A review of Kaon physics

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Summary

50 years of watching an s going into a d

- $\boxed{\text{OUTSTANDING}}$ $\Rightarrow$ The physics of the $\boxed{\Delta I = 1/2 \text{ rule}}$
- $\boxed{\text{SUPPRESSED}}$ $\Rrightarrow$
  - Direct CP violation: $\boxed{\varepsilon'/\varepsilon}$
  - Rare decays: $\boxed{s \to d + \begin{cases} \nu\bar{\nu} \\ \ell^+\ell^- \\ \gamma \end{cases}}$
- $\boxed{\text{FORBIDDEN}}$ $\Rightarrow$ Lepton-flavor violation

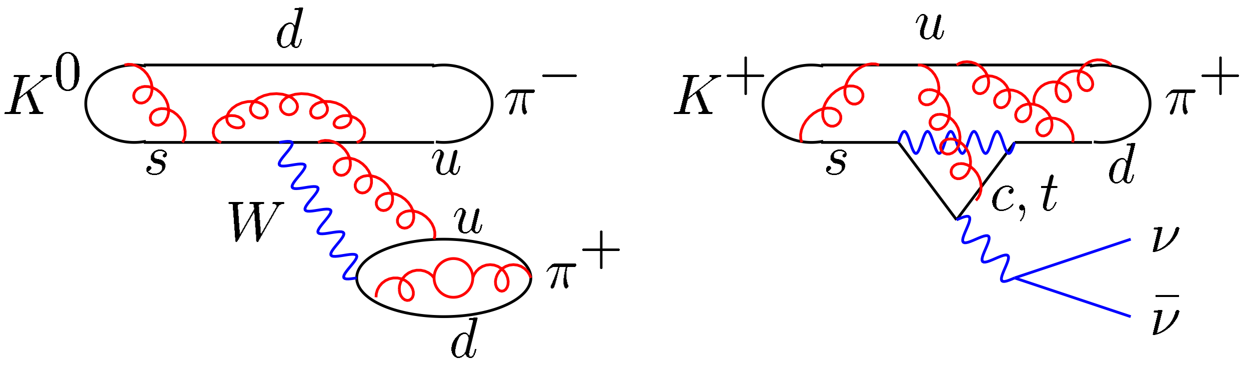
The $\Delta I = 1/2$ selection rule

$$\mathcal{A}_0 \equiv K \to (\pi\pi)_{I=0} \qquad \mathcal{A}_2 \equiv K \to (\pi\pi)_{I=2}$$

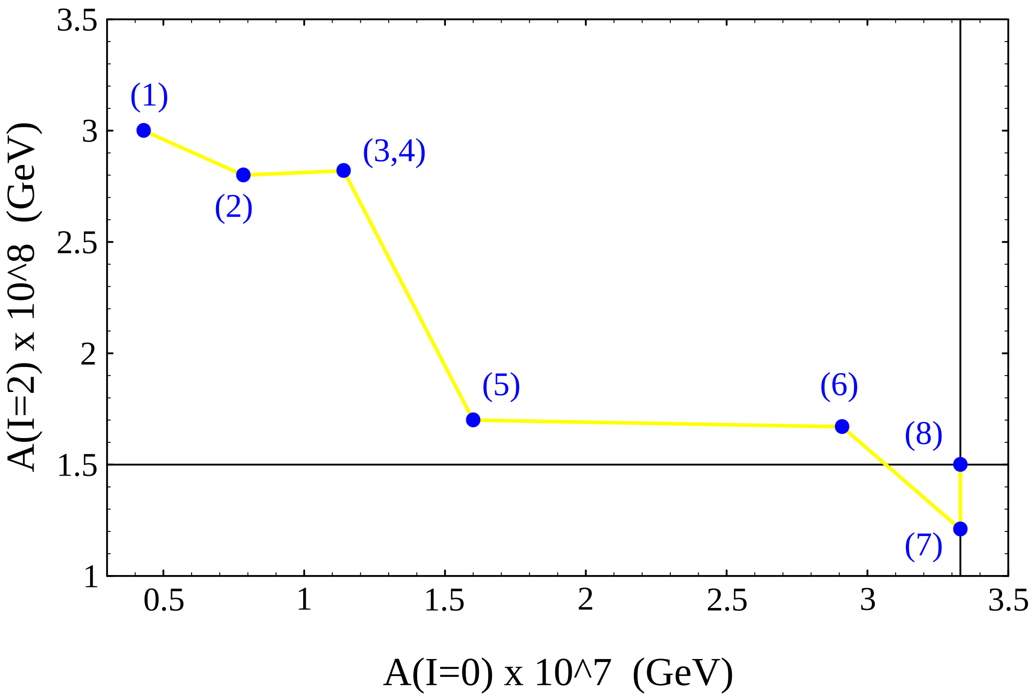
(Gell-Mann and Pais, 1954)

$$\boxed{|\mathcal{A}_0|/|\mathcal{A}_2| = 22.2}$$

Problem: how to go from $\sqrt{2} \to 22$?



Anatomy of the $\Delta I = 1/2$ rule in the χQM



$1 \to 4$: Perturbative QCD and factorization ($\approx m_t$)

$4 \to 5$: Non-factorizable $\langle \alpha_s GG/\pi \rangle$ corrections $O(p^2)$ ($\approx \Lambda_\chi, m_\pi$)

$5 \to 6$: Chiral loops ($\overline{MS}$ subtracted) ($\approx \Lambda_\chi, m_\pi$)

$6 \to 7$: $O(p^4)$ local counterterms ($\overline{MS}$) ($\approx \Lambda_\chi, m_\pi$)

$7 \to 8$: Isospin breaking ($\pi - \eta - \eta'$) ($\approx \Delta m_{\pi - \eta}$)

CP violation

$$CP\ |K^0\rangle = |\bar{K}^0\rangle$$

$$\begin{aligned} K_1 &= (K^0 + \bar{K}^0)/\sqrt{2} \quad \text{CP even} \quad \to \pi\pi \\ K_2 &= (K^0 - \bar{K}^0)/\sqrt{2} \quad \text{CP odd} \quad \to \pi\pi\pi \end{aligned}$$

$$\begin{aligned} K_S &= (K_1 + \varepsilon K_2)/\sqrt{1+|\varepsilon|^2} \\ K_L &= (K_2 + \varepsilon K_1)/\sqrt{1+|\varepsilon|^2} \end{aligned}$$

Definitions

Direct CP violation in $K \to \pi\pi$: $\boxed{|\eta_{00}/\eta_{+-}| \neq 1}$

$$\eta_{00} \equiv \frac{\langle \pi^0\pi^0 | \mathcal{H}_W | K_L \rangle}{\langle \pi^0\pi^0 | \mathcal{H}_W | K_S \rangle} \simeq \varepsilon - 2\, \varepsilon'$$

$$\eta_{+-} \equiv \frac{\langle \pi^+\pi^- | \mathcal{H}_W | K_L \rangle}{\langle \pi^+\pi^- | \mathcal{H}_W | K_S \rangle} \simeq \varepsilon + \varepsilon'$$

$$\frac{\varepsilon'}{\varepsilon} = \frac{1}{\sqrt{2}} \left\{ \frac{\langle (\pi\pi)_{I=2} | \mathcal{H}_W | K_L \rangle}{\langle (\pi\pi)_{I=0} | \mathcal{H}_W | K_L \rangle} - \frac{\langle (\pi\pi)_{I=2} | \mathcal{H}_W | K_S \rangle}{\langle (\pi\pi)_{I=0} | \mathcal{H}_W | K_S \rangle} \right\}$$

The Effective Hamiltonian

$$\mathcal{H}_{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V^*_{us} \sum_i \Big[z_i(\mu) + \tau\, y_i(\mu) \Big] Q_i(\mu)$$

$$\tau = -V_{td} V^*_{ts} / V_{ud} V^*_{us}$$

$$
\begin{aligned}
Q_1 &= (\overline{s}_\alpha u_\beta)_{\mathrm{V-A}}\,(\overline{u}_\beta d_\alpha)_{\mathrm{V-A}} \\
Q_2 &= (\overline{s}u)_{\mathrm{V-A}}\,(\overline{u}d)_{\mathrm{V-A}} \\
Q_{3,5} &= (\overline{s}d)_{\mathrm{V-A}} \sum_q (\overline{q}q)_{\mathrm{V\mp A}} \\
Q_{4,6} &= (\overline{s}_\alpha d_\beta)_{\mathrm{V-A}} \sum_q (\overline{q}_\beta q_\alpha)_{\mathrm{V\mp A}} \\
Q_{7,9} &= \tfrac{3}{2}\,(\overline{s}d)_{\mathrm{V-A}} \sum_q \hat{e}_q\,(\overline{q}q)_{\mathrm{V\pm A}} \\
Q_{8,10} &= \tfrac{3}{2}\,(\overline{s}_\alpha d_\beta)_{\mathrm{V-A}} \sum_q \hat{e}_q (\overline{q}_\beta q_\alpha)_{\mathrm{V\pm A}}
\end{aligned}
$$

Some more definitions

$$\boxed{\frac{\varepsilon'}{\varepsilon} = \frac{G_F\,\omega}{2|\varepsilon|\,\mathrm{Re}\,A_0}\,\mathrm{Im}\,\lambda_t\;\left[\Pi_0 - \frac{1}{\omega}\,\Pi_2\right]}$$

$$\Pi_0 = \frac{1}{\cos\delta_0}\sum_i y_i\,\langle Q_i\rangle_0$$

$$\Pi_2 = \frac{1}{\cos\delta_2}\sum_i y_i\,\langle Q_i\rangle_2 + \omega\sum_i y_i\,\langle Q_i\rangle_0\,\Omega_{\eta+\eta'}$$

$$\mathrm{Im}\,\lambda_t \equiv \mathrm{Im}\,V_{td}V_{ts}^*$$

Back-of-the-Envelope Estimate

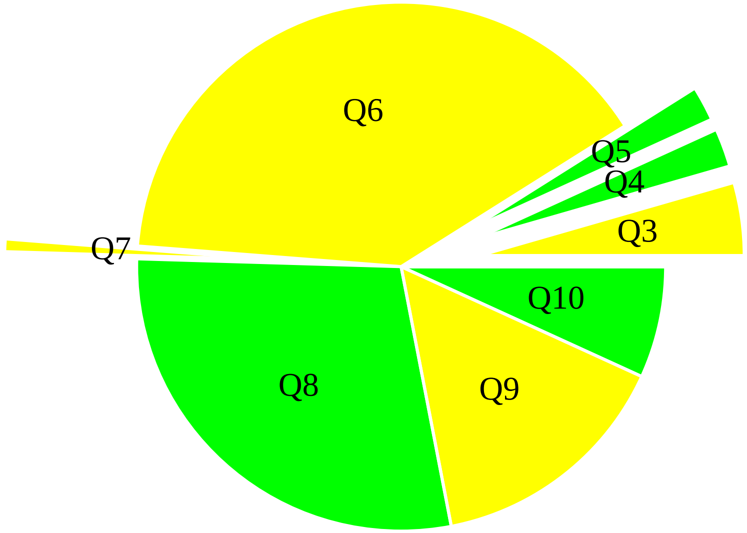
$$\frac{G_F \omega}{2|\varepsilon| \, \mathrm{Re}\, A_0} \simeq 10^3 \ \mathrm{GeV}^{-3} \qquad \mathrm{Im}\, \lambda_t \simeq 10^{-4}$$

$$\Pi_{0,2} \simeq \frac{\alpha_s}{\pi} [m_K]^3 \simeq 10^{-2} \ \mathrm{GeV}^3$$

$$\boxed{\varepsilon'/\varepsilon \simeq 10^{-3}}$$

ε'/ε in the VSA

Gluonic Penguins

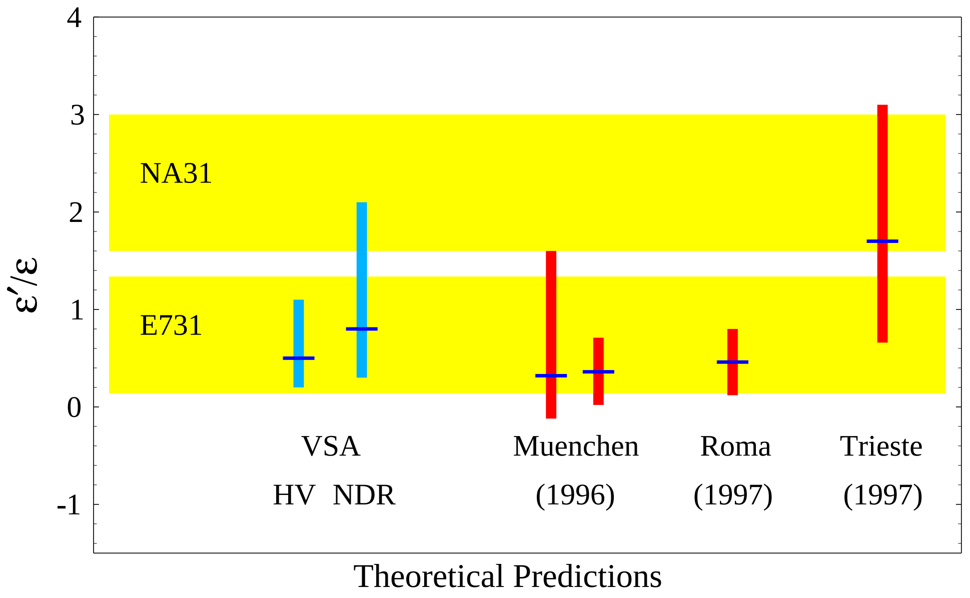


Electroweak Penguins and Box

In Yellow (Green) the Positive (Negative) contributions.

PRE-DICTIONS

(Before February 1999)



NA31: $(23 \pm 6.5) \times 10^{-4}$ | E731: $(7.4 \pm 6.0) \times 10^{-4}$

Phenomenological $1/N$: Current-current matrix elements from the $\Delta I = 1/2$ rule. Large N_c for L-R penguins (München).

1/N: Cutoff regularization of chiral loops, partial $O(p^4)$ contributions (Dortmund).

1/N: Cutoff function of the QCD matching scale. No scheme dependence (Y-L Wu, 2001).

1/N and ENJL: Scalar, vector and axial-vector resonances, good scale stability, chiral limit (Bijnens, Prades, 1999-00).

Sum Rules and LMD: $\hat{B}_K$ and $\left\langle Q_{7,8} \right\rangle_2$ at the NLO in $1/N$ in the chiral limit (Knecht, Peris and De Rafael, 2000).

Dispersive + data: $\left\langle Q_{7,8} \right\rangle_2$ from spectral functions (τ decays), chiral limit (Donoghue and Golowich, 1999; Narison, 2000; Bijnens et al. 2001)

FSI Omnès: Resummation of final state rescattering via dispersive analysis and large N_c (Pallante, Pich, Scimemi, 2000).

Lattice: $K \to \pi$ matrix elements of four-quark operators. Use chiral theory to obtain $K \to \pi\pi$ (Roma).

Chiral Quark Model: All matrix elements at $O(p^4)$ in terms of $\langle \bar{q} q \rangle$, $\langle \frac{\alpha_s}{\pi} GG \rangle$, M, determined via the $\Delta I = 1/2$ rule (Trieste).

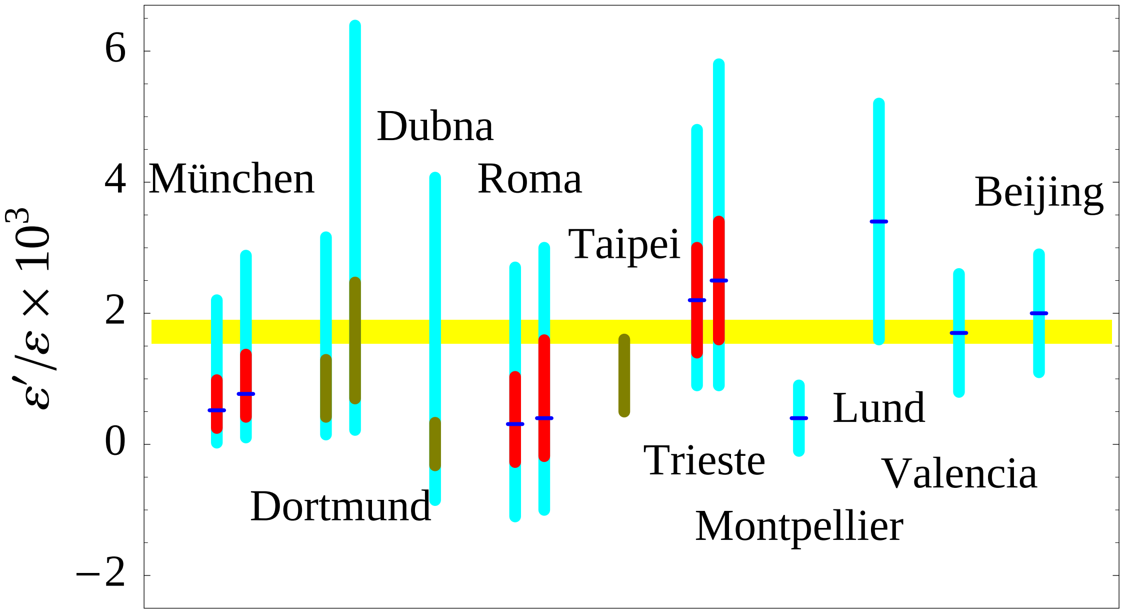
Phenomenological NJL: Chiral loops up to $O(p^6)$. It includes scalar, vector and axial-vector resonances (Dubna, 1999).

Generalized Factorization: Effective Wilson coefficients, matched with factorized matrix elements (H-Y Cheng, 1999).

Linear σ-model: $m_\sigma = 500 - 900$ MeV: ε'/ε and A_0 cannot be reproduced simultaneously (Keum et al., Harada et al., Bloch et al., 1999).

POST-DICTIONS

(After February 1999)



NA31-E731-KTeV-NA48: $(17.2 \pm 1.8) \times 10^{-4}$

Computing with what we know

Dual picture

$\mathcal{H}_{\text{eff}}$ vs. χPT

$\swarrow$ $\searrow$

semi-leptonic decays	long-distance dominated
$K_L \to \pi^0 \bar{\nu}\nu$	$K \to e^+ e^-$

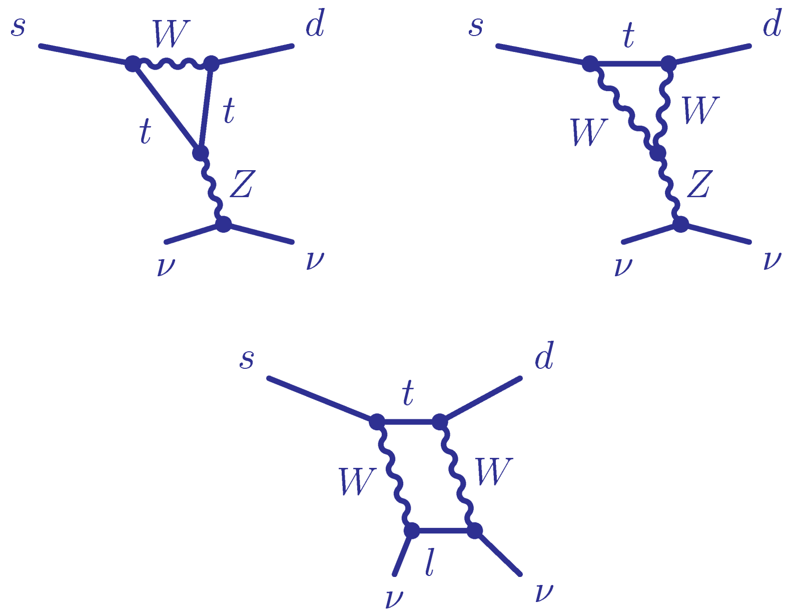
Rare decays

approximately 100%

$$K^+ \to \begin{cases} \mu^+\nu_\mu \\ \pi^+\pi^- \\ 3\pi \\ \pi^0\ell\nu_\ell \end{cases} \quad K_S^0 \to \begin{cases} \pi^+\pi^- \\ \pi^0\pi^0 \end{cases} \quad K_L^0 \to \begin{cases} 3\pi \\ \pi\ell\nu_\ell \end{cases}$$

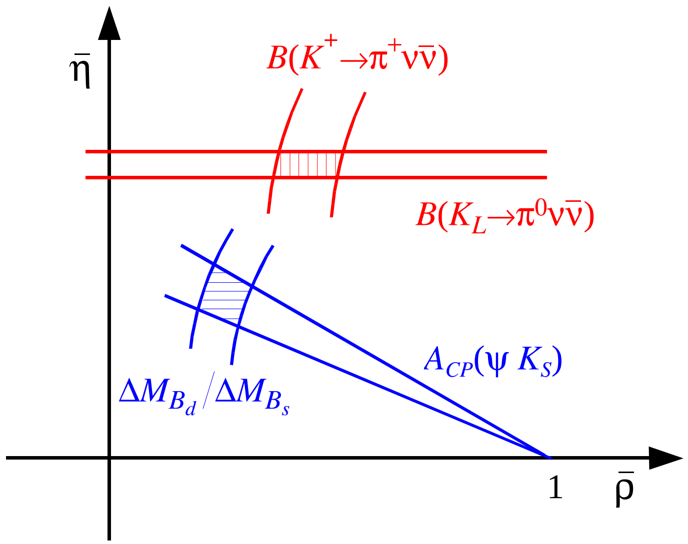
what is left we call it rare decays

GOLDEN MODES



$$\mathcal{H}_{K\to\pi\bar{\nu}\nu} \approx G_F\alpha\left[\lambda_c F_c + \lambda_t F_t\right]\bar{s}_L\gamma^\mu d_L\,\bar{\nu}_L\gamma_\mu\nu_L$$

	$K^+ \to \pi^+ \nu\bar\nu$	$K_L \to \pi^0 \nu\bar\nu$
	CP conserving	CP violating
CKM	V_{td}	$\mathrm{Im} V_{ts}^* V_{td} \sim J_{CP} \sim \eta$
contributions	top and charm	only top
scale dep. (BR)	$\pm 20\%$ (LO) $\to \pm 5\%$ (NLO)	$\pm 10\%$ (LO) $\to \pm 1\%$ (NLO)
BR (SM)	$(0.8 \pm 0.3) \cdot 10^{-10}$	$(2.6 \pm 1.2) \cdot 10^{-11}$
exp.	$\left(1.5^{+3.4}_{-1.2}\right) \cdot 10^{-10}$ BNL 787	$< 5.9 \cdot 10^{-7}$ KTeV



G. Buchalla, hep-ph/0110313

future: KEK-E391a

$K \to \pi \ell^+ \ell^-$

CP-conserving $\left\{ \begin{array}{l} K^+ \to \pi^+ \ell^+ \ell^- \\ K_S \to \pi^0 \ell^+ \ell^- \end{array} \right.$

$\boxed{\text{LD dominated}}$ $\Rightarrow$ test for χPT

CP-violating $K_L \to \pi^0 \ell^+ \ell^-$

$\boxed{\text{SD dominated}}$ $\Rightarrow$ test for $\mathcal{H}_{\text{eff}}$

$K \to \ell^+\ell^-$

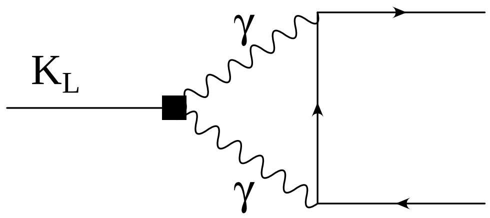
dominated by LD

$\boxed{\text{absorptive}}$ $\Rightarrow$ 2γ discontinuity

$\boxed{\text{dispersive}}$ $\Rightarrow$ more uncertain

$K \to e^+e^-$ $\Rightarrow$ large IR logs

$K \to \mu^+\mu^-$ $\Rightarrow$ no large IR $\Rightarrow$ ρ



Progress in Kaon rare decays

decay modes		PDG-86	PDG-96	PIC-01
$K_L^0 \to \mu^+\mu^-$	in 10^{-9}	9.1 ± 1.9	7.2 ± 0.5	7.18 ± 0.17
$K_L^0 \to \pi^0 e^+ e^-$	in 10^{-10}	< 23000	< 43	< 5.1
$K_S^0 \to \pi^0 e^+ e^-$	in 10^{-7}	-	< 11	< 1.4
$K^+ \to \pi^+ \nu\bar{\nu}$	in 10^{-10}	< 1400	< 24	$1.57^{+1.75}_{-0.82}$
$K_L^0 \to \pi^0 \nu\bar{\nu}$	in 10^{-7}	-	< 580	< 5.9
$K_L^0 \to \mu^\pm e^\mp$	in 10^{-12}	< 6000000	< 33	< 4.7
$K_L^0 \to \pi^0 \mu^\pm e^\mp$	in 10^{-10}	-	-	< 4.4
$K^+ \to \pi^+ \mu^+ e^-$	in 10^{-11}	< 500	< 21	< 2.8

T. K. Komatsubara, hep-ex/0112016

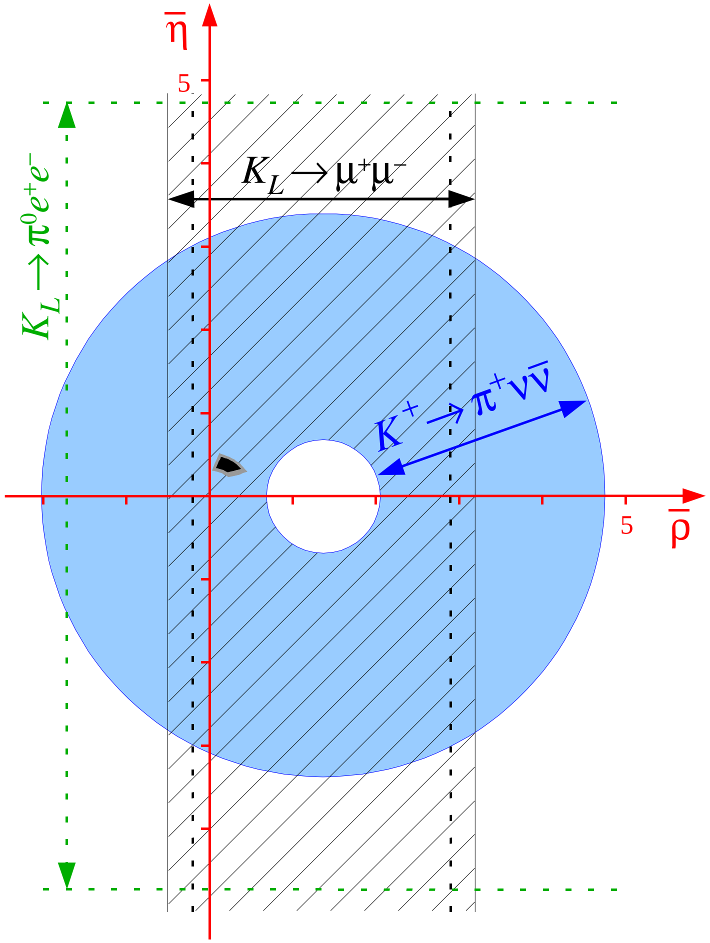
Transverse polarizations: $K \to \pi\mu\bar{\nu}$

- $\mathcal{P}_T$ is T-odd
- FSI small, SM $\leq 10^{-7}$
- Physics beyond SM $\Rightarrow$ $10^{-3} - 10^{-4}$

E246: $(-0.33 \pm 0.37 \pm 0.09)$ %

$\sigma \to 0.003$ (with 99-00 data set)

Kaon physycs and the unitary triangle



G. Isidori, hep-ph/0110255