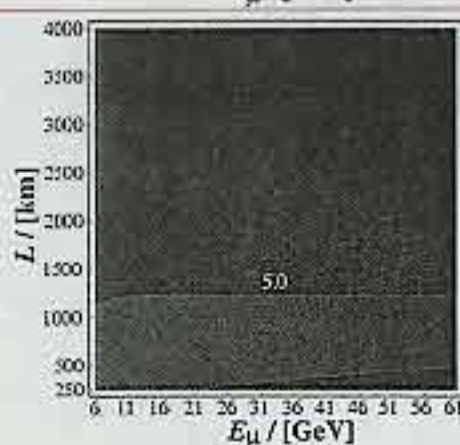
$$\delta = \frac{\pi}{6}$$



$$\delta = \frac{\pi}{2}$$

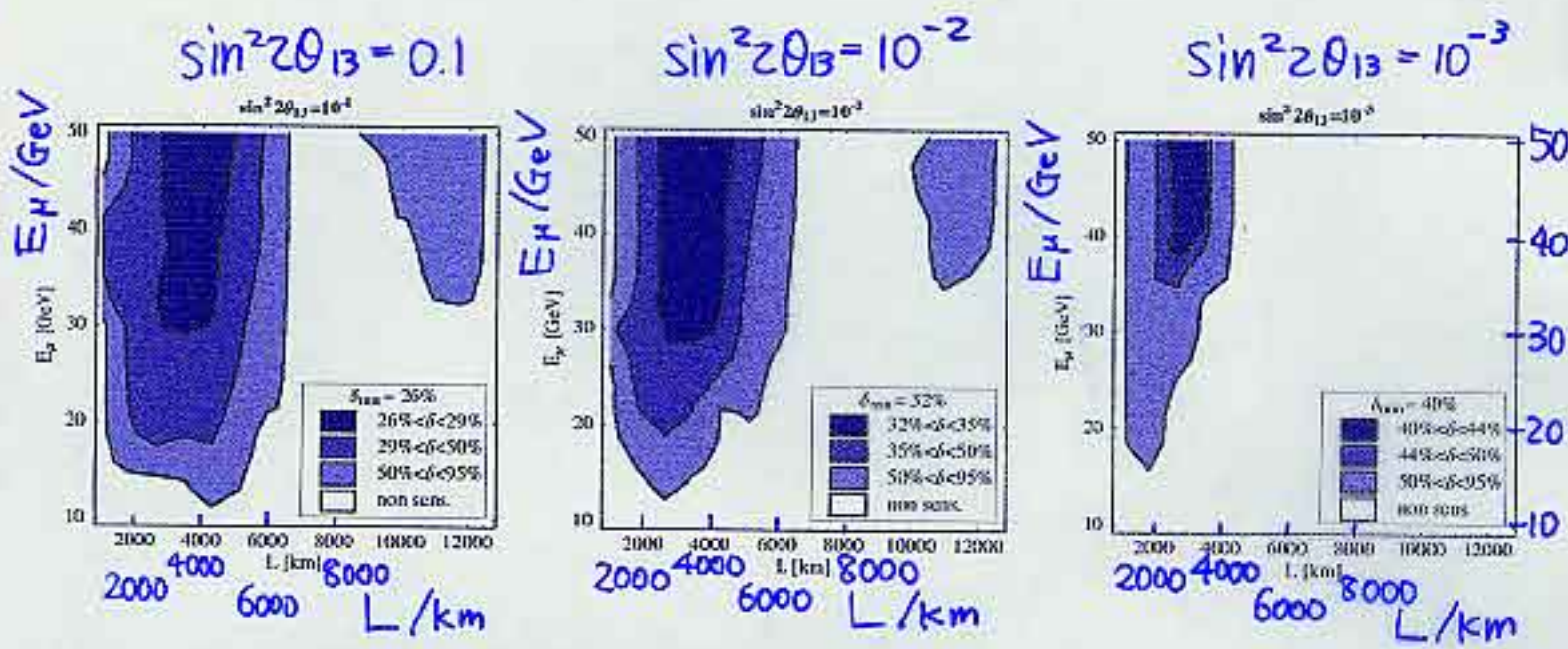


$$\delta = \frac{5\pi}{6}$$

Figure 10: Same as Fig.9, but for  $\delta m_{21}^2 = 5 \times 10^{-5} \text{ eV}^2$ .

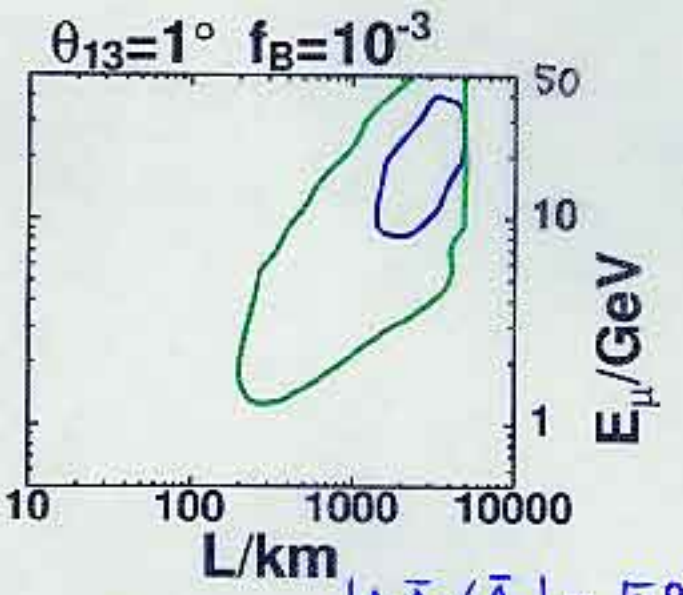
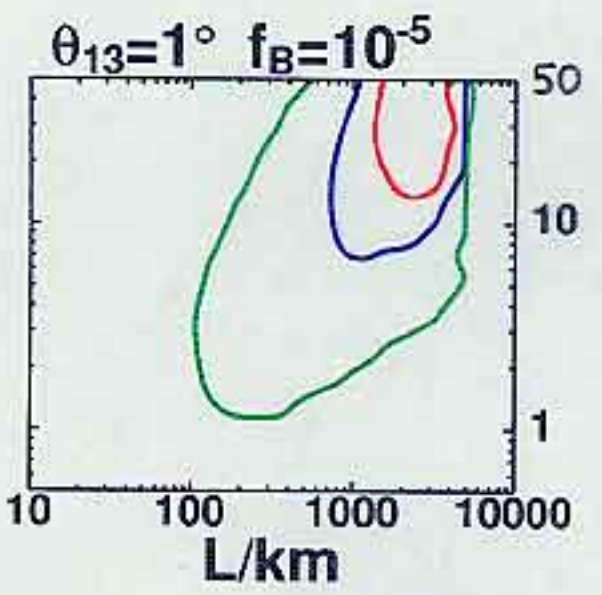
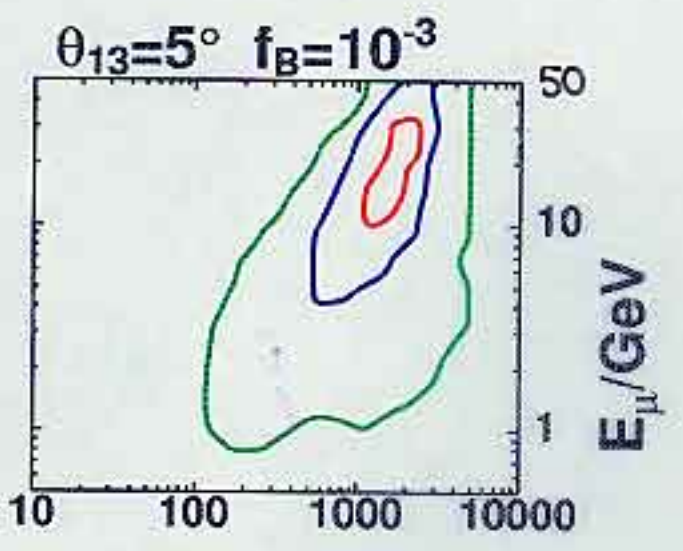
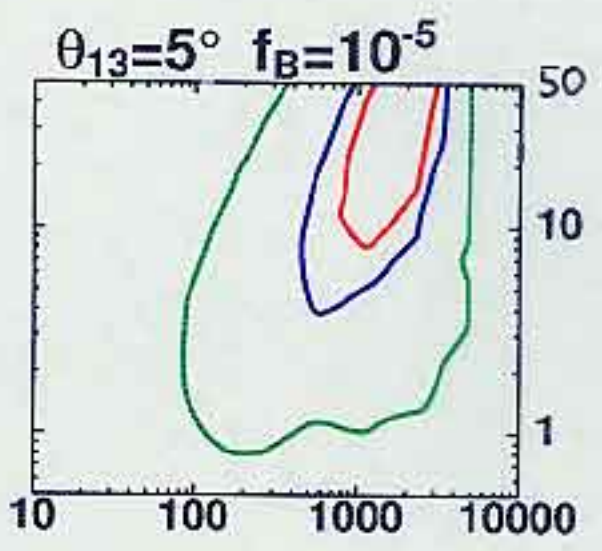
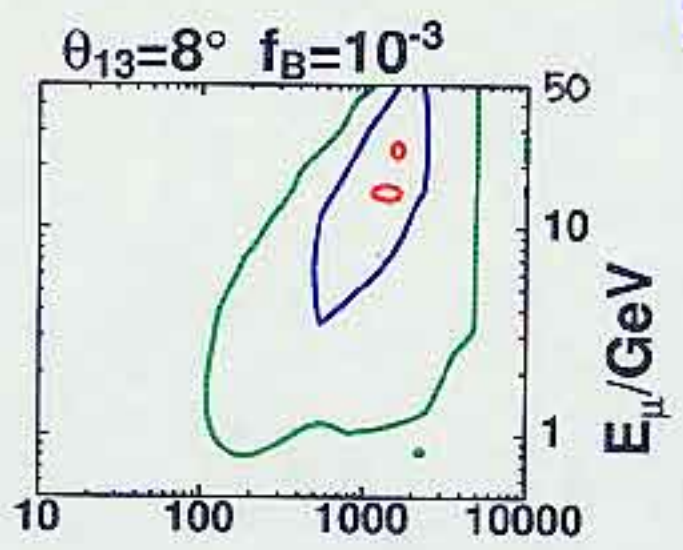
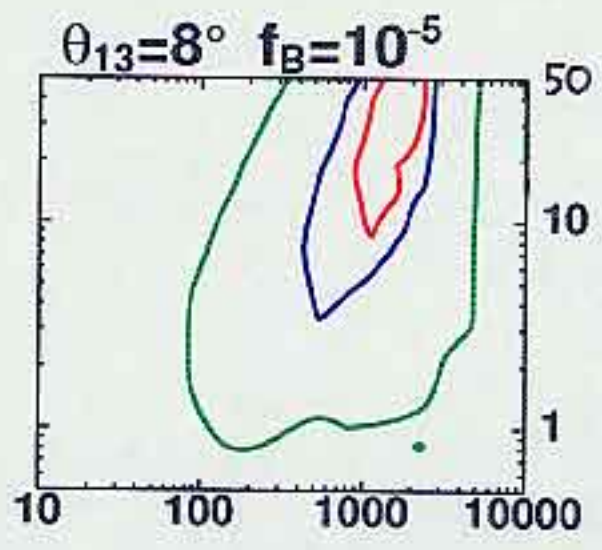


sensitivity to  $\delta$



**Figure 13:** Results of fits of the CP phase  $\delta_{CP}$  as function of the baseline  $L$  and the muon energy  $E_\mu$  for  $\Delta m_{31}^2 = 3.5 \cdot 10^{-3} eV^2$ ,  $\Delta m_{21}^2 = 10^{-4} eV^2$ ,  $\theta_{23} = \pi/4$ ,  $\theta_{12} = \pi/4$ ,  $N_\mu m_{kt} = 2 \cdot 10^{21}$  kt year and three values of  $\sin^2 2\theta_{13}$  ( $10^{-1}$ ,  $10^{-2}$ ,  $10^{-3}$ ). Dark shading indicates the preferred regions. The quantity  $\delta$  plotted here is the percentage of the  $\delta_{CP}$  parameter space  $[-\pi/2, \pi/2]$  which is compatible with the simulated experimental data at the  $3\sigma$  confidence level. The contour lines correspond to  $\delta = 50\%$  and  $\delta = 95\%$ . In the white shaded region no information on the CP phase can be obtained.

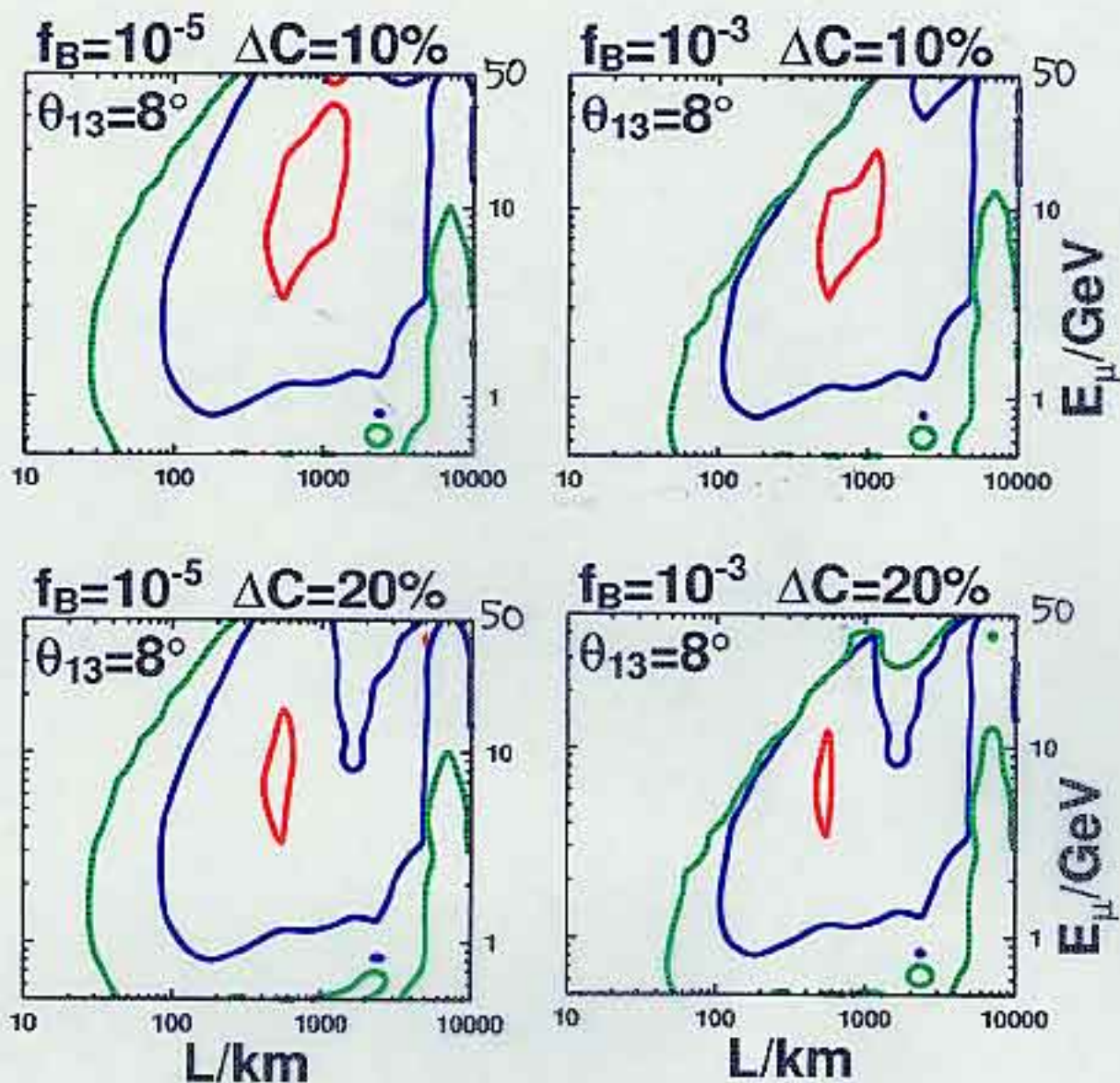




60kt ———  
 100kt ———  
 1000kt ———

detector size  $|\Delta \bar{A}/\bar{A}| = 5\%$   
 as a function of {baseline L  
 muon energy  $E_\mu$ }  
 to reject a hypothesis  $\delta=0$   
 at  $3\sigma$  CL Pinney-O.Y. PR D64('01)093008





100kt ———  
 1000kt ———  
 10000kt ———

$$\Delta C = \left| \frac{\Delta \bar{A}}{\bar{A}} \right|$$

### Fig.8

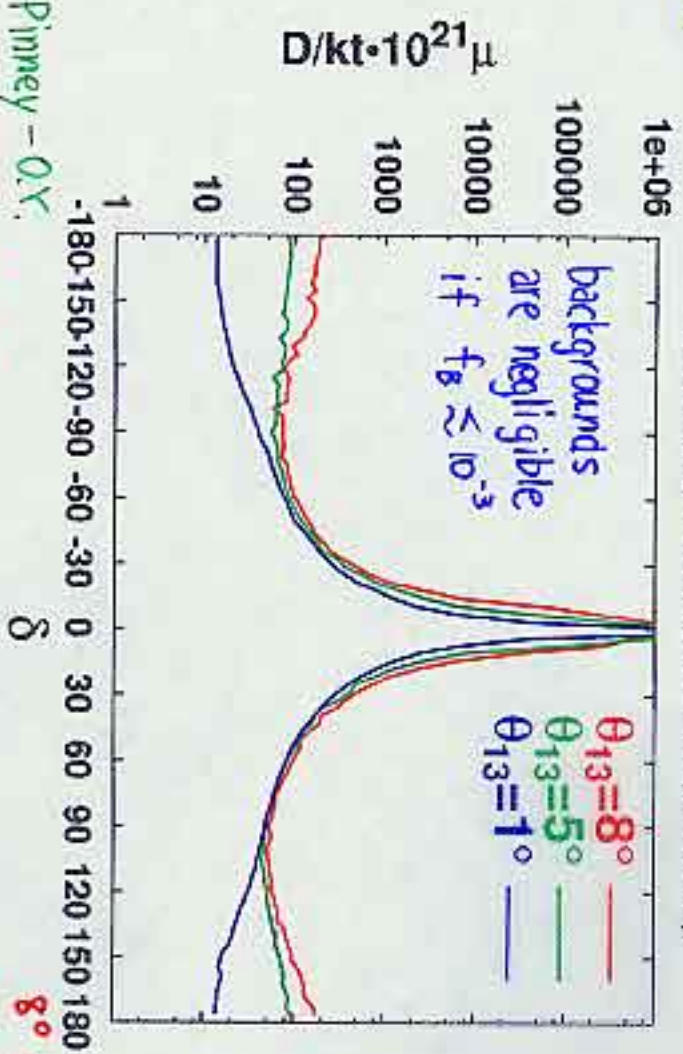
detector size  
 as a function of { baseline  $L$   
 muon energy  $E_\mu$  }

to reject a hypothesis  $\delta=0$

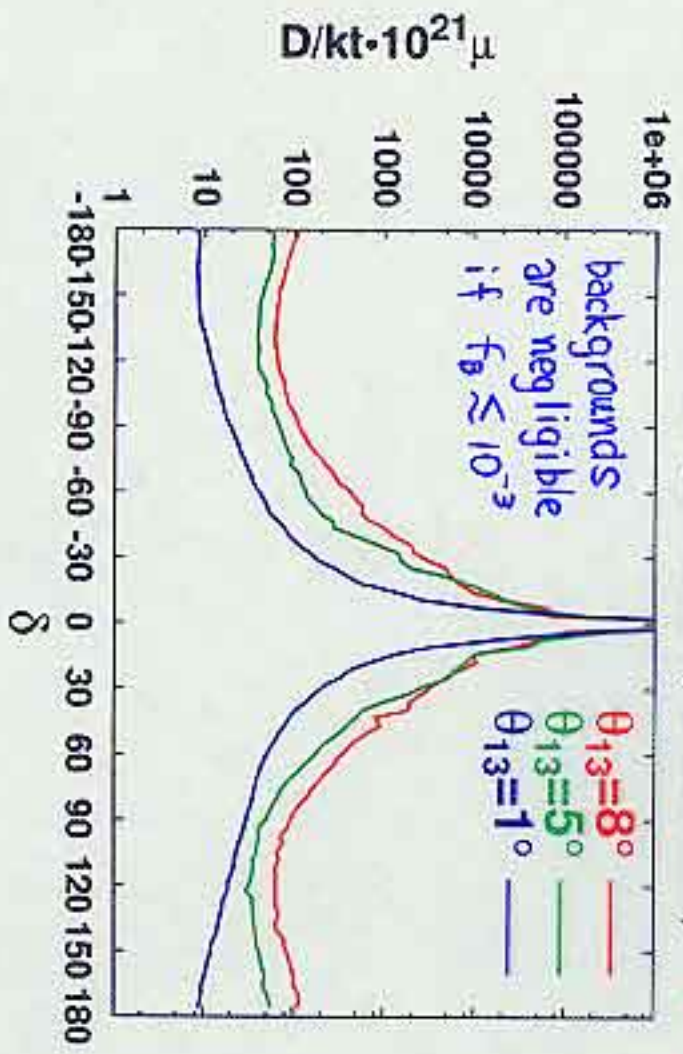
at  $3\sigma_{CL}$  Pinney-O.Y. PRD64(01)093008



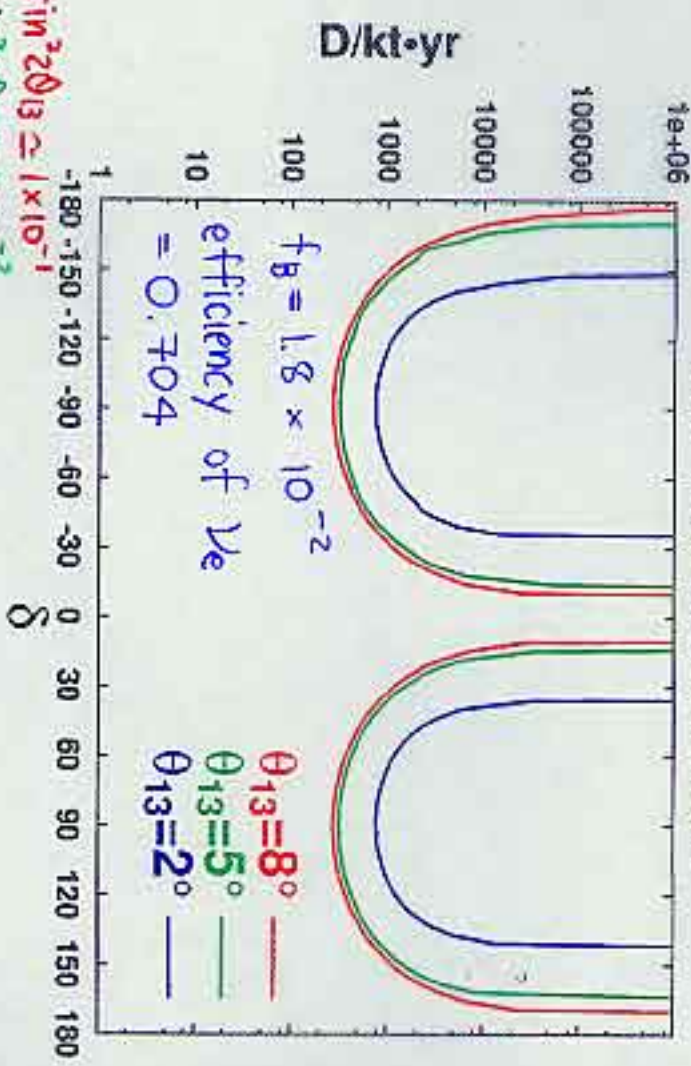
nufact (E=20GeV, L=1000km)



Pimney - O.Y.  
 PR D64 (v1) 073008  
 nufact (E=50GeV, L=3000km)

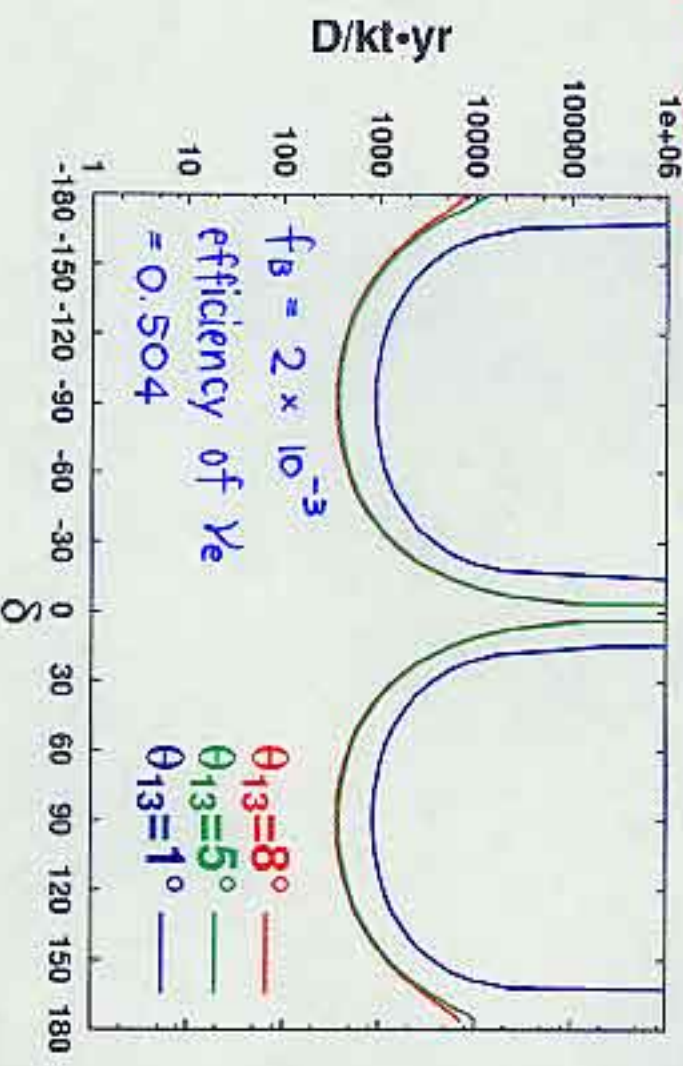


JHF 4MW NBB 1R e-like



$8^\circ \sin^2 2\theta_{13} \approx 1 \times 10^{-1}$   
 $5^\circ \sin^2 2\theta_{13} \approx 3 \times 10^{-2}$   
 $1^\circ \sin^2 2\theta_{13} \approx 1 \times 10^{-3}$

JHF 4MW NBB  $\pi^0$  cut





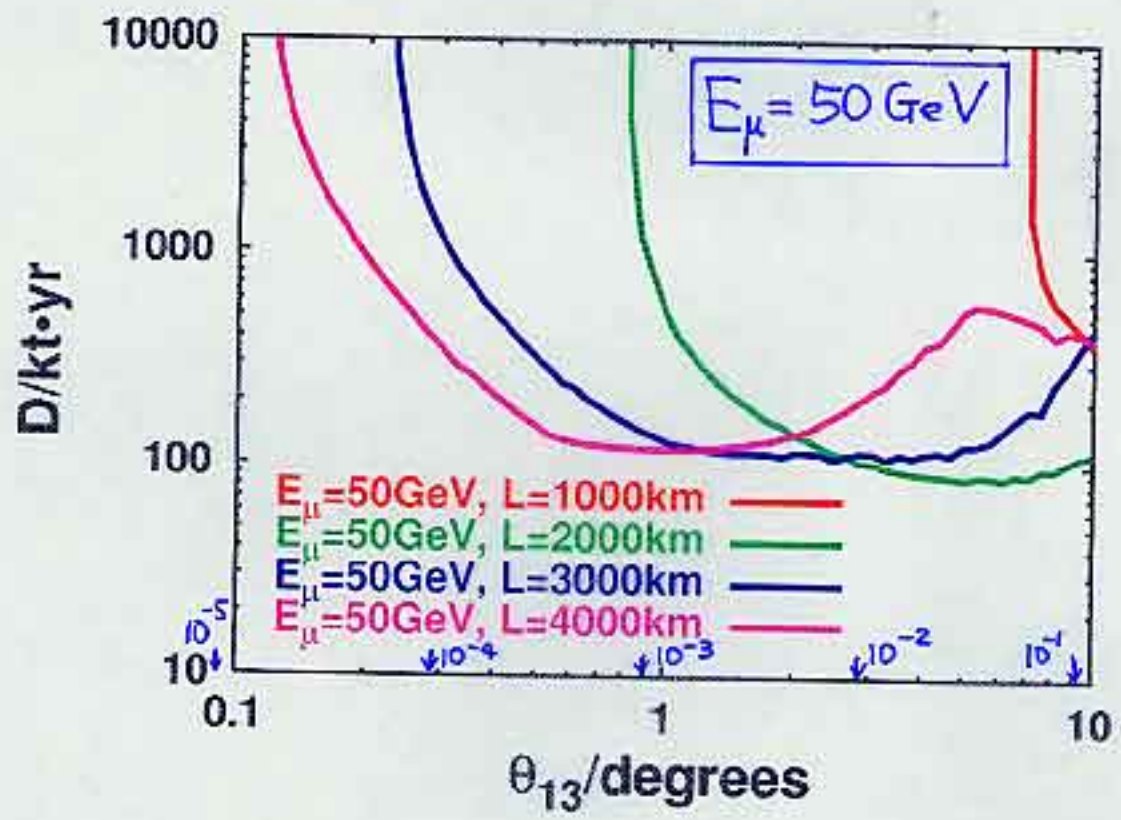
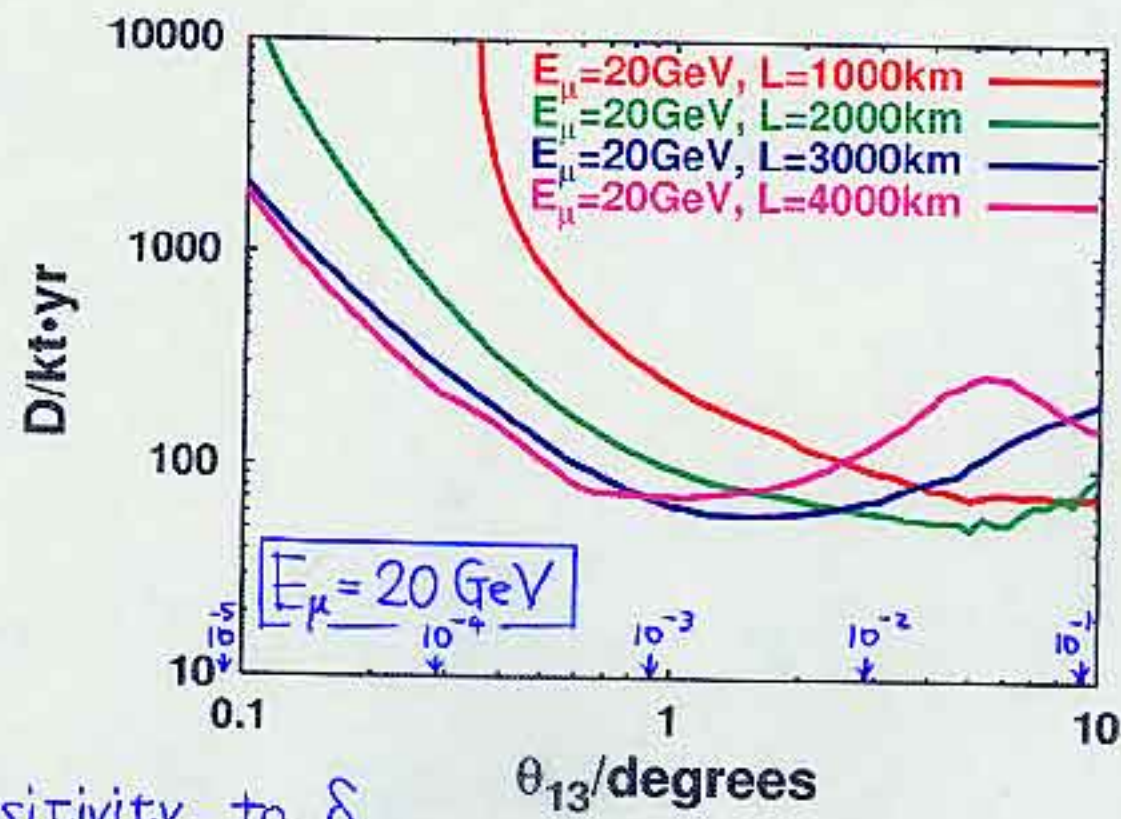


Fig. 1. Data size (kt·yr) required to reject a hypothesis of  $\bar{\delta} = 0$  at  $3\sigma$  when the true value is  $\delta = \pi/2$ , in the case of a neutrino factory with  $10^{21}$  useful muon decays per year and a background fraction  $f_B = 10^{-3}$ .

$f_B = 10^{-3}$

J. Pinney hep-ph/0106210

## General features

if  $f_B = 10^{-3}$  &  $\sin^2 2\theta_{13} \sim 0.1$

then lower  $E_\mu$  &  $L$  preferred

PY  $E_\mu \sim 20 \text{ GeV}$   
 $L \sim 1000 \text{ km}$

if  $|\Delta A/A| \leq 10\%$

← This may be too pessimistic

cf. Geller - Hara @nufact01  
 hep-ph/0111342

$|\Delta P/P| \lesssim 5\%$

then lower  $E_\mu$  &  $L$  preferred

KOS  $E_\mu \lesssim 7 \text{ GeV}$ ,  $L \sim 500 - 700 \text{ km}$

PY  $E_\mu \sim 15 \text{ GeV}$ ,  $L \sim 1000 \text{ km}$

Huber  $E_\mu \sim 25 \text{ GeV}$ ,  $L \sim 1500 \text{ km}$

Otherwise  $E_\mu \sim 50 \text{ GeV}$ ,  $L \sim 3000 \text{ km}$

is the optimum.

NB The results are in qualitative agreement, but more detailed analysis seems to be needed to reach quantitative agreement.



The behavior of  $\Delta\chi^2_{\text{indirect}}$  &  $\Delta\chi^2_3$  at large  $E_\mu$

$$\left. \begin{array}{l} \Delta\chi^2_{\text{KOS}} \propto \\ \Delta\chi^2_{\text{PY indirect}} \propto \end{array} \right\} \frac{\tilde{J}^2}{E_\mu} \left( \sin\delta + \text{const.} \cos\delta \frac{\Delta m^2_{32} L}{E_\mu} + \dots \right)^2$$

where as  $E_\mu \rightarrow \text{large}$

after correlations of errors are taken into account

$$\tilde{J} \equiv \frac{C_{13}}{8} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23}$$

→ • sensitivity is lost at large  $E_\mu$   
 (consistent with the claim by  
 Lipari PR D64 (01) 033002)

• both  $\Delta\chi^2_3$  &  $\Delta\chi^2_{\text{indirect}}$   
 are sensitive mainly to  $\sin\delta$   
 and a little to  $\cos\delta$

cf. (the case ( $E_\mu = 50 \text{ GeV}$ ,  $L = 3000 \text{ km}$ ))  

$$\frac{\Delta m^2_{32} L}{E_\mu} \sim 1$$
 is expected to be sensitive  
 both  $\sin\delta$  &  $\cos\delta$

\* J. Sato claims: main contribution is from  $\sin \delta$  main contrib. is from  $\cos \delta$  22

$$\Delta \chi^2 = F(\sin \delta) + \cancel{G(\cos \delta)}$$

↓

$$\Delta \tilde{\chi}^2 = F(\sin \delta)$$

(this corresponds mainly to CP-even processes)

\* Optimum set  $(\tilde{E}_\mu, \tilde{L})$  obtained from  $\Delta \tilde{\chi}^2$  is different from  $(E_\mu, L)$  obtained from  $\Delta \chi^2$ .

\* I claim that we are assuming the framework of  $N_\nu = 3$  from the beginning and we should not lose useful information by discarding  $G(\cos \delta)$ .

→ The correct optimum set is  $(E_\mu, L)$ .



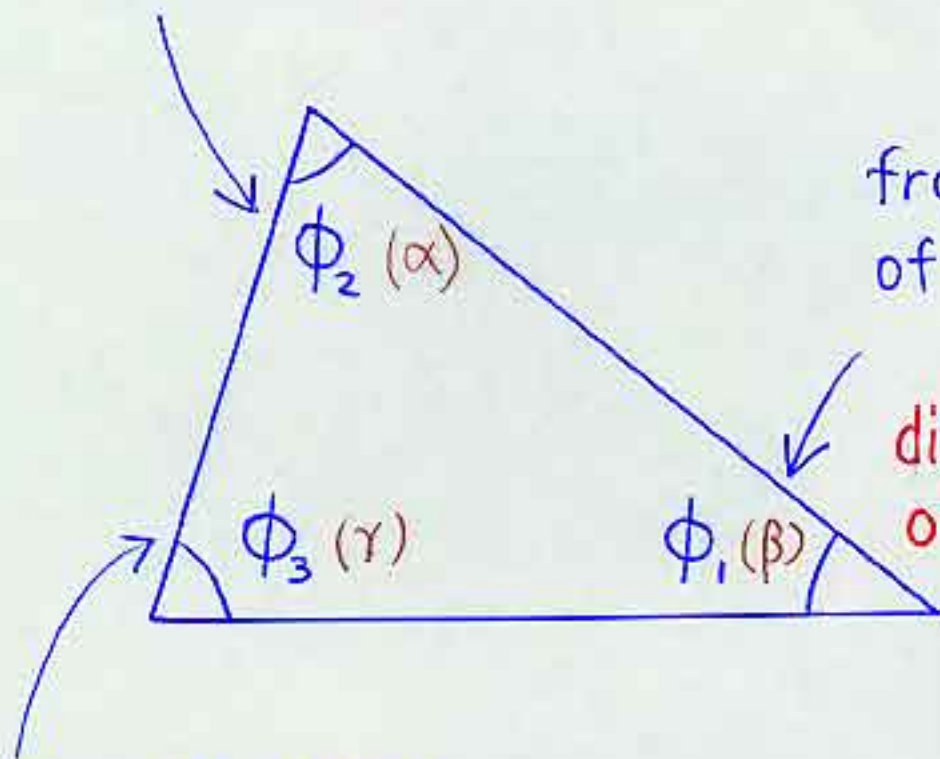
NB B factory

(Thanks to N. Kitazawa)<sup>23</sup>

from measurements

of  $B \rightarrow 2\pi$  : not necessarily CP-odd

indirect measurement of  $\phi$



from measurements  
of  $(B^0 \rightarrow J/\psi K_S)$   
 $(\bar{B}^0 \rightarrow J/\psi K_S)$

direct measurement  
of  $\phi$

from measurements

of  $B^\pm \rightarrow D_1^0 K^\pm$  : not necessarily CP-odd

indirect measurement of  $\phi$

$$\tan \phi_3 = - \frac{t_{12} \sin \delta}{t_{12} \cos \delta + t_{23} S_{13}}$$

$$\tan \phi_2 = - \frac{t_{12} t_{23} \sin \delta}{t_{12} t_{23} \cos \delta - S_{13}}$$

$$\phi_1 = \pi - \phi_2 - \phi_3$$

$\phi_j$  ( $j=1,2,3$ ) do depend on  $\cos \delta$  as well as  $\sin \delta$ .

parameter degeneracy ( $2^3$  fold)

\*  $(\delta, \theta_{13})$  ambiguity

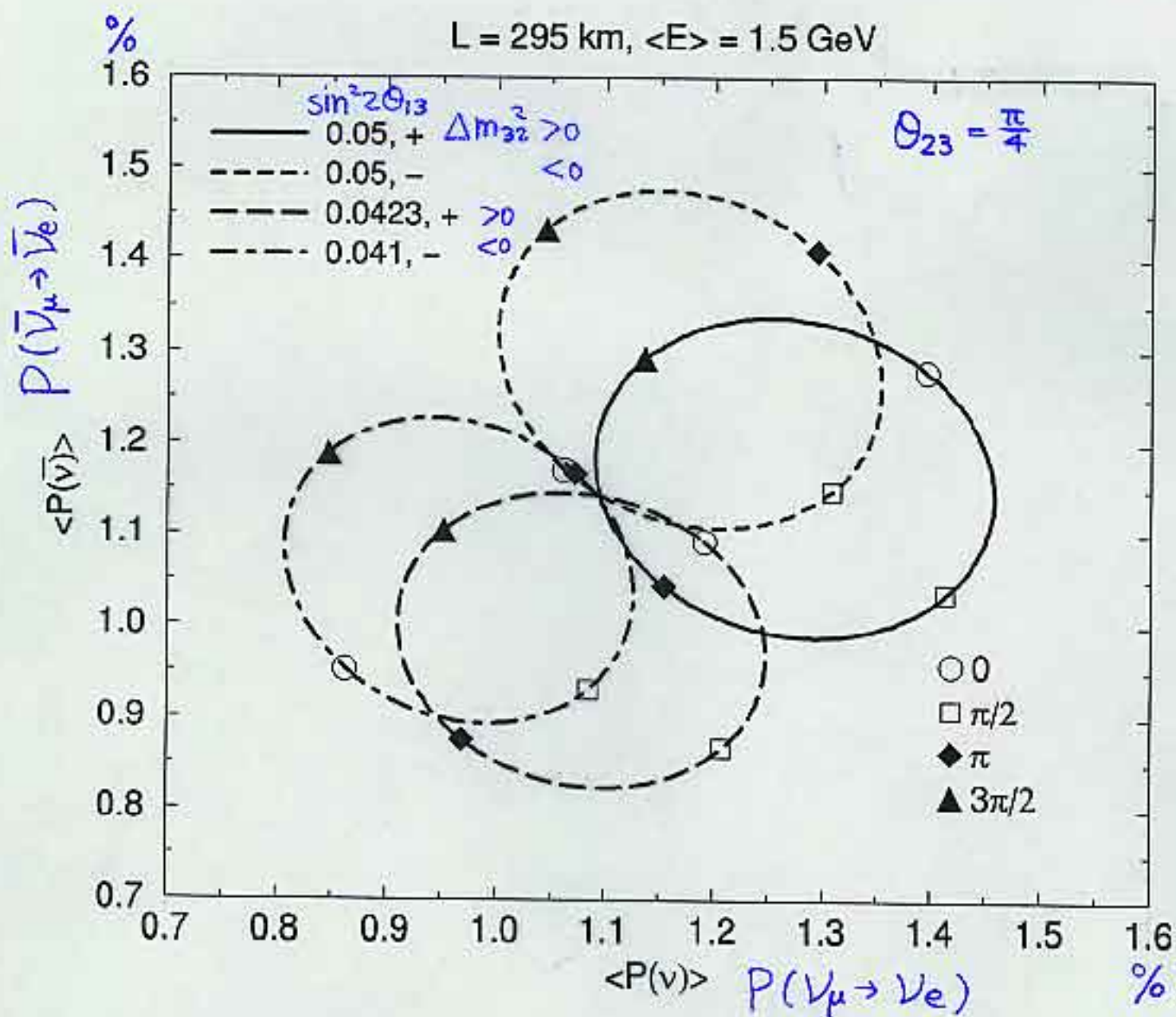
Burguet-Castell et al. NP B608('01)301

\* sign  $(\Delta m_{32}^2)$  ambiguity

Minakata-Nunokawa JHEP 0110('01)001

\*  $(\theta_{23}, \frac{\pi}{2} - \theta_{23})$  ambiguity

Barger - Marfatia - Whisnant, hep-ph/0112119



Minakata-Nunokawa

hep-ph/0111131



### 3. Summary

$D \equiv$  detector size to reject  $\delta=0$   
@  $3\sigma_{CL}$

JHF phase II (4MW)

$$D_{JHF} \gtrsim 1 \text{ Mt} \cdot \text{yr} \text{ for } \sin^2 2\theta_{13} \gtrsim 0.005$$

$\nu$  factory

$$D_{\nu \text{ factory}} \gtrsim 100 \text{ kt} \cdot 10^{21} \mu$$

$$\text{for } \sin^2 2\theta_{13} \gtrsim 10^{-4}$$

optimal set

$$(E_{\mu}, L) \sim \left\{ \begin{array}{ll} (50 \text{ GeV}, 3000 \text{ km}) & f_B \lesssim 10^{-5} \\ (20 \text{ GeV}, 1000 \text{ km}) & f_B \sim 10^{-3} \end{array} \right\}$$

$f_B$ : background fraction

### My personal impression

1st stage JHF phase I (2007 ~ )

measurement of  $\theta_{13}$

2nd stage JHF phase II (? ~ )

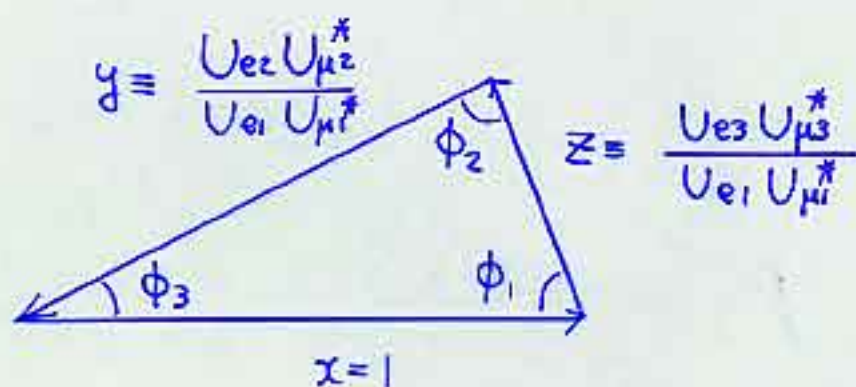
measurement of  $\delta$

(if  $\theta_{13}$  turns out to be small  
3rd stage  $\nu$  factory (?? ~ )  
measurement of  $\delta$ )

## Future problems

- direct measurement of CP?
- test of unitarity

Farzan - Smirnov hep-ph/0201105



determination of  $y$ ,  $z$  and one of the angles  $\phi_j$  ( $j=1,2,3$ ) gives a check of unitarity.



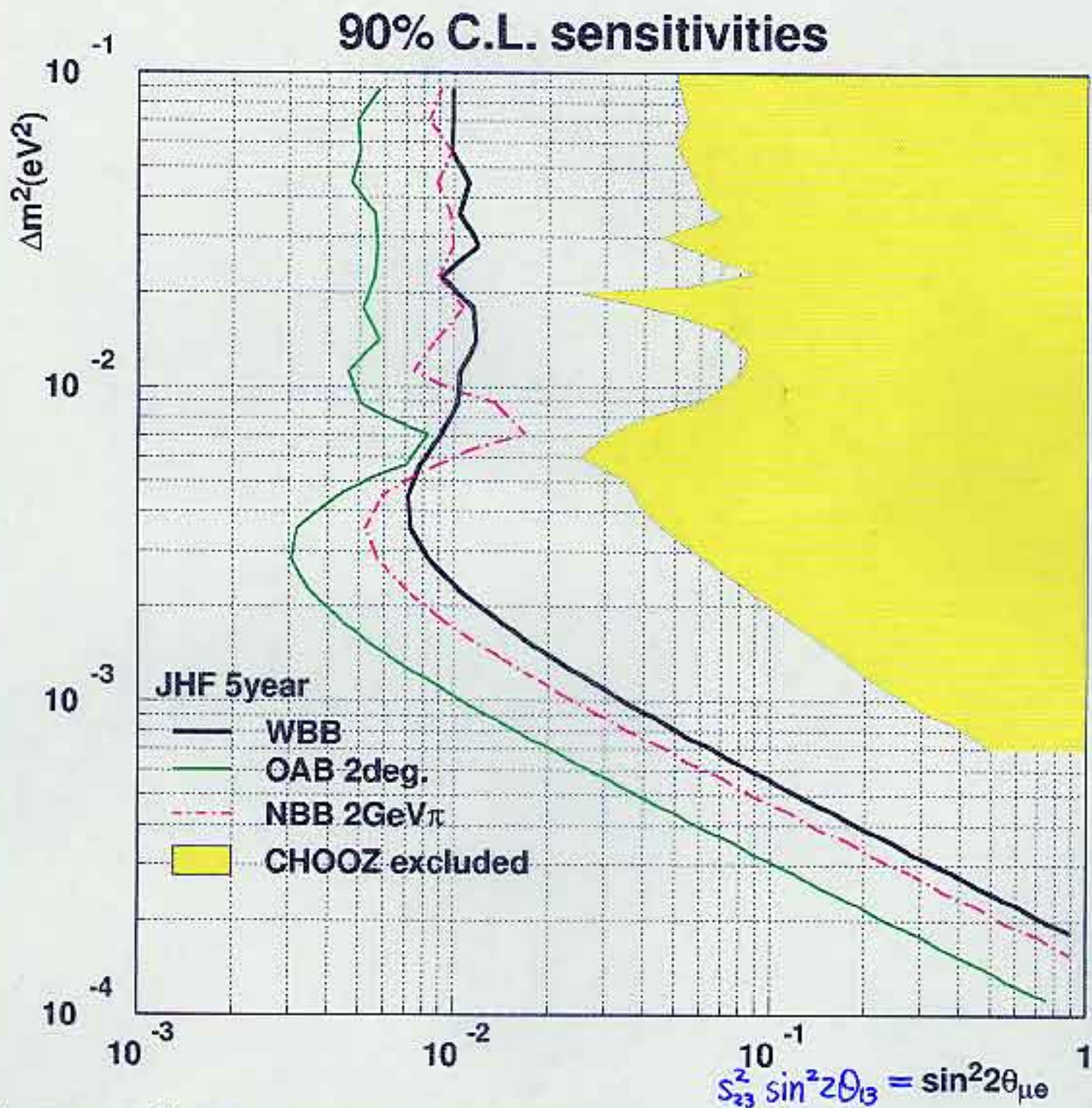
References on  $\nu$  factories before manufacture

- [1] B. Autin et al., CERN-SPSC-98-30;  
 A. De Rujula, M.B. Gavela and P. Hernandez, Nucl. Phys. **B547** (1999) 21;  
 S. Dutta, R. Gandhi and B. Mukhopadhyaya, hep-ph/9905475;  
 V. Barger, S. Geer and K. Whisnant, Phys. Rev. **D61** (2000) 053004;  
 M. Tanimoto, Phys. Lett. **B462** (1999) 115;  
 A. Kalliomaki, J. Maalampi and M. Tanimoto, Phys. Lett. **B469** (1999) 179;  
 A. Romanino, hep-ph/9909425;  
 M. Koike and J. Sato, Phys. Rev. **D61** (2000) 073012;  
 A. Donini, M.B. Gavela, P. Hernandez and S. Rigolin, hep-ph/9909254;  
 hep-ph/9910516;  
 V. Barger, S. Geer, R. Raja and K. Whisnant, hep-ph/9911524;  
 D. Ayres et al., physics/9911009.
- [2] V. Barger, S. Geer and K. Whisnant, Phys. Rev. **D61** (2000) 053004.
- [3] A. Cervera, A. Donini, M.B. Gavela, J. J. Gomez Cadenas, P. Hernandez, O. Mena and S. Rigolin, Nucl. Phys. **B579** (2000) 17.
- [4] V. Barger, S. Geer, R. Raja and K. Whisnant, Phys. Rev. **D62** (2000) 013004; Phys. Lett. **B485** (2000) 379; hep-ph/0007181.
- [5] V. Barger, S. Geer, R. Raja and K. Whisnant, Phys. Rev. **D62** (2000) 073002;
- [6] A. Donini, M. B. Gavela, P. Hernandez and S. Rigolin, Nucl. Phys. **B574** (2000) 23.
- [7] K. Dick, M. Freund, M. Lindner, and A. Romanino, Nucl. Phys. **B562** (1999) 29;  
 A. Romanino, Nucl. Phys. **B574** (2000) 675;  
 M. Freund, P. Huber and M. Lindner, Nucl. Phys. **B585** (2000) 105.
- [8] M. Freund, M. Lindner, S.T. Petcov and A. Romanino, Nucl. Phys. **B578** (2000) 27.
- [9] Neutrino Factory and Muon Collider Collaboration (D. Ayres *et al.*), physics /9911009;  
 C. Albright *et al.* hep-ex/0008064.
- [10] M. Campanelli, A. Bueno and A. Rubbia, hep-ph/9905240;  
 A. Bueno, M. Campanelli and A. Rubbia, Nucl. Phys. **B573** (2000) 27; Nucl. Phys. **B589** (2000) 577.
- [11] J. Sato, hep-ph/0006127;  
 M. Koike and J.Sato, Phys. Rev. **D61** (2000) 073012.
- [12] O. Yasuda, hep-ph/0005134.
- [13] J. Burguet-Castell, M.B. Gavela, J.J. Gomez-Cadenas, P. Hernandez, O. Mena, hep-ph/0103258.



sensitivity to  $\theta_{13}$  of JHF

Y. Itow et al. hep-ex/0106019



4: The 90% C.L. sensitivity contours for 5 years exposure of WBB, OA ions. The 90% C.L. excluded region of CHOOZ is plotted as a comparison maximum mixing of  $\sin^2 \theta_{23} = 0.5$  is assumed to convert from  $\sin^2 2\theta_{13}$  to :