# MSSM Higgs Phenomenology in the Decoupling Limit

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#### Outline

- MSSM Higgs Sector at Tree Level
  - masses, couplings, and the approach to decoupling
- MSSM Higgs Sector at One-Loop (and beyond)
  - $h^{\,0}$  mass bound, radiatively-corrected couplings, and the decoupling limit revisited
- Precision Higgs physics at future colliders
  - phenomenology of the decoupling limit

This talk is based on work that appears in: M. Carena, H.E. Haber, H.E. Logan and S. Mrenna, hep-ph/0106116 [*Phys. Rev.* **D65** (2002), in press].

### Challenge of the Decoupling Limit

- Given a non-minimal Higgs sector, the decoupling limit corresponds to the parameter regime in which all but one CP-even Higgs scalar is significantly heavier than the Z. The properties of the lightest CP-even Higgs boson are nearly indistinguishable from those of the Standard Model (SM) Higgs boson.
- The decoupling limit is very general. Many models with non-minimal Higgs sectors possess a decoupling limit. The MSSM Higgs sector is one such example.
- Discovery of the SM-like Higgs boson is not sufficient to reveal the underlying electroweak symmetry breaking dynamics.
- It is crucial to find evidence for Higgs physics beyond the SM Higgs boson. Either one must directly discover the non-minimal Higgs states (perhaps difficult, if they are too heavy), or one must detect deviations from SM Higgs predictions.
- ullet In the latter case, precision Higgs measurements are essential for detecting deviations from the SM of branching ratios, coupling strengths, cross-sections, etc.

### MSSM Higgs sector at Tree-Level

Five physical Higgs scalars:

- $H^{\pm}$ : a charged Higgs pair
- $h^0$ ,  $H^0$ : two CP-even Higgs scalars  $(m_h \leq m_H)$
- $A^0$ : a CP-odd Higgs scalar

All Higgs masses and couplings can be expressed in terms of two parameters usually chosen to be  $m_A$  and  $\tan\beta \equiv v_u/v_d$ . The CP-even Higgs mixing angle,  $\alpha$ , is given by

$$\cos^2(\beta - \alpha) = \frac{m_h^2(m_Z^2 - m_h^2)}{m_A^2(m_H^2 - m_h^2)}.$$

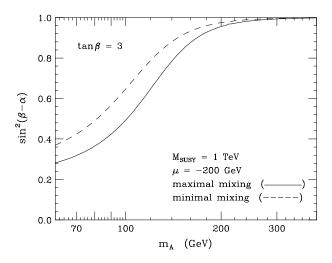
When  $m_A\gg m_Z$ , the  $h^0$  couplings are identical to those of the Standard Model Higgs boson, while the other Higgs states  $H^0$ ,  $A^0$  and  $H^\pm$  are all heavy and roughly degenerate in mass. This is called the decoupling limit.

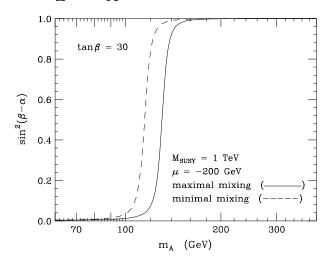
### Decoupling Limit of the MSSM Higgs Sector

In the limit  $m_A\gg m_Z$ , tree-level Higgs masses are given by:

$$\begin{split} m_h^2 &\simeq m_Z^2 \cos^2 2\beta \,, \\ m_H^2 &\simeq m_A^2 + m_Z^2 \sin^2 2\beta \,, \\ m_{H^\pm}^2 &= m_A^2 + m_W^2 \,, \\ \cos^2(\beta - \alpha) &\simeq \frac{m_Z^4 \sin^2 4\beta}{4m_A^4} \,, \\ \cot \alpha + \tan \beta &= -\frac{2m_Z^2}{m_A^2} \tan \beta \cos 2\beta + \mathcal{O}(m_Z^4/m_A^4) \,. \end{split}$$

Thus,  $m_A \simeq m_H \simeq m_{H^\pm}$ , up to corrections of  $\mathcal{O}(m_Z^2/m_A)$ , and  $\cos(\beta - \alpha) = 0$  up to corrections of  $\mathcal{O}(m_Z^2/m_A^2)$ .





The effective low-energy theory below the scale  $m_A$  is a theory with an effective Higgs sector consisting of a SM-like CP-even Higgs boson,  $h^0$ .

### **MSSM** Tree-Level Higgs Couplings

Higgs couplings to gauge bosons: suppression factors

$$egin{array}{lll} & \cos(eta-lpha) & \sin(eta-lpha) & ext{no angle factor} \\ & H^0W^+W^- & h^0W^+W^- & \\ & H^0ZZ & h^0ZZ & \\ & ZA^0h^0 & ZA^0H^0 & ZH^+H^-, \gamma H^+H^- & \\ & W^\pm H^\mp h^0 & W^\pm H^\mp H^0 & W^\pm H^\mp A^0 & \end{array}$$

CP-even Higgs couplings to fermion pairs [relative to  $m_f/v$ ]

$$h^0 b \bar{b} : -\frac{\sin \alpha}{\cos \beta} = 1 \sin(\beta - \alpha) - \tan \beta \cos(\beta - \alpha),$$

$$h^0 t \bar{t} : \frac{\cos \alpha}{\sin \beta} = 1 \sin(\beta - \alpha) + \cot \beta \cos(\beta - \alpha),$$

$$H^0 b \bar{b}$$
:  $\frac{\cos \alpha}{\cos \beta} = \cos(\beta - \alpha) + \tan \beta \sin(\beta - \alpha)$ ,

$$H^0 t \bar{t} : \frac{\sin \alpha}{\sin \beta} = \cos(\beta - \alpha) - \cot \beta \sin(\beta - \alpha).$$

In particular, tree-level couplings of  $h^0$  are precisely those of the Standard Model Higgs boson when  $\cos(\beta - \alpha) = 0$ .

## Radiatively-corrected MSSM Higgs mass bound

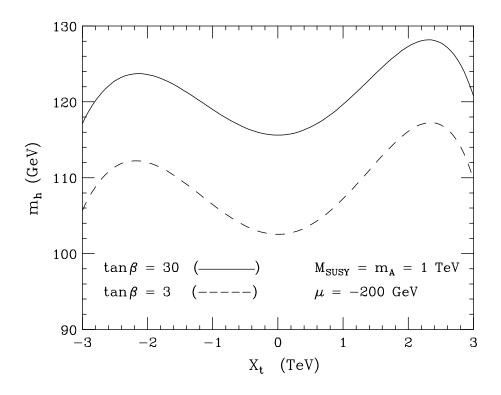
Due to supersymmetric relations among couplings, one finds that  $m_h \leq m_Z$  (a result already ruled out by LEP data). But, this inequality receives quantum corrections. The Higgs mass can be shifted due to loops of particles and their superpartners (an incomplete cancelation, which would have been exact if supersymmetry were unbroken):

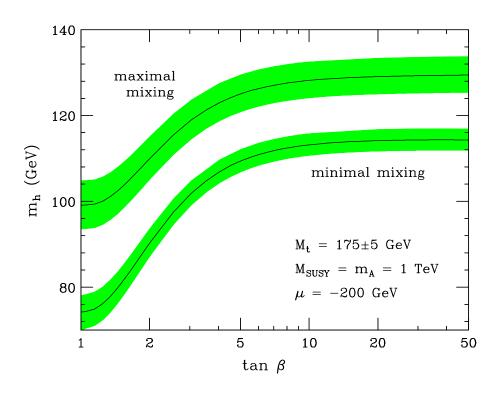


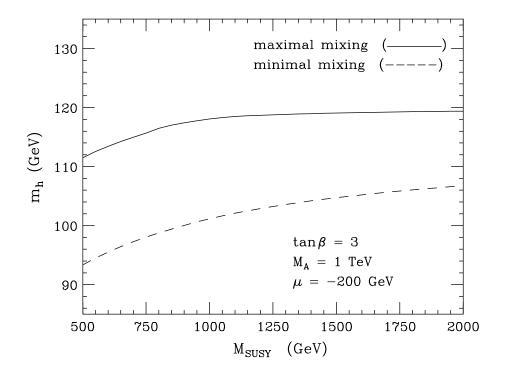
$$m_h^2 \lesssim m_Z^2 + \frac{3g^2 m_t^4}{8\pi^2 m_W^2} \left[ \ln \left( \frac{M_S^2}{m_t^2} \right) + \frac{X_t^2}{M_S^2} \left( 1 - \frac{X_t^2}{12M_S^2} \right) \right] ,$$

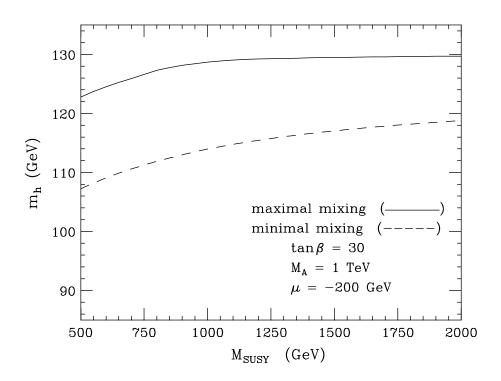
where  $X_t \equiv A_t - \mu \cot \beta$  governs stop mixing and  $M_S^2$  is the average stop squared-mass.

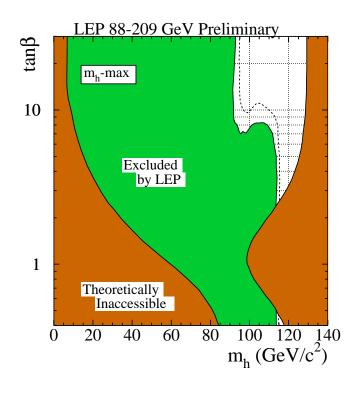
End result:  $m_h \lesssim 130$  GeV [assuming that the top-squark mass is no heavier than about 2 TeV].

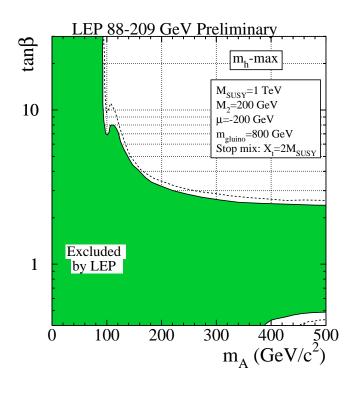












#### Present status of the LEP Higgs Search [95% CL limits]

- ullet Standard Model Higgs boson:  $m_H > 113.5 \; {
  m GeV}$
- ullet Charged Higgs boson:  $m_{H^\pm} > 78.5~{
  m GeV}$
- MSSM Higgs:  $m_h > 91.0$  GeV;  $m_A > 91.9$  GeV

At large  $\tan\beta$ , supersymmetric radiative corrections can also have a significant impact on the Higgs branching ratios. Example: the dominant decay mode  $h\to b\bar{b}$  is suppressed in some regions of MSSM Higgs parameter space.

### Leading radiative corrections to $h^0$ couplings

For Higgs couplings to vector bosons, the dominant corrections arise from corrections to  $\cos(\beta - \alpha)$ .

For Higgs couplings to fermions, in addition to corrections to  $\cos(\beta-\alpha)$ , one must also consider Yukawa vertex corrections, which modify the effective Lagrangian that describes the coupling of the Higgs bosons to the third generation quarks:

$$-\mathcal{L}_{\text{eff}} = \epsilon_{ij} \left[ (h_b + \delta h_b) \bar{b}_R H_d^i Q_L^j + (h_t + \delta h_t) \bar{t}_R Q_L^i H_u^j \right]$$
$$+ \Delta h_b \bar{b}_R Q_L^k H_u^{k*} + \Delta h_t \bar{t}_R Q_L^k H_d^{k*} + \text{h.c.},$$

resulting in a modification of the tree-level relation between  $h_t \ [h_b]$  and  $m_t \ [m_b]$  as follows:

$$m_b = \frac{h_b v}{\sqrt{2}} \cos \beta \left( 1 + \frac{\delta h_b}{h_b} + \frac{\Delta h_b \tan \beta}{h_b} \right) \equiv \frac{h_b v}{\sqrt{2}} \cos \beta (1 + \Delta_b) ,$$

$$m_t = \frac{h_t v}{\sqrt{2}} \sin \beta \left( 1 + \frac{\delta h_t}{h_t} + \frac{\Delta h_t \cot \beta}{h_t} \right) \equiv \frac{h_t v}{\sqrt{2}} \sin \beta (1 + \Delta_t).$$

The dominant contributions to  $\Delta_b$  are  $\tan \beta$ -enhanced. In particular, for  $\tan \beta \gg 1$ ,  $\Delta_b \simeq (\Delta h_b/h_b) \tan \beta$ ; whereas  $\delta h_b/h_b$  provides a small correction to  $\Delta_b$ . In the same limit,  $\Delta_t \simeq \delta h_t/h_t$ , with the additional contribution of  $(\Delta h_t/h_t) \cot \beta$  providing a small correction.

#### Explicit expressions

For  $\tan \beta \gg 1$ ,

$$\Delta_b \simeq \left[ \frac{2\alpha_s}{3\pi} \mu M_{\tilde{g}} I(M_{\tilde{b}_1}^2, M_{\tilde{b}_2}^2, M_{\tilde{g}}^2) + \frac{h_t^2}{16\pi^2} \mu A_t I(M_{\tilde{t}_1}^2, M_{\tilde{t}_2}^2, \mu^2) \right] \tan \beta$$

$$\Delta_t \simeq -\frac{2\alpha_s}{3\pi} A_t M_{\tilde{g}} I(M_{\tilde{t}_1}^2, M_{\tilde{t}_2}^2, M_{\tilde{g}}^2) - \frac{h_b^2}{16\pi^2} \mu^2 I(M_{\tilde{b}_1}^2, M_{\tilde{b}_2}^2, \mu^2) ,$$

and the function I is defined by:

$$I(a, b, c) = \frac{ab \ln(a/b) + bc \ln(b/c) + ca \ln(c/a)}{(a-b)(b-c)(a-c)},$$

Note that I is manifestly positive and I(a, a, a) = 1/(2a).

The  $\tau$  couplings are obtained by replacing  $m_b$ ,  $\Delta_b$  and  $\delta h_b$  with  $m_{\tau}$ ,  $\Delta_{\tau}$  and  $\delta h_{\tau}$ , respectively. At large  $\tan \beta$ ,

$$\Delta_{\tau} \simeq \left[ \frac{\alpha_1}{4\pi} M_1 \mu I(M_{\tilde{\tau}_1}^2, M_{\tilde{\tau}_2}^2, M_1^2) - \frac{\alpha_2}{4\pi} M_2 \mu I(M_{\tilde{\nu}_{\tau}}^2, M_2^2, \mu^2) \right] \tan \beta ,$$

where  $\alpha_2 \equiv g^2/4\pi$  and  $\alpha_1 \equiv g'^2/4\pi$  are the electroweak gauge couplings. One expects that  $|\Delta_\tau| \ll |\Delta_b|$ .

# Radiative corrections to $cos(\beta - \alpha)$ and implications for decoupling

Writing the CP-even Higgs mass matrix:

$$\mathcal{M}^{2} = \begin{pmatrix} m_{A}^{2} s_{\beta}^{2} + m_{Z}^{2} c_{\beta}^{2} & -(m_{A}^{2} + m_{Z}^{2}) s_{\beta} c_{\beta} \\ -(m_{A}^{2} + m_{Z}^{2}) s_{\beta} c_{\beta} & m_{A}^{2} c_{\beta}^{2} + m_{Z}^{2} s_{\beta}^{2} \end{pmatrix} + \delta \mathcal{M}^{2},$$

and noting that  $\delta \mathcal{M}_{ij}^2 \sim \mathcal{O}(m_Z^2)$ , and  $m_H^2 - m_h^2 = m_A^2 + \mathcal{O}(m_Z^2)$ , one obtains for  $m_A \gg m_Z$ 

$$\cos(\beta - \alpha) = c \left[ \frac{m_Z^2 \sin 4\beta}{2m_A^2} + \mathcal{O}\left(\frac{m_Z^4}{m_A^4}\right) \right] ,$$

where

$$c \equiv 1 + \frac{\delta \mathcal{M}_{11}^2 - \delta \mathcal{M}_{22}^2}{2m_Z^2 \cos 2\beta} - \frac{\delta \mathcal{M}_{12}^2}{m_Z^2 \sin 2\beta}.$$

These formulae exhibit the expected decoupling behavior for  $m_A\gg m_Z$ . But, they also allow for an unexpected  $m_A$ -independent decoupling corresponding to c=0. Assuming large  $\tan\beta$ , a solution to c=0 arises for:

$$\tan \beta \simeq \frac{2m_Z^2 - \delta \mathcal{M}_{11}^2 + \delta \mathcal{M}_{22}^2}{\delta \mathcal{M}_{12}^2}.$$

# Leading corrections to Higgs-fermion couplings

$$h^{0}b\bar{b}: \qquad -\frac{\sin\alpha}{\cos\beta} \left(\frac{1}{1+\Delta_{b}}\right) \left[1-\Delta_{b}\cot\alpha\cot\beta\right] + \frac{\delta h_{b}}{h_{b}}(1+\cot\alpha\cot\beta) \right] ,$$

$$H^{0}b\bar{b}: \qquad \frac{\cos\alpha}{\cos\beta} \left(\frac{1}{1+\Delta_{b}}\right) \left[1+\Delta_{b}\tan\alpha\cot\beta\right] + \frac{\delta h_{b}}{h_{b}}(1-\tan\alpha\cot\beta) \right] ,$$

$$h^{0}t\bar{t}: \qquad -\frac{\cos\alpha}{\sin\beta} \left[1-\frac{1}{1+\Delta_{t}}\frac{\Delta h_{t}}{h_{t}}(\cot\beta+\tan\alpha)\right] ,$$

$$H^{0}t\bar{t}: \qquad \frac{\sin\alpha}{\sin\beta} \left[1-\frac{1}{1+\Delta_{t}}\frac{\Delta h_{t}}{h_{t}}(\cot\beta-\cot\alpha)\right] .$$

where  $\Delta_b \sim \alpha_s \tan \beta f(M_S)$ , and  $f(M_S)$  is a dimensionless function of supersymmetric masses.

### The Decoupling Limit Revisited

Working to first order in  $cos(\beta - \alpha)$ , and using

$$\tan \alpha \tan \beta = -1 + (\tan \beta + \cot \beta) \cos(\beta - \alpha) + \mathcal{O}(\cos^2(\beta - \alpha)),$$

it follows that

$$g_{hbb} \simeq g_{h_{\rm SM}bb} \left[ 1 + (\tan \beta + \cot \beta) \cos(\beta - \alpha) \right]$$

$$\times \left( \cos^2 \beta - \frac{1 + \delta h_b/h_b}{1 + \Delta_b} \right).$$

Note that  $(\tan\beta+\cot\beta)\cos(\beta-\alpha)\simeq\mathcal{O}(m_Z^2/m_A^2)$ , even if  $\tan\beta$  is very large (or small). Thus, the deviation from decoupling limit vanishes as  $m_Z^2/m_A^2$  for all values of  $\tan\beta$ .

Similarly,

$$g_{htt} \simeq g_{h_{\text{SM}}tt} \left[ 1 + \cos(\beta - \alpha) \left( \cot \beta - \frac{1}{1 + \Delta_t} \frac{\Delta h_t}{h_t} \frac{1}{\sin^2 \beta} \right) \right].$$

The deviation from the decoupling limit is suppressed at large  $\tan \beta$ .

### A SM-Like $H^0$

If  $\sin(\beta-\alpha)=0$ , then  $g_{HVV}=g_{h_{\rm SM}VV}$  [V=W or Z]. Expanding about  $\sin(\beta-\alpha)=0$  yields:

$$g_{Hbb} \simeq g_{h_{\rm SM}bb} \left[ 1 - (\tan \beta + \cot \beta) \sin(\beta - \alpha) \right]$$

$$\times \left( \cos^2 \beta - \frac{1 + \delta h_b/h_b}{1 + \Delta_b} \right).$$

In this case,  $g_{Hbb} = g_{h_{\rm SM}bb}$  only if  $|(\tan \beta + \cot \beta) \sin(\beta - \alpha)| \ll 1$ , which need not be satisfied if  $\tan \beta$  is very large (or small).

Similarly,

$$g_{Htt} \simeq g_{h_{\text{SM}}tt} \left[ 1 - \sin(\beta - \alpha) \left( \cot \beta - \frac{1}{1 + \Delta_t} \frac{\Delta h_t}{h_t} \frac{1}{\sin^2 \beta} \right) \right].$$

The parameter region where  $\sin(\beta-\alpha)\simeq 0$  corresponds to large  $\tan\beta$  and  $m_A<(m_h)_{\rm max}$ . In this region,  $H^0$  has SM-like couplings to vector bosons and up-type fermions. However,  $\tan\beta\sin(\beta-\alpha)$  need not be small in this regime, and so the Hbb couplings deviate from the corresponding SM value.

### Summary: the Approach to Decoupling

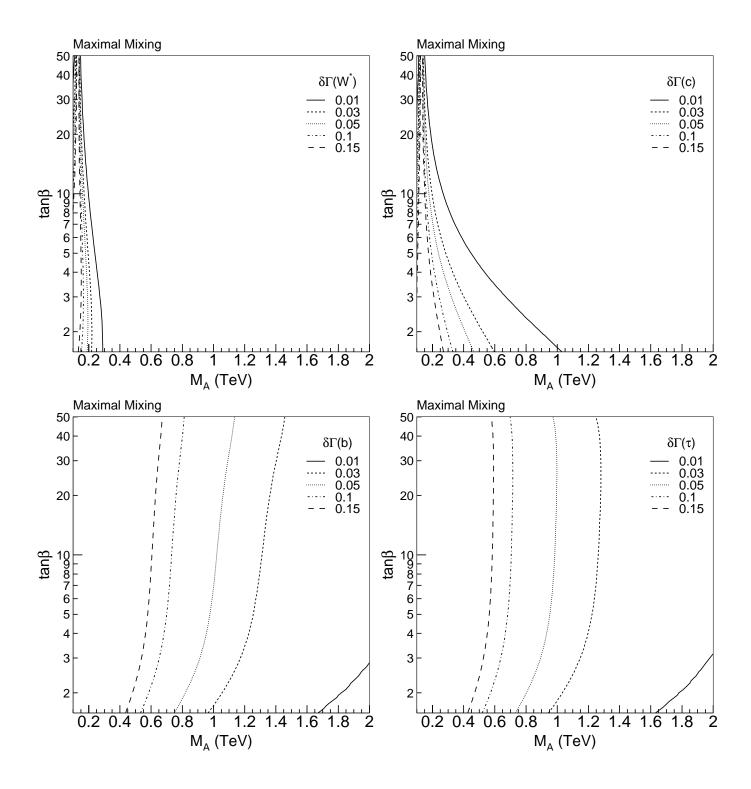
If we only keep the leading  $\tan \beta$ -enhanced radiative corrections, then

$$\begin{split} & \frac{g_{hVV}^2}{g_{h_{\rm SM}VV}^2} \simeq 1 - \frac{c^2 m_Z^4 \sin^2 4\beta}{4m_A^4} \,, \\ & \frac{g_{htt}^2}{g_{h_{\rm SM}tt}^2} \simeq 1 - \frac{c m_Z^2 \sin 4\beta \cot \beta}{m_A^2} \,, \\ & \frac{g_{hbb}^2}{g_{h_{\rm SM}bb}^2} \simeq 1 + \frac{4c m_Z^2 \cos 2\beta}{m_A^2} \left[ \cos^2 \beta - \frac{1}{1 + \Delta_b} \right] \,. \end{split}$$

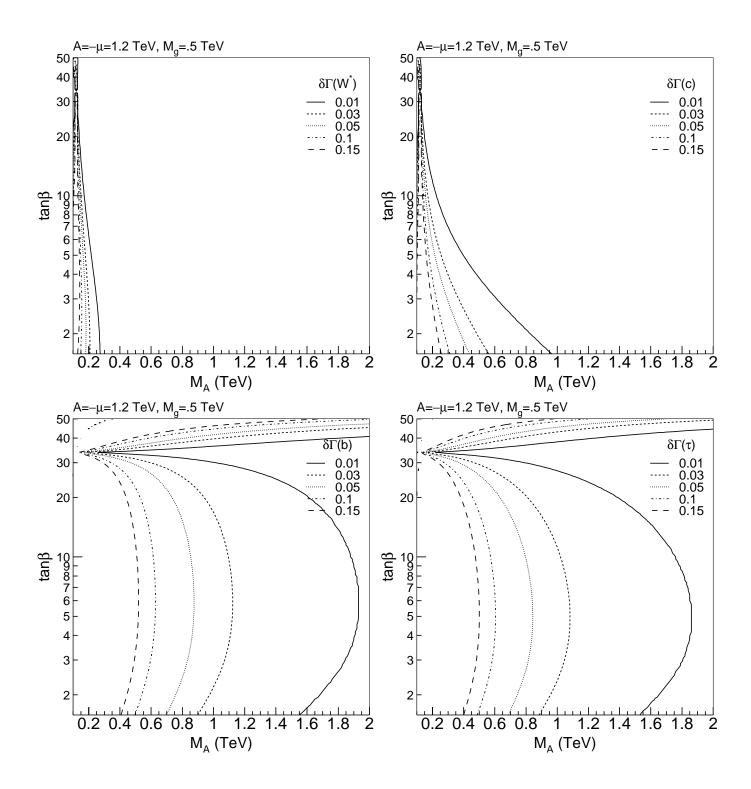
The approach to decoupling is fastest for the  $h^0$  couplings to vector bosons and slowest for the couplings to down-type quarks.

If c=0, which can occur at large  $\tan\beta$ , then we have  $m_A$ -independent decoupling.

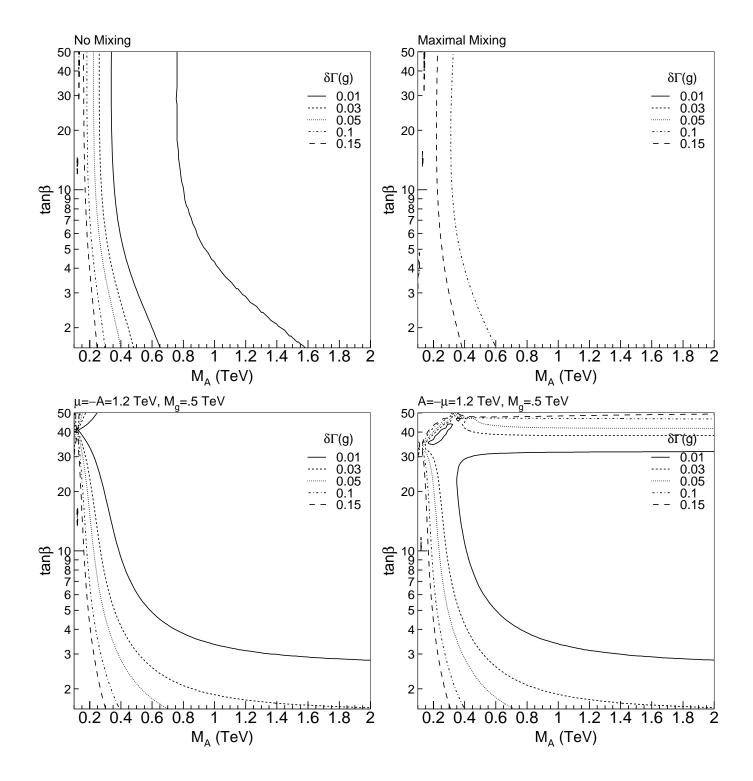
For loop-induced Higgs decays, such as  $h^0 \to gg$  or  $\gamma\gamma$  there are two decoupling limits of relevance:  $m_A \gg m_Z$  and  $M_{\rm SUSY} \gg m_Z$ . If only the former holds, then squark-loops can generate a small deviation in the loop-induced partial widths from the corresponding SM values.



Deviations of Higgs partial widths from their SM values in the maximal-mixing scenario.



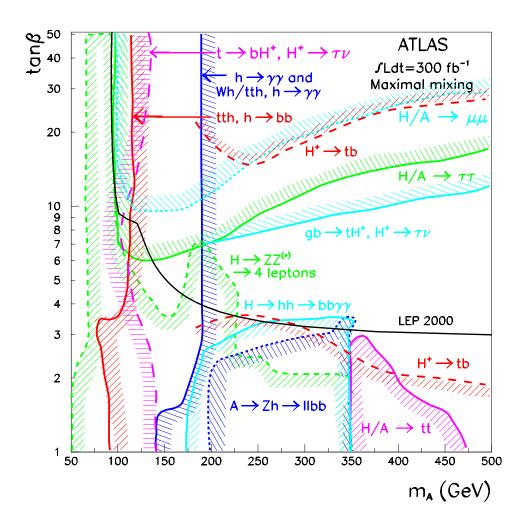
Deviations of Higgs partial widths from their SM values in the large  $\mu$  and  $A_t$  scenario, with  $A_t=-\mu=1.2$  TeV.



Deviations of the partial width  $\Gamma(g)$  from its SM value. Part of the effect shown is due to supersymmetric loop contributions to  $h^0 \to gg$ .

## **Precision Higgs Boson Measurements**

The LHC will provide the first set of Higgs boson measurements, and will achieve some degree of accuracy.



In the decoupling regime, only  $h^0$  will be discovered for moderate values of  $\tan \beta$ .

However, a robust program of precision Higgs physics requires a lepton collider.

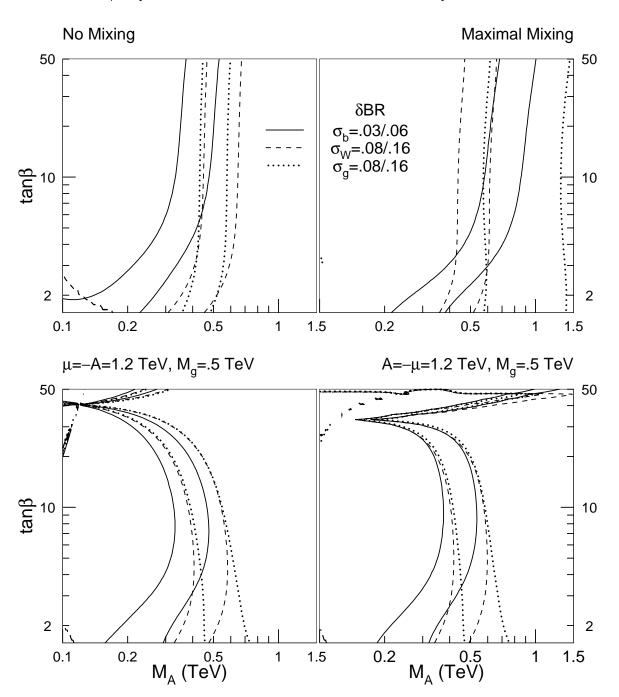
Expectations for precision of LC measurements of branching ratios (BRs) [Battaglia and Desch]

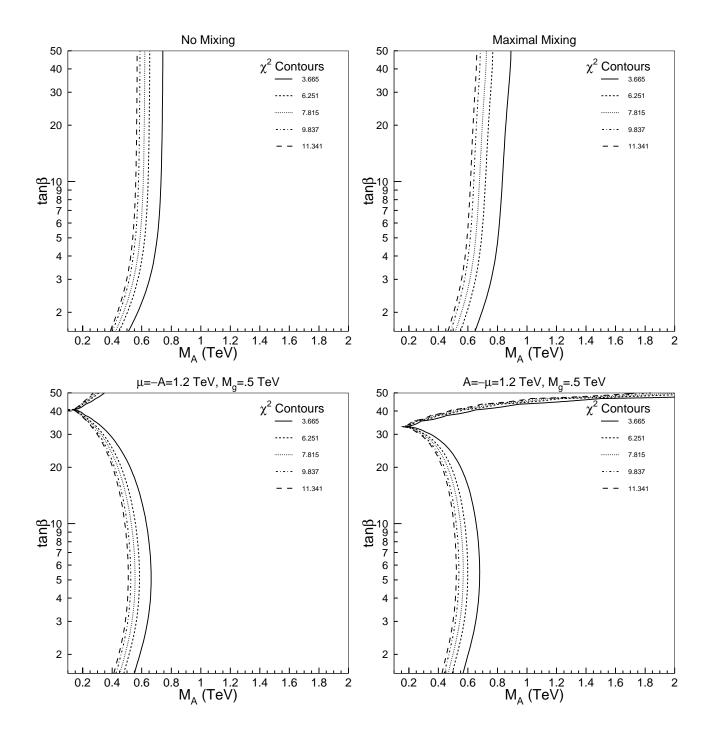
final state	experimental	theoretical
$-bar{b}$	4.4%	3.5%
WW	2.4%	
ZZ	2.4%	
$c \bar{c}$	7.4%	24%
$ au^+ au^-$	6.6%	
gg	7.4%	3.9%
$\underline{}$	10%	2.5%

Expected uncertainty of measurements of squared couplings (equivalently partial widths) for a 120 GeV SM-like Higgs boson assuming 500 fb $^{-1}$  at  $\sqrt{s}=500$  GeV, except for the measurement of  $g_{htt}^2$  which assumes 1000 fb $^{-1}$  at  $\sqrt{s}=800$  GeV.

#### Implications for the MSSM Higgs sector

Contours of  $\delta BR \equiv [BR_{MSSM} - BR_{SM}]/BR_{SM}$  in the  $m_A$ —tan  $\beta$  plane for different MSSM parameter scenarios.





Contours of  $\chi^2$  for Higgs boson decay observables. The contours correspond to 68, 90, 95, 98 and 99% confidence levels (right to left) for the three observables  $g_{hbb}^2$ ,  $g_{h\tau\tau}^2$ , and  $g_{hqq}^2$ .

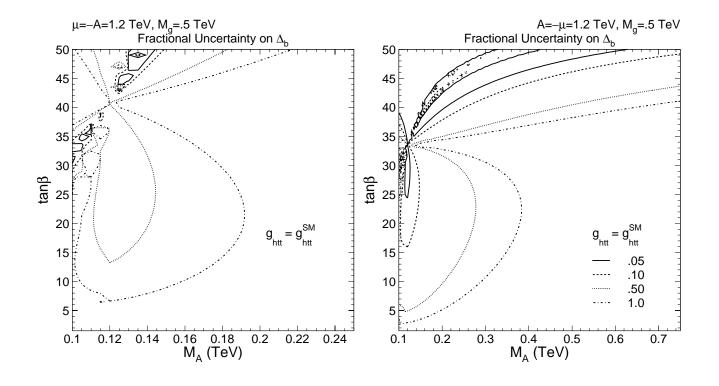
### Extracting the SUSY parameter $\Delta_b$

Consider the ratio of couplings

$$\frac{\hat{g}_{hbb} - \hat{g}_{h\tau\tau}}{\hat{g}_{htt} - \hat{g}_{hbb}} = \frac{\hat{g}_{Hbb} - \hat{g}_{H\tau\tau}}{\hat{g}_{Htt} - \hat{g}_{hbb}} \simeq \frac{\frac{\Delta_b - \Delta_\tau}{1 + \Delta_\tau} - \frac{\delta h_b}{h_b} + \left(\frac{1 + \Delta_b}{1 + \Delta_\tau}\right) \frac{\delta h_\tau}{h_\tau}}{1 - \left(\frac{1 + \Delta_b}{1 + \Delta_t}\right) \frac{\Delta h_t}{h_t} \cot \beta + \frac{\delta h_b}{h_b}},$$

where  $\hat{g}_{\phi ff} \equiv g_{\phi ff}/g_{h_{SM}ff}$  [ $\phi \equiv h$ , H]. If desired,  $t\bar{t}$  couplings can be replaced by  $c\bar{c}$  couplings. Keeping only the leading  $\tan\beta$ -enhanced one-loop terms, and assuming  $|\Delta_{\tau}| \ll 1$ ,

$$\frac{\hat{g}_{hbb} - \hat{g}_{h\tau\tau}}{\hat{g}_{htt} - \hat{g}_{hbb}} = \frac{\hat{g}_{Hbb} - \hat{g}_{H\tau\tau}}{\hat{g}_{Htt} - \hat{g}_{hbb}} \simeq \frac{\Delta_b - \Delta_\tau}{1 + \Delta_\tau} \simeq \Delta_b ,$$



### **Conclusions**

- ullet The decoupling limit presents a severe challenge for future Higgs studies. A program of precision Higgs measurements will begin at the LHC, but will only truly blossom at a future high energy  $e^+e^-$  collider.
- It is essential to find evidence for departures from Standard Model Higgs predictions. Such departures will reveal crucial information about the nature of the electroweak symmetry breaking dynamics.
- In the MSSM, deviations from the decoupling limit provide useful information about the non-minimal Higgs sector and can yield indirect information about the MSSM parameters. At large  $\tan \beta$ , there can be additional sensitivity to MSSM parameters via enhanced radiative corrections.
- Precision Higgs measurements can provide critical tests of the supersymmetric interpretation of new physics beyond the Standard Model.