Theory of Extracting $|V_{cb}|$ and $|V_{ub}|$

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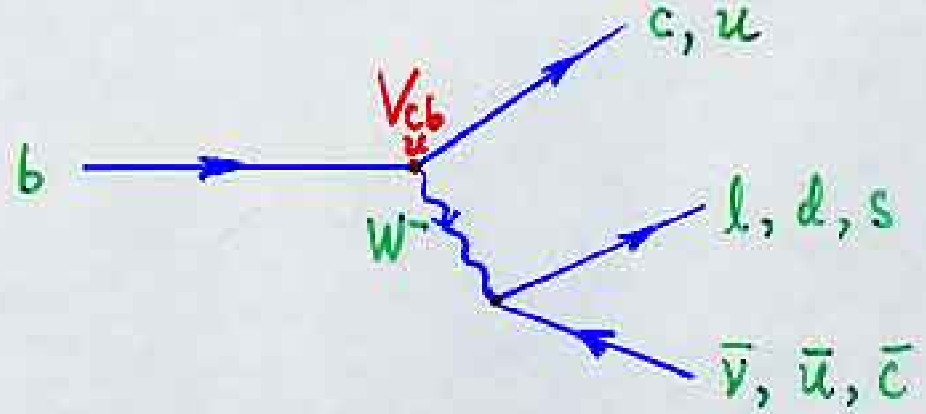
How to measure $|V_{cb}|$ and $|V_{ub}|$?



Semileptonic B decays
- exclusive $B \to D^{(*)} l \bar{\nu}$, $B \to \pi(\rho) l \bar{\nu}$
- inclusive $B \to X_c l \bar{\nu}$, $B \to X_u l \bar{\nu}$

Nonleptonic B decays
- exclusive $B \to D_s \pi$
- semi-inclusive $B \to D_s X_u$
- inclusive $B \to u \bar{c} s$

$$\text{observable} = |V_{\substack{cb\\u}}|^2 \cdot T$$

Theory may be involved in it. theory

- Routine observable : branching fraction, lifetime
- Theory-motivated observable : "theoretically clean" "model-independent" in some sense

Exclusive semileptonic B meson decays

$|V_{cb}|$ from $B \to D^{(*)} \ell \bar{\nu}$

$|V_{ub}|$ from $B \to \pi(\rho) \ell \bar{\nu}$

$|V_{cb}|$ from $B \to D^* l \bar{\nu}$

Theory-motivated observable: $\left.\frac{d\Gamma}{dw}\right|_{w=1}$ $\quad w = v_B \cdot v_{D^*}$

instead of routine observable: $\Gamma = Br(B \to D^* l \bar{\nu})/\tau_B$

$$\frac{d\Gamma}{dw} = \left(|V_{cb}|\, F(w)\right)^2 \frac{G_F^2 M_B^5}{48\pi^3} r^3 (1-r)^2 \sqrt{w^2-1}\,(w+1)^2$$
$$\times \left[1 + 4\frac{w}{w+1}\frac{1-2wr+r^2}{(1-r)^2}\right] \qquad r = \frac{M_{D^*}}{M_B}$$

Motivation:

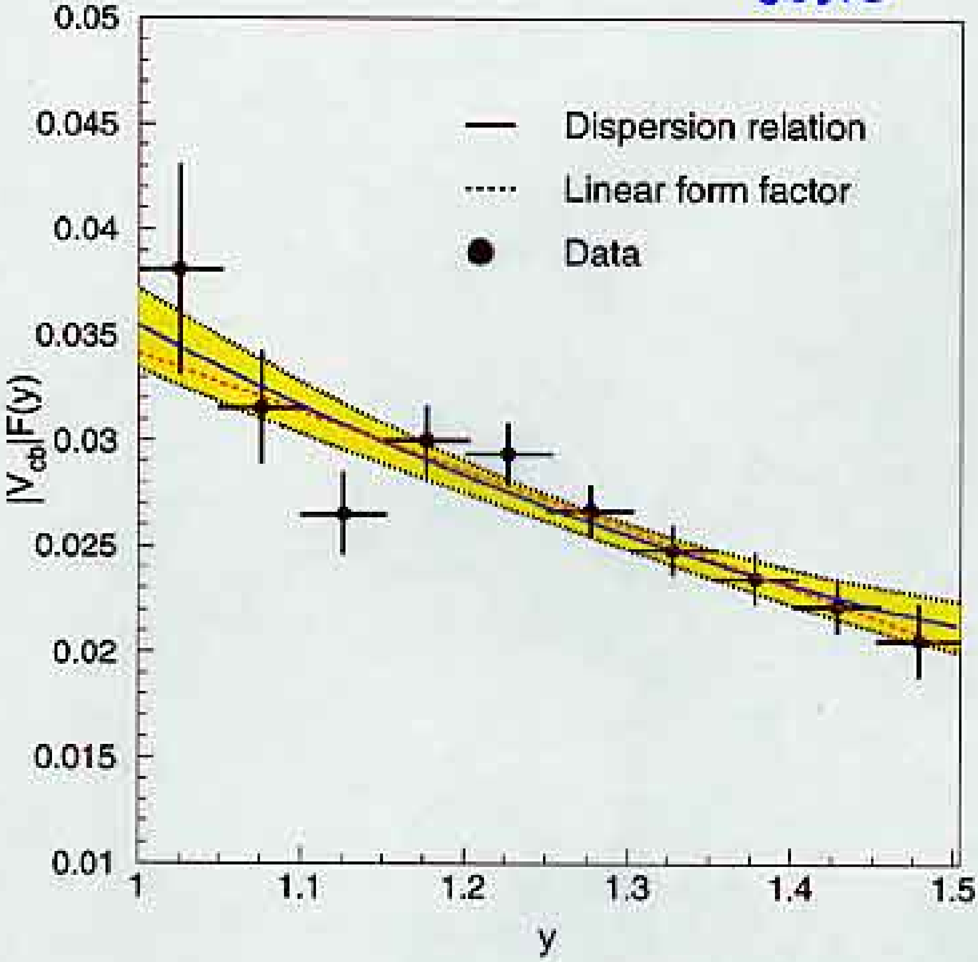
heavy quark spin-flavor symmetries $\Rightarrow F(1) = 1$
for $m_b, m_c \to \infty$

However $\left.\frac{d\Gamma}{dw}\right|_{w=1} = 0$

Procedure:

- converting $\frac{d\Gamma}{dw}$ into $|V_{cb}|F(w)$
- ploting $|V_{cb}|F(w)$ as a function of w
- extrapolating to the zero recoil point, $w=1$

Belle



$\bar{B}^0 \to D^{*+} e^- \bar{\nu}$

Theoretical uncertainties

- $F(1)$ normalization

Radiative and power corrections

$$F(1) = \eta_A \left(1 + \frac{0}{m_Q} + \frac{\delta_2}{m_Q^2} + \frac{\delta_3}{m_Q^3} + \cdots \right)$$

Luke's theorem

$$\frac{\delta_2}{m_Q^2} \sim \frac{\Lambda_{QCD}^2}{m_c^2} \sim 10\% \quad \begin{cases} \text{phenomenological models} \\ \text{quenched lattice QCD} \end{cases}$$

- $F(w)$ shape

Analyticity and unitarity constraints

Dispersion relation constrained form factor

$$\frac{|V_{cb}|F(1)_{\text{dispersive}} - |V_{cb}|F(1)_{\text{linear}}}{|V_{cb}|F(1)_{\text{linear}}} = 5\%$$

(Belle fit)

$|V_{cb}|$ from $B \to D \ell \bar{\nu}$

Theory-motivated observable : $\left.\frac{d\Gamma}{dw}\right|_{w=1}$ $\quad w = v_B \cdot v_D$

$$\frac{d\Gamma}{dw} = \left(|V_{cb}|\, F_D(w)\right)^2 \frac{G_F^2 M_B^5}{48\pi^3} r^3 (1+r)^2 (w^2-1)^{3/2} \qquad r = \frac{M_D}{M_B}$$

similar to $B \to D^* \ell \bar{\nu}$, but ...

$$F_D(1) = \eta_V \left(1 + \frac{\delta_1}{m_Q} + \frac{\delta_2}{m_Q^2} + \frac{\delta_3}{m_Q^3} + \cdots\right)$$

$$\frac{\delta_1}{m_Q} \sim \frac{\Lambda_{QCD}}{m_c} \sim 30\% \text{ correction}$$

worse than $B \to D^* \ell \bar{\nu}$

$|V_{ub}|$ from $B \to \pi l \bar{\nu}$

Routine observable : $\dfrac{Br(B \to \pi l \bar{\nu})}{\tau_B} = \Gamma(B \to \pi l \bar{\nu})$

$$\frac{d\Gamma}{dq^2} = |V_{ub}|^2 \frac{G_F^2}{24\pi^3} \left(E_\pi^2 - m_\pi^2\right)^{3/2} f_+^2(q^2)$$

$$\langle \pi(p') | \bar{u} \gamma^\mu b | B(p) \rangle = f_+(q^2)(p+p')^\mu + f_-(q^2)(p-p')^\mu$$

$q = p - p'$

Need theory for the form factor $f_+(q^2) \Big\langle \begin{matrix} \text{normalization} \\ \text{shape} \end{matrix}$

- quark model
- lattice QCD
- light-cone sum rules
- perturbative QCD

Difficult to quantify the theoretical uncertainties

similar for $B \to \rho l \bar{\nu}$

Theory of inclusive semileptonic B decays $B \to X_q l \bar{\nu}$

$q = u, c$

Hadronic tensor

$$W_{\mu\nu} = -\frac{1}{2\pi}\int d^4y\, e^{iq\cdot y} \langle B(P)|[j_\mu(y), j_\nu^\dagger(0)]|B(P)\rangle$$

$$= -g_{\mu\nu}W_1 + \frac{P_\mu P_\nu}{M_B^2}W_2 - i\varepsilon_{\mu\nu\alpha\beta}\frac{P^\alpha q^\beta}{M_B^2}W_3 + \frac{q_\mu q_\nu}{M_B^2}W_4 + \frac{P_\mu q_\nu + q_\mu P_\nu}{M_B^2}W_5$$

momentum transfer $q = k_l + k_\nu$

$$j^\mu(y) = \bar{q}(y)\gamma^\mu(1-\gamma_5)b(y)$$

Starting point: light-cone expansion

$$M_B \gg \Lambda_{QCD} \Rightarrow y^2 \to 0$$

$$\langle B(P)|[j^\mu(y), j^{\nu\dagger}(0)]|B(P)\rangle = \sum_n C_n^{\mu\nu}(y,P)(y^2)^n$$

Light-cone approach to inclusive B decays

[CJ, Paschos]

At leading twist

Structure functions $\left.\begin{array}{l} W_1(\nu, q^2) \\ W_2(\nu, q^2) \\ W_3(\nu, q^2) \\ W_4(\nu, q^2) \\ W_5(\nu, q^2) \end{array}\right\}$ a single universal function $f(\xi)$

Distribution function

$$f(\xi) = \frac{1}{4\pi}\int \frac{d(y\cdot P)}{y\cdot P}\, e^{i\xi y\cdot P}\langle B|\bar{b}(0)\not{y}\, U(0,y)\, b(y)|B\rangle\Big|_{y^2=0}$$

$$W_1 = 2\left[f(\xi_+) + f(\xi_-)\right]$$

$$W_2 = \frac{8}{\xi_+ - \xi_-}\left[\xi_+ f(\xi_+) - \xi_- f(\xi_-)\right]$$

$$W_3 = -\frac{4}{\xi_+ - \xi_-}\left[f(\xi_+) - f(\xi_-)\right]$$

$$W_4 = 0\,, \quad W_5 = W_3$$

$$\xi_\pm = \frac{\nu \pm \sqrt{\nu^2 - q^2 + m_q^2}}{M_B} \qquad \nu = \frac{q\cdot P}{M_B}$$

What is known about the distribution function?

- Normalization $\int_0^1 d\xi\, f(\xi) = 1$ (exactly)

  because b-quark number conservation $\rightarrow$

  $$\langle \bar{B}(P) | \bar{b} \gamma^\mu b | \bar{B}(P) \rangle = 2P^\mu$$

- Shape

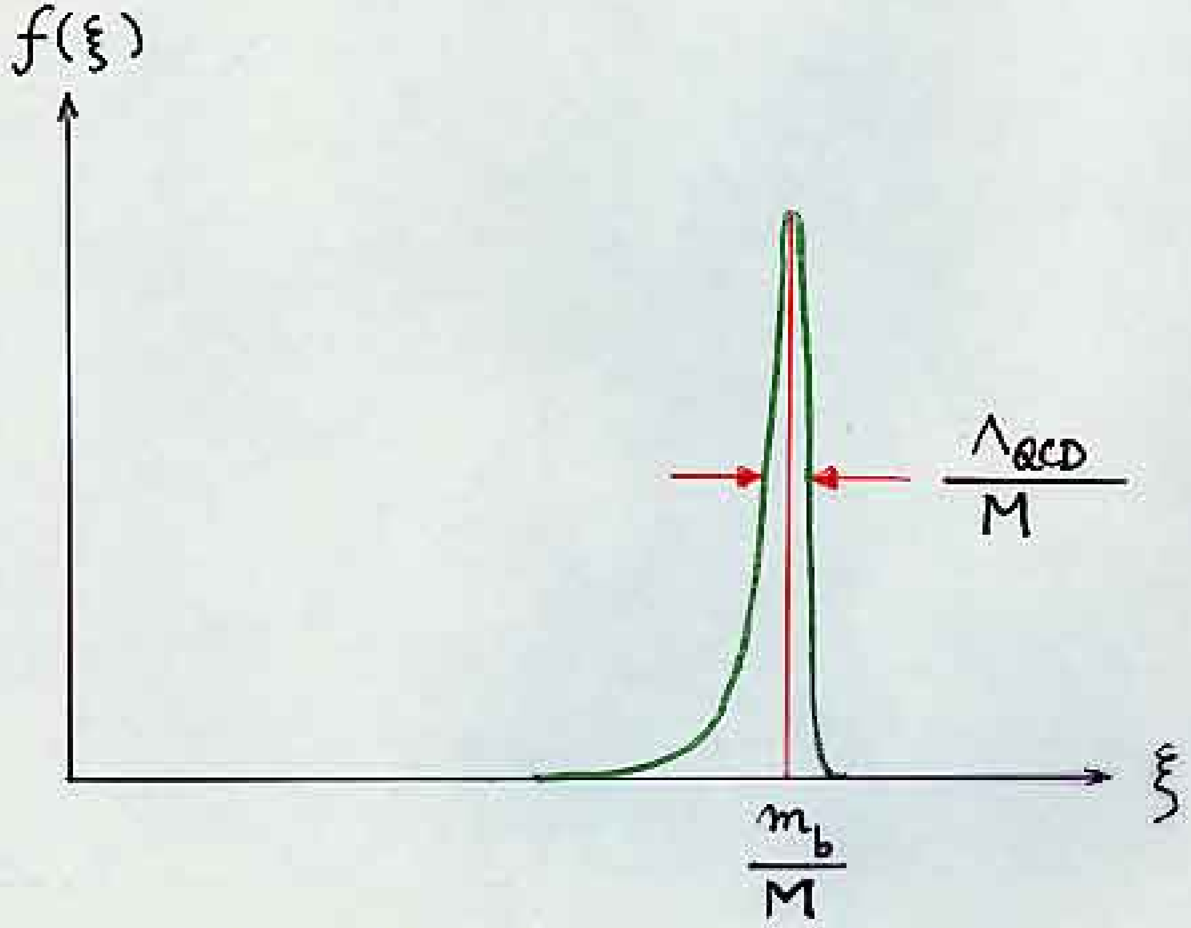
  – free quark limit $f_{\text{free}}(\xi) = \delta\left(\xi - \frac{m_b}{M_B}\right)$

  – mean $\langle \xi \rangle = \frac{m_b}{M_B}\left[1 - \frac{5}{6m_b^2}(\lambda_1 + 3\lambda_2)\right] \approx \frac{m_b}{M_B}$

  – variance $\langle (\xi - \langle \xi \rangle)^2 \rangle = -\frac{\lambda_1}{3M_B^2} \sim \left(\frac{\Lambda_{QCD}}{M_B}\right)^2$

Sharply peaked around $\xi = \langle \xi \rangle \approx \frac{m_b}{M_B}$

Width $\sim \frac{\Lambda_{QCD}}{M_B}$



- different forms for different b-hadrons
- sharp peak around $\xi = \frac{m_b}{M} \to 1$

  width $\sim \frac{\Lambda_{QCD}}{M}$

  M the mass of the b-hadron

Comparison with heavy quark expansion approach

HQE ≡ heavy quark expansion approach [Chay, Georgi, Grinstein, Bigi, et al.]

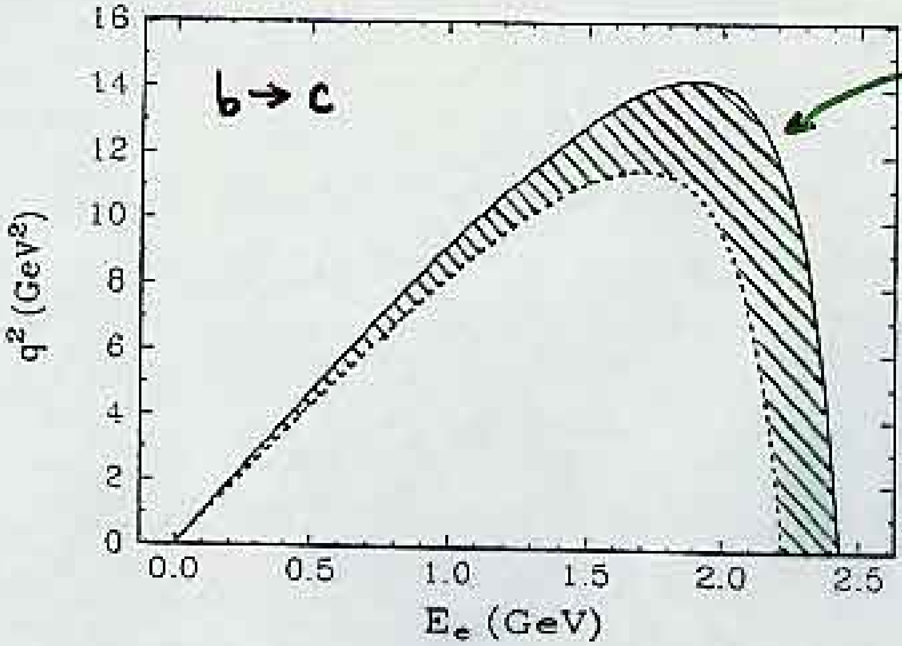
LC ≡ light-cone approach

- Starting point
  - $\boxed{\text{HQE}}$: local operator product expansion $y \to 0$

    $$T[j^{\mu\dagger}(y), j_\nu(0)] = \sum_i C_i^{\mu\nu}(y)\, O_i(0)$$
  - $\boxed{\text{LC}}$: non-local light-cone expansion $y^2 \to 0$
- Assumption of quark-hadron duality
  - $\boxed{\text{HQE}}$: YES. using quark phase space
  - $\boxed{\text{LC}}$: NO. using hadron phase space
- Singularity in lepton energy spectrum
  - $\boxed{\text{HQE}}$: YES
  - $\boxed{\text{LC}}$: NO
- Distribution function
  - $\boxed{\text{HQE}}$: partial resummation of heavy quark expansion → a different distribution function ("shape function")

    $$f(\omega) = \frac{1}{2M_B}\langle B|\bar{h}_v\, \delta(\omega + i n\cdot \hat{D})\, h_v|B\rangle$$

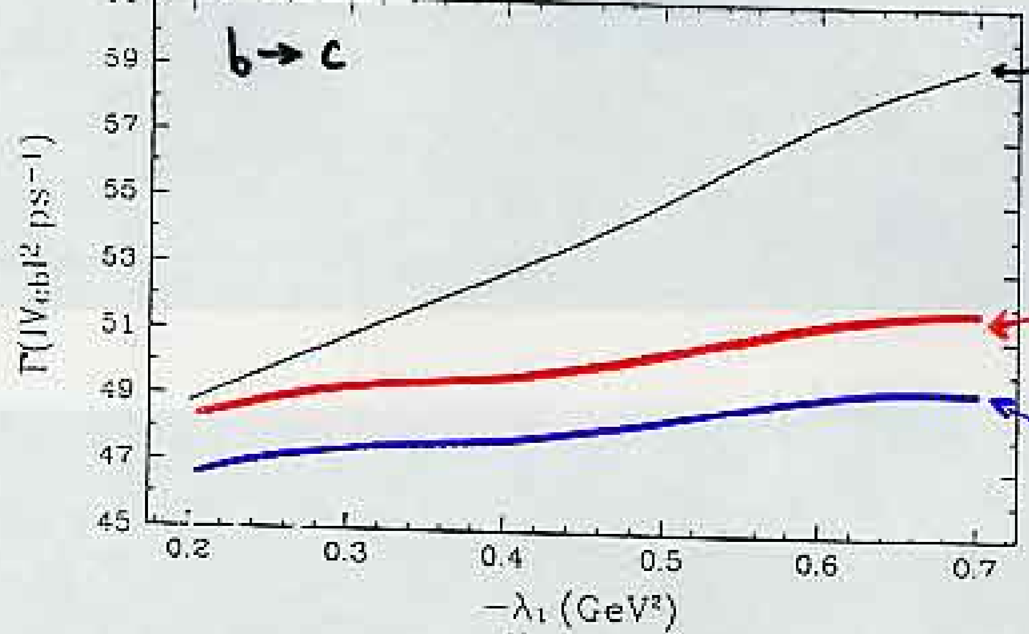
    $\frac{\Lambda_{QCD}}{m_b}$ correction to leading contribution in terms of $f(\omega)$
  - $\boxed{\text{LC}}$: $\frac{\Lambda^2_{QCD}}{M_B^2}$ correction to leading contribution in terms of $f(\xi)$



gap between the quark-leve[l] and the hadron-level phase spaces

$$B \to X_c \, l \, \bar{\nu}$$

semileptonic decay width of the B meson as a function of $-\lambda_1$

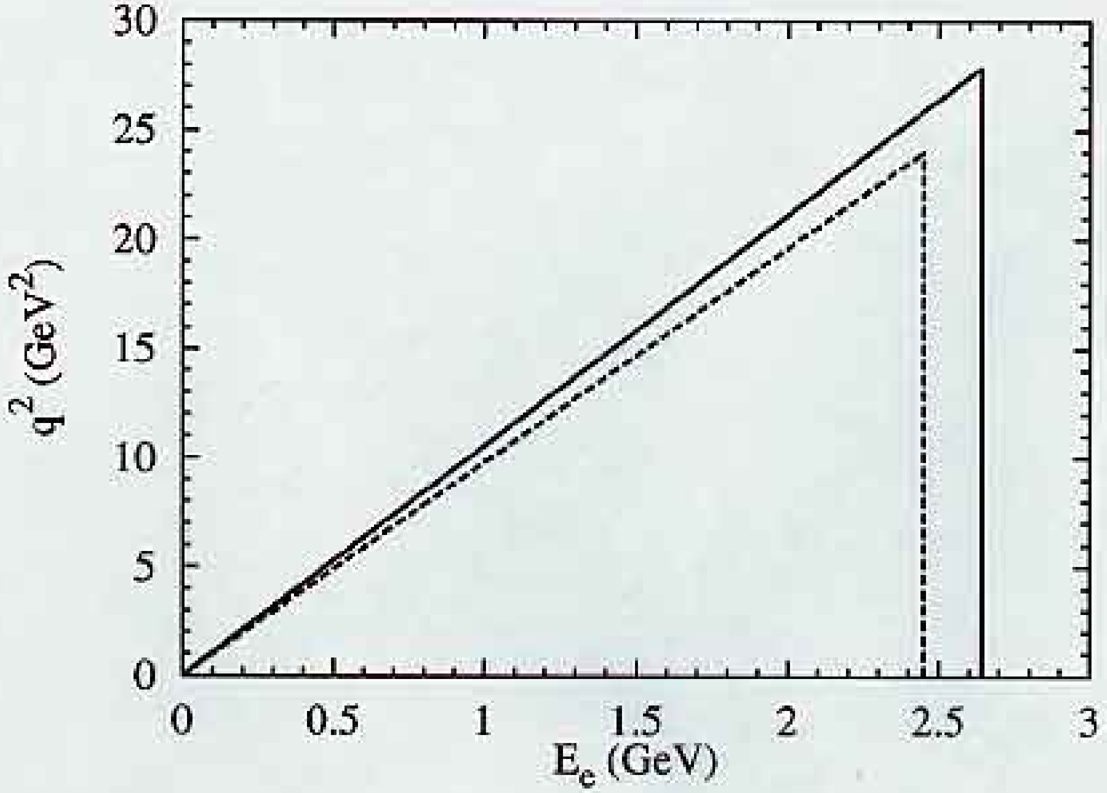


LC approach

free quark deca[y]

heavy quark expansion

$$-\lambda_1 = 2m_b^2 K(B)$$



The $B \to X_u \ell \nu$ Phase Space

Manifestation of quark-hadron duality violation

HQE uses quark phase space

LC uses hadron phase space

Compare —

for $B \to X_c l \bar{\nu}$ $\quad \dfrac{\Gamma_{LC} - \Gamma_{HQE}}{\Gamma_{HQE}} \approx 14\%$

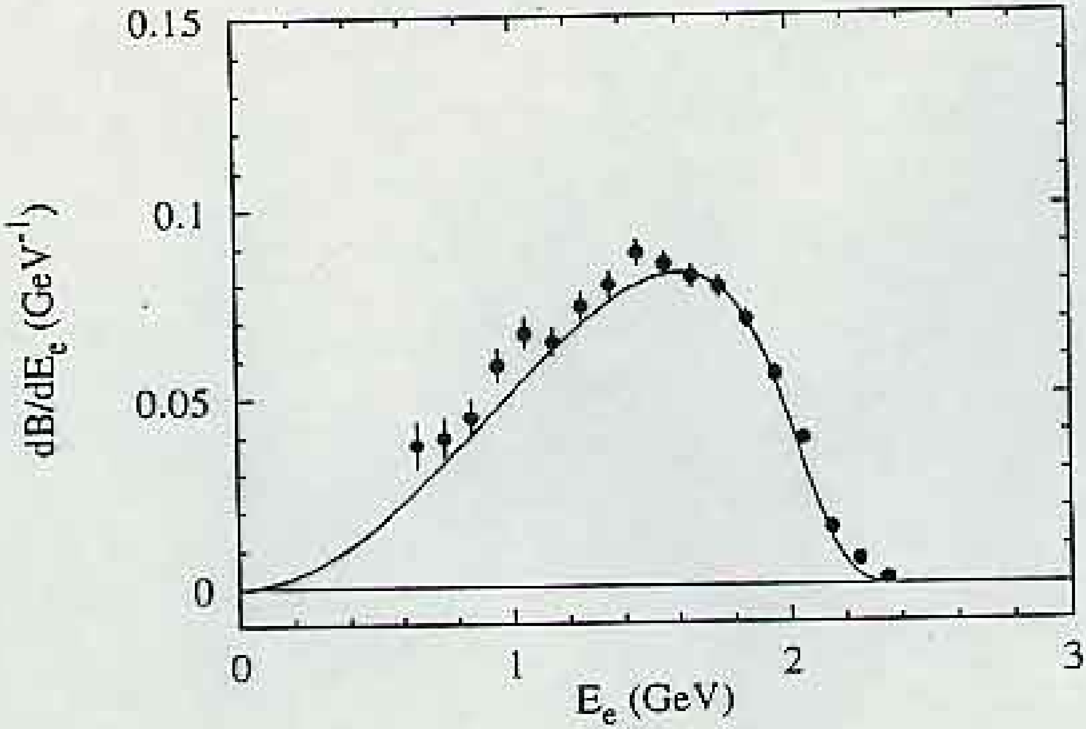
for $B \to X_u l \bar{\nu}$ $\quad \dfrac{\Gamma_{LC} - \Gamma_{HQE}}{\Gamma_{HQE}} \approx 12\%$

HQE misses the rate due to phase space extension.

Conclusion:

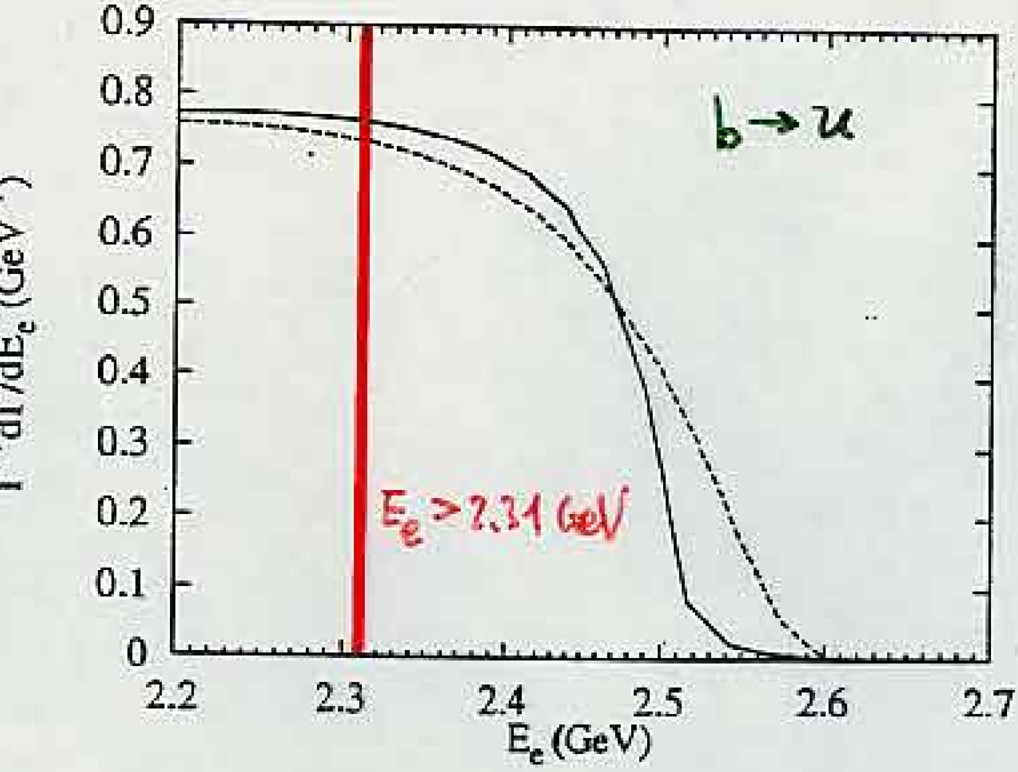
- Significant duality violation in HQE
- Important to include phase space effect

electron energy spectrum (LC)
in $B \to X_c e \bar{\nu}_e$



CLEO data

$b \to u$ electron energy spectrum in the endpoint region (LC)



useful for the determination of $|V_{ub}|$

Inclusive semileptonic B meson decays

$|V_{cb}|$ from $B \to X_c \ell \bar{\nu}$

$|V_{ub}|$ from $B \to X_u \ell \bar{\nu}$

Traditional method

Routine observable:

$$\frac{Br(B \to X_c l \bar{\nu})}{\tau_B} = \Gamma(B \to X_c l \bar{\nu}) = |V_{cb}|^2 \cdot \gamma_c$$

theory

$$\frac{Br(B \to X_u l \bar{\nu})}{\tau_B} = \Gamma(B \to X_u l \bar{\nu}) = |V_{ub}|^2 \cdot \gamma_u$$

Problem in theoretical calculation of γ_c, γ_u

- HQE: quark-hadron duality
- LC: detailed shape of distribution function unknown
  (though not sensitive to it)

Particular problem in $|V_{ub}|$ determination

$$\frac{\Gamma(B \to X_u l \bar{\nu})}{\Gamma(B \to X_c l \bar{\nu})} \sim 1\%$$

Very large $B \to X_c l \bar{\nu}$ background !

Using kinematic cut to suppress background

kinematic cut	fraction of events
$E_l > \frac{M_B^2 - M_D^2}{2M_B}$	$\sim 10\%$
$q^2 > (M_B - M_D)^2$	$\sim 20\%$
$M_X < M_D$	$\sim 80\%$

M_X cut is most efficient $\to$ small extrapolation

Better method for $|V_{ub}|$

Theory-motivated observable: $\int_0^1 d\xi_u \frac{1}{\xi_u^5} \frac{d\Gamma}{d\xi_u}$ $\quad \xi_u = \frac{q^0 + |\vec{q}|}{M_B}$

Motivation: b-quark number conservation $\Rightarrow$

$$\int_0^1 d\xi_u \frac{1}{\xi_u^5} \frac{d\Gamma}{d\xi_u} = |V_{ub}|^2 \frac{G_F^2 M_B^5}{192\pi^3} \quad \text{at leading twist}$$

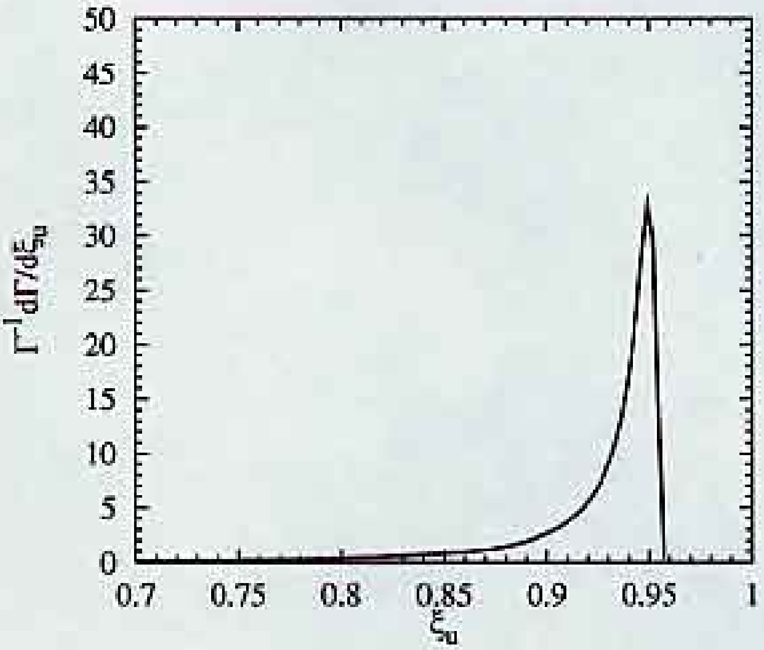
Semileptonic sum rule [CJ]

However, large $B \to X_c \ell \bar{\nu}$ background

Procedure:

- using cut $M_X < M_D$
- measuring weighted spectrum $\frac{1}{\xi_u^5} \frac{d\Gamma}{d\xi_u}$
- extrapolating to entire phase space

ξ_u spectrum at hadron level



Theoretical uncertainties

- Higher twist correction

$$\int_0^1 d\xi_u \frac{1}{\xi_u^5} \frac{d\Gamma}{d\xi_u} = |V_{ub}|^2 \frac{G_F^2 M_B^5}{192\pi^3} (1 + \Delta HT) \quad \Delta HT \sim \frac{\Lambda_{QCD}^2}{M_B^2}$$

$\Delta HT = 0.012$ from HQET [CJ, PL, B520 (2001) 92]

$\sim 1\%$ error on $|V_{ub}|$ due to higher twist correction

- $f(\xi)$ shape

$\sim 6\%$ error on $|V_{ub}|$ due to extrapolation

Improvement:

- Constraints from experimental data (distribution, moment, ⋯)
- Universal $f(\xi)$ from $B \to X_s \gamma$
- Lattice QCD

Summary

- The main theoretical uncertainties in extracting $|V_{cb}|$ and $|V_{ub}|$ are from QCD.

- $|V_{cb}|$ from inclusive semileptonic width $\Gamma(B \to X_c l \bar{\nu})$ has a theoretical uncertainty comparable to $|V_{cb}|$ from exclusive semileptonic spectrum $\frac{d\Gamma}{dw}(B \to D^* l \bar{\nu})$ if the fundamental uncertainty due to the assumption of quark-hadron duality is avoided and the phase space effect is included.
  This is achievable with the light-cone approach to inclusive B decays.

- More precise $|V_{ub}|$ can be extracted using the semileptonic sum rule.