

# NEUTRINO PHENOMENOLOGY WITH SOFT BILINEAR $R_p$ VIOLATION

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## OUTLINE

- MSSM Superpotential with  $R_p$
- BASIS INVARIANTS
- NEUTRINO MASSES : ESTIMATES
- MODEL
- PHENOMENOLOGY\*

## R-violating Superpotential

- The superpotential of the MSSM with lepton number violation is

$$W = \mu^I H_u L_I + \lambda^{IJk} L_I L_J E_k^c + \lambda'^{Ipq} L_I Q_p D_q^c + h_u^{pq} H_u Q_p U_q^c \quad (1)$$

where  $L_I = (H_d, L_i)$

$$\mu_I = (\mu_0, \mu_i), \frac{1}{2}h_e^{jk} = \lambda^{0jk}, \lambda^{ijk} = \lambda^{ijk}, \lambda'^{0pq} = h_d^{pq}, \lambda'^{ipq} = \lambda^{ipq}.$$

$$\frac{1}{2}h_\tau = \lambda_\tau^{01}, \lambda_b^0 = h_b,$$

- In a generic basis, we write the superpotential for one generation as

$$W = \mu^I H_u L_I + \lambda^{IJ} L_I L_J E^c + \lambda^I L_I Q D^c + h_u H_u Q U^c \quad (2)$$

and the soft SUSY breaking terms as

$$\begin{aligned} V_{soft} = & m_u^2 H_u^\dagger H_u + L^{I\dagger} [\tilde{m}_L^2]_{IJ} L^J + B^I H_u L_I \\ & + A_t H_u Q U^c + A_I^b L^I Q D^c + A_\tau^{IJ} L_I L_J E^c + h.c. \end{aligned}$$

$$B_I \neq B\mu_I \quad (3)$$

## Basis Invariants

$$\delta_\mu^i = \frac{\vec{\mu} \cdot \lambda^i \cdot \vec{v}}{|\vec{\mu}| m_i^e} \rightarrow \frac{\mu_i}{\mu_4} \quad (2)$$

$$\delta_{\lambda'}^{ipq} = \frac{\vec{\lambda}'^{pq} \cdot \lambda^i \cdot \vec{v}}{m_i^e} \rightarrow \lambda'^{ipq} \quad (3)$$

$$\delta_B^i = \frac{\vec{B} \cdot \lambda^i \cdot \vec{v}}{|\vec{B}| m_i^e} \rightarrow \frac{B_i}{B_4} \quad (4)$$

$$\delta_\lambda^{ijk} = \frac{\vec{v} \cdot \lambda^i \lambda^k \lambda^j \cdot \vec{v}}{m_i^e m_j^e} \rightarrow \lambda^{ijk} \quad (5)$$

We calculate the neutrino mass matrix elements by:

- basis-independent calculation of one-loop diagrams propagating MSSM mass eigenstates
- bilinear R-parity violating masses are included in the mass insertion approximation ( $\nu$  masses are small, can be done perturbatively)

↳ direct constraint on RPV from  $\bar{\nu}$  data.

$\delta_\mu$  is equivalent to

$$\sin \beta = \frac{H_I \times v_I}{|\mu| |\nu|}$$

v's { Hall, Suzuki  
Nowakowski, Pilafidis  
Banks et al,  
Nardi  
de Gouvea, White  
Valle et al

Tree level neutrino mass is non-zero if  $\delta_\mu \neq 0$ :

$$m_\nu^{tree} = -(\vec{\mu} \cdot \hat{L}_i) \sum_{\alpha} \frac{Z_{\alpha 3}^* Z_{\alpha 3}^*}{m_{\chi_\alpha}} (\vec{\mu} \cdot \hat{L}_j), \quad (6)$$

which gives a mass

$$m_3^{tree} = \sum_{i,\alpha} (\delta_\mu^i)^2 Z_{\alpha 3}^{*2} |\mu|^2 / m_{\chi_\alpha} \quad (7)$$

$$\hat{\nu}_3^{tree} = \frac{\delta_\mu^i}{\delta_\mu} \hat{L}_i, \quad (8)$$

where  $\delta_\mu = \sqrt{\sum_i (\delta_\mu^i)^2}$ ,  $\hat{L}_i = \frac{\lambda^i \cdot \vec{v}}{|\lambda^i \cdot \vec{v}|}$

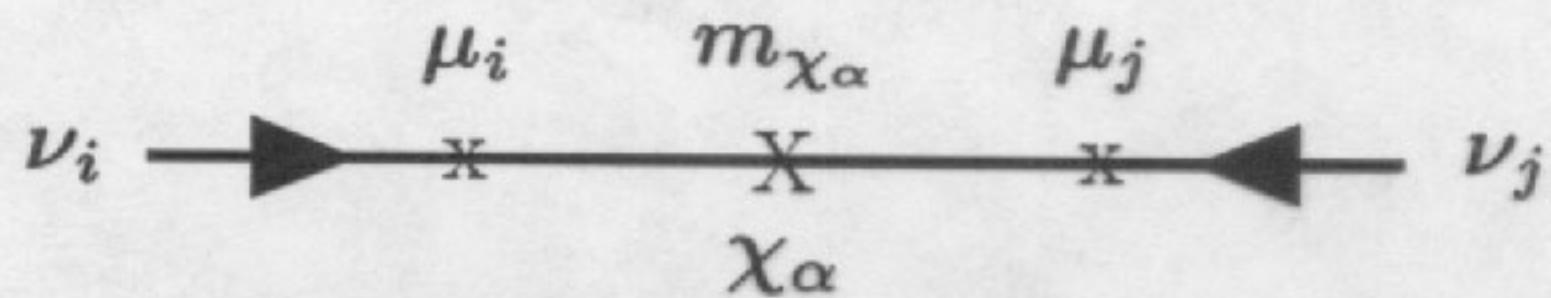


Figure 1: Tree-level neutrino mass in the mass insertion approximation

## Diagrams

- two scalar-fermion-fermion couplings at the vertices of the loop
- two  $\Delta L = 1$  interactions
- loop can contain coloured or colour-singlet charged or neutral particles
- either two gauge couplings, one gauge and one Yukawa/trilinear, or two Yukawa/trilinear couplings at the vertices
- neutral loop can only have two gauge couplings
- charged loop cannot have two gauge couplings as  $\Delta L = 2$  is forbidden on a charged line.
- charged loop can have one gauge and one Yukawa at the vertices. Need gaugino-lepton mixing on fermion line.

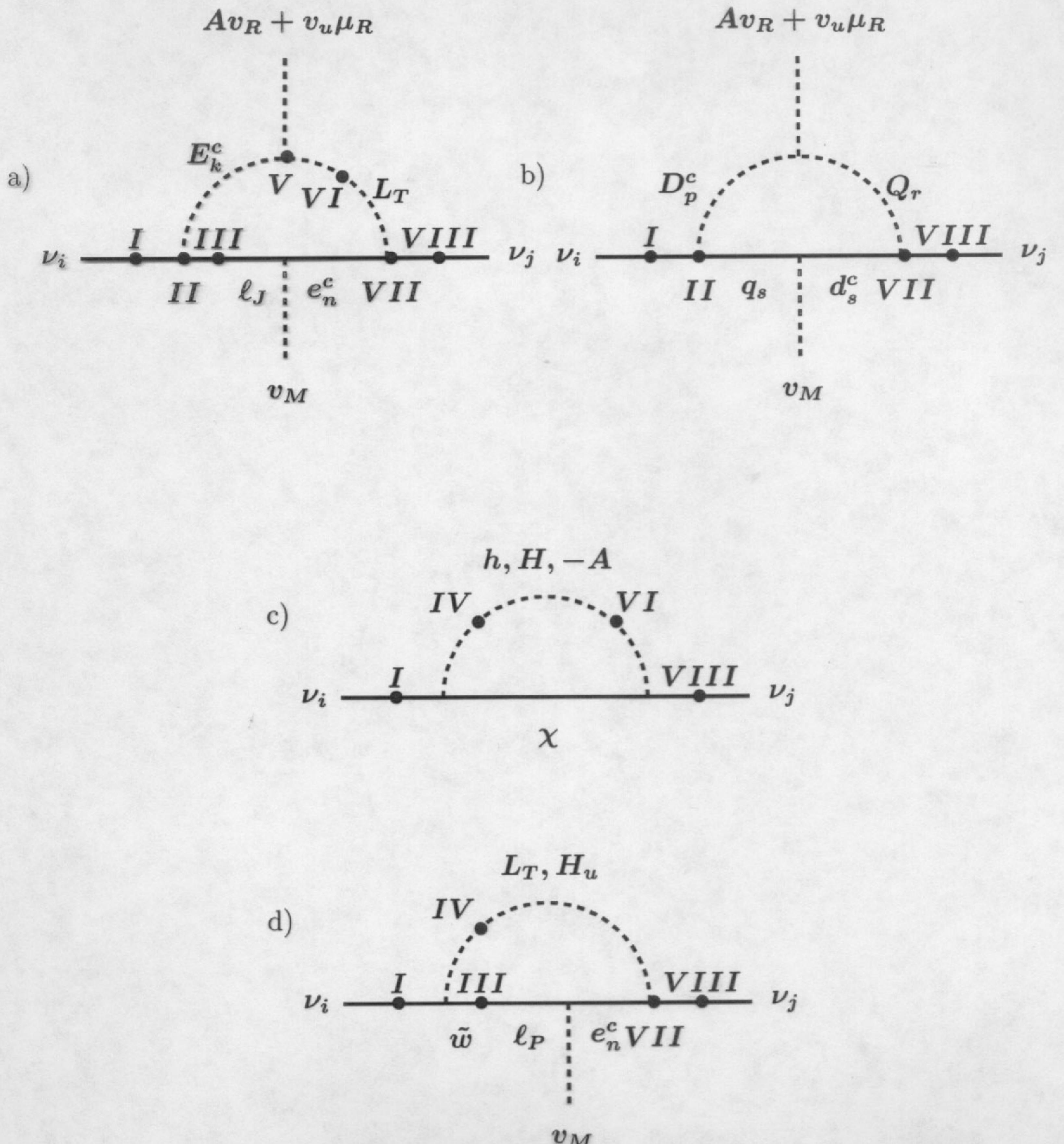


Figure 2: Schematic representation of one-loop diagrams contributing to neutrino masses. The blobs indicate possible positions for  $\mathcal{R}_p$  interactions, which can be trilinears (at positions II and VII) or mass insertions. The misalignment between  $\vec{\mu}$  and  $\vec{v}$  allows a mass insertion on the lepton/higgsino lines (at points I, III, or VIII) and at the  $A$ -term on the scalar line (position V). The soft  $\mathcal{R}_p$  masses appear as mass insertions at positions VI and IV on the scalar line. Figure a) corresponds to the charged loop with trilinear couplings  $\lambda$  (or  $h_e$ ) at the vertices. Figure b) is the coloured loops with trilinear  $\lambda'$  or yukawa  $h_b$  couplings. Figure c) is the neutral loops with two gauge couplings, and figure d) corresponds to the charged loop with one gauge and a Yukawa coupling. This diagram occurs if gauginos mix with charged leptons—that is if  $\delta_{..} \neq 0$ . See also figure 3.

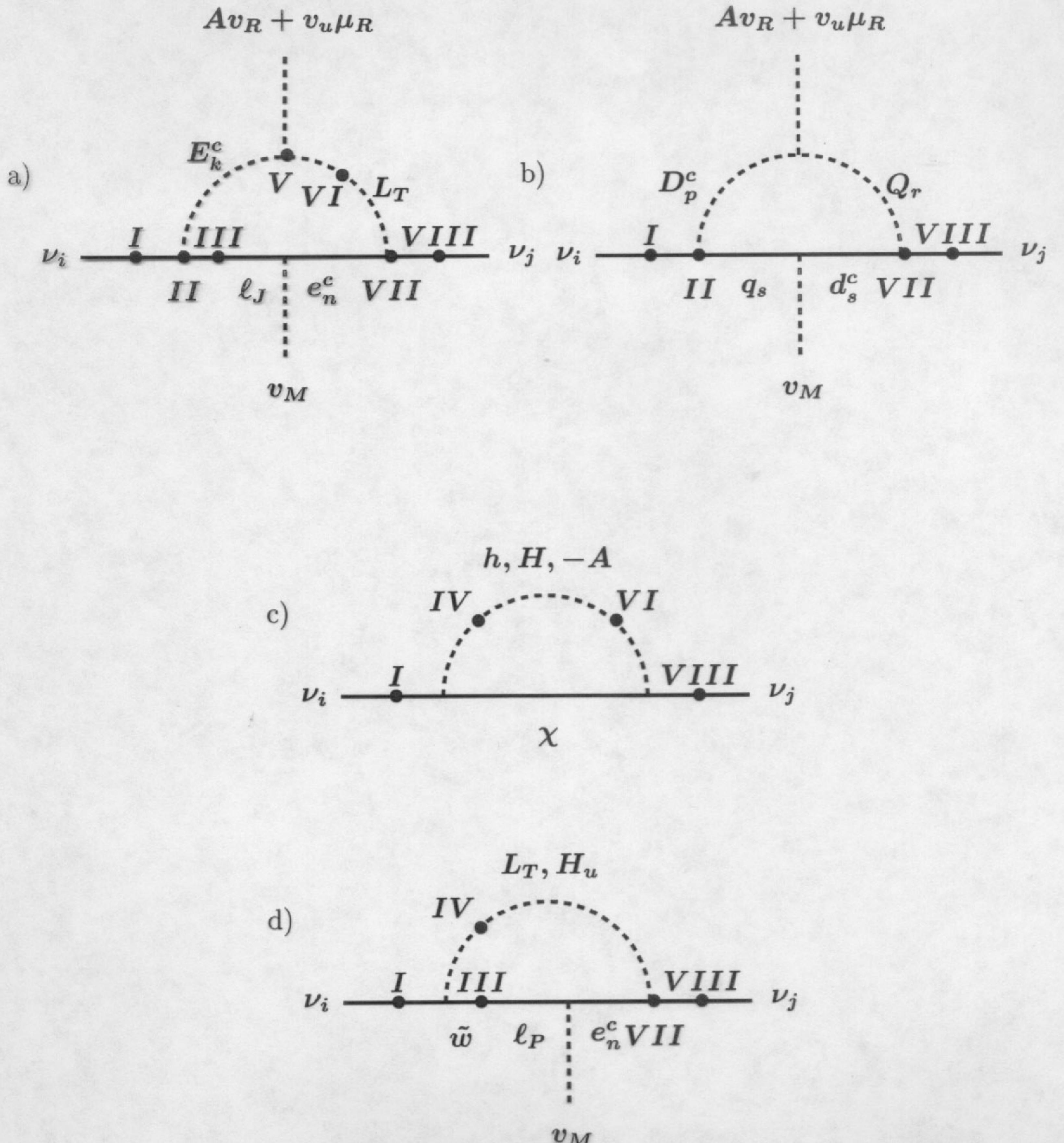


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No.	diagram	position of $\not{R}_p$	$16\pi^2 m_{SUSY} [m_\nu]^{ij}$
1	a	II VII	$\delta_\lambda^{ink} \delta_\lambda^{jkn} m_{e_n} m_{e_k}$
2	b	II VII	$3\delta_{\lambda'}^{iqq} \delta_{\lambda'}^{jqq} (m_{d_q})^2$
3	c	IV VI	$g^2 \delta_B^i \delta_B^j m_\chi m_{SUSY} / 4$
4	b	I VII + II VIII	$3(\delta_\mu^i \delta_{\lambda'}^{jqq} + \delta_\mu^j \delta_{\lambda'}^{iqq})(m_{d_q})^2 h_d^q$
5	a	II VI	$\delta_\lambda^{ijk} m_{e_k} \delta_B^k (m_{e_j} h_e^j - m_{e_i} h_e^i)$
6	a	I VII + II VIII	$(\delta_\mu^i \delta_\lambda^{jkk} + \delta_\mu^j \delta_\lambda^{ikk})(m_{e_k})^2 h_e^k$
7	a	I V	$\delta_\mu^i \delta_\mu^j ((m_{e_j} h_e^j)^2 + (m_{e_i} h_e^i)^2)$
8	a	II V	$\delta_\lambda^{ijk} \delta_\mu^k m_{e_k} (h_e^i m_{e_i} - h_e^j m_{e_j})$
9	a	III V	$\delta_\mu^i \delta_\mu^j m_{e_j} m_{e_i} h_e^i h_e^j$
10	a	III VIII	$\delta_\mu^i \delta_\mu^j ((m_{e_i} h_e^i)^2 + (m_{e_j} h_e^j)^2)$
11	a	I VI	$\delta_\mu^i \delta_B^j (m_{e_j} h_e^j)^2 + \delta_\mu^j \delta_B^i (m_{e_i} h_e^i)^2$
12	a	III VII	$\delta_\lambda^{jin} \delta_\mu^n m_{e_n} (m_{e_j} h_e^j - m_{e_i} h_e^i)$
13	a	III VI	$(\delta_B^i \delta_\mu^j h_e^j h_e^i m_{e_i} m_{e_j} + \delta_B^j \delta_\mu^i h_e^i h_e^j m_{e_j} m_{e_i})$
14	d	III IV	$g(\delta_B^i \delta_\mu^j (m_{e_j})^2 + \delta_B^j \delta_\mu^i (m_{e_i})^2)$
15	d	III VIII	$g \delta_\mu^i \delta_\mu^j ((m_{e_i})^2 + (m_{e_j})^2)$
16	d	I III	$g \delta_\mu^i \delta_\mu^j ((m_{e_i})^2 + (m_{e_j})^2)$
17	d	I VII	$g m_{e_k} m_{SUSY} (\delta_\mu^i \delta_\lambda^{jkk} + \delta_\mu^j \delta_\lambda^{ikk})$ $3 g m_{d_k} m_{SUSY} (\delta_\mu^i \delta_{\lambda'}^{jkk} + \delta_\mu^j \delta_{\lambda'}^{ikk})$
18	d	III VII	zero for degenerate sleptons
19	c	I VI + IV VIII	$g^2 m_{SUSY}^2 (\delta_B^i \delta_\mu^j + \delta_\mu^i \delta_B^j) / 4$
20	d	I V	$g \delta_\mu^i \delta_\mu^j ((m_{e_i})^2 + (m_{e_j})^2)$
21	d	I IV	$g(\delta_\mu^i \delta_B^j (m_{e_j})^2 + \delta_\mu^j \delta_B^i (m_{e_i})^2)$

Table 2: Estimated contributions to  $[m_\nu]^{ij}$  from all the diagrams. In the second two columns is the label of the diagram of figure 2, and the position on the diagram of the two  $\Delta L = 1$  interactions. Column four is the “basis independent” estimated contribution to the neutrino mass matrix in the flavour basis. All indices other than  $i$  and  $j$  are summed.

# Phenomenology

## Neutrinos

Experiment	$\Delta m^2$ (eV $^2$ )	$\sin^2 2\theta$	$\tan^2 \theta$
Atmospheric	$(2 - 5) \times 10^{-3}$	$0.88 - 1$	—
MSW-LMA	$(2 - 70) \times 10^{-5}$	$0.6 - 1$	$(2 - 40) \times 10^{-1}$
MSW-SMA	$(0.4 - 1) \times 10^{-5}$	$10^{-3} - 10^{-2}$	$(1 - 30) \times 10^{-4}$
MSW-LOW	$4 \times 10^{-10} - 2 \times 10^{-7}$	$0.7 - 1$	$(1 - 80) \times 10^{-1}$
Vacuum	$(1 - 6) \times 10^{-10}$	$0.5 - 1$	$(1 - 90) \times 10^{-1}$
Just-so	$(4 - 10) \times 10^{-12}$	$0.5 - 1$	$(3 - 30) \times 10^{-1}$
CHOOZ	$> 3 \times 10^{-3}$	$< 0.22$	

Table 1: Allowed mass squared differences and mixing angles for MSW-LMA, MSW-SMA, MSW-LOW, Vacuum and Just-so oscillation solutions. Chooz bound also shown. SNO data included.

The  $3 \times 3$  rotation matrix from neutrino flavour ( $f$ ) to mass ( $m$ ) eigenstates can be parametrised by three rotations:  $V_{fm} = R_{23}(\theta_{23})R_{13}(\theta_{13})R_{12}(\theta_{12})$ . Neglecting Majorana and Dirac phases,

$$V_{fm} \equiv \begin{bmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23} & c_{12}c_{23} - s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{23}s_{12} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - c_{23}s_{12}s_{13} & c_{13}c_{23} \end{bmatrix}. \quad (9)$$

**Model B**

$$m_\nu = m_\mu^{(0)} \hat{\delta}_\mu^i \hat{\delta}_\mu^j + m_{\mu B}^{(0)} (\hat{\delta}_\mu^i \hat{\delta}_B^j + \hat{\delta}_B^i \hat{\delta}_\mu^j) + m_B^{(0)} \hat{\delta}_B^i \hat{\delta}_B^j \quad (14)$$

where

$$m_\mu^{(0)} = |\vec{\delta}_\mu|^2 m_{SUSY}$$

$$m_{\mu B}^{(0)} = \frac{\alpha}{16\pi} |\vec{\delta}_\mu| |\vec{\delta}_B| m_{SUSY} = \sqrt{\frac{\alpha}{16\pi} m_\mu^{(0)} m_B^{(0)}}$$

$$m_B^{(0)} = \frac{\alpha}{16\pi} |\vec{\delta}_B|^2 m_{SUSY}$$

$\alpha = \frac{g^2}{4\pi}$ ,  $\hat{\delta}_\mu$  and  $\hat{\delta}_B$  are unit vectors in  $\{L^i\}$  space.

$\vec{\delta}_\mu$  and  $\vec{\delta}_B$ , are not required to be orthogonal. An orthonormal basis would be  $\hat{\delta}_\mu$  and  $\hat{\delta}_{B_\perp}$  where

$$\delta_{B_\perp}^i = \delta_B^i - (\delta_B \cdot \hat{\delta}_\mu) \hat{\delta}_\mu^i \quad . \quad (15)$$

So  $m_\nu$  can be written

$$m_\nu \equiv m_\mu \hat{\delta}_\mu^i \hat{\delta}_\mu^j + m_{\mu B} (\hat{\delta}_\mu^i \hat{\delta}_{B_\perp}^j + \hat{\delta}_{B_\perp}^i \hat{\delta}_\mu^j) + m_B \hat{\delta}_{B_\perp}^i \hat{\delta}_{B_\perp}^j$$

$$\cos \rho = \hat{\delta}_\mu \cdot \hat{\delta}_B$$

$m_\nu$  has two non-zero eigenvalues:

- Hierarchical:  $\Delta m_{atm}^2 = m_3^2$  and  $\Delta m_{sol}^2 = m_2^2$
- Pseudo-Dirac:  $\Delta m_{atm}^2 = m_3^2, m_2^2$  and  $\Delta m_{sol}^2 = m_3^2 - m_2^2$ .

Suppose  $(\delta_\mu^1, \delta_\mu^2, \delta_\mu^3)$  is rotated by an angle  $\gamma$  with respect to  $(V_{13}, V_{23}, V_{33})$

$$\frac{\alpha}{16\pi} |\delta_B|^2 m_{SUSY} \sin^2 \rho = (\cos^2 \gamma m_2 + \sin^2 \gamma m_3) \quad (16)$$

$$|\delta_\mu|^2 m_{SUSY} \left(1 - \frac{\alpha}{16\pi}\right) = \frac{m_2 m_3}{\cos^2 \gamma m_2 + \sin^2 \gamma m_3} \quad (17)$$

$$\cot \rho = \frac{\cos \gamma \sin \gamma (m_2 - m_3)}{\cos^2 \gamma m_2 + \sin^2 \gamma m_3} - \sqrt{\frac{\alpha}{16\pi} \frac{m_2 m_3}{(\cos^2 \gamma m_2 + \sin^2 \gamma m_3)^2}} \quad (18)$$

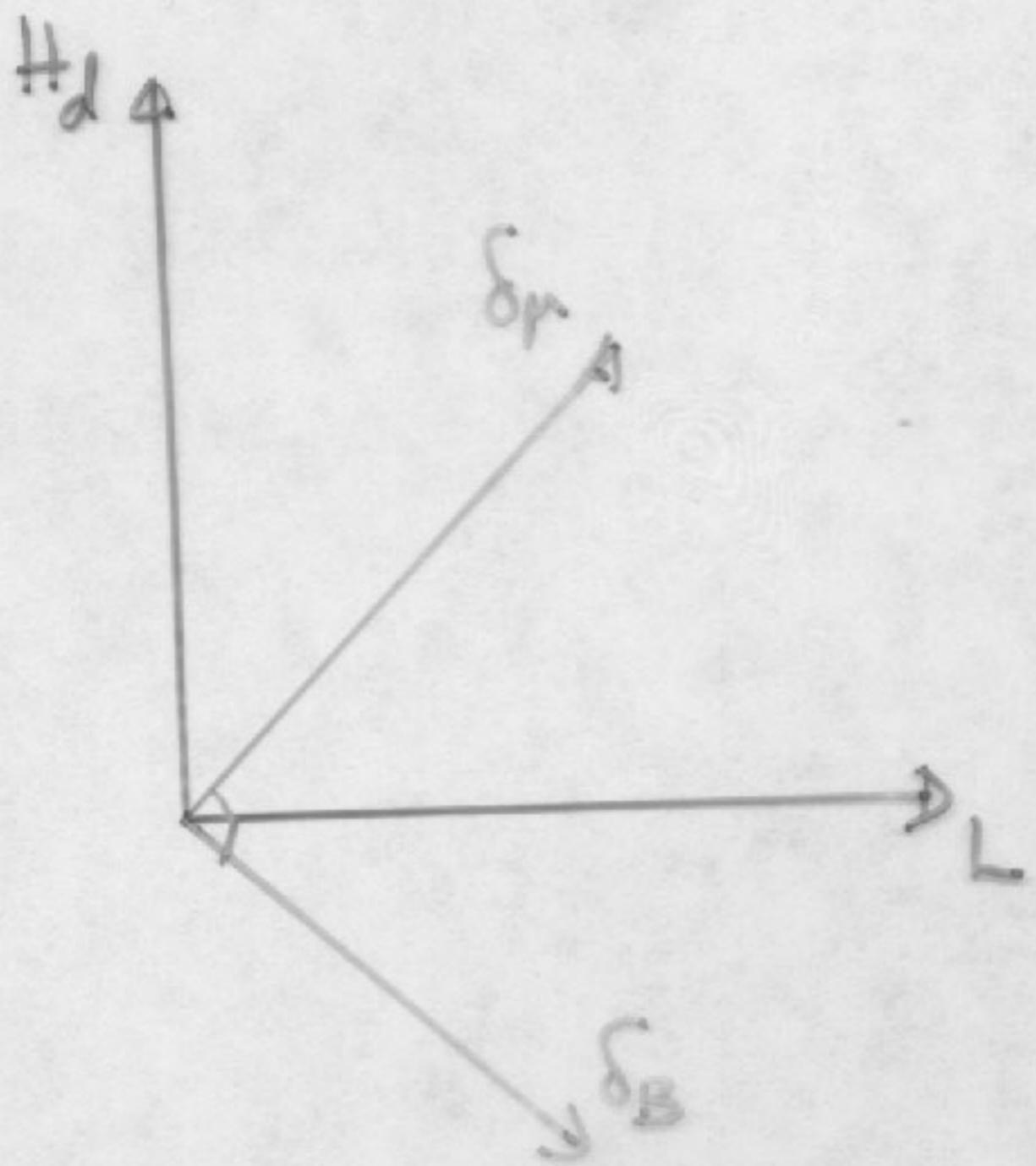
$$\frac{1}{|\delta_\mu|} \delta_\mu^f = \cos \gamma V_{f3} + \sin \gamma V_{f2} \quad (19)$$

$$\frac{1}{|\delta_{B_\perp}|} \delta_{B_\perp}^f = -\sin \gamma V_{f3} + \cos \gamma V_{f2} \quad (20)$$

Degenerate  
spectrum

$$m_2 \sim m_3 \sim \sqrt{\Delta m_{\text{atm}}^2}$$

$$\Rightarrow g \sim \pm \pi/2$$



$$\rightsquigarrow \frac{\alpha}{16\pi} |\delta_B|^2 \simeq |\delta_\mu|^2 \simeq \frac{\sqrt{\Delta m_{\text{atm}}^2}}{m_{\text{susy}}}$$

Hierarchical  
spectrum

CHOOZ requires

$$V_{e3} = \cos \gamma \hat{\delta}_{\mu}^e - \sin \gamma \hat{\delta}_{B\perp}^e \lesssim .1.$$

- LMA:  $V_{e2} \sim 1/\sqrt{2}$

so  $\delta_{\mu}^e \simeq \frac{1}{\sqrt{2}} \sin \gamma$ , and  $\delta_{B\perp}^e \simeq \frac{1}{\sqrt{2}} \cos \gamma$

$V_{e3}$  is small due to a cancellation between  $\delta_{\mu}^e$  and  $\delta_{B\perp}^e$ .

- SMA: both  $\delta_{\mu}^e$  and  $\delta_{B\perp}^e$  may be small.

Cosmological attractive: would allow a BAU produced before the electroweak phase transition to survive.

Primordial asymmetry stored in a conserved quantum number to survive.

For  $B/3 - L_e$  to be effectively conserved before the phase transition,

$$B_4 \mu^e - \mu_4 B^e \lesssim 2 \times 10^{-7} |\mu| |B| \quad (21)$$

For  $\delta_{\mu} \sim 10^{-6}$ ,  $\delta_B \sim 10^{-4}$ , to generate the atmospheric and SMA solar masses, this is satisfied for  $\hat{\delta}_{\mu}^e \lesssim .1$  or  $\hat{\delta}_B^e \lesssim 2 \times 10^{-3}$ .

- For a hierarchical spectrum, for small values of angle  $\gamma$

$$\cos^2 \gamma m_2 \sim \sin^2 \gamma m_3$$

- $|\delta_\mu| \sim m_3 \rightarrow (\Delta m_{atm}^2)^{1/2}$        $\Theta_{atm} \stackrel{\text{def.}}{=} \left( \delta_\mu^\mu, \delta_\mu^\tau \right)$
  - $|\delta_B| \sim m_2 \rightarrow (\Delta m_{solar}^2)^{1/2}$ .
- For all other values of  $\gamma$
- $|\delta_B| \rightarrow (\Delta m_{atm}^2)^{1/2}$        $\Theta_{atm} \stackrel{\text{def.}}{=} \left( \delta_{B\perp}^\mu, \delta_{B\perp}^\tau \right)$
  - $|\delta_\mu| \rightarrow (\Delta m_{solar}^2)^{1/2}$ .

Solar+CHOOZ+SuperK

$m_3 \rightarrow \delta_B$

$\delta_B \cdot 10^{-5}$

- LMA-MSW
- + MSW-SMA
- △ MSW-LOW
- VAC
- \* Just-So

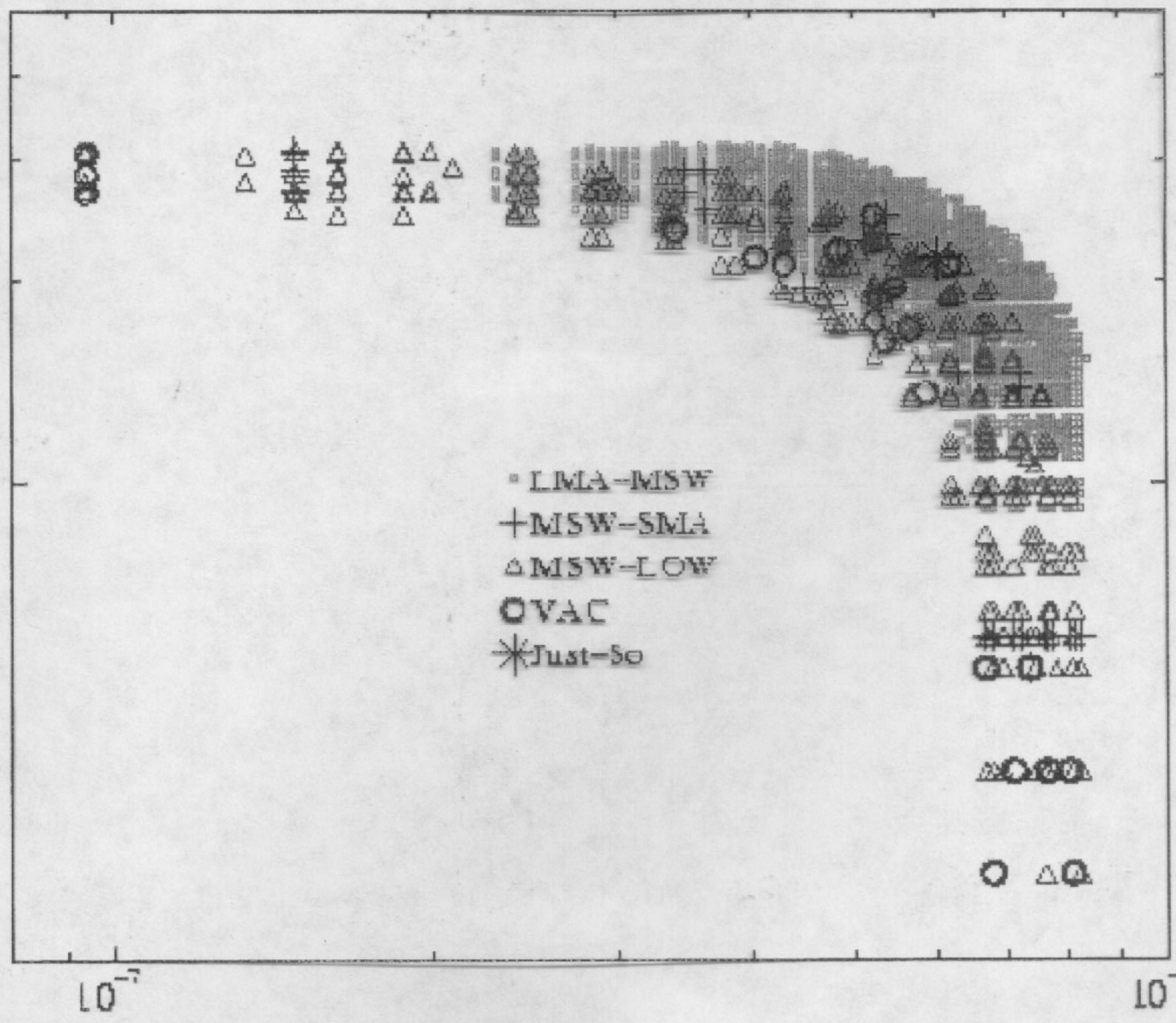
$m_3 \rightarrow \delta_\mu$

$10^{-7}$

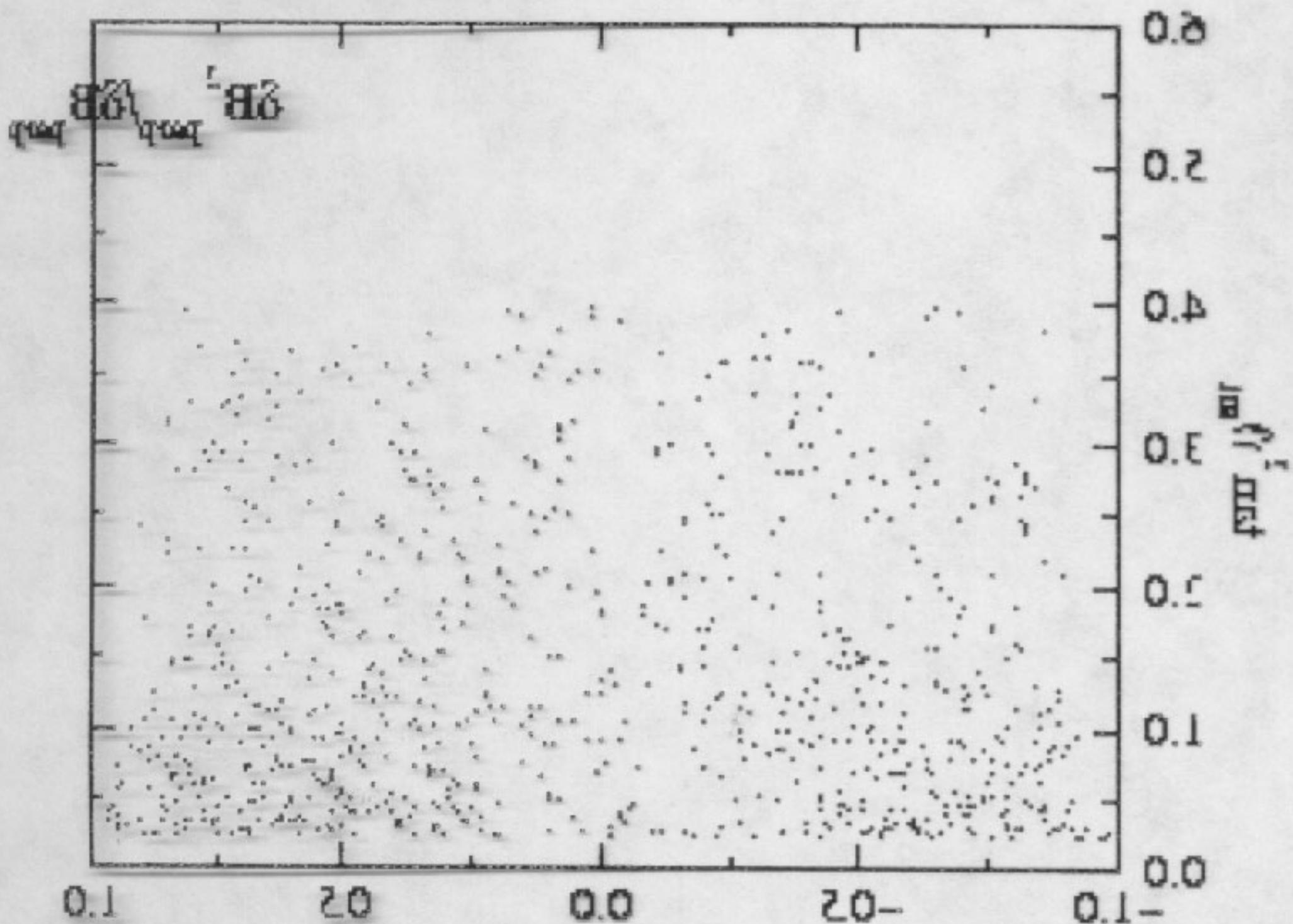
$10^{-6}$

$\delta_\mu$

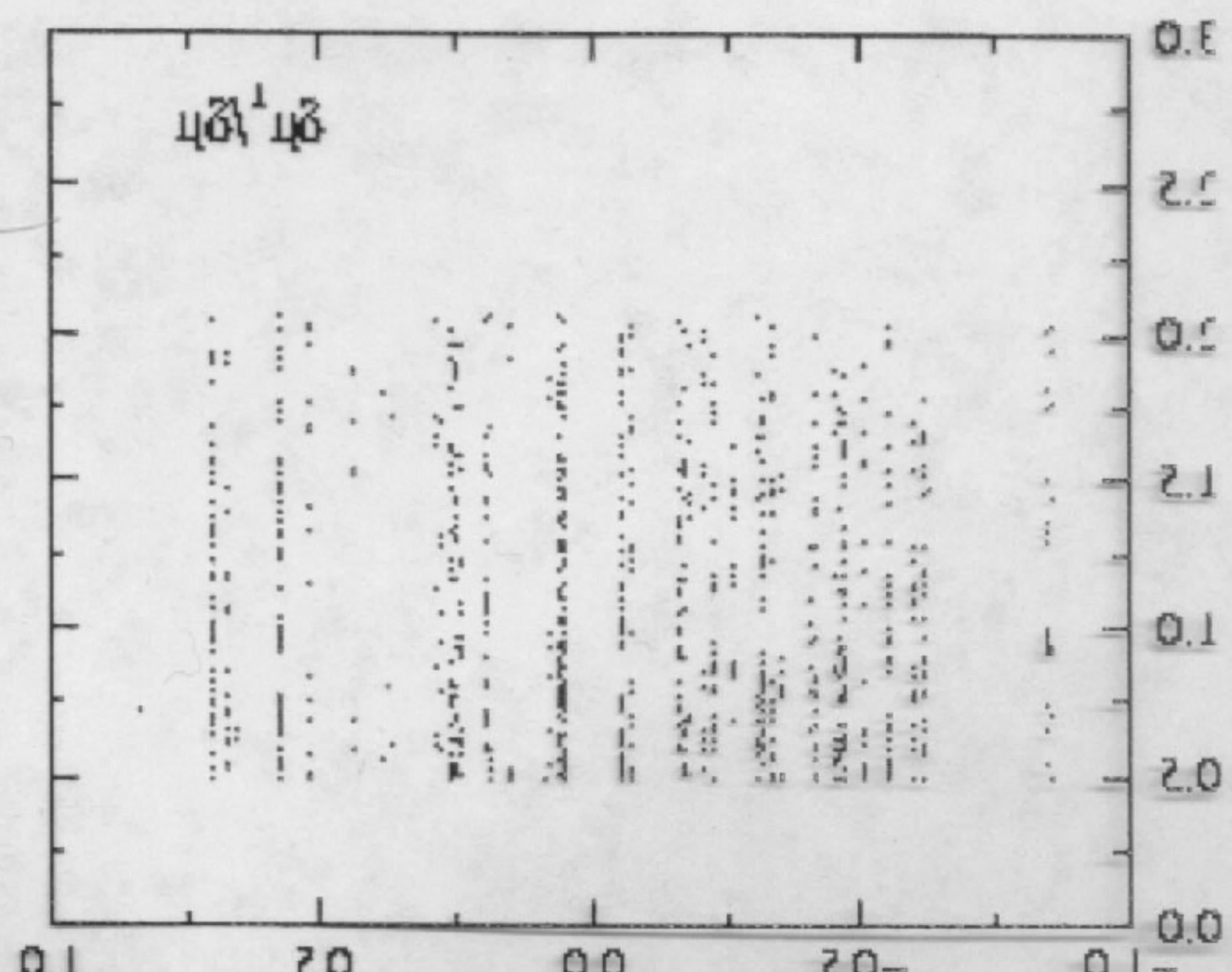
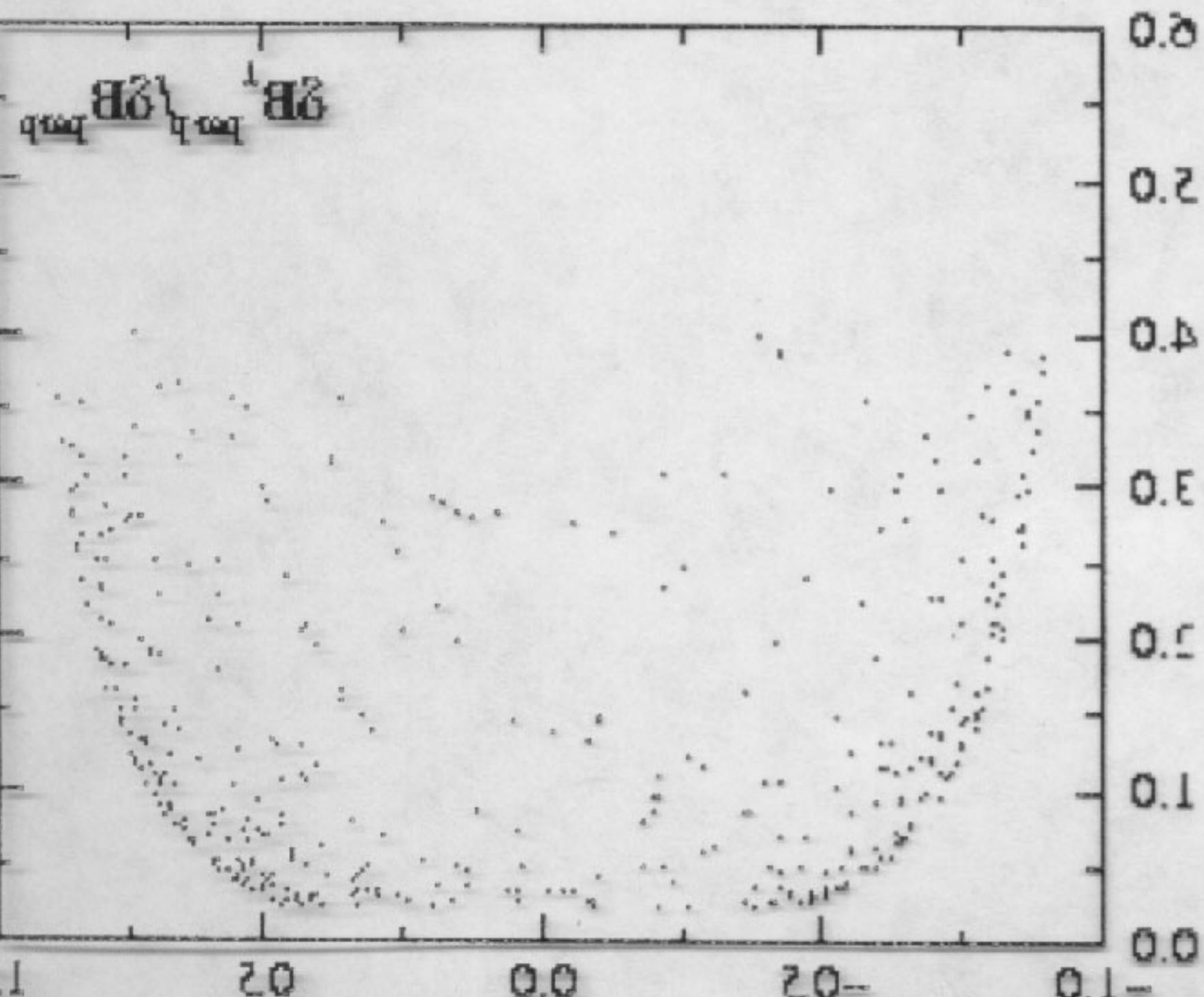
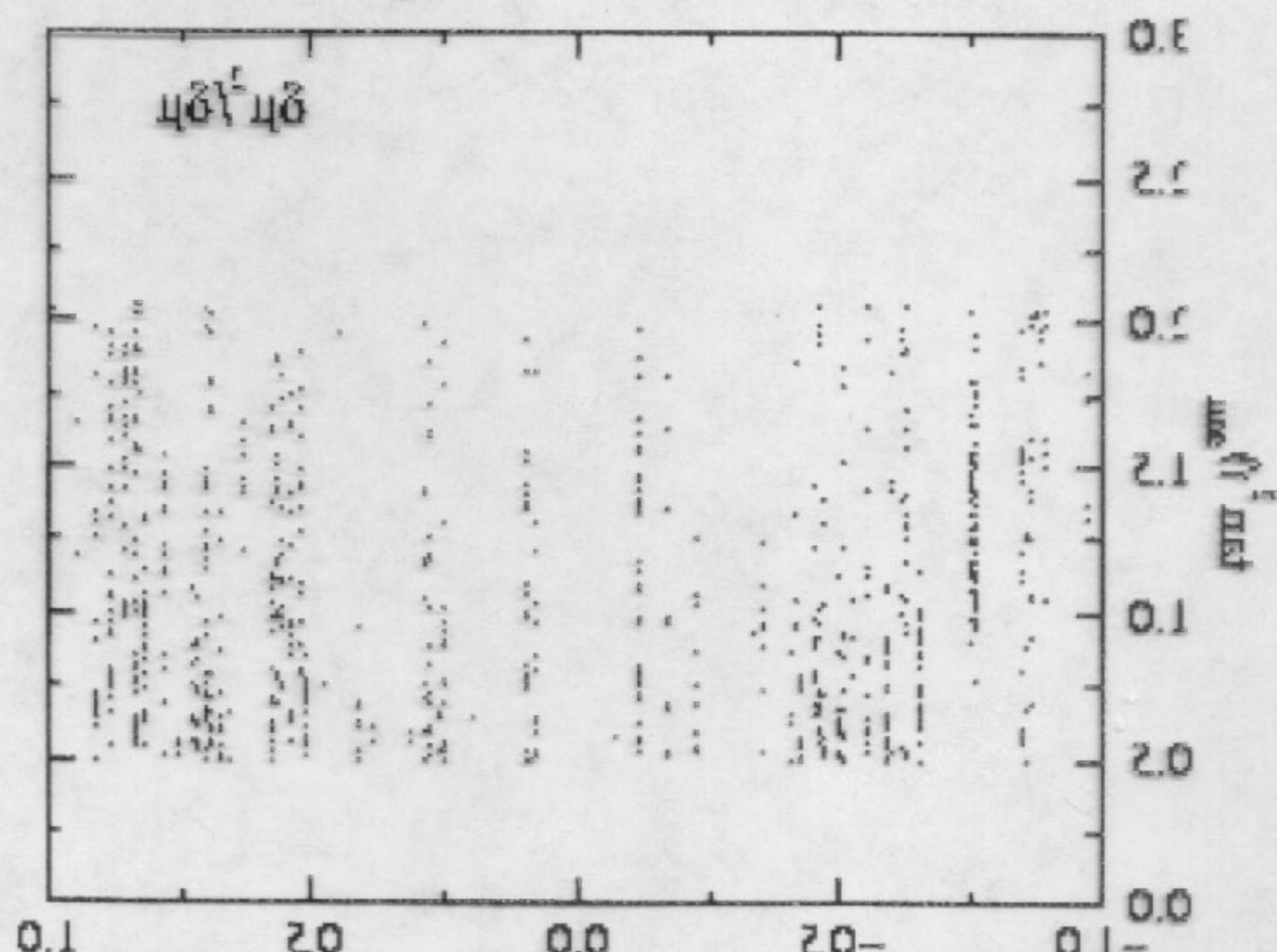
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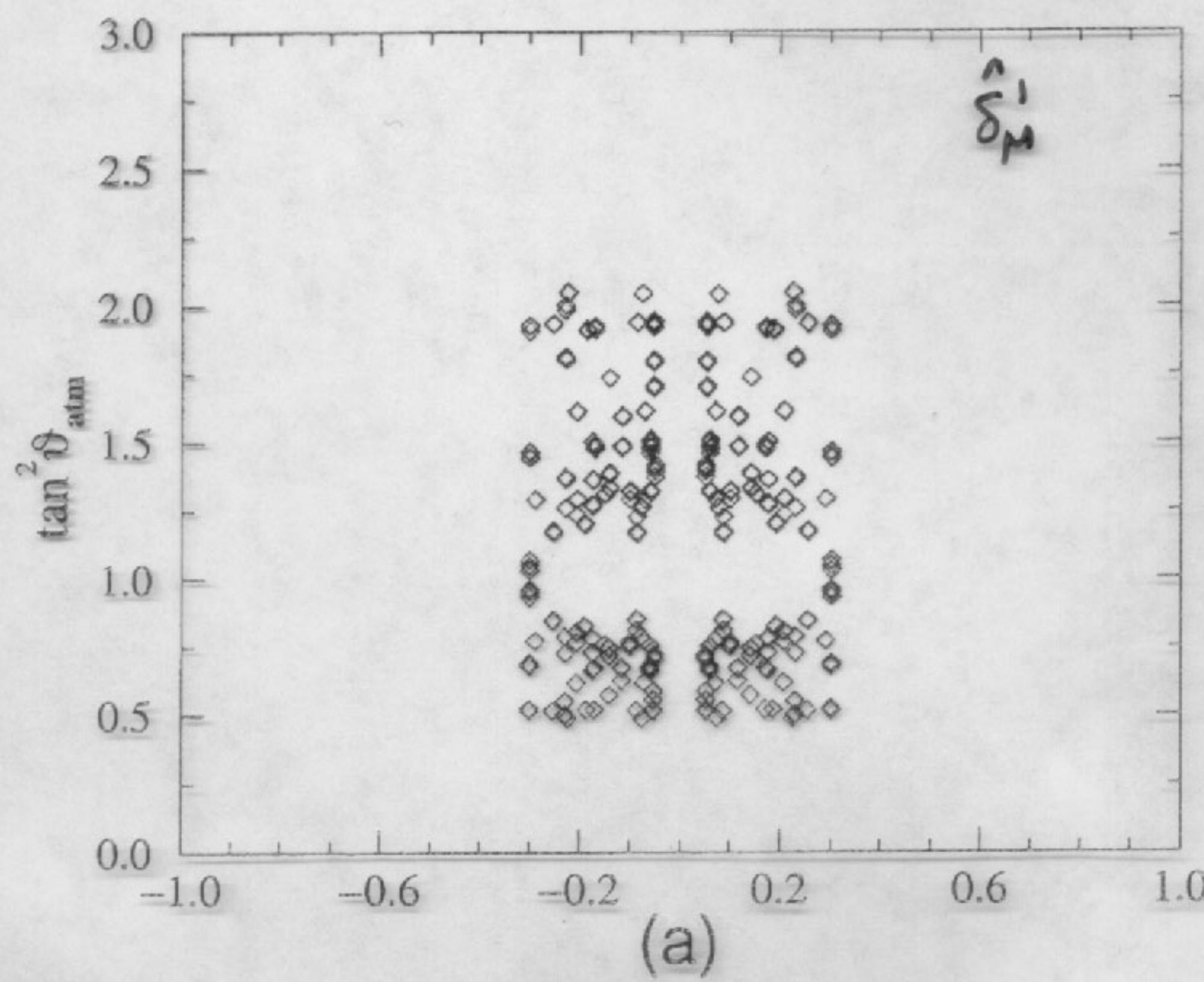
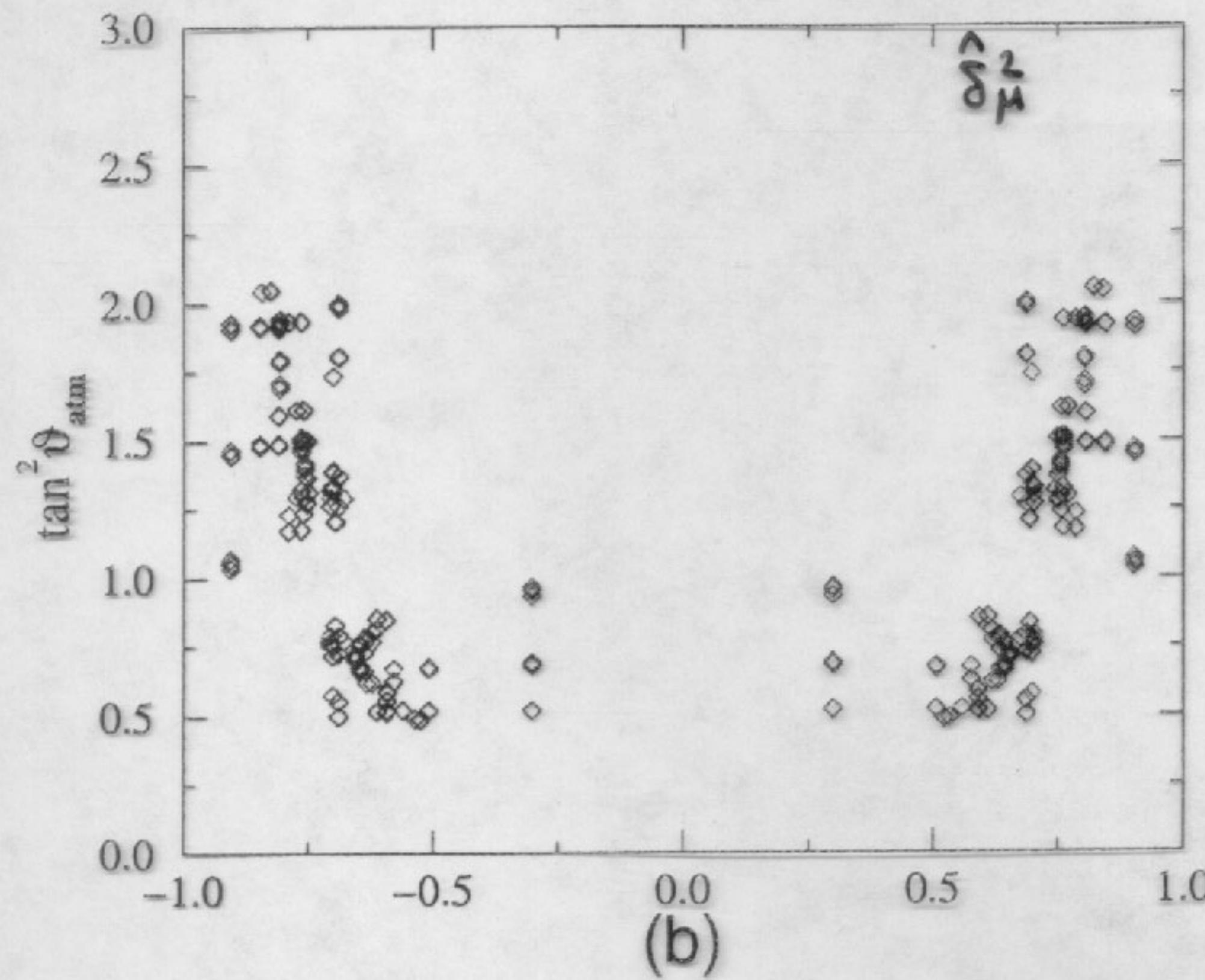
LMA+CHOOZ+Zenker



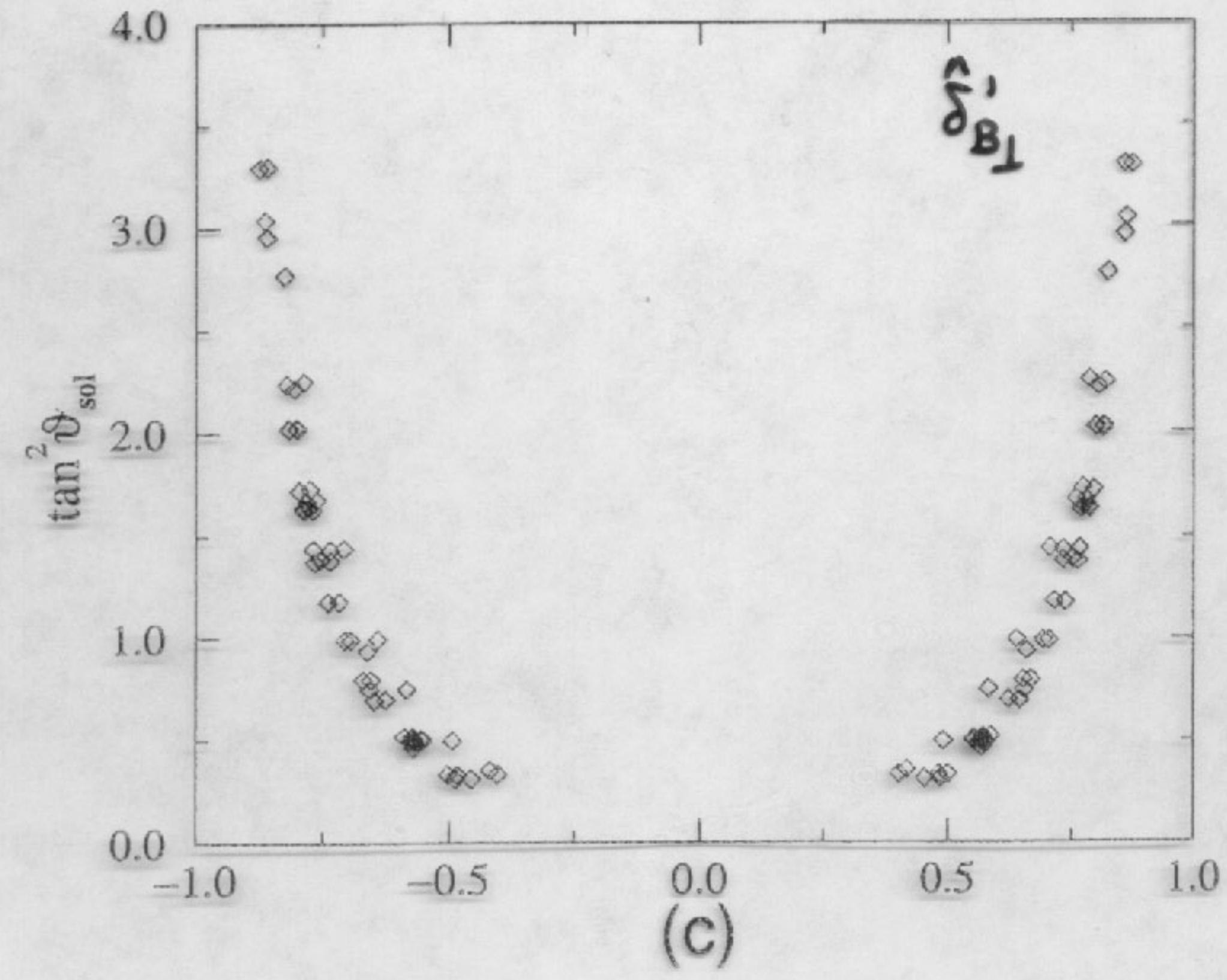
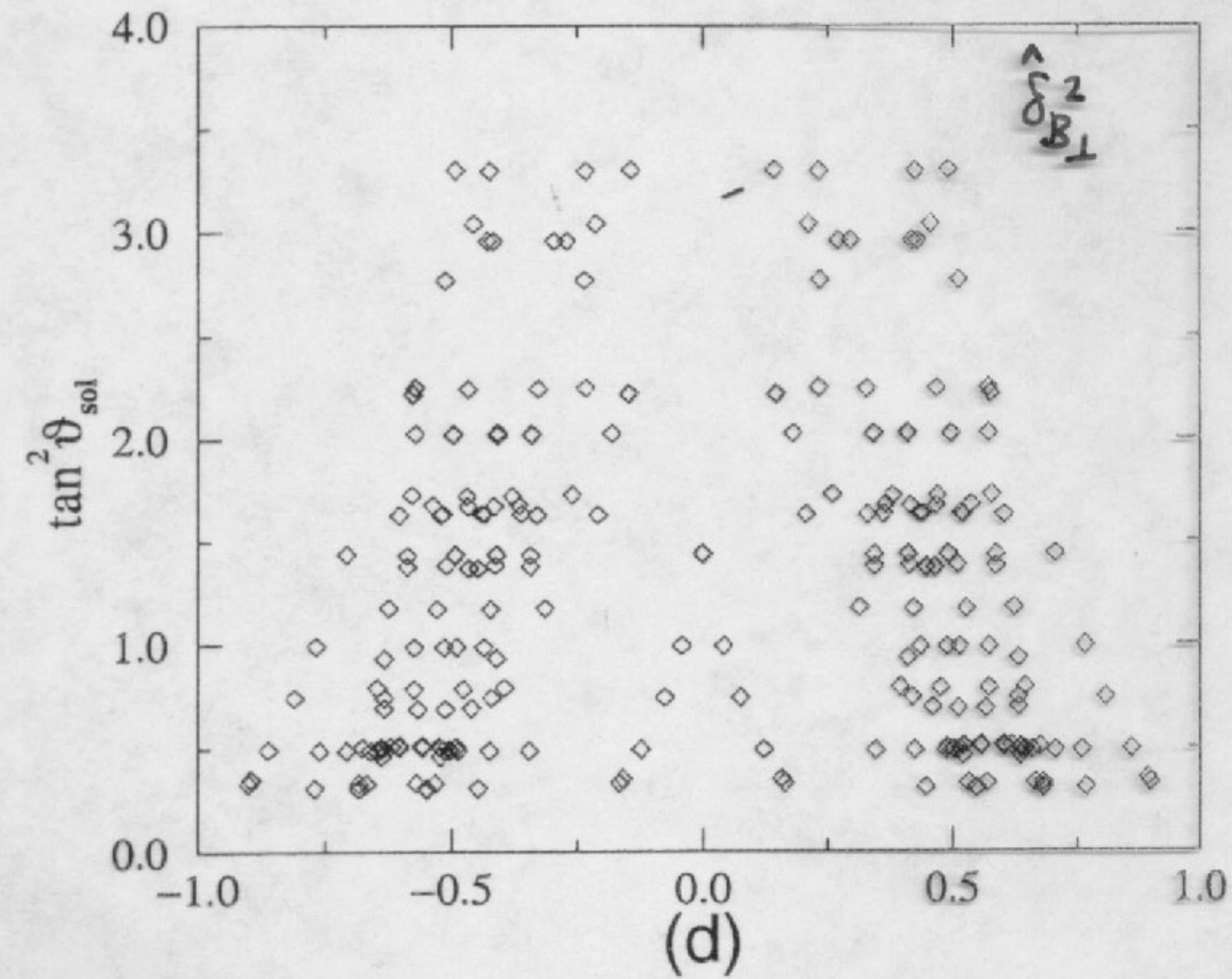
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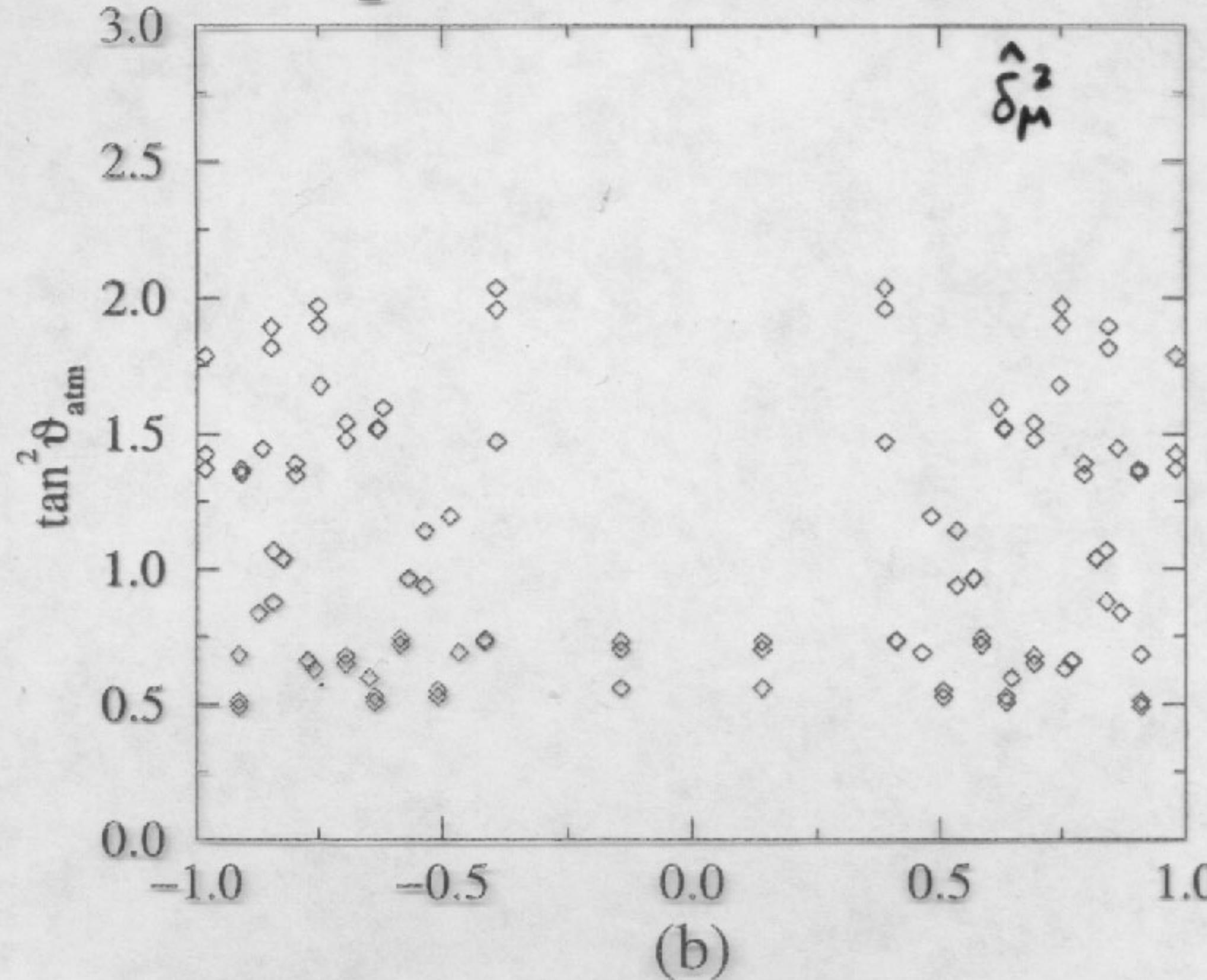
LOW+CHOOZ+SuperK



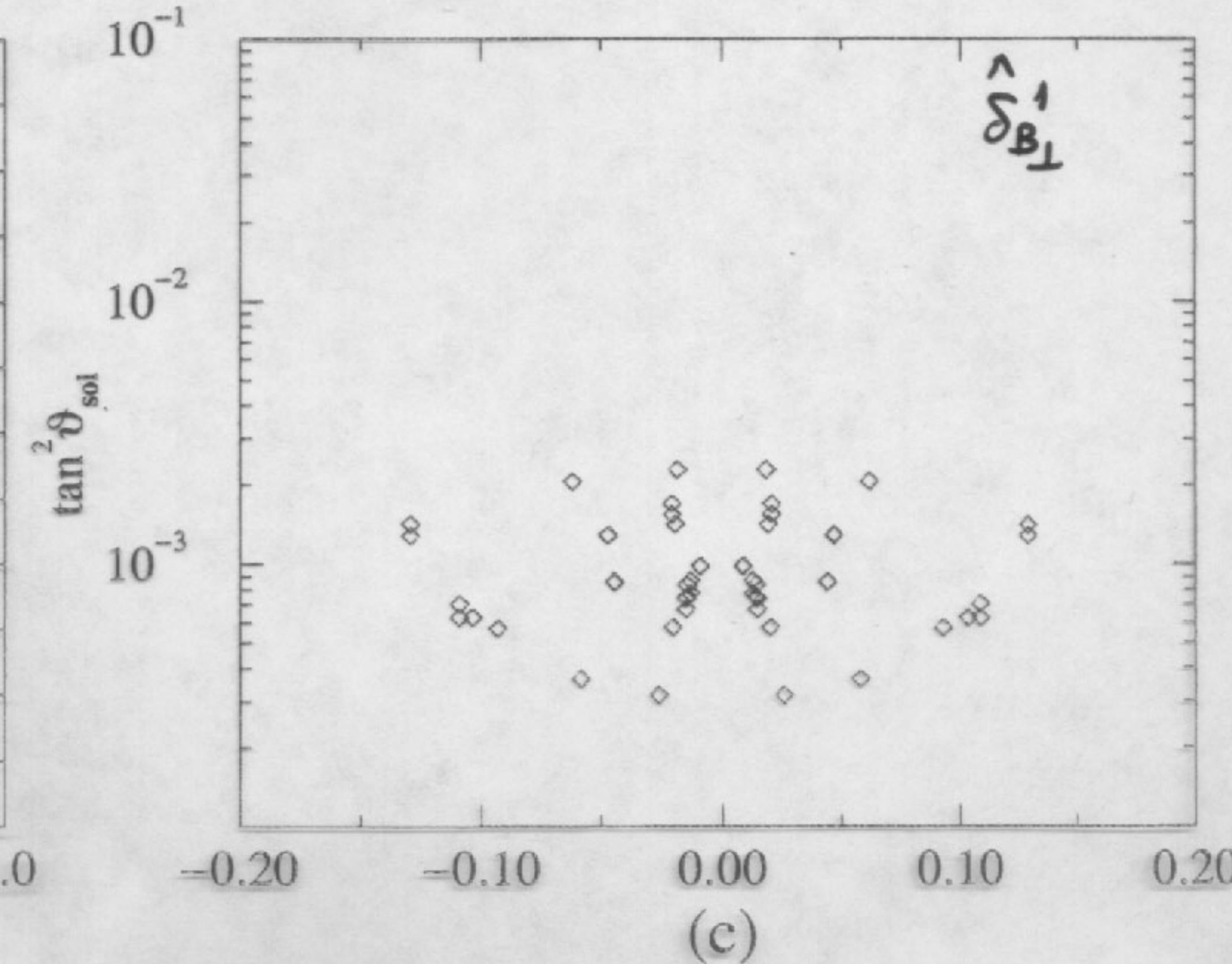
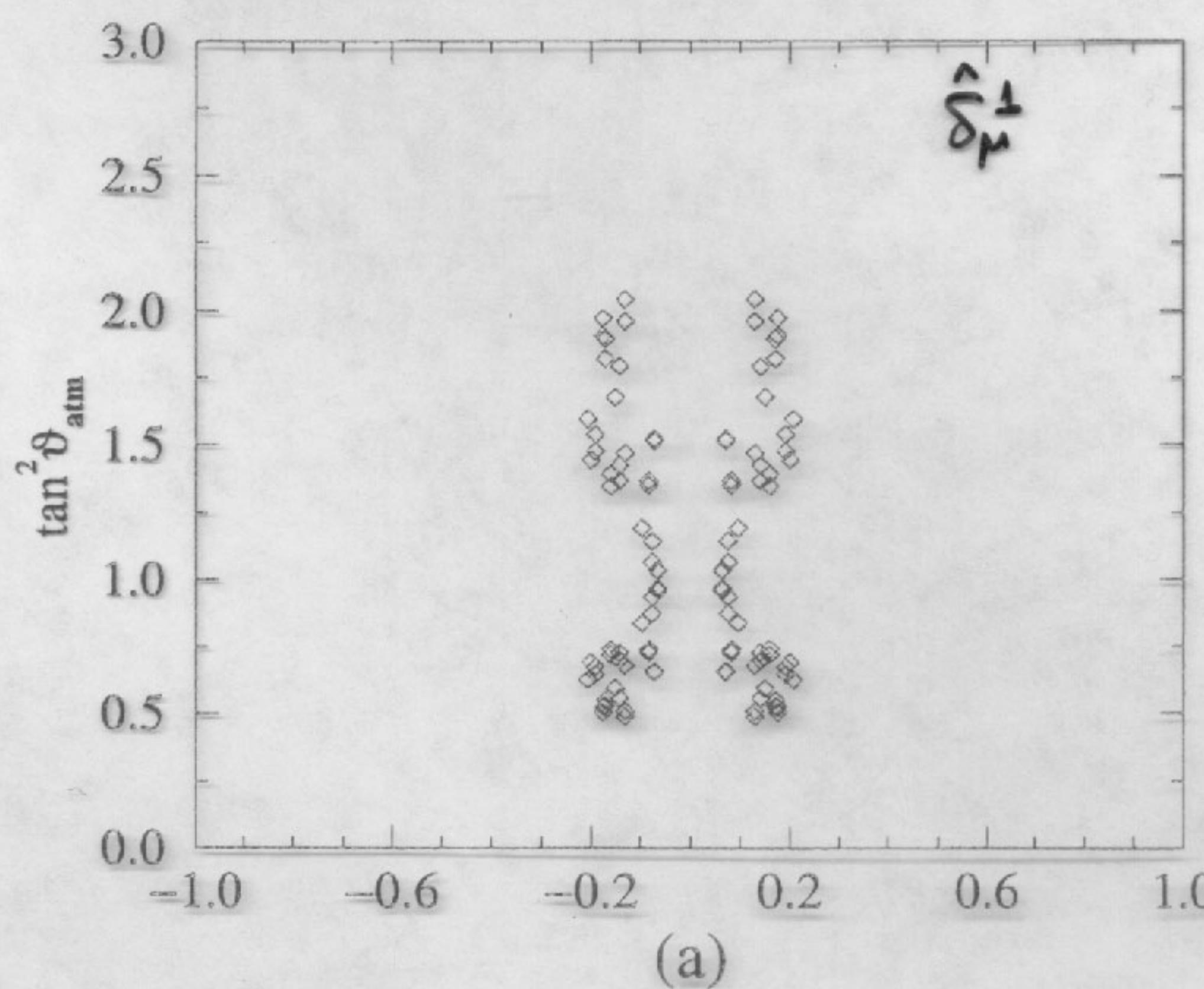
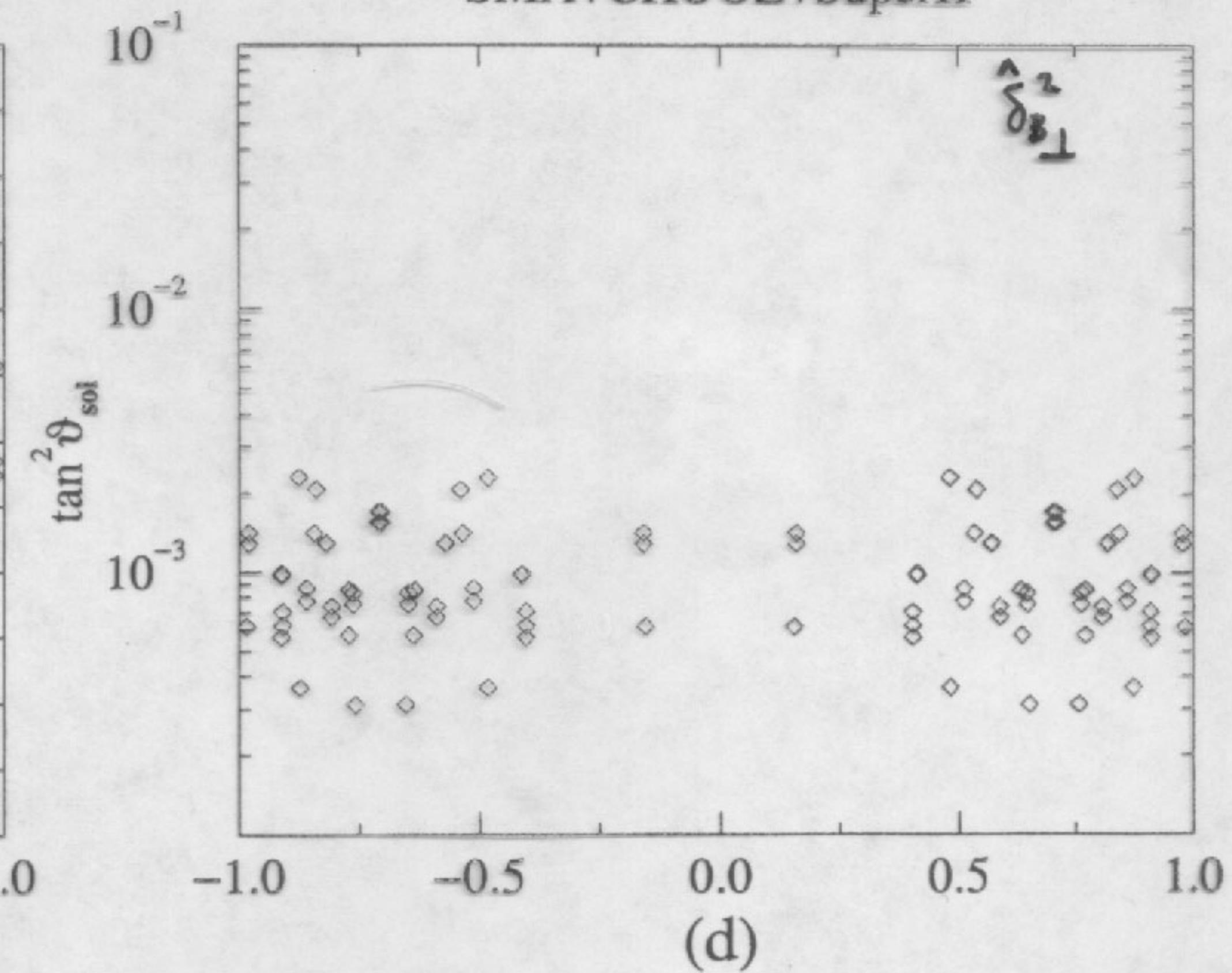
LOW+CHOOZ+SuperK



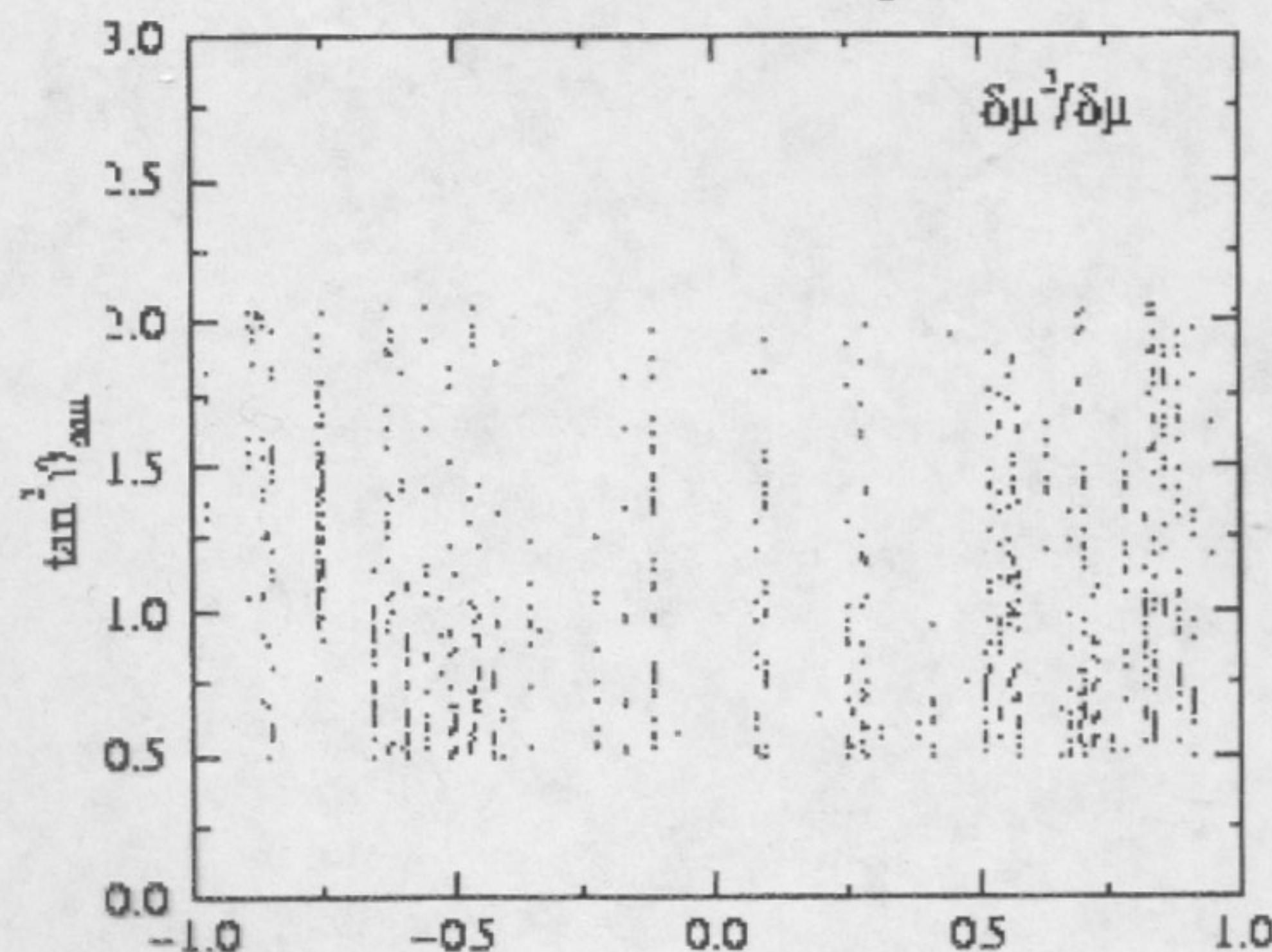
SMA+CHOOZ+SuperK



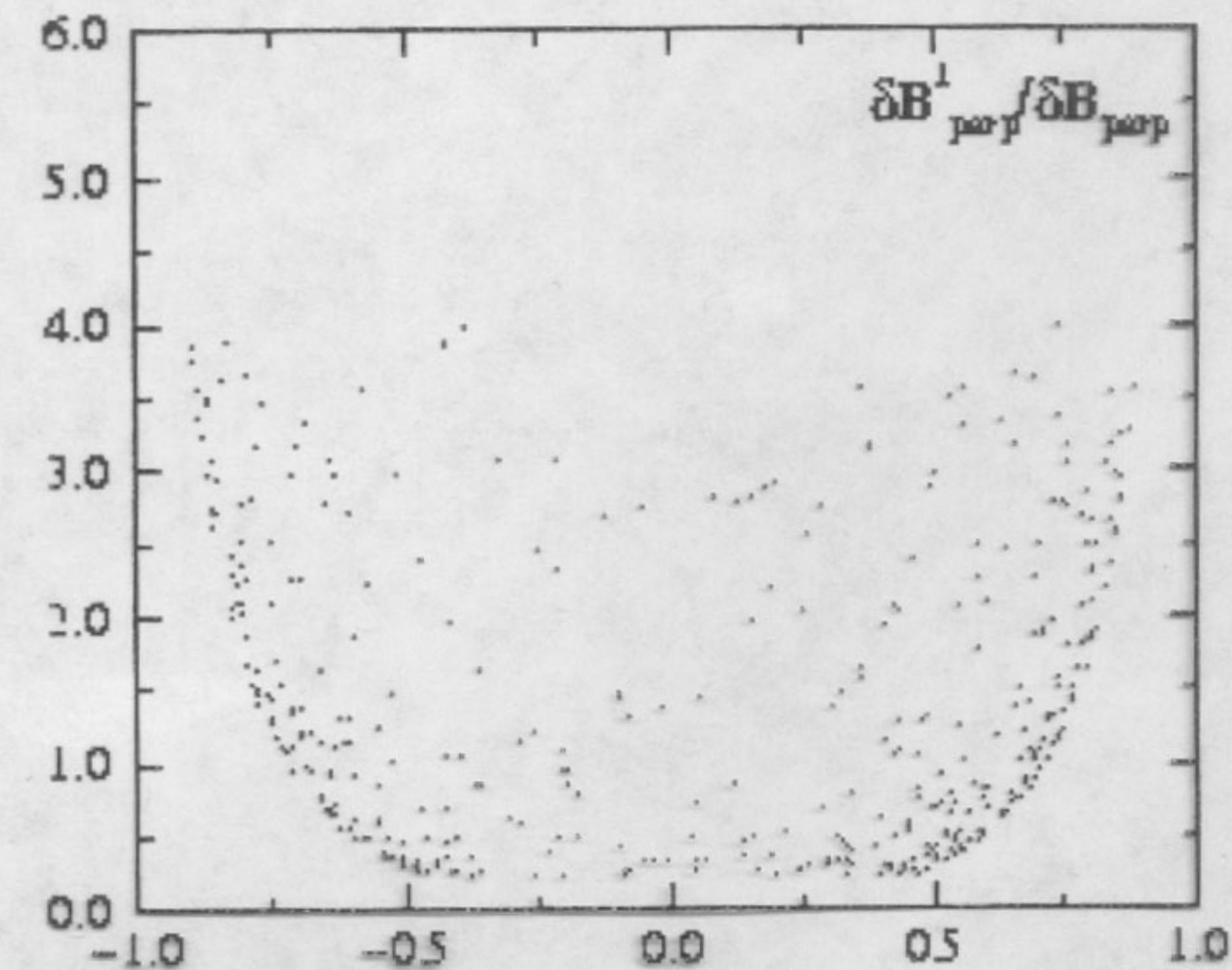
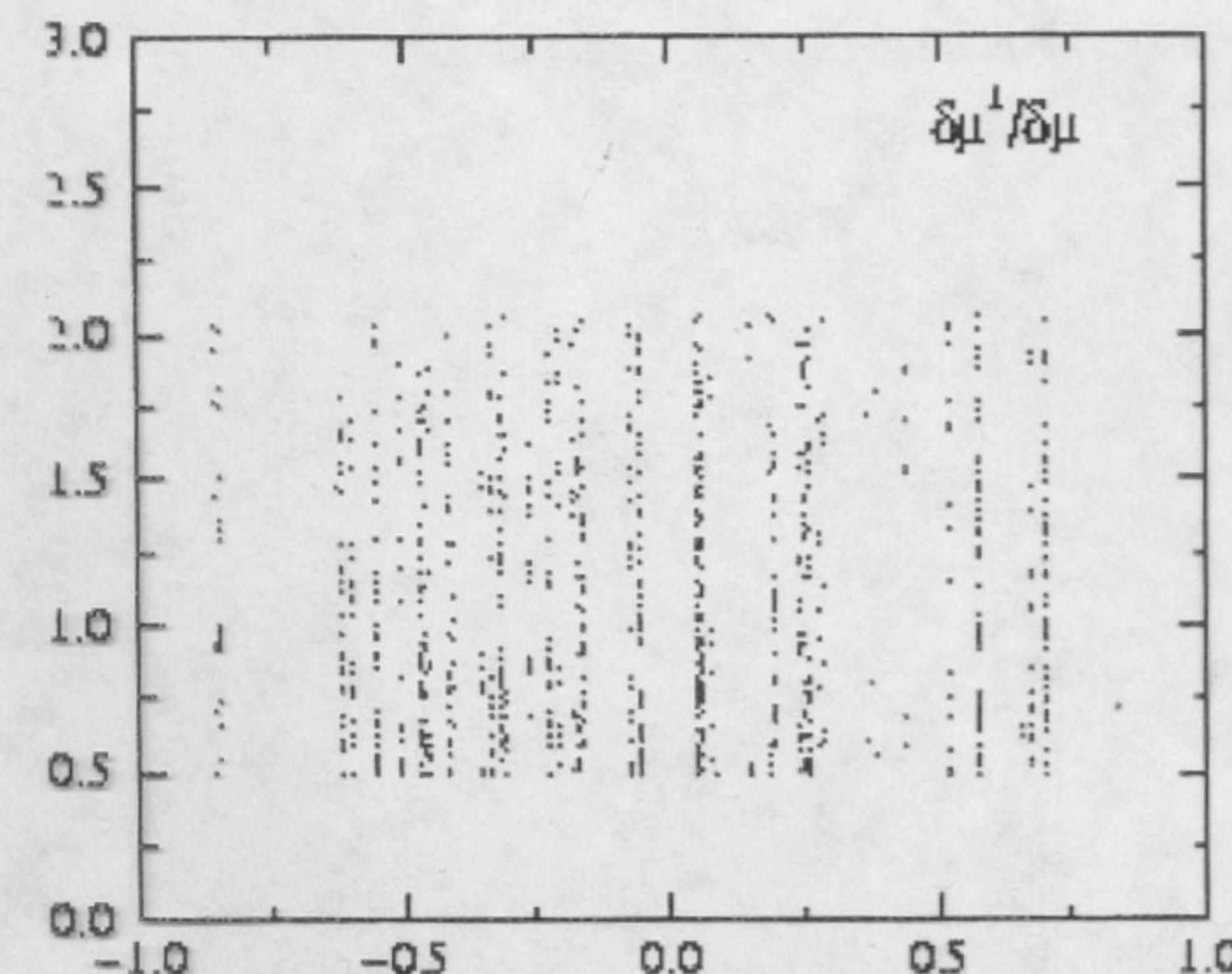
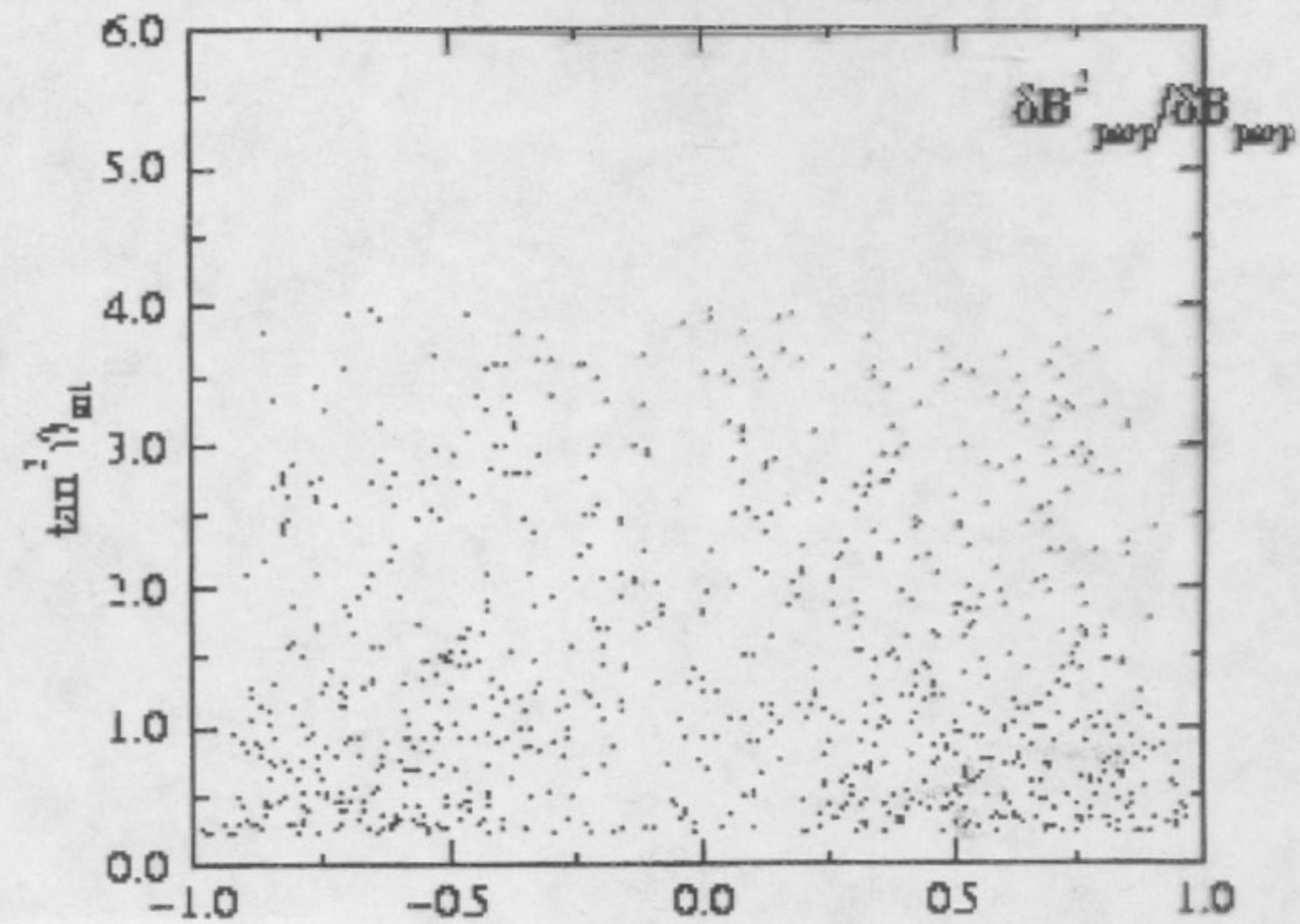
SMA+CHOOZ+SuperK



LMA+CHOOZ+SuperK

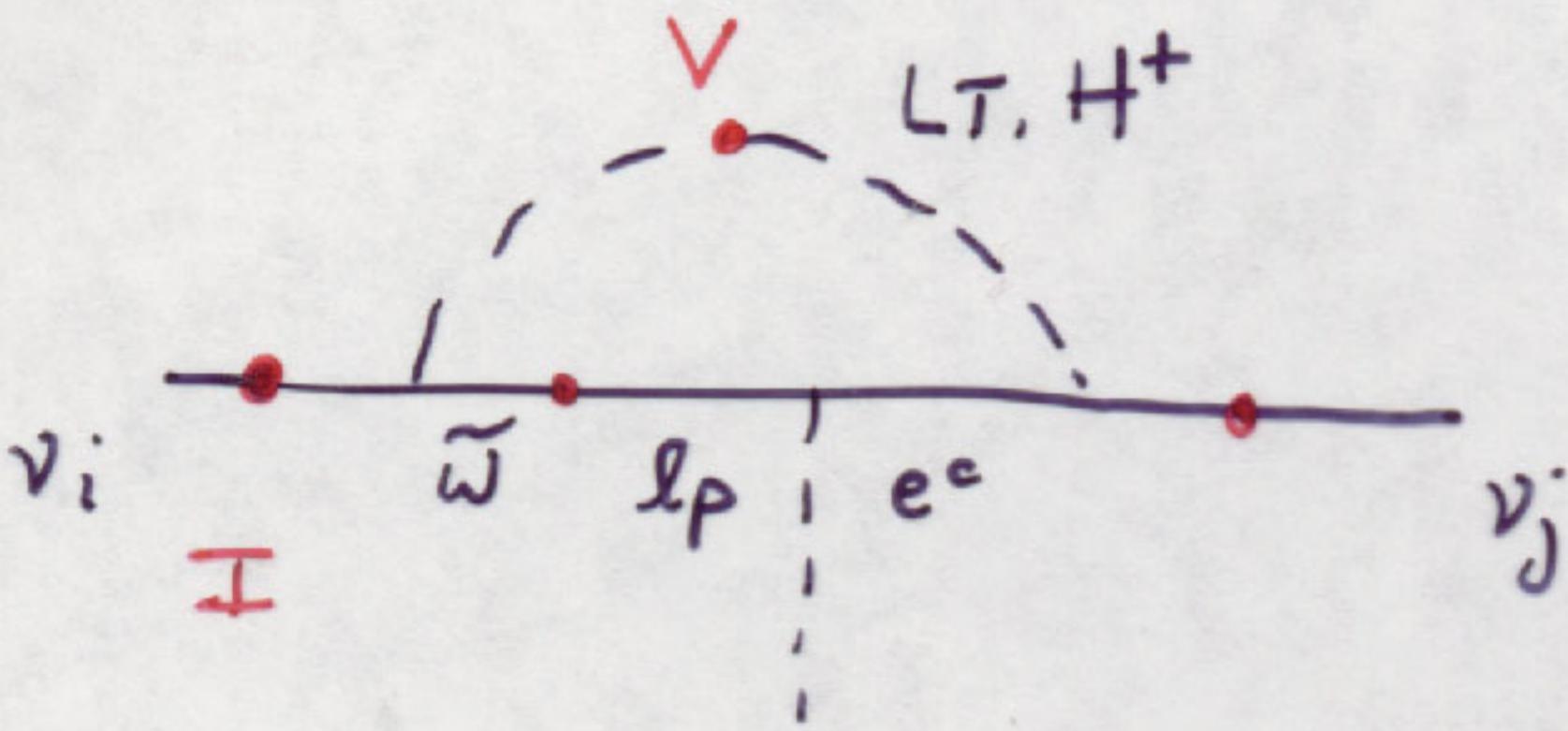


LMA+CHOOZ+SuperK



## Model C

Only  $\delta_\mu \neq 0$ .



To fit atmospheric  $\nu$  data

$$m_3 : |\vec{\delta_\mu}| \sim \left[ \frac{\Delta m_{\text{atm}}^2}{m_{\text{susy}}} \right]^{1/2}$$

$$\delta_\mu^\mu \sim \delta_\mu^\tau \gg \delta_\mu^e$$

(SK + Chooz)

$$m_2 : (I, V) \sim g \delta_\mu^i \delta_\mu^j \tan \beta \frac{m_{e_j}^2}{16\pi^2 m_{\text{susy}}}$$

$$\Rightarrow m_2 \lesssim m_\nu^{\text{tree}} \left( \frac{\tan \beta h_\tau^2}{8\pi^2} \right)^2$$

↳ for  $\tan \beta \sim 50$

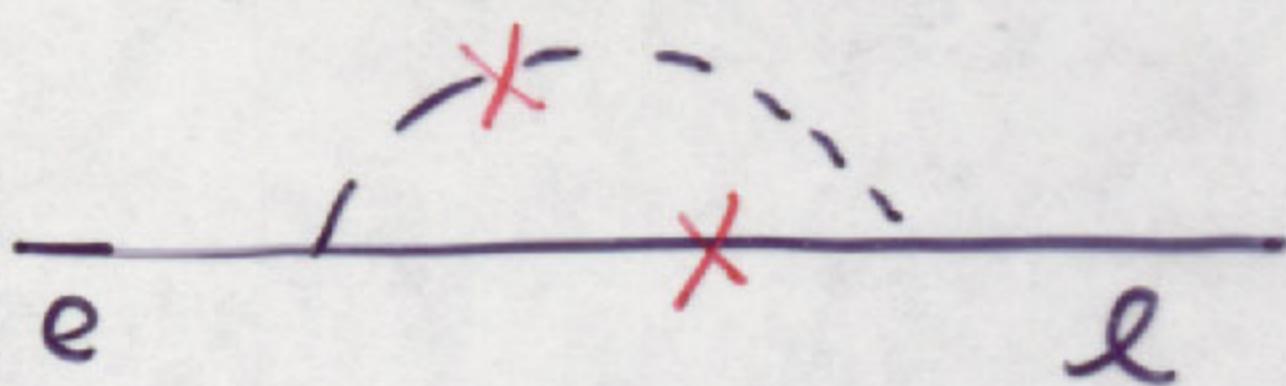
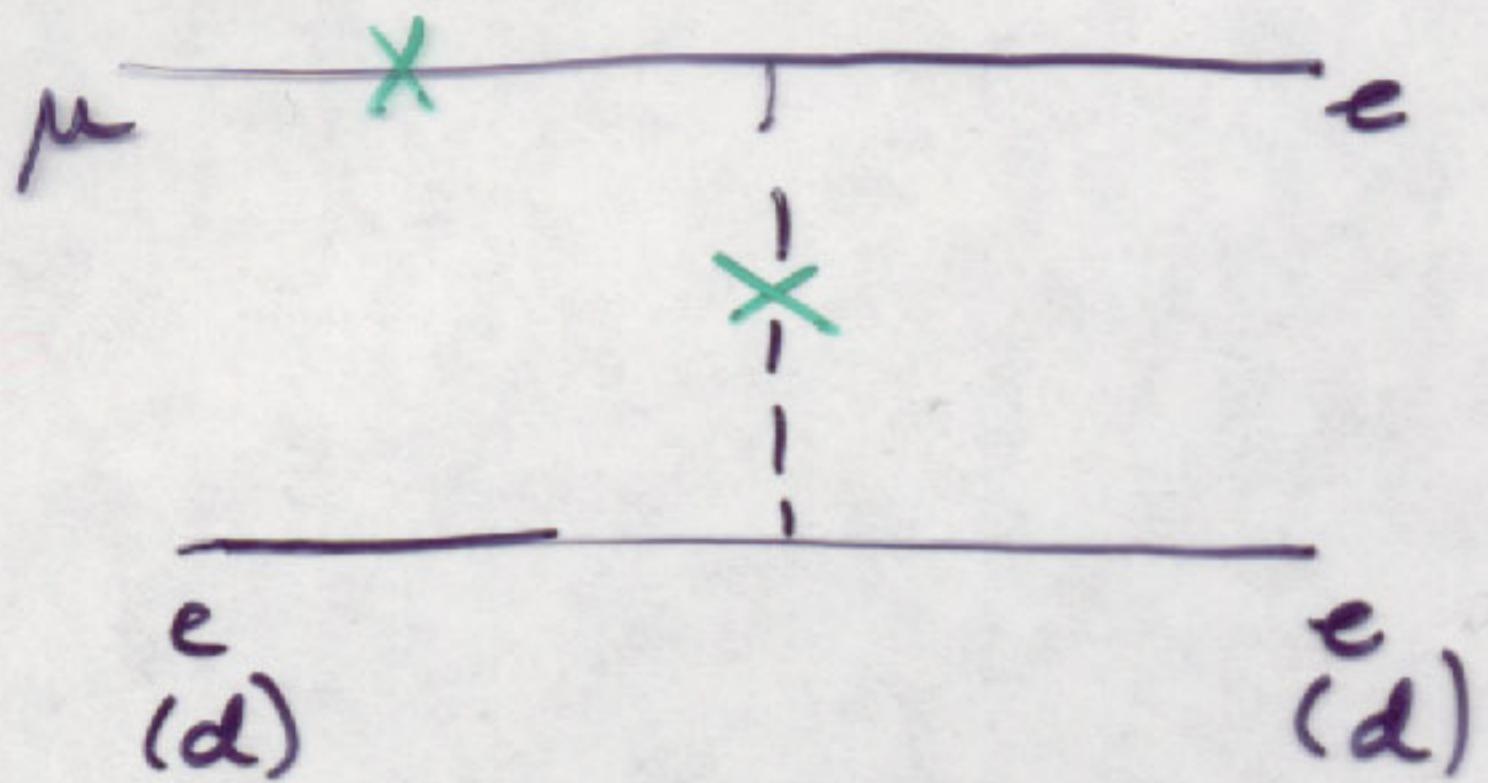
$$m_2 \sim 10^{-5} m_\nu^{\text{tree}}$$

but only small solar angles can be obtained.

## LFV low Energy Processes

$(\mu \rightarrow e \gamma)$      $(\mu \rightarrow eee)$     ( $\mu$ -e conversion)

Tree level

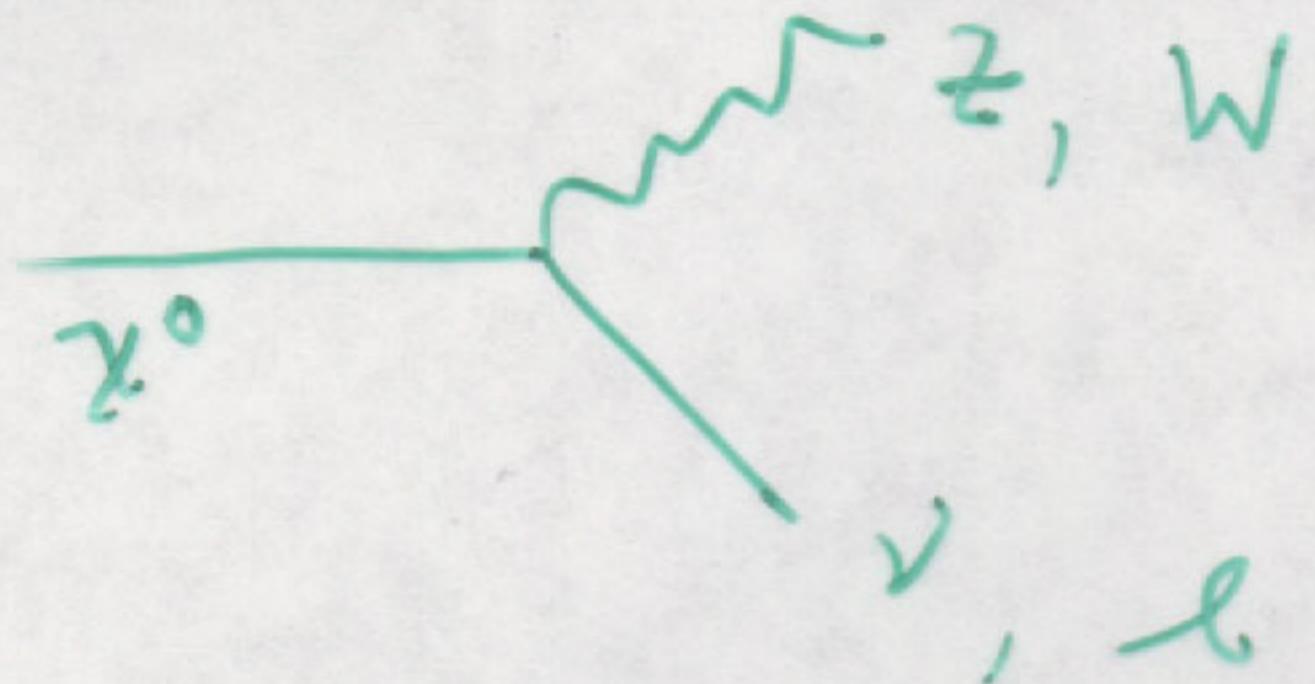


$$A \sim \begin{pmatrix} \delta_\mu^i \delta_B^j \\ \delta_\mu^i \delta_\mu^j \\ \delta_B^i \delta_B^j \end{pmatrix}$$

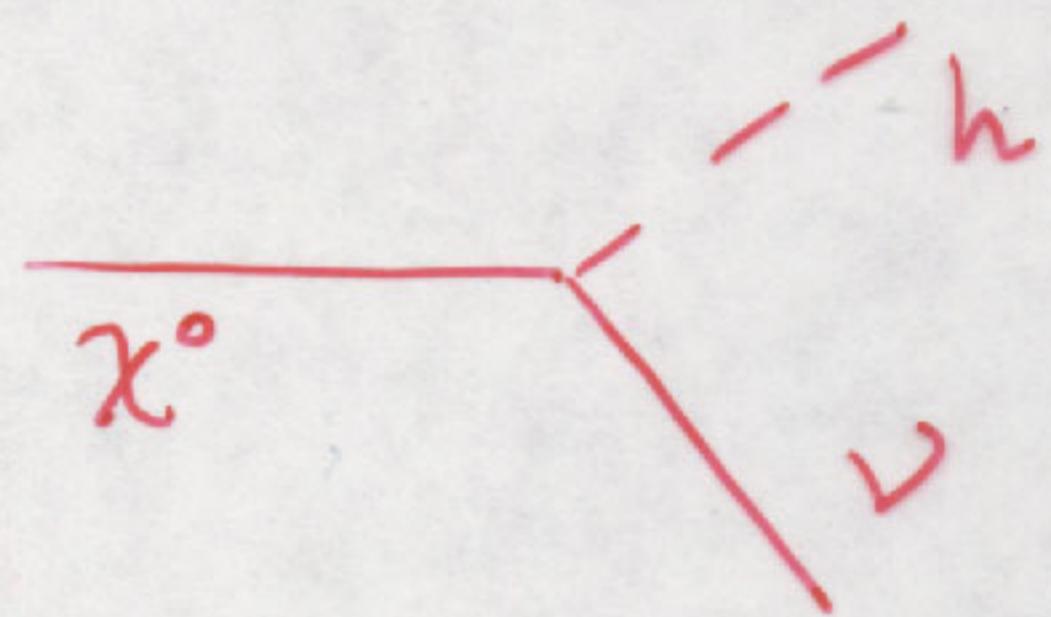
$$\Rightarrow BR \simeq \frac{1}{G_F^2} \left( \frac{\delta \delta}{m_{\text{susy}}^2} \right)^2 \sim 10^{-18}$$

Ratios of BR will be defined according to each combined constraint of Super K + Chooz + 0.

LSP decays



Mukhopadhyaya  
Vissam, Roy



Hempfling

In GMSB

$$\tilde{\gamma} \rightarrow \gamma G$$

Carena, Pokorski, Wagner

Higgs decays

Top, stop decays

# Conclusions

- \* We can accomodate the  $\nu$  oscillation scenario in a model with only soft R given atmospheric, solar & CHOOZ constraints.
- \* Low energy LFV processes have BR below current and expected experimental limits.
- \* Can have signatures at colliders from LSP, Higgs, top, stop decays.