Perturbative QCD Effects in B Decays and Phenomenological Implications

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Heavy Meson Decay

$$P_\mu = m_b v^\mu + k^\mu$$

Meson Frame $\simeq$ Heavy Quark Frame $+ \mathcal{O}\left(\frac{\Lambda_{QCD}}{m_b}\right)$

Non Perturbative initial state effect $\rightarrow$ Fermi Motion described by the Shape Function

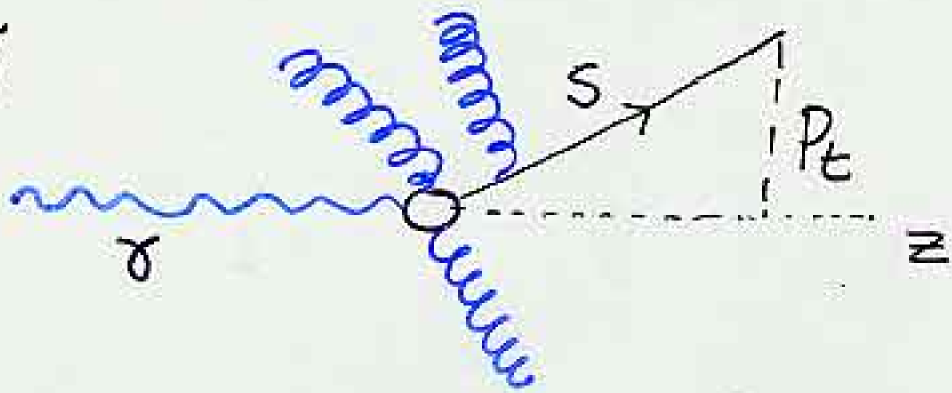
Decays

$$b \rightarrow c + W$$

$$b \rightarrow u + W, \quad b \rightarrow s + \gamma$$

relevant in the threshold region f.ex. near the endpoint of the photon spectrum

$b \to s + \gamma$



Transverse Momentum

of s with respect to the γ axis

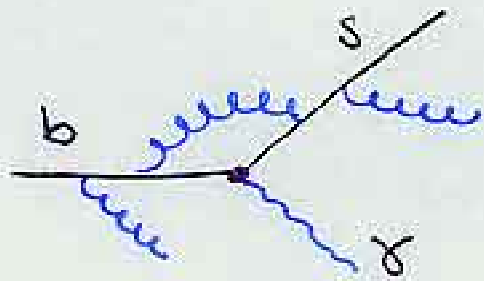
Three Different Effects

- Initial non perturbative bound state effect: Fermi Motion

$$P_F \sim \Lambda_{QCD}$$

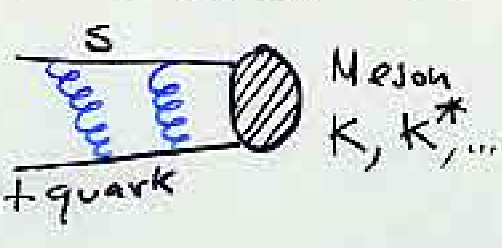
- Perturbative Effects

$$P_P$$



- Final non perturbative hadronization

$$P_H \sim \Lambda_{QCD}$$



$$P_t = P_F + P_H + P_P$$

Fermi Motion
Hadronization
$\simeq \Lambda_{QCD}$
cannot be separated

if $P_P \gg \Lambda_{QCD}$ perturbative effects dominate

¡Why?!

Transverse Momentum Distribution (in $b \to s\gamma$)

- transverse momentum of the produced hadrons with respect to the photon direction in the B rest frame is determined by the photon momentum

- - Transverse Momentum is complementary to the threshold distribution
  TM is sensitive to the ortogonal plane to the decay axis.

  - This separation is for the perturbative as well as for non-perturbative parts equally valid $\Rightarrow$ information can be derived concerning the non perturbative effects.

How to Calculate p_t Distributions

IR singularities

$E_g \to 0$

$\theta \to 0$

collinear sing.

$O(\alpha_s)$

$$M(\epsilon,t) = \frac{A_1\alpha_s}{\epsilon t} + \frac{\alpha_s S_1(t)}{\epsilon} + \frac{\alpha_s C_1(t)}{t} + \alpha_s F_1(\epsilon,t)$$

$$t = \frac{1-\cos\theta}{2} \qquad \epsilon = \frac{E_g}{m_b/2}$$

A_1, S_1, C_1, F_1 IR finite functions

$$C_1 = \int_0^1 C_1(\epsilon)\, d\epsilon = -\frac{3}{4}\frac{C_F}{\pi}$$

$$S_1 = \int_0^1 S_1(t)\, dt = -\frac{C_F}{\pi}$$ ← New → massive partons

$$f(x) = \frac{1}{\Gamma_0}\frac{d\Gamma}{dx} \qquad x = \frac{p_{tp}^2}{m^2}$$

$$f(x) = \delta(x) - \frac{C_F}{2\pi}\alpha_s\left(\frac{\ln x}{x}\right)_+ + \alpha_s \underbrace{\left(-\frac{5}{4}\right)}_{C_1+S_1/2}\left(\frac{1}{x}\right)_+$$

Higher Orders

- $\alpha_s \to \alpha_s(m^2 x)$
- $A_1\alpha_s \to A_1\alpha_s + A_2\alpha_s^2$

} one gluon distribution with next to leading accuracy

since $\alpha_s^n \ln^n \sim O(1)$

- no reason to stop the series at any given order ⟶ Resummation

Resummation

at $O(\alpha_S^n)$

$$\alpha_S^n \left[\ln_S^n \ln_C^n + \ln_S^{n-1} \ln_C^n + \ln_S^n \ln_C^{n-1} + \dots \right]$$

It is convenient to go to the impact parameter $\vec{b}$ space

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{d\vec{b}}(\vec{b}) = \int_{-\infty}^{+\infty} d\vec{p}_t \, e^{i\vec{p}_t\cdot\vec{b}} \frac{1}{\Gamma_0}\frac{d\Gamma}{d\vec{p}_t}(\vec{p}_t)$$

since in QCD (as in QED)

The Amplitudes in the IR limit do factorize

$$\frac{1}{\Gamma_0}\frac{d^n\Gamma_n(k_1 \dots k_n)}{dk_1 \dots dk_n} \simeq \frac{1}{n!}\prod_{i=1}^{n}\frac{1}{\Gamma_0}\frac{d\Gamma_i}{dk_i}(k_i)$$

as well as the Kinematical constraints in the impact parameter space

$$e^{i\vec{p}_t\cdot\vec{b}}\,\delta(\vec{p}_t + \vec{k}_{1t} + \vec{k}_{2t} + \dots \vec{k}_{nt}) \to e^{i\vec{k}_{1t}\cdot\vec{b}}\, e^{i\vec{k}_{2t}\cdot\vec{b}} \dots e^{i\vec{k}_{nt}\cdot\vec{b}}$$

Result

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{db} = C(\alpha_S)\, f(b;\alpha_S) + R(b;\alpha_S)$$

Coefficient Function process dependent

universal function

$f = \exp[L g_1(\beta_0\alpha_S L) + g_2 + \dots]$

process dependent finite

$$\boxed{\frac{d\Gamma}{d\vec{p}_{t\gamma}} = \int \frac{d\vec{b}}{4\pi^2} e^{-i\vec{p}_t\cdot\vec{b}} \frac{d\Gamma}{d\vec{b}}}$$

Impact Parameter

$$\omega = \beta_0 \alpha_s \ln \frac{Q^2 b^2}{b_0^2}$$

$$g_1(\omega) = \frac{A_1}{2\beta_0 \omega}\left[\ln(1-\omega) + \omega\right]$$

$$g_2(\omega) = -\frac{A_2}{2\beta_0^2}\left[\frac{\omega}{1-\omega} + \ln(1-\omega)\right] + \frac{A_1\beta_1}{2\beta_0^3}\left[\frac{\ln(1-\omega)}{1-\omega} + \frac{\omega}{1-\omega} + \frac{\ln^2(1-\omega)}{2} \ldots\right.$$

$$+ \frac{B_1}{\beta_0}\ln(1-\omega)$$

$$LO \to f(b) = e^{L g_1(\omega)}$$

$$NLO \to f(b) = e^{L g_1(\omega) + g_2(\omega)}$$

$\omega \to 1$ singular $\left(p_t \to \frac{1}{b} \simeq \Lambda_{QCD}\right)$

Threshold Distribution

photon energy near the endpoint of the spectrum

$$E_\gamma^{max} = \frac{m_b}{2} \qquad f(z) = \frac{1}{\Gamma_B}\frac{d\Gamma}{dz} \qquad z = \frac{2E_\gamma}{m_b}$$

Known

$$g_1(\lambda) \neq g_1(\omega) \qquad g_2(\lambda) \neq g_2(\omega) \qquad \lambda \sim \ln N$$

$$f(N) = \int dz\, z^{N-1} f(z)$$

Singularities

- $\lambda \to \frac{1}{2}$ $\quad m_X^2 \sim m\,\Lambda_{QCD}$
  ↳ non perturbative Fermi Motion (soft effects)
  Fermi Motion ⟺ Shape Function can be factorized
- $\lambda \to 1$ $\quad m_X^2 \sim \Lambda_{QCD}^2$
  ↳ final state hadronization

Shape Function ≡ Structure function for heavy flavours

factorizes initial state (Fermi M / soft gluons) effects

[in threshold distribution initial bound state and hadronization effects have two different scales ⟺ separation

Conclusions

- We have computed next-to-leading transverse momentum distributions in $b \to s\gamma$ decays
- Non-perturbative effects do appear in the resummed distribution peculiarly related to both initial and final states
- It seems not possible to define the analogue of the shape function for the transverse momentum distributions as it is for the threshold distributions

Future Perspectives

- The inclusion of non perturbative effects calls for an extended shape function involving possibly a different effective theory containing transverse degrees of freedom
- Once the $C(\alpha_S)$ and $R(b;\alpha_S)$ are completely evaluated a comparison with the experimental data will be possible